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Understanding Coastal Morphodynamics Using Stability Methods

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ABSTRACT

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Stability methods, as they are applied in describing the initiation, growth and long term evolution of morphological features, are discussed. In particular, their use in describing large-scale, long-term rhythmic morphological features is highlighted. The analysis of such models indicates that many rhythmic bottom features arise from an inherent instability of a morphodynamical system, rather than being forced by external conditions. A synopsis of their theoretical basis is given, and the assumptions commonly pertaining to their use are described. These models, which can be applied more efficiently than many other process-oriented models, are categorized, and the kind of information that they can provide is also described. Finally, their relation to other areas and techniques of long-term, aggregated scale morphodynamics is discussed, and their usefulness to and applicability by the practitioner is summarized.

ADDITIONAL INDEX WORDS: *Morphology, rhythmic features, sea bed, waves, tides, stability analysis, nonlinear modeling, mathematics, numerical models.*

INTRODUCTION

Rhythmic patterns of the seabed and shoreline are a common feature of sandy beaches, upper and lower shorefaces, and the continental shelf. Examples of such nearshore phenomena, which can often be detected by eye, are ripples (ALLEN 1984; SLEATH, 1984), undulations of the shoreline known as beach cusps (KOMAR, 1998; KUENEN, 1948), bars in the nearshore zone (HOLMAN and BOWEN, 1982; HOMMA and SONU, 1963) and channel-shoal patterns in tidal basins (CLEVERINGA, 1999; EHLERS, 1988). The bedforms that exist further offshore, like shoreface-connected sand ridges (SWIFT *et al.,* 1978; VAN DE MEENE and VAN RIJN, 2000), tidal sand waves (HUNTLEY *et al.,* 1993; LANCKNEUS and DE MOOR, 1991) and tidal sand banks (OFF, 1963; STRIDE, 1982), require the use of ship or satellite measurements for identification (see VOGELEZANG *et al.,* 1997, for examples).

A common property of all these morphological features or patterns is that they are repetitive in space and time, so that a typical wavelength, amplitude and migration speed can be assigned to them. It is, in particular, this observed spatial rhythmicity that suggests that these features are genetically linked.

An example of rhythmic, nearshore bedforms can be seen in Figure 1, which shows a video image of the coastal area near Duck, North Carolina. The white areas indicate the locations of the bars where the incoming waves break. In this case the bars appear to be crescentic with typical alongshore wavelengths of 50 m. Such video images are made on a regular basis at a large number of different locations, as part of the ARGUS project initiated by R. A. Holman (see *e.g.* LIPPMANN *et al.,* 1993). Another example on a much larger scale is shown in Figure 2. Here a series of bedforms on the continental shelf (tidal sand banks and ridges) and the shoreface (shoreface-connected ridges) are apparent. In a similar vein, Figure 3 shows bathymetric data from Rotterdam Harbor, where sand waves and long bed waves can be seen. Lastly, in Figure 4 we show rhythmic features on a far smaller scale in the form of ripples on a beach.

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Figure 1. Video image of nearshore morphology (crescentic bars) observed just south of the pier at the U.S. Army Field Research Facility at Duck, North Carolina. The white areas indicate the locations where incoming waves break at the bars. Image reproduced by kind permission of Prof. Rob Holman of Coastal Imaging Lab., College of Oceanic and Atmospheric Sciences, Oregon State University.

These and other rhythmic bedforms are summarized in Table 1, along with their observed spacings, typical heights, migration speeds (if they have one) and typical evolution timescales.[1] It can be seen that the order of magnitude of these scales varies widely.

The major objective of the present paper is to give an explanation of stability methods, without recourse to many technical details, and to give the non-specialist an overview of the subject and of the capabilities of these types of methods. In the next section such an explanation is attempted. In § 4 the relation between these methods and other aggregated-scale methods for coastal evolution is described, and finally a discussion and conclusions are presented.

BEDFORMS

Origin of Bedforms

Bedforms are generated, or at least influenced, by the prevailing water motion. This is due to the fact that waves and currents are usually sufficiently strong to erode sediment from the bottom. Subsequent transport and deposition then leads to morphological evolutions (see Dyer, 1986; Fredsøe and Deigaard, 1993, for the general concepts of sediment transport). Thus most rhythmic features are a manifestion of the coupling between water motion, sediment transport and bottom changes, although some bedforms may be geological relicts (McBride and Moslow, 1991). See Blondeaux (2001) for an overview of much of the mechanics of the generation of coastal bedforms.

Observations indicate that the resulting morphological patterns strongly depend on the hydrodynamical conditions. For example, offshore sandwaves and sandbanks only occur in areas where depth-averaged tidal currents are stronger than 0.5 m/s (Belderson, 1986). Likewise, shoreface-connected sand ridges are only present on storm-dominated inner continental shelves (Swift *et al.,* 1978). Closer to the coast, for instance on the upper shoreface, the patterns are highly variable in time (see *e.g.,* Lippmann and Holman, 1990), and are related to the (sometimes rapidly) changing characteristics of incoming waves. For instance, during mild weather conditions bars on dissipative beaches can have an alongshore rhythmic structure (see Figure 1), whereas during storms the crests tend to become shore-parallel. Analysis of

[1] These correspond to the times over which these features typically persists. The actual formation times in the field may be different.

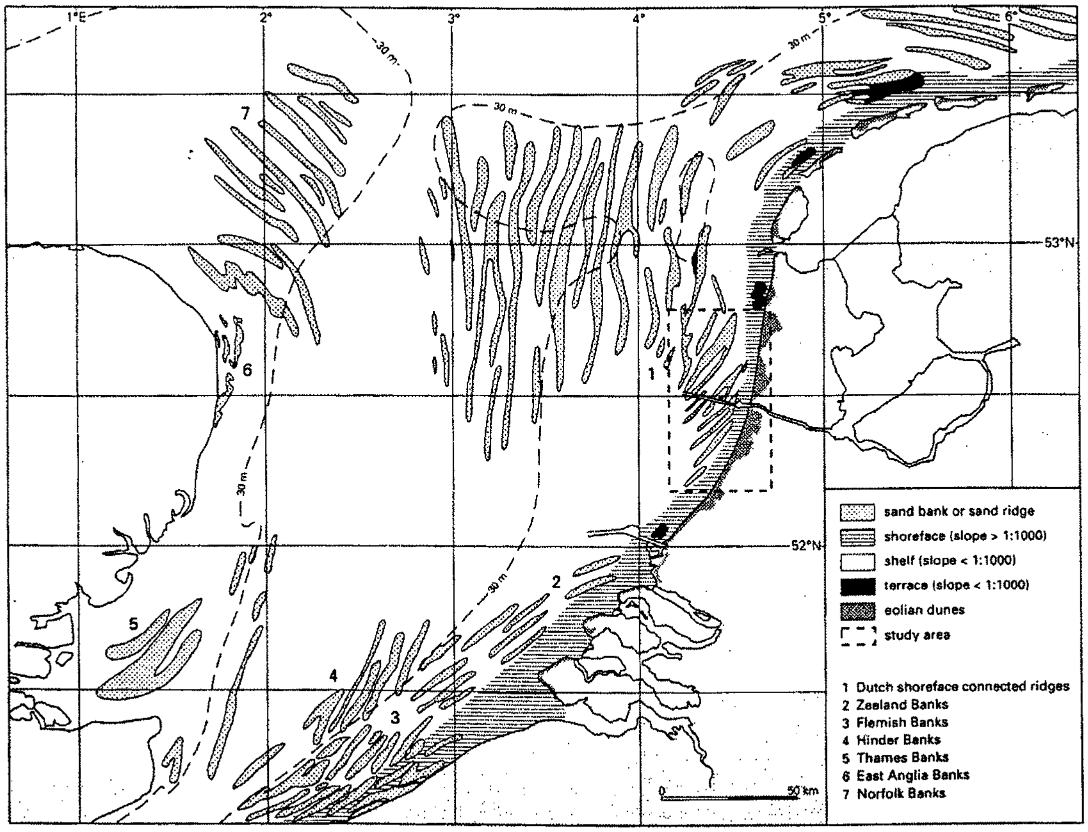

Figure 2. Sand banks, sand ridges and shoreface-connected sand ridges in the North Sea (from Van de Meene and van Rijn, 2000).

field data along the Dutch coast (Wijnberg and Terwindt, 1995) and near the Duck site in North Carolina (Lee *et al.,* 1998) has revealed that these bars can migrate seaward on time scales of 5–15 years; at other sites or under different circumstances bars may then migrate back onshore, or simply diffuse away after migrating offshore (see Lee *et al.* (1995) and Elgar and Guza (2001)).

Although, as noted, rhythmic bedforms occur due to prevailing water motions, it is important to appreciate that they are not, in general, generated by correspondingly rhythmic or periodic patterns in the circulation; rather, any such periodic variations in hydrodynamical conditions can be seen as a reaction or adjustment to the evolving bedform. This is discussed further later on.

Modeling of Rhythmic Bedforms

There are many practical problems (*e.g.* dredging, sand mining[2]) that require knowledge about the dynamics of rhythmic bedforms. Prediction of their formation and subsequent evolution is important for coastal zone management (*e.g.* erosion and accretion of beaches, navigation of ships). Further, more specific examples of these issues will be presented later on in this paper.

To understand and forecast rhythmic morphological features three main types of models are used. The first type is that of data-driven models. These models are based on empirical relations between a series of state variables (*e.g.* bar amplitudes) and follow from the analysis of field observations. Examples are discussed in Larson *et al.* (2003). A disadvantage of this approach is that no or only limited use is made of the known physics of the bars and water motion. Another problem is that such models are only expected to yield valuable predictions as long as the statistical properties of the system and forcing conditions remain unchanged. Hence these models are not suitable to investigate the effect of interferences on the behavior of a morphodynamic system.

The second type is that of full-scale, process-oriented numerical models. These models are based on the physical equations of conservation of mass and momentum, and attempt to describe many (or in some cases all) of the important constituent processes in coastal seas: waves, currents, sediment transport, bed level changes and the dynamic coupling between the bottom and the water motion. Prototype examples of such models are the coastal area models discussed by De Vriend *et al.* (1993b) (see also De Vriend and Ribberink,

[2] Sand extraction pits of dimensions similar to those of tidal sand banks do not exist yet, but in The Netherlands there is a school of thought that regards such pits as capable of meeting the increasing demand of filling sand.

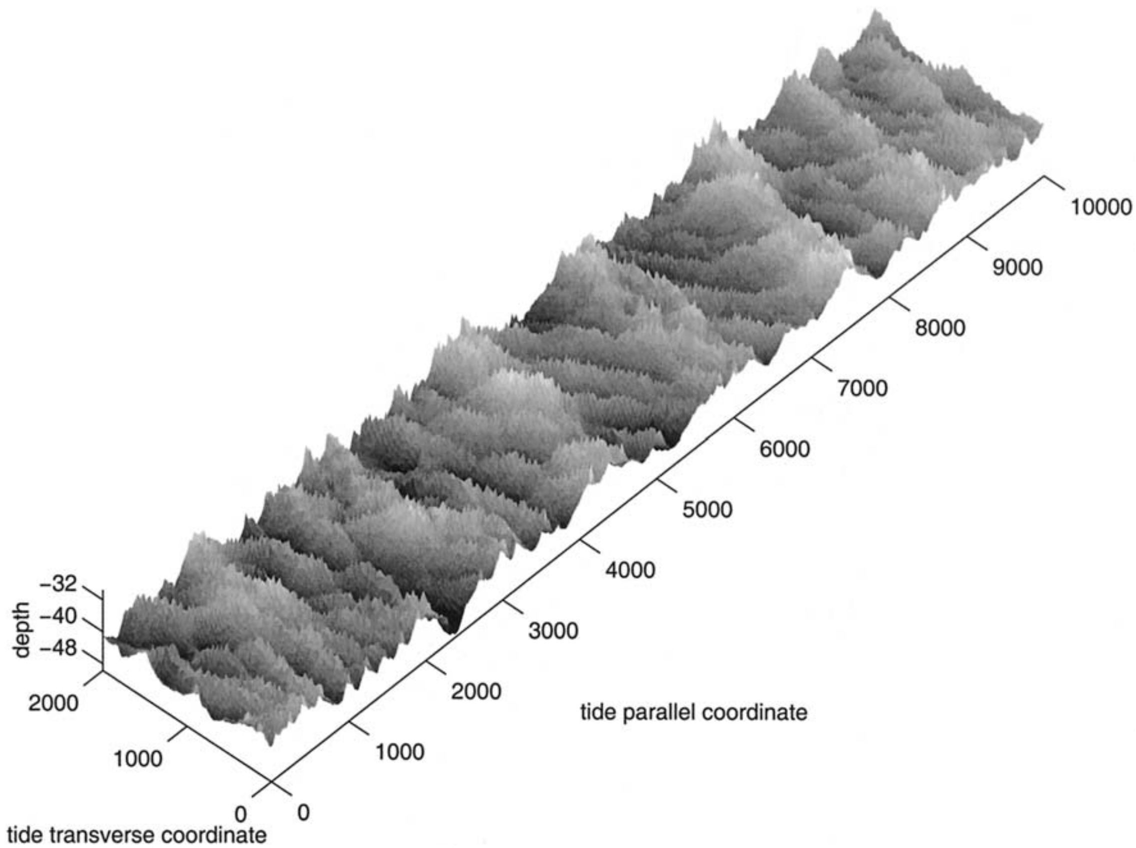

Figure 3. Bathymetric data from the entrance channel of the Rotterdam Harbor. Courtesy Dr. M.A.F. Knaapen. Sand waves and long bed waves are clearly observed.

1996 and CAYOCCA, 2001). Although such studies demonstrate that these models are able to simulate some of the observed rhythmic features in coastal seas and tidal basins, serious problems are encountered if they are used for predictions over periods that are large compared to the characteristic timescale of morphological phenomena. This is a consequence of the fact that the models lack sufficient knowledge of the constituent shorter scale processes, such as wave breaking, mixing and sediment transport. These effects occur on a broad range of temporal and spatial scales and have strong nonlinear interactions, but are often only described parametrically in order to keep the model manageable. Another disadvantage of process-oriented models is related to the variability in and errors associated with the input and boundary conditions, which enter process-oriented models through explicit forcing terms in the equations of motion, such as those of wind etc., and through conditions at open boundaries. A further disadvantage of process-oriented models is that they require extensive computer memory and long CPU-times, which makes them less suited for carrying out sensitivity studies.

Alternatively, specialized models for rhythmic bedforms have been developed and analyzed. These models, generally referred to as stability models, are also based on the physical equations, so they can also suffer from the same problems as process-oriented models concerning lack of knowledge of small-scale processes and therefore long-term predictability. However, they are used to isolate a certain phenomenon or feature (the bedform) and assume a simplified geometry and utilize simplified boundary and initial conditions, so that they can be solved very efficiently using mathematical methods[3] and can therefore be used in sensitivity studies to reveal such a lack of knowledge, and thereby refined by excluding unimportant effects and improving the representation or the parameterization of important ones. They therefore do not have all the same constraints as the full numerical process-oriented models, and their simplicity yields basic knowledge about the physical mechanisms that govern the formation and maintenance of bedforms.

This basic knowledge can also reveal an understanding of an inherent predictability horizon if the stability model exhibits, say, chaotic behavior. This knowledge is extremely relevant for practical purposes, because it gives clues as to how to interpret and understand both field data and the output of complex, numerical models (such evidence is more difficult

[3] Verification of numerical models by stability models and vice versa is a rather neglected area of investigation.

Figure 4. Asymmetric ripples on an intertidal beach in the Dutch Coast, probably generated by unidirectional currents (photo by A. Falqués).

to extract from the numerical models themselves). Hence stability models are helpful tools in making substantial progress in the development and status of long-term morphological models.

Finally, these stability models can be used to obtain quantitative predictions of both transient behavior (which describes adjustment processes, *e.g.* due to construction works) and nontransient behavior (which describe the "free" behavior of the system) by systematically varying the initial conditions and the model parameters respectively. However, their main benefit has so far been realized in examining so-called "free" behavior, *i.e.,* the inherent instability of certain morphodynamical regimes.

Forced vs. Free Features

In the literature these idealized models are and have been used to test two hypotheses: The first is that the spatial scale of the bedforms is determined by the spatial scale of the dominant water motion; the second is that the bedforms are a result of an inherent instability mechanism of the coupled water-bottom system. In other words, to see if the bedform is *forced* or *free,* in the terminology used by DE VRIEND (2003).

Forced Features

In studies of this type a feedback from the bottom to the water motion is not necessary to understand the initial formation mechanism. Examples of such studies are those of BOWEN and INMAN (1971) on edge waves and crescentic bars, LAU and TRAVIS (1973) on partially standing normally incident waves and submarine longshore bars, GUZA and BOWEN (1975) on edge waves and beach cusps (see also DYER, 1986; HOLLAND and HOLMAN, 1996) and HOLMAN and BOWEN (1982) on bars and bumps in the nearshore zone related to incoming waves and edge waves. Some of these types of stud-

Table 1. *Characteristic wave-lengths, heights, migration speeds and evolution timescales of rhythmic bedforms in coastal seas.*

Bedform	Spacing	Height	Speed	Time-scale
Ripples	(0.1–1) m	(0.01–0.1) m	—	Hours
Beach cusps	(1–100) m	(0.1–1) m	—	Hours–days
Nearshore bars	(50–500) m	(1–5) m	(0–100) m/yr	Days–weeks
Shoreface-connected sand ridges	(5–8) km	(1–5) m	(1–10) m/yr	Centuries
Sandwaves	(300–700) m	(1–5) m	(1–10) m/yr	Decades
Tidal sand banks	(5–10) km	(5–15) m	—	Centuries
Long bed waves	1.5 km	5 m	Unknown	Unknown

ies do include feedback from the bedforms to the hydrodynamics (see *e.g.* BOCZAR-KARAKIEWICZ and DAVIDSON-ARNOTT, 1991), but, as mentioned earlier, this is not *necessary* to the basic mechanism. Note that human interference, in the form of, say, beach structures, is another form of forced motion (see DE VRIEND, 2003).

Free Features

In the second hypothesis there is no direct relation between the spatial scales of the dominant water motion and that of the resulting morphological patterns, and the feedback from the bottom to the water motion is essential to understand the mechanism. These studies consist of a stability analysis of a simple equilibrium solution of the equations of motion (often a flat bed or an alongshore uniform beach profile) with respect to small periodic bottom perturbations with arbitrary spatial scales. Initially, such perturbations will decay or grow exponentially and both the growth rate and spatial pattern of the perturbations are solutions of an eigenvalue problem. The pattern with the largest growth rate is called the *preferred* or *fastest-growing* mode and its characteristics are usually compared with field observations. The first study of this kind was due to HINO (1974a, b), following earlier suggestions by SONU (1973); HINO showed that bars in the nearshore zone can be explained as a morphodynamic instability of a simple equilibrium involving the wave-driven set-up and longshore current in the breaker zone. Later on this model was substantially modified and improved by CHRISTENSEN *et al.* (1994) and FALQUÉS *et al.* (1996). The agreement between model predictions and field data was satisfactory, taking into account the many simplifications underlying the model. Models based on the same hypothesis have also been developed for tidal sandbanks and sandwaves (HUTHNANCE (1982), PATTIARATCHI and COLLINS (1987), HULSCHER *et al.* (1993), HULSCHER (1996b)), for sea ripples (BLONDEAUX, 1990; VITTORI and BLONDEAUX, 1990, 1992), for beach cusps (WERNER and FINK, 1993, in which a network model is used to examine the development of cusps, which shows that they can develop as a result of self-organization) and for channel-shoal formation in tidal embayments (SCHUTTELAARS and DE SWART, 1996, 1997).

Despite this general progress in understanding the dynamics of rhythmic bottom features many questions remain unanswered. This motivated the setting up of a separate research topic within the EC MAST-III PACE project, which was devoted to rhythmic features. These investigations are not recounted here; the reader is referred to FALQUÉS *et al.* (1997), FALQUÉS *et al.* (1999), CALVETE *et al.* (1999), CALVETE *et al.* (2001c) and WALGREEN *et al.* (2002) for work on shoreface-connected ridges; HULSCHER (1996b), KOMAROVA and HULSCHER (2000) and HULSCHER and VAN DEN BRINK (2001) for work on sand banks and sand waves on the continental shelf; and VITTORI *et al.* (1999) for an investigation of the occurrence and growth of nearshore crescentic bar systems.

Other work on this kind of approach to solving morphodynamical problems includes: SCHUTTELAARS and DE SWART (1996), SCHUTTELAARS and DE SWART (1997), DE SWART and BLAAS (1998), SCHUTTELAARS and DE SWART (2000), VAN LEEUWEN and DE SWART (2001) and SCHRAMKOWSKI *et al.* (2002), who examine tidal embayments and the growth of bedforms there; DEIGAARD *et al.* (1998), who examine the growth of bedforms associated with alongshore surf zone bars-related investigations have been undertaken by FALQUÉS *et al.* (2000), DAMGAARD *et al.* (2002) and CABALLERIA *et al.* (2002); CHRISTENSEN *et al.* (1994), FALQUÉS *et al.* (1996), PLANT and HULSCHER (2000), MURRAY *et al.* (2001) and RIBAS *et al.* (2003) examine other nearshore instabilities.

MORPHODYNAMICAL INSTABILITY THEORY

Morphodynamical and Hydrodynamical Time Scales

The essence of coastal morphodynamics is the interaction between water motions and a changing topography. Let the water variables (flow $\vec{u}(\vec{x}, t)$ and free surface $\zeta(\vec{x}, t)$) be indicated by $u(\vec{x}, t)$, where t means time and $\vec{x}$ is the spatial coordinate vector, which may be two- or three-dimensional, and let the topographic variable be indicated by $h(\vec{x}, t)$. Then, a morphodynamical system can be described symbolically as:

$$\frac{\partial u}{\partial t} = f(u, h, t) \tag{1}$$

$$\frac{\partial h}{\partial t} = g(u, h) \tag{2}$$

where (1) describes the hydrodynamics on a topography given by $h(\vec{x}, t)$ and (2) describes the topographic evolution driven by the water motion defined by $u(\vec{x}, t)$, in which g represents the divergence of the total load volumetric sediment flux. This parameterization is done so that the dependence of g on the flow $\vec{u}$ can be made explicit; this is important because the sediment moves under the action of the current at the bed. These equations are not partial differential equations in t alone, but rather f and g involve partial derivatives with respect to one, two or three spatial coordinates. Usually, (1) consists of the momentum and mass conservation equations for the fluid, as well as, sometimes, a concentration equation. Both (1) and (2) are supplemented with boundary conditions, such as no-normal-flow conditions at closed boundaries, or periodic conditions at pairs of open boundaries.

The dependence of f on t allows for a non-steady external forcing. However, many stability studies deal with an external forcing (like incident wind waves or tidal currents, for instance) which can be parameterized through some average. Therefore, we omit this explicit time-dependence in f.

To proceed further, (1) and (2) are made non-dimensional in order to make their magnitudes explicit. This is frequently achieved by introducing: L_H, a horizontal length scale related to the extent of the features in question; L_V, a vertical length scale related to the water depth; and V_0, a velocity related to the ambient current. This defines a hydrodynamic time scale $T_h = L_H/V$, and a morphodynamic time scale: T_m. A comparison of the two timescales reveals that:

$$\frac{T_h}{T_m} \ll 1. \tag{3}$$

If the non-dimensionalization is performed, and a scaling based on (3) introduced ($\tau = (T_h/T_m)t$, where τ represents a slow, morphodynamic timescale) then (1)–(2) read:

$$\frac{T_h}{T_m}\frac{\partial u}{\partial \tau} = f(u, h) \tag{4}$$

$$\frac{\partial h}{\partial \tau} = g(u, h). \tag{5}$$

where all quantities are now assumed to be non-dimensional. Because of (3), (4)–(5) can be approximated as

$$f(u, h) = 0 \quad \text{and} \tag{6}$$

$$\frac{\partial h}{\partial \tau} = g(u, h). \tag{7}$$

This is called the quasi-steady hypothesis for fluid motions and it means that the fluid adjusts instantaneously to the changes in the bottom configuration. It thus filters out hydrodynamic instabilities because those "fast" variables are now "slaved" to the slow bottom evolution.

Basic State

Most stability analyses in morphodynamics are aimed at explaining the occurrence of certain patterns on the topography and on the flow. These patterns do not have importance only in themselves; their investigation is a way of gaining insight into the fundamental physics of the system. The natural way to proceed is to consider a simple state, the so-called *basic state,* where these patterns are not present[4] and to analyze what happens when a small perturbation on the topography and/or the flow is superimposed. If the perturbation tends to grow it will give rise to a new situation where those patterns will be present. Its growth usually reveals a positive feedback within the system, which can be considered to be the cause of such rhythmic features. The basic state represents the mean dynamic balance in the absence of rhythmic features. This balance is quite often achieved by a steady or equilibrium solution (*e.g.* steady wave-driven longshore current) but it is sometimes time-dependent (*e.g.* tidal current). For simplicity, in order to illustrate the concepts without unnecessary complications the former situation will be considered here. Thus, the basic state will be assumed to be a steady topographic configuration H and a steady configuration of the fluid U such that, according to (6)–(7):

$$f(U, H) = 0 \tag{8}$$

$$g(U, H) = 0 \tag{9}$$

It is important to appreciate that the success of a stability analysis is dependent on being able to find a suitable basic state. This is not always a trivial task. In some cases it is possible to determine a functional relationship between u and h in the basic state. In other circumstances it is necessary to solve the equation system numerically to determine this state, and systems where no basic state (at least no physically meaningful one) exists may also be encountered. Clearly, in a method based on idealized geometries and isolation of features, a basic state that is relatively easy to define is in keeping with the level of complexity of the model. Typically, the basic state is spatially invariant (homogeneous) in at least one direction.

Linear Stability Analysis

We shall now look into the dynamics of any small perturbation superimposed on the basic state. Essentially, this entails substitution of quantities

$$u = U + u' \quad \text{and} \tag{10}$$

$$h = H + h' \tag{11}$$

into (6)–(7), where U and H satisfy (8)–(9). A key point in linear stability analysis is that $|u'| \ll |U|$ and $|h'| \ll |H|$. This assumption allows linerization of (6)–(7), by linearizing u' (h') with respect to U (H), so that a considerable simplification is obtained. The restriction implied by this assumption is that we now only consider the initial evolution of these features. There is no exact definition of the duration for which the linear theory is appropriate, but a commonly used measure is the e-folding time (time over which the amplitude increases by a factor e—approximately 2.7). Obviously, this is can only provide the practitioner with an order of magnitude estimate. The relevance of the spatial information obtained from the linear analysis is discussed later.

This linearization thus gives us equations that describe the initial evolution of the perturbations to a good degree of approximation:

$$0 = L_{11}u' + L_{12}h' \tag{12}$$

$$\frac{\partial h'}{\partial t} = L_{21}u' + L_{22}h' \tag{13}$$

where L_{ij} are linear operators, so that $L_{ij}u'$ ($L_{ij}h'$) involve only linear terms in $u'(h')$ and their partial derivatives. It is important to note that that the linear equations emerge from consideration of a nonlinear system by assuming forms (10) and (11) and linearizing.

We now define $\vec{x} = (x, y, z)$, where z is the vertical coordinate. We also assume that the basic state is homogeneous in one direction:[5] y. This means that the dependence on y and t is exponential. Thus, the topographic variable (which does not depend on the vertical coordinate z) will read

$$h(x, y, t) = H(x) + \epsilon Re\{\hat{h}(x)e^{i(ky-\omega t)}\}. \tag{14}$$

Similar expressions hold for the flow variables, where now a vertical dependence many be included.

Note that the small parameter ϵ in (14) is included here to state explicitly that the perturbation is assumed to be small: $\epsilon \ll 1$. Here, the wavenumber k indicates the spatial periodicity in the y direction of the perturbed configuration and ω is its frequency in time. Usually, k is assumed to be real

[4] A basic state is an equilibrium (steady) solution of the dynamical equations. In linear models it is this simple basic state that is almost always studied because it is the simplest and the most intuitively obvious. In nonlinear studies (SCHUTTELAARS and DE SWART, 1997) other basic states may be studied.

[5] If the basic state is also invariant in the x direction, then $\hat{h}(x) = \exp(ilx)$.

and $\omega = \omega_r + i\omega_i$ is complex ($\omega_r = Re\omega;\ \omega_i = Im\omega$). This describes a perturbation with wavelength $2\pi/k$, a phase speed (migration speed of the topographic features) given by ω_r/k and a growth rate ω_i. If $\omega_i > 0$ ($\omega_i < 0$), then the solution is exponentially growing (decaying) in time and if $\omega_i = 0$ it is neutral, *i.e.,* it propagates like a wave without growth or decay.

Hence (12)–(13) become

$$0 = \hat{L}_{11}(k)\hat{u} + \hat{L}_{12}(k)\hat{h} \tag{15}$$

$$i\omega\hat{h} = \hat{L}_{21}(k)\hat{u} + \hat{L}_{22}(k)\hat{h} \tag{16}$$

For each k, there is a set of normal modes, that is, a set of eigenvalues ω_n and eigenfunctions $(\hat{u}_n(x, z), \hat{h}_n(x))$ where n is a mode number. Each particular solution will grow (decay) in time depending on its growth rate ω_{ni}. The spatial structure of the bottom and the corresponding velocities are given by any such solution:

$$Re\{\hat{h}(x)e^{iky}\}, \qquad Re\{\hat{u}(x, z)e^{iky}\} \tag{17}$$

with an arbitrary amplitude, which is not determined by the eigenproblem. Any solution of the linearized governing equations can be expanded as a combination of such normal modes with the appropriate wavenumbers and amplitudes, which depend on the initial conditions. If for any choice of k and n there is no solution with positive growth rate, the basic state is stable and no rhythmic patterns will emerge from this basic configuration. Otherwise, if there is at least one linear mode with $\omega_i > 0$, the basic state is unstable[6] and a rhythmic (in the y direction) pattern determined by the eigenfunction corresponding to this particular ω will emerge. This kind of analysis is often referred to as *normal mode analysis.*

Solving the system (15)–(16) usually requires a numerical solution; it is unusual for a simple analytical and realistic solution to be found, at least without using solution matching within the solution domain, or more abstruse transcendental functions.

Fastest Growing Modes

Typically, a linear stability analysis will reveal one of three possibilities: (1) no positive growth rates: this indicates that the basic state is stable with respect to the perturbations imposed. If rhythmic features are present, it may indicate that they result from some periodicity or rhythmicity imposed extraneously (*i.e.,* they are not a free instability of the system), or, perhaps, that the dynamical system considered is not appropriate or complete; (2) positive growth rates for any value of k; or (3) positive growth rates for some finite range of values of k. In the latter two cases the growth rate for a particular mode may be plotted as a function of k to reveal the wavenumber k_c (and wavelength, $\lambda_c = 2\pi/k_c$) and mode number n_c with the largest growth rate: the *fastest growing mode* (FGM). Random (periodic) initial disturbances will excite all the linear modes of the system with all wavenumbers. But from this mixture the FGM will prevail after some time,[7] so this mode is assumed to be the indicator of the observed feature spacings (*i.e.,* the distance between sand wave crests, or the distance between the horns of crescentic bars) and of their shape (*i.e.,* elongated, rounded, shore-normal, orientation with respect to the tidal current *etc.*). In most cases, such an FGM can indeed be found. However, there are circumstances in which no such mode is predicted because the growth rate is monotonically increasing with k. This can be so, for instance, if the mechanism of bed-slope correction is not included in the sediment conservation equation (2), since the tendency for sediment (sand) to move down slope under gravity acts to prevent very small wavelength disturbances with steep slopes developing.

The FGM will have a frequency ω_{nc} associated with it. If its real part is zero then it indicates that the feature does not propagate. The information that a linear (temporal) stability analysis can provide is summarized in Table 2. This actually amounts to a lot of useful information from a fairly standard method. Accordingly, much use has been made of this approach in examining the growth of bed forms, and with much success.

Table 2. *The information obtainable from a linear stability analysis.*

Parameters Provided by a Linear Stability Analysis	Description of Parameter
Instability (/stability)	Indicates whether (or not) the basic state is adequate, the model catches the essential physics and the feature arises from a free instability (of small amplitude)
Wavelength	Gives the predicted spacing between the features
Growth rate	Gives a spin-up time for the growth of the features
Propagation rate	Gives the speed at which the features move
Spatial structure of normal modes	Gives the shape and the orientation of the bed-form with respect to the periodic direction

Nonlinear Stability Analysis

The prediction of instability given by the linear analysis reveals the presence in the system of a physical mechanism which tends to produce a certain pattern. However, the rest of the predictions of linear stability analysis are always subject to the validity of the small amplitude assumption, which, strictly speaking, will be violated by the exponential growth of the (linear) bedform. Nevertheless, linear theory *may* give predictions concerning their long-term behavior, in terms of the migration speed of the bedform, or its shape, but this depends on the type of feature being examined and the degree of nonlinearity in the dynamics. Frequently, however, migra-

[6] It is often said that the growing solution is unstable, but in fact, it is the basic state that is unstable to this particular solution.

[7] The limitations of this statement are discussed in the section about nonlinear stability analysis.

Table 3. *The information obtainable from a nonlinear stability analysis.*

Parameters Provided by a Nonlinear Stability Analysis	Description of Parameter
(Amplitude) height	(Half) The distance from crest to trough of the bedform
Actual spatial structure	The shape and spacing of the bedforms to be compared with the features in Nature
Actual flow	Flow characteristics to be compared with velocity measurements
Type of instability	Indicates the qualitative type of the bedform regime, which may constitute a series of bifurcations
Long-term evolution	The sometimes complex evolution of the bedform beyond linear theory. Can be used to verify other aggregated-scale models.

tion speeds and spacings of the bedforms compare well with those observed, and are therefore a robust output of the linear analyses. In other cases, the characteristics of the finite-amplitude features may be very different from the predictions of linear theory. Moreover, linear theory does not give any information on the actual amplitude of the features. This information is essential for fundamental reasons (*e.g.* to determine the dynamical properties of the complex morphodynamical system) as well as practical ones (predictions concerning how big a bedform is likely to get; reliable long-term predictions; the identification of a long-term predictability horizon).

To determine this kind of information, therefore, requires a nonlinear analysis, capable of describing the nonlinear interaction and competition between different normal modes. The purpose of nonlinear stability analysis is to provide predictions of the characteristics that the linear analysis cannot. These are summarized in Table 3.

Types of Nonlinear Stability Model

A nonlinear analysis implies dealing with the full equations (6)–(7). According to Craik (1985) (chapter 3) there exist three classes of approach. His first and third classes (derivation of integral bounds and direct numerical simulations) are not considered herein, the first because of its complexity and the often weak bounds derivable, and the third because it is not treated as stability analysis at all here. The second group is described as weakly nonlinear theories, which are based on an assumption that the bed form, though of finite-amplitude, is still small in some sense, and may therefore be expandable in terms of some small parameter. Many of these approaches were pioneered in river dynamics (see also De Vriend *et al.,* 1993a). This class is augmented here by another, based on the full nonlinear equations, which are nonetheless expandable in terms of some suitable expansion of linear eigenfunctions, both stable and unstable. Both approaches have their uses, although the latter is clearly more general, being applicable in highly nonlinear regimes. Below we give a very brief synopsis of these theories.

Types of Nonlinear Stability

It is important to reiterate that linear theory only describes the initial evolution process. In the longer term the exponential growth of the amplitude of the perturbations (both in the bed and in the water motion) cause nonlinear terms to become at least as important as the linear ones. Hence the stability analysis has to be extended into the nonlinear regime. But here extensive use *must* be made of the results of the linear analysis. Basically two following situations can appear: (1) The system is weakly nonlinear. This means that most of the perturbations have negative growth rates; only a narrow spectrum of perturbations has small positive growth rates (*e*-folding time scale is large compared to characteristic morphological time scale). This is the situation in which a model parameter R (*e.g.* describing the state of the system) is slightly above its critical value R_c: for $R < R_c$ the basic state is stable. (2) The system is strongly nonlinear. This implies that there is a wide spectrum of perturbations with positive growth rates and these rates are not small.

A further subdivision must then be made: (a) The spectrum of perturbations is continuous. This is so in the example that we discussed so far (see (14)); because the unbounded y direction means that the wavenumber k can have any value. (b) The spectrum of perturbations is discrete. This is so if the eigenvalue problem (12)–(13) is solved on a bounded domain.

Therefore 4 cases can be distinguished:

System is Weakly Nonlinear; Spectrum is Discrete

Morphodynamic example: Vittori and Blondeaux (1990), ripple formation due to sea waves (see also Schuttelaars and De Swart, 1997, on channels and shoals in tidal embayments). In this case a perturbation analysis can be performed in the small parameter $\epsilon = (R - R_c)/R_c$. At order ϵ the linear dynamics are recovered with characteristic wavenumber k_c; at the next order nonlinear terms in the equations of motion produce solutions proportional to $2k_c$ and $0k_c$. Then, at order ϵ^3, a solution proportional to k_c itself is generated. Since this is physically equivalent to forcing an oscillator at its eigenfrequency, without any additional condition unbounded solutions will occur. Since such solutions are non-physical, they must be removed and this can be done by imposing a solvability condition (also called the Fredholm alternative, see Nayfeh (1981) for details). This process generates a so-called Landau equation, which describes the evolution of a (complex) amplitude of the perturbation A with respect to a "slow" time $T = \epsilon^2 t,$

$$\frac{dA}{dT} = a_1 A - a_3 |A|^2 A \qquad (18)$$

with A a complex amplitude of the perturbation, $T = \epsilon^2 t$ a slow time coordinate, a_1 related to the largest eigenvalue of the linear problem (positive real part) and a_3 the so-called Landau coefficient, which depends on the system under investigation. The important point to appreciate is that the same *generic* equation is found for any system. Solutions depend on the real part of a_3. When the real part of a_3 is positive, positive solutions tend to a new equilibrium characterized by a finite amplitude; then if ω_r is nonzero the perturbation represents a travelling finite amplitude wave. If the real part of a_3 is negative then explosive behavior occurs (*i.e.* the amplitude becomes unbounded in a finite time). Formally the analysis has to be continued to higher orders, which, physically, may mean a rapid transition to a highly complex state.

A special type of dynamics occurs when free modes of the linear system are already forced at order ϵ^2. Then a solvability condition must be imposed at this order. A morphodynamic example is that of the brick-pattern ripples induced by sea waves (see VITTORI and BLONDEAUX, 1992, for further details).

System is Weakly Nonlinear; Spectrum is Continuous

To the best of our knowledge this type of analysis has not been applied to coastal morphodynamical studies. Basically, the analysis is identical to previous case, but the important difference is that there is now a narrow *spectrum* of perturbations, each with its own complex frequency ω. Physically this implies that a wave group is described which in general has dispersive properties: phase velocity and group velocity differ. Again a *generic* result is found from the perturbation analysis in the small parameter $(R - R_c)/R_c$: the Ginzburg-Landau equation. This is a generalization of Landau equation, but with more degrees of freedom, and therefore many more different types of solutions—the amplitude can be steady, but also periodic, quasi-periodic and even chaotic, see SCHIELEN *et al.* (1993) and references herein.

System is Strongly Nonlinear; Spectrum is Discrete

Morphodynamic example: channel-shoal dynamics in a tidal embayment by SCHUTTELAARS and DE SWART (1997) and shoreface-connected sand ridges by CALVETE *et al.* (2002). In these cases the solution is expanded using a truncated series of eigenfunctions of the linear problem. Substituting this into the equations of motion and projecting the latter onto the eigenfunctions of the adjoint problem yields equations describing the amplitudes of eigenfunctions. This method is called Galerkin projection (see *e.g.* HOLMES *et al.,* 1996, for further details). The final system consists of ordinary differential equations for amplitudes of the bottom modes and algebraic equations for the amplitudes of the fast variables, which describe the flow over the topography.

In both models, results indicate finite-amplitude behaviour in the mode amplitudes. The nonlinear stability analysis for channels and shoals in tidal embayments (see SCHUTTELAARS and DE SWART, 1997) has yielded new results that could not be obtained with the linear stability analysis (see SCHUTTELAARS and DE SWART, 1996): forms become migratory and a splitting of channels is found. In the nonlinear model of shoreface connected ridges (see CALVETE *et al.,* 2002), the long-term bottom pattern shows the observed asymmetries of the ridges with steep bottom gradients on the downstream side and small scale bottom perturbations, as well. In the latter case, the long-term dynamics are dominated by the initially fastest growing mode (see CALVETE *et al.,* 2001b), the migration speed of the finite-amplitude perturbations appears to be unaffected by nonlinear effects and the alongshore spacing between ridges corresponds to the one predicted by the linear problem.

System is Strongly Nonlinear; Spectrum is Continuous

Example: KOMAROVA and NEWELL (2001). There has been no morphodynamic study on this type of system, but in other disciplines of fluid dynamics (*e.g.* convection) important work along these lines has been done (see NEWELL *et al.,* 1990; PASSOT and NEWELL, 1994). Their analysis results in a so-called order parameter equation. This is one, generic, strongly nonlinear partial differential equation which is valid for all wavenumbers. It can describe many observed "defect" features in patterns, such as dislocations and disclinations (*i.e.,* sudden disappearances or joinings of crest-lines). Furthermore the order parameter equation reduces to earlier generic equations (such as the Ginzburg-Landau equation) in the correct limits. Since defects occur in many observed morphologic patterns (*e.g.* sea ripples, offshore sand waves) the applicability of order parameter equations to morphodynamic systems deserves further attention.

RELATION TO OTHER AREAS OF AGGREGATED-SCALE MORPHODYNAMICAL MODELING

Earlier the basic information provided by stability analyses was detailed. It can be appreciated that this type of information can be viewed as a kind of *meta*-information; the stability model is designed to isolate and investigate one type of feature, to give the kind of information shown in Table 2 and 3. In this section we briefly discuss some other areas of morphodynamic modeling and their relation to stability methods, and attempt to answer two questions: In what way are stability methods useful for this area? and: How can research in these areas be useful to stability methods?

Process-Oriented Modeling

The morphodynamical equations on which stability analyses are based are also solved in numerical models. In such circumstances, some of the drawbacks previously noted in the numerical approach no longer exist because the issues concerning the long-term predictability are the same for both approaches. Comparison with such numerical models is therefore desirable. So far, this aspect of intercomparison has largely been ignored, perhaps because a stability analysis is designed, in part, to circumvent expensive numerical calculations by assuming a simple (normal mode) decomposition. However, the special solutions provided by a stability analysis can, and perhaps should, be used for numerical model verification. This kind of comparison has a number of uses: it can be used directly to verify a numerical model, by, for instance, measuring numerical growth rates of small features; a linear stability analysis may also reveal a suitable offshore boundary location, which can result in large computational savings.

Recently, DAMGAARD *et al.* (2002) compared results from a linear stability analysis with a morphodynamic simulation of

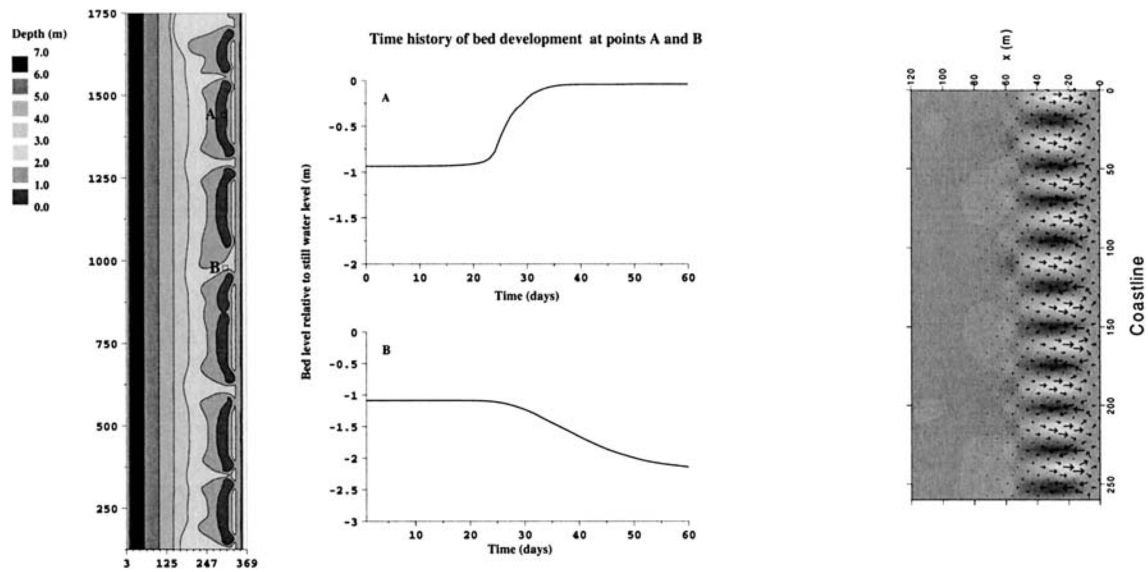

Figure 5. Left: The result of a fully nonlinear morphodynamic simulation of surf zone rip channel growth under normal wave incidence (adapted from DAMGAARD *et al.,* 2002); Right: The result of a similar fully nonlinear simulation of surf zone hydrodynamics (shallow water) coupled to topographic evolution looking at transverse bars (also normal incidence; shoals in white and troughs in dark colors respectively) (see CABALLERIA *et al.,* 2002). In both images, the coastline is situated on the right side of the plot. Both these simulations were performed as part of the EU SASME project.

rip current channel growth from a fully nonlinear numerical model. Rip channel spacings predicted by the nonlinear numerical (finite difference) model and by the linear stability model are similar, and there is a very good correspondence between initial growth rates in both models. The results of the fully nonlinear morphodynamical simulation can be seen in Figure 5, in which the rip channels can clearly be discerned. In related work CABALLERIA *et al.* (2002) have conducted fully nonlinear simulations of the growth to finite amplitude of transverse bars (both rip channels and transverse bars can be viewed as bedforms falling under the general heading of "nearshore bars" in Table 1). In Figure 5 we can also see the finite amplitude form of these morphodynamical simulations. Stability models can also sometimes be used to investigate whether a numerical model is exhibiting real (physical) behavior, or whether an observed response is a numerical artifact. Stability models can also reveal the extent of the regimes (in some parameter space) over which certain kinds of nonlinear analyses (such as weakly nonlinear ones) may be relevant (see also VAN LEEUWEN and DE SWART, 2001).

Depth of Closure

Stability models can also be used to investigate boundary effects and their importance. One example of this that may have wider relevance is that of depth of closure. Viewed from the point of view of theoretical morphodynamics, depth of closure can be considered as the depth (or offshore distance) at which a shoreface profile is morphodynamically inactive. It is well known that this depth is dependent on both the timescale considered and on the type of bed form. For instance, for sand banks and sand waves, the whole continental shelf can be considered active, and a long enough timescale may allow variations due to changes in background (forcing) conditions to have an effect, such as a change in sea level or, on a shorter timescale, a particularly large storm. It is therefore forcing dependent. However, over a suitable timescale the identification of a depth of closure for, say, bar activity on a beach face can provide useful information for numerical models, which need a morphodynamically inactive offshore boundary.

Stability analyses can provide estimates of depth of closure, simply by testing the sensitivity of stability models to the offshore numerical boundary: *i.e.,* when results begin to change significantly as the offshore boundary is moved onshore then we are in the region of *a* depth of closure. This depth will not necessarily be the one useful to practising engineers if they are primarily interested in event-driven depths, but on a longer time scale it will be relevant.

Data-Driven Modeling

Data-driven modelling seeks to derive a mathematical model from the data; stability analysis seeks to derive an aggregated scale mode (evolution equations) from a basic physical analysis. They are therefore proceeding toward a similar goal from different directions. In a stability analysis we use our understanding of a given physical system to write dynamical equations, which we then perturb and then analyse in order to find FGM(s). The opposite approach starts with a sequence of measurements (time series), which is analyzed to extract the dynamical information that the measurements contain. We thus want to see what the data can tell us about the dynamics. In particular, we introduce some tool that can sometimes be used to suggest what types of

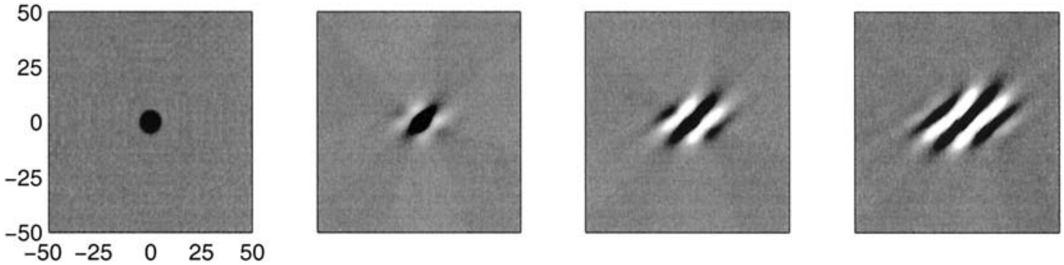

Figure 6. Evolution of a 10 × 10 km² Gaussian pit, subject to an asymmetrical tide containing an M_0 (0.95 ms^{-1}) and an M_4 (0.05 ms^{-1}), from left to right and vice versa. Top view of a 100 × 100 km² region at the evaluation times (from left to right): $\tau = 0$, $\tau = 2$, $\tau = 4$ and $\tau = 6$ ($\tau = 1$ corresponds to a dimensional time of 45–450 years). Troughs are depicted black, crests white and the undisturbed sea bed is gray. The pit elongates, deepens and slowly migrates, whereas around it a bank pattern emerges that slowly grows, spreads and migrates as well.

equations are appropriate to compare the predictions made by mathematical models to measurements collected in the field.

For instance, the principal oscillation pattern (POP) analysis (HASSELMAN, 1988; JANSEN, 1997) assumes linear evolution equations (linear dynamics such as those examined in a linear stability analysis). POP analysis therefore bears a close relation to linear stability analysis, and the analysis reduces to an eigenproblem for growing or decaying (or, theoretically, neutral) modes. However, when POP analysis is applied in a prediction scheme the unstable modes are discarded as numerical artifacts. This also means that the analysis will have a limited timescale of application, since all modes decay, albeit slowly perhaps. Conversely, in linear stability analysis we use only the unstable modes to describe the initial evolution of the bed form, so that if only stable modes are predicted then the basic state is not expected to develop rhythmic bed forms. It seems possible that real, growing modes might sometimes be missed in a POP prediction, if features are incipient. POP analysis can also reveal several types of oscillations, such as travelling or standing waves. Linear dynamics, however, may poorly describe a given process when equations of complex non-linear dynamics (*e.g.* hysteresis, time-varying parameters) should be sought.

The non-linear generalization of POPs is called principal interaction patterns (PIPs). They assume a nonlinear evolution model, and here nonlinear stability analysis can suggest such models for use in PIP analysis: see § 3.5.2. See LARSON *et al.* (2002) for an overview of these methods and their relevance within PACE.

Stability-type models can be combined with data assimilation, to find reliable estimates for the model parameters.

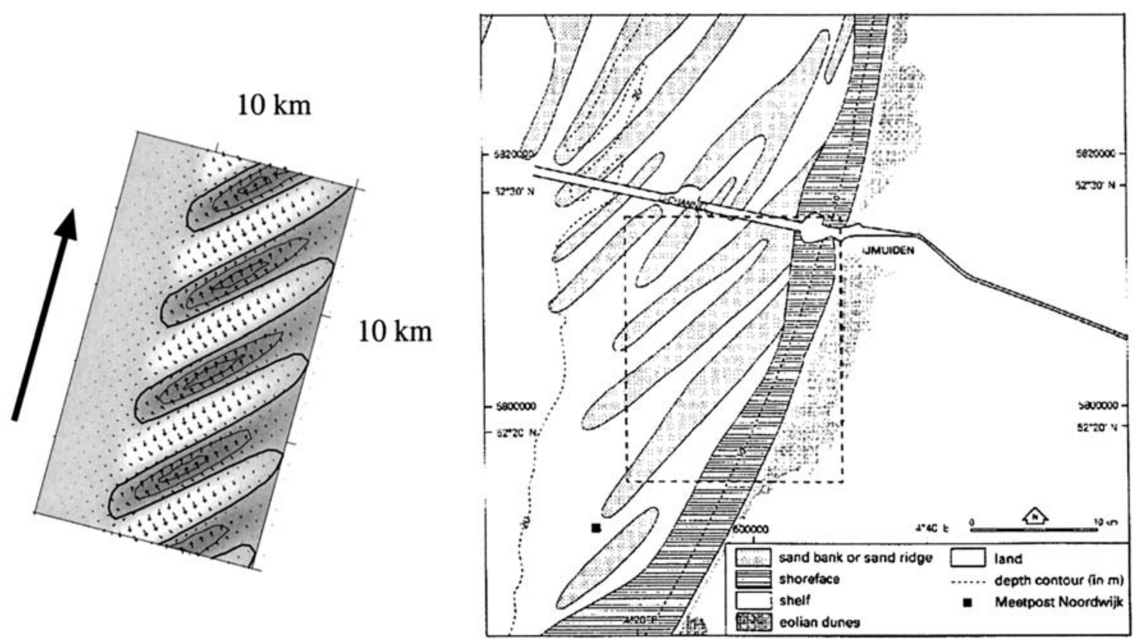

Figure 7. Shoreface-connected ridges off the Dutch coast (right image) contrasted with those predicted by a stability model (see CALVETE *et al.,* 2001b, c).

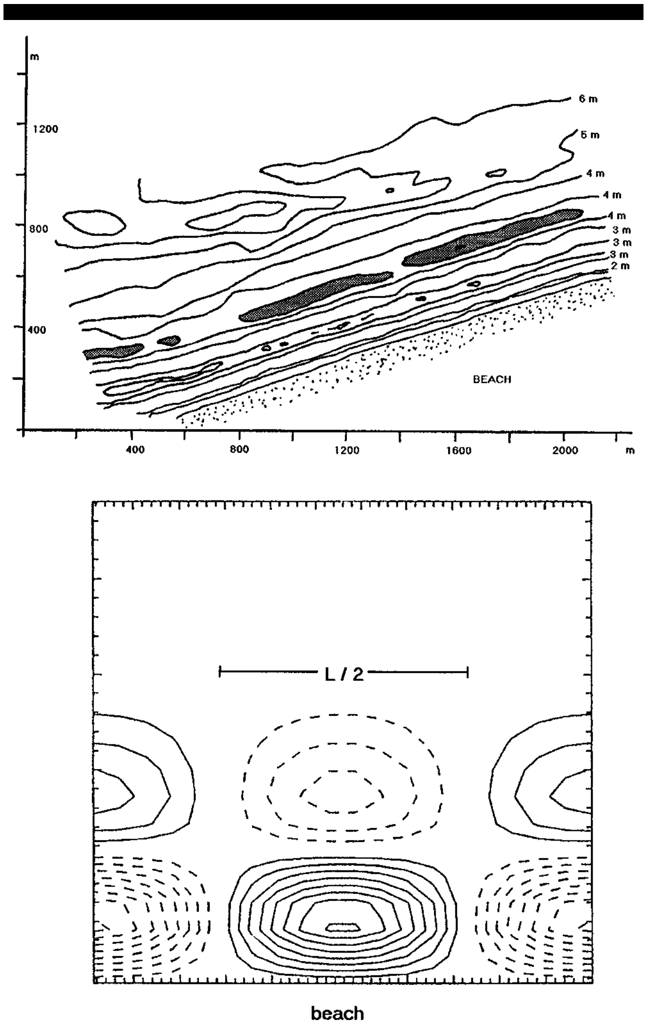

Figure 8. Top: bottom topography measured by PRUSZAK *et al.* (1997) at Lubiatowa beach (adapted from PRUSZAK *et al.* (1997)). Bottom: typical crescentic bottom forms as predicted by the theory.

For alternate bars in rivers, the previously mentioned Ginzburg-Landau equation (see SCHIELEN *et al.,* 1993) has been tuned with laboratory data, and the resulting model turned is accurate in predicting bed behaviour for a far longer time than the tuning period (see MORALISSEN *et al.,* 2003). Subsequently, this technique has been applied to sand waves, in which a Landau-equation was assumed to describe the bed dynamics. Here data assimilation was the only way to find the coefficients, the resulting model described the bed changes after dredging the tops quite well (see KNAAPEN and HULSCHER, 2002). The latter shows that stability models can successfully be used for data-driven modelling.

Behavior-Oriented and Parametric Modeling

These models are distinguished from process-based models, of which stability models are an example, by being based in some degree on observed behaviour of some aspect of coastal zone evolution, or some observed relationship between a series of state variables. HANSON *et al.* (2002) give an overview of these methods (including process-based models). Being motivated by certain observations, behavior-oriented models are designed to model a certain aspect of coastal evolution, like, for instance, one-line models or profile evolution models. This much they often have in common with stability models, so there seems to be some scope for the use of stability methods to predict schematized patterns (a spacing, an orientation, *etc.*—see Table 2), which could then be a starting point for behavior-oriented modeling. An obvious example is that of shoreline sand waves, which are amenable to modeling by process-based, and *in particular* stability models (because they form an identifiable, rhythmic feature), but are also treated by one-line models, based on complex diffusion equations. The two approaches are completely different in detail, but they both model essentially the same thing. The stability model could, conceptually, provide the diffusion coefficient; see HANSON *et al.* (2003).

DISCUSSION AND CONCLUSIONS

It is a remarkable feature of rhythmic patterns that they are often so apparent by inspection of, say, a photograph or satellite image; indeed, it is field data that has been the source of inspiration to develop and analyze stability models. It is relatively easy to identify the spacing between each of the features, their orientation and extent.

Observing the *temporal* development of these features, however, is usually far more difficult. Some features, particularly smaller scale ones like small cuspate features near the shore, can be observed to grow or decay over short time spans (days or even hours), and are visible to the naked eye, but most have time scales that require very long data sets for verification. In some cases, such as tidal sand banks, these would take a very long time to collect. Reconstruction from geological records would seem to be a possible useful tool in such circumstances (see TRENTESEAUX *et al.,* 1999).

The concept of chronology, *i.e.* the degree to which the order of forcing events affects the development of a system, may have relevance for stability analyses of morphodynamic systems too, despite being linked to the forcing itself (SOUTHGATE, 1995). The order of events determines the basic state characteristics, which in turn control the developments of the perturbations; in other words, different sequences of forcing events lead to the analysis of the stability of different attractors in the phase space.[8]

The question becomes more subtle when different forcing events characterized by equal "average" properties are considered, and only the "average" behavior of the perturbations is of interest. Any attempt to reach general conclusions inevitably is frustrated by the complexity of the problem and only particular cases can be considered. In VITTORI and BLONDEAUX (1997) the stability of a random wave approaching the coast with respect to edge wave perturbations was considered. The incoming wave can be thought of as the su-

[8] It should also be noted, however, that the importance of chronology has yet to be established; see VAN DER MOLEN and DE SWART (2001).

perposition of different wave components whose amplitudes are given by the wave spectrum and with random phases, so that different forcings are obtained with different phases, even though all the sequences are characterized by the same "average" specific energy or equivalently by the same "average" energy flux.

The analysis described (see VITTORI and BLONDEAUX, 1997) leads to an equation for the amplitude of the perturbations that is of Landau type (18), in which the linear coefficient a_1 of (18) is now time dependent, because of the randomness of the forcing, thus allowing the characterization of a chosen sequence of events. The resulting behavior is of a net growth with a random fluctuation imposed on it; ensemble averaging of a series of realizations reveals a smaller final amplitude than for the equivalent non-random case, thus implying that the randomness of the forcing makes the system more table. This analysis only considers a hydrodynamic problem in order to remove the additional complexity related to the different time scales characterizing the flow and bottom developments. However the analysis provides results which are felt to have a general relevance.

There has been a recent focus of interest on situations where a dynamic equilibrium is subjected to a large impact at one point in space. These are usually due to human interferences such as a once-only or a repeatedly nourished (forced) beach, and sand mining (see ROOS *et al.,* 2001), and bed subsidence due to gas mining (see FLUIT and HULSCHER, 2002; ROOS and HULSCHER, 2002).

Ostensibly, events like these are different from those examined in the usual stability analysis because they offset a previous equilibrium locally; traditionally, stability methods have been used to explain the occurrence of existing natural bedforms by examining the morphology where they are absent. In fact, the same methods may be applied to these events, which is not surprising when one views the beach before human interference as the basic state, and the removed or added material as a (finite amplitude) initial perturbation. Such sand mining and beach nourishment provide us with spatial structures the growth of which we may be able to measure within decades. Thus growth rates can be compared with theoretical values.

This can be seen in Figure 6, where a growing sand bank pattern radiates away from the sandpit center. This is a linear approximation (initial development), which has similarities with a linear stability analysis of sand banks (see HUTHNANCE (1982), HULSCHER *et al.* (1993)). These linear analyses of sand and gas mining are based on a small perturbation on a flat bed. This can be regarded as a first step towards an investigation of a realistic perturbation in a fully developed sea bed. The latter may be a flat bed or a bank pattern bed, depending on their stability properties within this model. Stable configurations are supposed to be realized in nature. However, growth towards such a stable configuration takes a long time for sand banks so that the study of human perturbations starting from an unstable equilibrium does not necessarily contradict reality, wherein natural and human-related perturbations may grow simultaneously. A nonlinear sandbank analysis will show the validity of this linear analysis. This knowledge is also needed for the study of the long-term impact of human actions: interactions between human perturbation and intrinsic behaviour are expected. These nonlinear aspects are under investigation. This technique has a high potential to link stability methods to applications, which are usually human-impact related.

These substantial movements of material might also have the effect of moving or flipping a previously stable equilibrium (*i.e.,* stable to small perturbations) into another, possibly also stable state. A classic example of this kind of mechanism, from hydrodynamic instability theory, is given in (DRAZIN and REID, 1981, page 375), in which a subcritical instability is described. Here, although linear instability is only encountered after the Reynold's number is increased beyond a critical value, R_c, for a range of values $R_G < R < R_c$ a finite amplitude perturbation can induce instability. The possibility of this happening in morphodynamics cannot be discounted.

As previously mentioned, stability analyses have proved successful in reproducing naturally observed phenomena (see *e.g.* CALVETE *et al.,* 2001b; FALQUÉS *et al.,* 1997, 1999; HULSCHER, 1996b; VITTORI and BLONDEAUX, 1997; VITTORI *et al.,* 1999). They have also proved successful in gaining an understanding of observed behavior and in providing a capability of predicting the kinematics and dynamics of the morphodynamical features in question.

Another important area has been the process of verification: see *e.g.* HULSCHER and VAN DEN BRINK (2001); CALVETE *et al.* (2001c) and CALVETE *et al.* (2001b); VITTORI *et al.* (1999). The first of these shows that stability models can be used to make predictions where sand banks and sand waves exist, co-exist or are absent. The model of HULSCHER (1996b) discriminates between those offshore bed states and gives physical parameters that theoretically explain their (non)occurrence. These model predictions are tested in the North-Sea basin and give encouraging results. In HULSCHER and VAN DEN BRINK (2001) the predicted areas of sand wave occurrence are directly compared with the observed areas. The model predictions were able to cover the observed locations. However, local flat beds within this predicted sandwave field were not predicted. This may be due to local changes in bed material, resulting in a much higher critical bed shear stress, which was not accounted for in the model. Accurate predictions on sandwave occurrence are important for, for instance, the design of pipeline trajectories (see NÉMETH *et al.,* 2002). These studies shown in detail how the practitioner can make use of these types of models and obtain useful information from them.

The studies on shoreface connected ridges (see CALVETE *et al.,* 2001b, c) have clarified that necessary conditions for shoreface-connected ridges to develop are a shelf with a large transverse slope and with the availability of medium to fine sand. Furthermore, the frequent occurrence of storms is required, with associated large wave orbital motions near the bottom and a mean alongshore flow. This is primary information to be used in complex models to model the effect of human interferences (constructions, pipelines, *etc*) in zones where shoreface-connected ridges are present. In addition, it is shown that shoreface-connected ridges are generated due to a free instability, and are not to be a geological relict. The overall success of these studies can best be appreciated by examining

Figure 7, in which ridges off the Dutch coast are compared with those predicted by the aforementioned stability model. The likenesses are startling.

The theoretical model of VITTORI *et al.* (1999) has been compared with different data, for example from Lubiatowa beach, Poland (PRUSZAK *et al.,* 1997, see Figure 8), and Niigata west beach, Japan (HOMMA and SONU, 1963). The theory predicts the appearance of longshore periodic forms with a pattern which both qualitatively and quantitatively agrees with observations (see Figure 8). For example, at Lubiatowa Beach the theory predicts a wavelength of 850 meters, a value which is close to the observed size of the longshore bottom forms (600–800 m). Similar results are found when the theoretical predictions are compared with measurements by HOMMA and SONU (1963).

We have discussed the relation between the various other modeling approaches and stability methods and the ways in which they can be used to benefit each other. It was explained that there are some strong similarities between some, and possible links and areas ripe for cross-fertilization between others. A tentative list of such areas might be:

- As described earlier, stability models often make use of the same dynamical equations as do numerical (so-called process-oriented) models. The results from the studies of idealized situations that stability models are designed for can reveal an important understanding and interpretation of observations *and* of the output from (numerical) process-oriented models. Studies comparing the two approaches would be fruitful, in understanding physics and verifying models. The studies of DAMGAARD *et al.* (2002), VAN LEEUWEN and DE SWART (2001) and CABALLERIA *et al.* (2002) are pioneering in this regard.
- As discussed, many observed rhythmic features form as inherent (free) instabilities of the morphodynamical system. In this sense they are *self-organizing,* and this interpretation should be explored further (see WERNER and FINK, 1993).
- Results obtained on nonlinear stability analyses can show qualitatively different behavior (patterns) from that predicted by linear stability analyses, to the extent that linear results may "look wrong", whereas those predicted by nonlinear analysis may be close to observations. In other words, the "failure" of a linear analysis does not necessarily mean that a nonlinear analysis will not be fruitful.
- In some cases nonlinear stability analyses can provide evolution equations as *generic* equations for data-driven models. This is particularly so in the cases of weakly nonlinear dynamics, and work on calculating coefficients from observations for input into such a model (Ginzburg-Landau) has been undertaken by KNAAPEN *et al.* (2001a) in river dynamics. This approach needs to be extended to coastal morphodynamics.
- The application of stability models to forced beaches and problems similar thereto, could prove fruitful, as discussed above. Work in this area on the effects of sand mining has already been begun (see FLUIT and HULSCHER, 2002; ROOS and HULSCHER, 2002). Such analyses may provide time-scales for the rearrangement of recharge material, and may provide guidance on the existence of other, stable beach and shelf states.
- Stability models could perhaps be used more as methods for improving or even designing behavior oriented models. The example given earlier was of shoreline sand waves, which would seem to be a good subject for stability analyses in their own right; there may be numerous other examples.

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