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Analysis of Field Data of Coastal Morphological Evolution over Yearly and Decadal Timescales. Part 2: Non-Linear Techniques

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ABSTRACT

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A number of techniques for non-linear analysis of time series data have been developed in recent years and applied in many environmental sciences. In this paper, some of these techniques are reviewed and their usefulness for coastal morphological data assessed, with examples in coastal morphology and related fields where these are available. The methods reviewed are time-delay embedding techniques, singular spectrum analysis, forecasting signatures, fractal analysis and neural networks.

It is expected that readers from diverse backgrounds such as statistics, environmental modeling, and data measurement would be interested in this subject. Accordingly, introductions have been provided for some concepts and background material. These include the general purposes of data analysis, the approaches traditionally used by statisticians and physical scientists, the general nature of coastal morphological data, and the distinction between linear and non-linear analysis methods.

It is concluded that the information potentially provided by these techniques is essential in understanding the generic behavior of coastal morphology on yearly and decadal timescales, and hence in constructing models and making forecasts of future morphological evolution. However, for these purposes, non-linear data analysis techniques usually require longer data series, and higher spatial and temporal resolution, than is available in present coastal morphological data sets. Data from remote sensing sources have the potential to meet these requirements.

ADDITIONAL INDEX WORDS: *Coastal morphology, non-linear data analysis, time series, time-delay embedding, singular spectrum analysis, forecasting signatures, fractal analysis, neural networks.*

INTRODUCTION

Observations of coastal morphology indicate complex behavior over a wide range of scales in time and space. Of particular practical, as well as scientific, interest are timescales of years and decades, and space scales up to several tens of kilometers in the longshore direction and up to several kilometers cross-shore (referred to as "long-term" and "large-scale" in this paper). These timescales correspond to the design lifetimes of many coastal engineering schemes, and the space scales to coastal "cells" (*i.e.* geographical coastal units with reasonably distinct longshore boundaries such as headlands, inlets or major human developments).

Most recent research into coastal physical processes has focused on shorter time and space scales. This has been motivated by an attempt to measure, understand and model these processes over timespans of up to a few tidal cycles and wave events. However, it is becoming increasingly recognized that the understanding and modeling of coastal morphodynamics over larger time and space scales is not simply a question of making extrapolations of these small-scale processes. There are fundamental limits to this type of extrapolation, as a result of internal dynamics ("chaotic" behavior) and lack of knowledge of detailed future sequences of hydrodynamic forcing conditions. These factors are in addition to the practical limitations of data availability and computational model accuracy and efficiency.

An understanding of coastal morphodynamics on the larger scales needs to be based on processes at those scales (De Vriend *et al.,* 1993). An important requirement is the collection and analysis of data, but, at present, there are far fewer data sets of coastal morphology at these scales compared to shorter scales. For practical applications, methods of data analysis have tended to be limited to simple, standard techniques such as linear regression and spectral analysis to detect overall trends and periodicities (in space and time) in the morphological response. At a research level, sophisticated lin-

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ear techniques of time series analysis, especially those based on Principal Component Analysis, have become widely adopted. These methods are reviewed in the companion paper, LARSON *et al.* (2003). However, the recognition that coastal systems are strongly non-linear indicates that data analysis should be taken a stage further, to include non-linear techniques. In recent years, there has been a tremendous growth in the development of such methods, designed for extracting the generic properties of the dynamics of non-linear systems, although they have rarely been applied to coastal morphology even at a research level. The aim of this paper is to identify and review the most useful of these methods, how they can be applied to long-term coastal morphodynamic data, and what type of information they can supply.

It is recognized that readers from a variety of backgrounds will be interested in this subject. These backgrounds can include:

- Generic approaches to analysis of time series of data
- Modeling of natural physical systems, including model comparison with field data
- Quantitative descriptive approaches to understanding coastal morphology
- Measurements of coastal morphology (including hydrodynamic forcing), and "traditional" data analysis methods.

Accordingly, several short introductory sections are provided to explain concepts and terminology that may be unfamiliar to some readers. These introductory sections cover:

- The general purposes for analyzing data from any natural system
- The different approaches to data analysis traditionally used by statisticians and physical scientists
- The general nature of long-term coastal morphological data
- The distinction between linear and non-linear analysis techniques

Following these introductory sections, the main part of the paper contains brief descriptions of the most relevant non-linear techniques for coastal morphology, including examples of applications in coastal morphology or other environmental sciences. However, detailed descriptions of the techniques and the framework of dynamical systems theory on which they are based are beyond the scope of this paper. The paper concludes with a summary and overall assessment of the role of non-linear data analysis techniques for understanding coastal morphology.

PURPOSES OF DATA ANALYSIS

General Purposes of Data Analysis

There are several purposes for which one may want to analyze a particular data set. A given analysis technique is usually more appropriate for some purposes than others, and one should not believe that methods exist that can be appropriately used for all purposes. At the outset of any analysis, therefore, it is essential to be clear about the objectives of the analysis. The following main purposes of data analysis can be identified, in approximate order of complexity:

- *Data Reduction.* The aim is to reduce highly multi-variate data sets to smaller numbers of variables, while minimizing the errors that will necessarily occur when any subsequent analysis is performed on the reduced data set.
- *Phenomenology.* For this purpose, the data are analyzed to reveal the modes of behavior that occurred during the period of measurements. For coastal data sets, this often concerns a joint analysis of time series of morphological variables and physical forcing conditions (*e.g.* nearshore wave properties). The result of this type of analysis thus is a quantitative description. There is no requirement that the measurements cover all possible states of the system.
- *Forecasting.* In this case, the aim is to make accurate forecasts of one or more of the system variables over relatively short future timescales, *i.e.* over time spans that are considerably shorter than the length of the data time series. There is no need (or wish) to understand the underlying physical processes. To put it simply, in forecasting one is looking for a sophisticated way of extrapolating the time series.
- *System Characterization.* Here the aim is to discover generic dynamical properties of the system that describe its long-term behavior. Such properties would include the types of attractor states and the conditions under which they occur, the effective dimensionality ("degrees of freedom") of the system, and the relative importance of internal dynamics and external forcing. In some cases, comparisons with other data sets may be made (*e.g.* between "forcing" data and "response" data) to provide evidence to support a particular physical theory. Ideally, the data should cover all possible states of the system.

The above types of data analysis are often combined. For example, data reduction often allows the other types of analysis to be performed more readily. Another example is that system characterization can build on several preceding analyses of the phenomenological type.

This paper focuses on non-linear analysis techniques. These are important for system characterization of natural systems (including coastal morphology) that are known, on physical grounds, to be governed by non-linear processes. Non-linear techniques can also be used for data reduction, phenomenology, and forecasting, but because of the limited quantity of data in even the best coastal morphology data sets, there may be little or no benefit over linear techniques (see, for example, PENLAND (1989), who discusses the merits of linear and non-linear techniques in meteorological applications).

Objectives of System Characterization

Because of the importance of non-linear techniques for system characterization, it is appropriate to indicate what are the characteristics of a dynamical system about which such analysis techniques can provide information. Essentially, the aim is to understand the generic long-term behavior of the system, rather than to make detailed predictions of coastal morphology or to understand the constituent hydrodynamic and sediment processes. It is to help answer questions like:

- What type of attractor state does the coastal morphodynamic system try to go towards (such as equilibrium, periodic and chaotic states)?
- How long does it take the system to reach this attractor state?
- What are the typical size and stable statistical properties of this state (*e.g.* the mean and variance of bedform amplitudes)?
- What is the dimension of the attractor?
- Over what time and space scales does this attractor state occur?
- How strong are direct responses to random temporal changes in the hydrodynamic forcing? Are they strong and/or frequent enough to prevent the system from reaching an attractor state?

These issues are of great practical importance for providing guidance about the best methods of making morphological predictions. For example:

- Many long-term modeling techniques assume movement towards an equilibrium state. However, if the system does not move towards equilibrium such models would not be applicable.
- In many cases, short-term variations of morphological behavior are as significant as the long term trends, and in some cases, more so. For making predictions of these variations, it is essential to understand their cause. For example, they could result from measurement errors, random variations of input conditions, or deterministic internal dynamical processes.
- Knowledge of the dimension of the attractor is relevant because it gives the minimum number of independent variables necessary to model the time evolution of the system, and can also indicate the type of attractor (FRAEDRICH, 1986).

The data requirements for a thorough system characterization can be severe. This is particularly true if the system is known to be governed by non-linear physical processes and to be spatially extended. Both apply to coastal morphology. The amount of data available in even the most comprehensive data sets is substantially less than that needed for some analysis techniques.

Before describing the most common and promising of these techniques for coastal morphology, one should be aware of two broad approaches to data analysis, which we term a "statistician's approach" and a "physical scientist's approach". We conclude that, because of the non-linear, spatially extended nature of coastal morphodynamics, and the limited data typically available, the "physical scientist's approach" is generally preferred for system characterization.

STATISTICAL AND PHYSICAL APPROACHES TO THE ANALYSIS OF DATA

Aims and Concepts

A *statistician's* approach is to deduce general properties of a natural system by data analysis alone, without reference to other sources of information about the system.

Table 1. *Some key words used in differnet senses by statisticians and physical scientists.*

Word	Statistician's Meaning	Physical Scientist's Meaning
PROCESS	A general relationship between data values	A physical mechanism that operates in the natural system
MODEL	A calculation procedure for parameter values of a statistical process	A calculation procedure for parameter values of a physical process
FORCING	A property of the data not described by identified statistical processes	External physical forces acting on the natural system
DYNAMICS	How the system evolves in time, interpreted in the context of statistical processes	How the system evolves in time, interpreted in the context of physical processes

A *physical scientist's* approach is to start with a hypothesis about the physical mechanisms that govern the natural system, and then to look for evidence from data analysis to support or contradict that hypothesis.

A physical scientist's approach will provide only circumstantial evidence for a particular hypothesis (*i.e.* another hypothesis might explain the data equally well or better). This is related to the idea that physical theories cannot be proved, but only falsified or have limits of applicability determined. ORESKES *et al.* (1994) discuss these issues in relation to the earth sciences. A statistician's approach, if it is possible to carry out, will therefore provide stronger evidence than the physical scientist's. However, the requirements for the statistician's approach, in terms of quantity and quality of data, are more severe, and in many natural systems these requirements cannot be met.

The difference in these approaches has led to important differences in meaning of some key words (Table 1), thereby carrying the risk of confusion. For example, a statistician's model would be regarded by a physical scientist as a data extrapolation or interpolation technique (albeit a sophisticated one in many cases). A physical scientist's model, on the other hand, would be regarded by a statistician as a source of additional constraints in a statistician's model. Another example is that the statistician's notion of "forcing" is often referred to as "noise" by physical scientists.

Strengths and Weaknesses of the two Approaches

Statistician's Approach

The main strengths of the statistician's approach are:

- It can provide positive evidence (as distinct from only circumstantial evidence) to distinguish generic properties of the data.
- It does not require prior knowledge of the system dynamics, and so is appropriate when the dynamics are poorly or not at all understood.

The main weakness is that the quantity, quality and other properties of the data needed for this approach can be difficult or impossible to achieve, especially for natural systems. Such requirements typically include:

- Long time series of data (typically several thousand data values or greater, although the number of data values depends on the technique used)
- Stationarity of mean and variance
- Equal sampling intervals
- Low-dimensionality (small number of degrees of freedom in the system)

Physical Scientist's Approach

The main strengths of the physical scientist's approach are:

- The available evidence to support a hypothesis consists of the physical theory by which the hypothesis was derived as well as the evidence from data analysis.
- The data requirements are determined by the nature of the hypothesis (and not the full range of alternative hypotheses) and are therefore less strict than for the statistician's approach.

The main weakness is that the evidence from data analysis is only circumstantial.

Relevance of the two Approaches to Coastal Morphological Data

Presently available time series of coastal morphological data from in-situ measurements usually fall well short of the requirements for a statistician's approach, in particular for discriminating any non-linear character of the dynamics. The best data sets have at most a few hundred values per time series, which is insufficient for establishing some statistical properties of a coastal morphological system. Furthermore, such data series usually have undesirable features such as irregular interval spacing (in space and/or time), non-stationarity of mean and variance, and external noise from measurement error and other sources. However, data sets from remote sensing sources, from ground, air or satellite, have the potential for supplying the large quantity of data, over wide areas, and at regular time intervals that could be used in a statistician's approach.

In those cases where non-linear analysis techniques can be used on short data series, it is often possible to construct alternative data series of the same length (sometimes referred to as "surrogate" data) using linear mechanisms which show equivalent statistical properties. However, these series are often constructed in an artificial manner, and physics-based knowledge would indicate that these linear mechanisms can be rejected as serious explanations of the physical processes. An example is provided by the forecasting signatures technique described later in the paper.

A physical scientist's approach to system characterization employs some understanding of the physical processes of coastal morphology. For this approach, the data requirements depend on the hypothesis being tested, and can be less severe than a statistician's approach. These hypotheses may, for example, require analysis of data to detect periodic features of morphology, statistical signatures of self-organized patterns or low-dimensional chaos, or correlations to strong forcing events.

NATURE OF COASTAL MORPHOLOGICAL DATA

Basic Form of Nearshore Morphological Data

The basic form of coastal morphological data consists of time series of bed levels at spatial locations covering the coastal area of interest (*i.e.* a complete description of the bathymetry in space and time). In this paper, the term "bed levels" refers to levels of the beach or nearshore zone relative to some vertical datum, irrespective of whether the levels are emerged or submerged. Mathematically, this basic form of the data can be expressed by the function z(x,y,t) in which:

z = bed level (m)
x,y = horizontal co-ordinates (m)
t = time (s)

In most applications, however, an analysis of this complete bathymetric data is either not possible or not sought. It is more usual to measure and/or analyze a spatial subset of the bathymetric data that represents some morphological features or geographical locations. Some typical data subsets are:

- *Profile Lines.* A common survey technique consists of measurements of bed levels along one or more profile lines perpendicular to the coast (Figure 1). The analysis of this data aims to reveal spatial and temporal patterns along these profiles. Evolution of cross-shore morphology can occur on all timescales from a few days to millennia. Often significant evolution of cross-shore morphology occurs on a shorter timescale than longshore morphology, so providing some justification for treating them independently in models. However, this difference of evolution rates does not always happen, particularly for more complex coastline configurations. Cross-shore evolution models applicable to different timescales are described in HANSON *et al.* (2003) and COWELL *et al.* (2003).
- *Bed Level Contours.* Data analysis is often performed for a single contour representing the shoreline (such as the mean sea level contour), but analyses of more than one contour are also sometimes made (Figure 2). Planshape models are based on the evolution of one or more nearshore contours (HANSON *et al.,* 2003).
- *Nearshore Bars.* Sometimes the interest is in the growth, decay and migration of nearshore bars. In this case, bathymetric data is used to define morphological variables that characterize the bar features (such as bar height, side slopes, and location) (Figure 3).
- *Other Nearshore Morphological Features.* The evolution of any large-scale nearshore morphological feature will be of scientific, and sometimes of practical, interest. These could include channels, tidal deltas, sand ridges, *etc.,* and be of natural or human origin. In some cases, they may not fit neatly into a descriptive classification scheme. As before, interest is focused on the bathymetric data that defines the morphological feature. Many morphological features show rhythmic patterns and are thought to be caused by a morphodynamic instability of the seabed (DODD *et al.,* 2003).
- *Small-Scale Bedforms.* A bathymetric data set with a sufficiently high spatial resolution can be analyzed to study

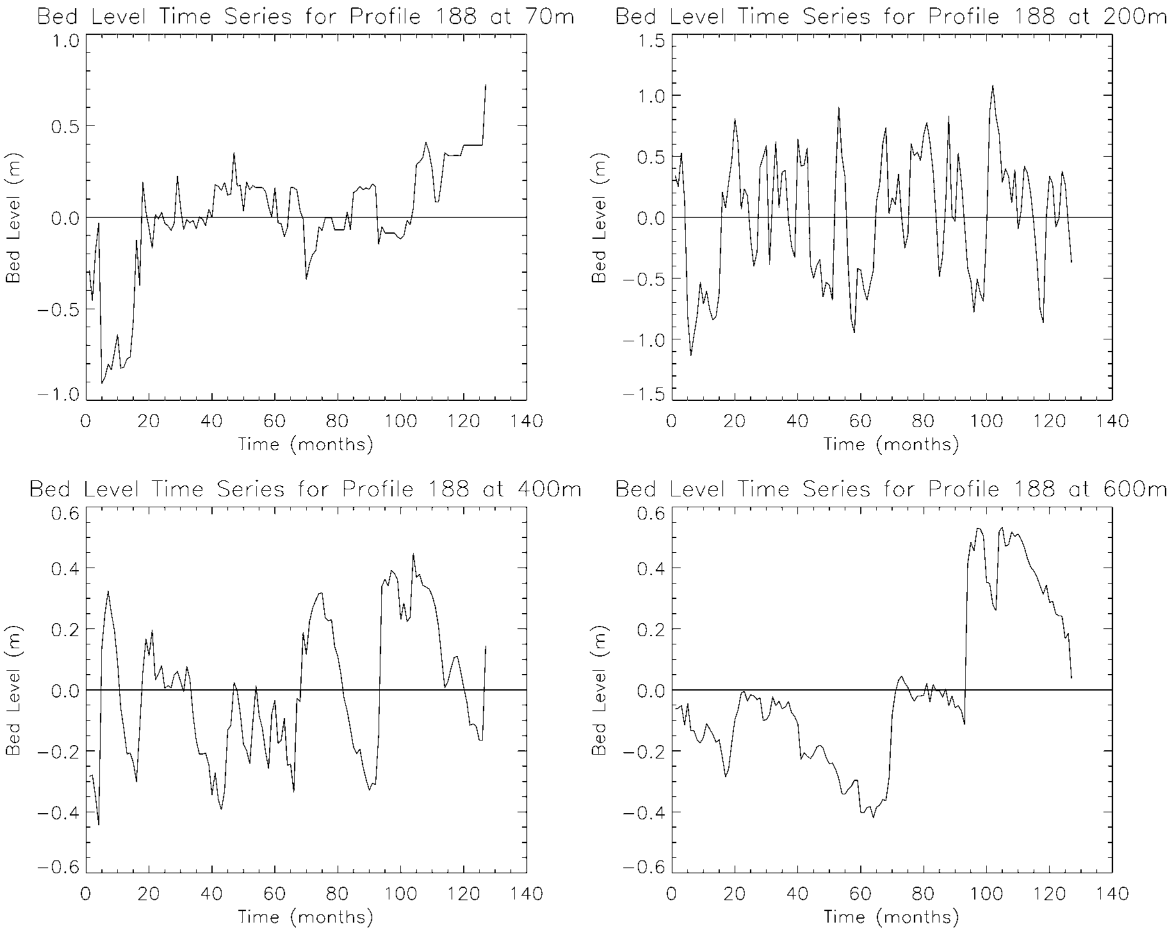

Figure 1. Example of time series of seabed elevation data at four locations on a cross-shore profile at Duck, NC, USA (1981–1991). Bed levels in each figure are relative to the mean of the data series. Approximate locations (relative to onshore datum) and mean depths (relative to mean sea level) as follows: Top Left, Shoreline, Distance 70 m, Depth 0 m. Top Right, Inner Bar, Distance 200 m, Depth 2.5 m. Bottom Left, Outer Bar, Distance 400 m, Depth 4 m. Bottom Right, Upper Shoreface, Distance 600 m, Depth 6 m.

the dynamics of small-scale bedforms such as sand ripples and beach cusps. In this paper, however, we will not consider these small-scale features further and focus instead on larger-scale features and bathymetric data with an appropriate (coarser) spatial resolution.

Desirable Features of Long-Term Morphological Data Sets

The desirable features of a long-term nearshore morphological data set depend very much on the purpose for which the analysis is being done and on the techniques used. However, some general desirable data properties can be identified that are common for all purposes and techniques. These properties are outlined here; the more detailed requirements for particular methods of data analysis are discussed later in "Non-Linear Data Analysis Methods".

a) *Temporal Coverage and Resolution.* Temporal coverage should be a few times longer than the maximum timespan of interest. To investigate the relationships between long-term trends and shorter-term variations, a temporal resolution of at least two orders of magnitude finer than the total length of the data set is needed. For long-term nearshore morphodynamics, typical values would be several decades or greater for temporal coverage, and less than one month for temporal resolution. Note that for data analyses for other purposes than system characterization, such as phenomenology, the requirements on temporal resolution are generally less severe.

b) *Spatial Coverage and Resolution.* The spatial extent of the data should cover the geographical region or morphological feature of interest. In the latter case, measurement locations may need to change (through time) to follow the feature as it moves. The spatial resolution should be sufficient to give several data values covering the smallest scale of interest.

c) *Uniformity of Sampling.* Many analysis techniques require data values to have constant sampling intervals in the independent variable (usually this is time or space). It is possible to interpolate the data (linearly or by other means) so that data values are at specified constant increments, but such a procedure alters the statistical properties of the data

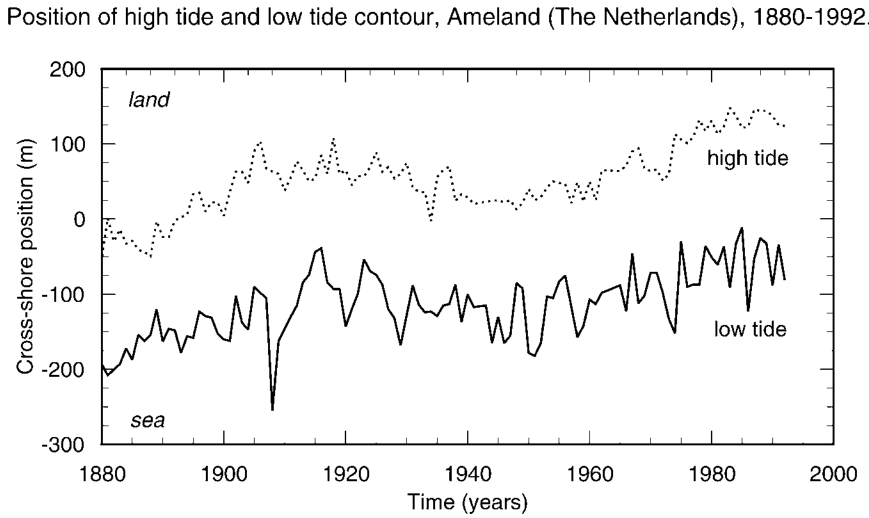

Figure 2. Example of time series of shoreline position at one longshore location on Ameland, the Netherlands (1880–1990).

series. This could be important, for example, in assessing the relative importance of randomness vs deterministic chaos as a mechanism for data variability. If interpolation is done, sensitivity tests should be carried out to assess differences in the statistical properties of interest for different choices of data interval and interpolation procedure. An alternative approach is to use "fuzzy" time co-ordinates in which unequal time intervals are treated as measurement errors in the time co-ordinates (BREEDON and PACKARD, 1992).

d) *Stationarity.* A stationary time series is one that has the same value of some statistical property (usually the mean or variance) for all subsets of the data series as for the whole series. A number of analysis techniques require the data series to be stationary in mean and/or variance, although all time series of natural systems will deviate from stationarity to some extent. The effects of non-stationarity can be estimated by performing sensitivity analyses on the data series before and after removing linear trends, or by performing analyses on subsets of the data.

e) *Measurement Accuracy.* The required accuracy is very much dependent on the application. Achievable accuracies with modern in-situ survey techniques are within a few centimeters for vertical and horizontal position fixing. This provides sufficiently low-noise data for most data analysis purposes concerned with large-scale and long-term morphodynamics.

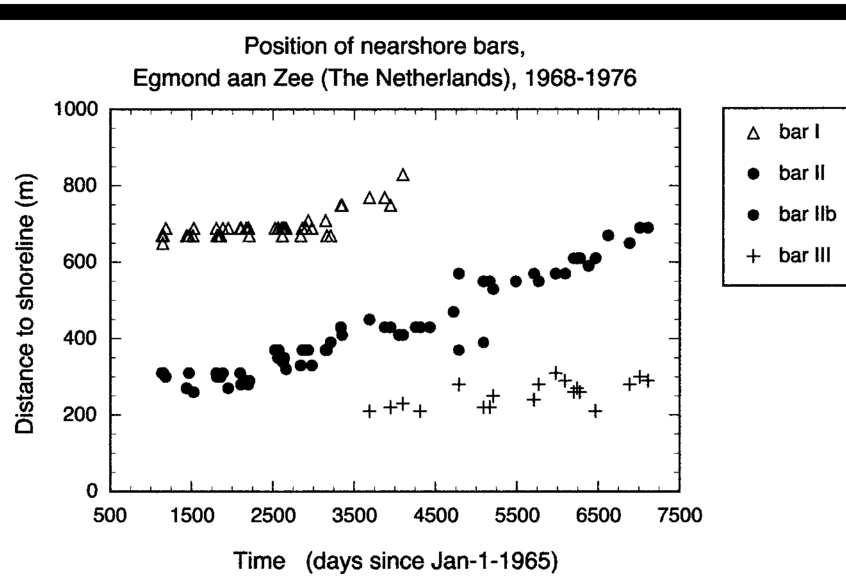

Figure 3. Example of time series of nearshore bar position at one longshore location at Egmond-aan-Zee, the Netherlands (1968–1976). Adapted from Kroon (1994).

f) *Concurrent Hydrodynamic Data.* The "physical scientist's" approach often requires an investigation of how morphological data are correlated to concurrent hydrodynamic data (usually wave, wind, current and water level conditions).

LINEAR VS NON-LINEAR DATA ANALYSIS

What is a Non-Linear Analysis Method?

It is not a trivial issue to give a clear-cut definition of non-linear analysis methods. We may define them by exclusion by stating they include all methods that are not linear, but this shifts the problem to a good definition of a linear method. This is not particularly easy either, since the borderline between linear and non-linear methods is blurred (CHATFIELD, 1996). For example, some non-linear methods are modifications of essentially linear methods, such as piece-wise linear regression. One common feature of the non-linear methods discussed in the following section is the use of time-lagged variables, although this is not a defining characteristic of non-linear methods generally. However, for the purpose of system characterization, an analysis method can be called non-linear if it is used as a tool for studying non-linear properties in a data series (*e.g.* JAFFE *et al.,* 1994).

When to Use Non-Linear Methods?

Whether to use linear or non-linear techniques for a given time series depends on the purpose of the analysis, typical characteristics of the time series, and background knowledge of the physical system that generated the time series. Features of a time series that suggest the use of non-linear techniques for system characterization include:

- The probability distribution of the data has a strong asymmetry
- The time series contains sudden occurrences of large values with irregular spacing in time (after having established that these are genuine, and not measurement error "spikes")
- The time series shows a change in variance over time
- The time series exhibits time irreversibility (*i.e.* properties of the series are different when analyzed in the reverse direction)
- Background knowledge indicates that the time series is generated by non-linear physical processes

How does this apply to coastal data? Firstly, the background knowledge of physical processes in the coastal morphodynamic system indicates that the system is non-linear, for example through the non-linear response of sediment transport rates to the prevailing hydrodynamic conditions. Furthermore, the system is spatially extended (and therefore inherently high dimensional), and contains feedback from morphodynamic response to the hydrodynamic conditions. Both these factors suggest that the morphodynamic response will have a strong internal dynamical character. However, direct responses to external dynamical forcing also have an important role; strong wave events (and groups of events) will have a strong effect on the morphodynamic response. An important task of non-linear analysis of morphological data, therefore, is to distinguish internal dynamical behavior from forced responses.

One feature often observed in coastal morphological time series that can be indicative of non-linear response, is time irreversibility. This is illustrated by the time series of beach levels on the Upper Shoreface at Duck, USA (Figure 1, bottom right). In this case, there is strong accretion during storm group events, as sediment is eroded from the bar regions and deposited further offshore. These strong accretional events are followed by gradual erosion as sediment is moved onshore under more moderate wave activity. There would be a quite different response if the time series were reversed.

NON-LINEAR DATA ANALYSIS METHODS

Introduction

In this section, we review some of the most common non-linear techniques that have been applied in other environmental sciences, and assess their relevance to coastal morphology. The main aim is system characterization, although we mention other aims for which the techniques are particularly useful. The data requirements for a non-linear statistician's approach are usually far higher than can be supplied by environmental data in general, and coastal morphology data in particular, so a physicist's approach tends to be preferred. However, we start the review with two methods that have the character of a statistician's approach.

Time-Delay Embedding Techniques (Grassberger-Procaccia Method)

Non-linear analysis of time series took a major step forward in the early 1980s with the concept of "time-delay embedding" (TDE), first put forward by PACKARD *et al.* (1980) and TAKENS (1981). The idea is that the character of the overall dynamics of multivariate systems is reflected in the response of just one variable (this is known as Takens' theorem). By using this concept, the time series of this one variable is sufficient to determine the properties of the whole system.

The most commonly used technique exploiting the time-delay embedding concept is by GRASSBERGER and PROCACCIA (1983) for calculating a quantity known as the correlation dimension of the attractor, denoted by D. The value of D will indicate the type of attractor (equilibrium, periodic, chaotic) and the number of independent variables needed to characterize the system. This is essentially a statistician's approach. It requires large data series, and for this reason we conclude that it is inappropriate for present coastal morphological data sets. However, the method is included here because it is commonly used for other dynamical systems, and forms the basis for more refined methods that can use less data and hence be more suitable for coastal morphology.

Briefly, the method works as follows. We start with a time series x_i with N values, and select an integer d (known as the embedding dimension) smaller than N. A point in the state space of dimension d is defined by the vector $\{x_1 \ldots x_d\}$, and the system evolves in time with successive values $\{x_2 \ldots x_{d+1}\}$, $\{x_3 \ldots x_{d+2}\}$ etc. up to $\{x_{N-d+1} \ldots x_N\}$. Grassberger and

Procaccia define the correlation dimension, D, of the resulting set of points in d-dimensional space analogously to the fractal box-counting dimension for a set of points in real space. They put forward a computationally efficient means for calculating D. The procedure is repeated for a range of values of d = 1, 2, 3 *etc.,* and a graph is made of the correlation dimension D vs the embedding dimension, d. Often it is found that D remains constant beyond a certain value of d, and this value of D then represents the correlation dimension of the attractor of the full system. If D does not "saturate" but rises indefinitely with increasing d, the system either has a correlation dimension higher than the highest d tested or shows completely random behavior. A non-integer value of D indicates that the dimension is fractal and the attractor is chaotic. The value of D indicates the number of independent variables needed to model the dynamical behavior of the system.

A major problem with the Grassberger-Procaccia method and related methods is that they require large data time series (of the order of 10^D values in a single time series (RUELLE, 1990)). Typically at least several thousand data values are required and these are usually not available from coastal bathymetric data sets. Another difficulty is that the data need to be equally spaced in time. There are some possible approaches to overcome these problems:

(1) In many cases, coastal bathymetric data is measured simultaneously (or nearly simultaneously compared with the time interval between surveys) at a number of different locations, for example, in measuring a series of beach profiles. The time series at each location may be too short to apply TDE techniques, but the total amount of data at all the locations could be sufficient. It may be possible to combine the data time series at different locations provided it can be demonstrated that the time series have been created by similar mechanisms, and that correlation calculations are not taken across the boundaries of different series (this type of approach has been suggested by SUGIHARA, 1994, for time series of heart rhythms from different patients). It can be argued on physical grounds that nearby locations have similar bed material and are exposed to similar hydrodynamic conditions. However, the question of what constitutes a "similar mechanism" remains, and can only be properly answered by appropriate statistical tests. One such method is described in DIKS *et al.* (1996). To the authors' knowledge, such a procedure has not yet been attempted for coastal bathymetric data.

(2) Coastal bathymetric data is rarely measured at equal time intervals, but most TDE methods require that data values are equally spaced in time. Some methods to address this problem are discussed earlier in "Desirable features of long term data sets: uniformity of sampling".

Despite these possibilities, TDE techniques generally require too much data for inherently high-dimensional systems such as coastal morphology. They have been applied to a range of natural high-dimensional systems, but with limited success. The high data requirements have prompted researchers to explore other methods that require less data.

Singular Spectrum Analysis

Singular Spectrum Analysis (SSA), or Singular System Analysis, is a method that uses TDE concepts but requires less data than the Grassberger-Procaccia method. A few hundred data points per time series may be sufficient. (VAUTARD *et al.,* 1992). The aim of SSA for system characterization is similar to that for TDE, namely to identify the type of attractor state and the number of independent variables needed to describe the system.

SSA is a particular application of the Empirical Orthogonal Function (EOF) technique (see LARSON *et al.,* 2001) that employs time-lagged variables and an embedding dimension. The method uses a data matrix whose components typically contain bathymetric values measured at a specific location lagged in time (VAUTARD *et al.,* 1992). In order words, the column vectors of the data matrix contain the measured time series successively delayed in time up to the maximum shift known as the embedding dimension (the term "window length" is also used in this context). The eigenvectors of the corresponding covariance matrix yield the dominant patterns in the time series under study. In some cases, measured time series from several locations are analyzed simultaneously leading to Multichannel SSA (MSSA). MSSA is mathematically equivalent to extended EOF (EEOF) analysis (see LARSON *et al.,* 2001), although EEOF is used for data sets with many points in space and only few lags in time.

Selecting the embedding dimension d is crucial in the analysis and SSA will not resolve periods longer than d. In practice, d should not exceed N/3 (VAUTARD *et al.,* 1992), where N is the number of values recorded in the time series (SSA is typically successful in resolving periods in the range d/5 to d).

SSA has the potential to be applied to long-term coastal morphological data for system characterization purposes. This does not yet appear to have been attempted, although such applications have been made in the fields of signal processing and climatology (*e.g.* FRAEDRICH, 1986). SSA is also useful as a technique for noise-reduction, detrending, and identification of oscillatory components. In addition, SSA can be employed for predictive purposes if it is combined with, for example, autoregressive (AR) models to form a linear forecasting algorithm. In such algorithms, the significant principal components are forecasted by fitting an AR model to each of these components.

An example is given here in which SSA is used for a phenomenological purpose, to extract long-term fluctuations in shoreline position. This analysis has been carried out for shoreline data at two different field sites, Ogata on the western side of Japan (facing the Japan Sea), and Duck on the eastern side of United States. The shorelines at these two sites both exhibit complicated fluctuations in time at many scales, including long-term trends and/or oscillatory behavior over periods much longer than the measurement time.

The beach at Ogata was surveyed weekly for about 12 years (1973–1985) and the shoreline position at equal time intervals was interpolated from the time series of profile measurements. Figure 4 (left) displays the measured shoreline position at Ogata Beach as a function of time (with re-

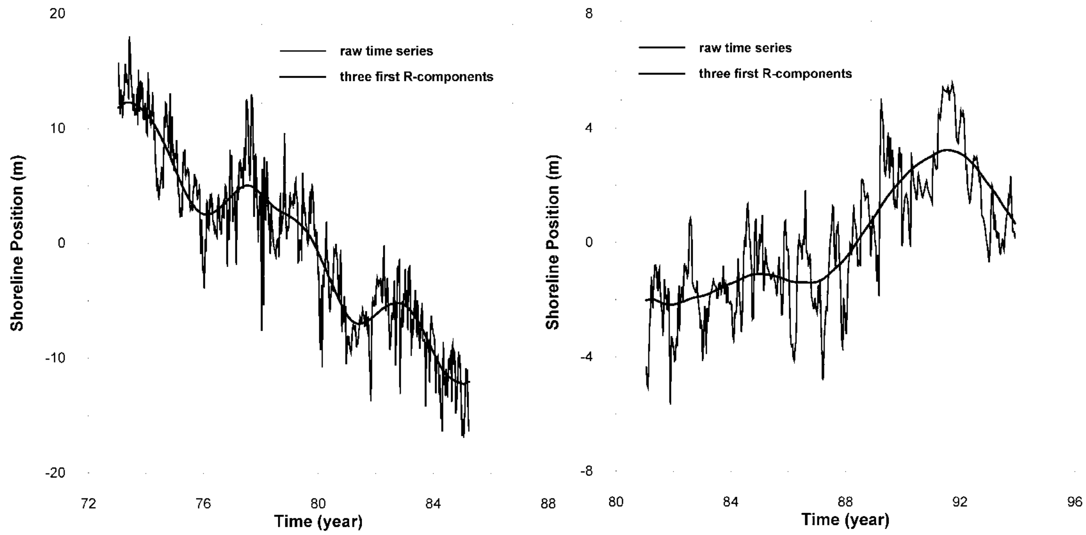

Figure 4. Singular Spectrum Analysis: Measured time series of shoreline position and SSA reconstructed components showing the three lowest components. (Left) from Ogata, Japan, and (Right) from Duck, USA.

spect to the mean shoreline position), clearly showing marked erosion and shoreline retreat. The other data set analyzed on shoreline evolution originated from Duck and encompassed 13 years (1981–1994; Profile 188), where profile surveying was done at approximately two-weekly intervals. Figure 4 (right) gives the time series of measured shoreline position at Duck, which indicated accretion and shoreline advance during the period of measurements.

Both time series included complex long-term trends that might be identified using SSA. If required, these trends could then be removed before continued analysis of the shorter term behavior. Figure 4 shows the sum of the three lowest reconstructed components from the SSA analysis for the Ogata and Duck data. SSA is a data-adaptive technique where the shape of the principal components are empirically determined from the data, implying that complex long-term trends may be identified even though the time span covering these trends is relatively short. However, giving the long-term trends a physical interpretation, or separating these trends from oscillations at shorter time scales, may not be trivial. The trend identified for the Ogata data seems to indicate a near linear decrease superimposed by an oscillation that appears to have a period of about 5 years. In the case of Duck, the overall advance of the shoreline that occurred during the first ten years seems to have been halted and in recent years there is a tendency of a general retreat.

Forecasting Signature Methods

Forecasting signature (FS) techniques attempt to determine a system's generic dynamic properties, and can make short-term predictions, but without providing knowledge of the underlying detailed physical mechanisms. However, it requires the use of trial models, which gives the method a physical scientist's character. In contrast with TDE methods, relatively modest amounts of data in a single time series are required (typically a few hundred).

As with TDE methods, FS methods exploit the idea of time-delay embedding by using an embedding dimension created from lagged sequences of data values. Briefly the method works as follows: The time series is split into two halves; the first half acts as a "library" of data values from which predictions are made, while the second half acts as a testing set to assess the accuracy of the predictions. A trial value of the embedding dimension, d, is selected, and vectors (*i.e.* "points" in d-dimensional space) are created from lagged data sequences from the library set, similarly to the procedure described above for TDE methods. A data value from the testing set is then selected (say, at time, t), and the aim is to determine how well predictions from the library set agree with the actual data values from the testing set at successive future times (*i.e.* at times $t+1$, $t+2$ *etc.*). To do this, a d-dimensional vector is defined by the selected value from the testing set and the $d-1$ previous values. From the library set the closest $d+1$ points to the chosen point are identified. Predictions are then made at successive time intervals by noting how the $d+1$ points from the library set evolve. The predicted vector is then calculated as a weighted average of the $d+1$ points at the new time level.

This procedure enables one to plot the accuracy of predictions against future time. Generally, such a plot would show high accuracy at small future times, with the prediction accuracy decreasing at longer future times. So far, we have investigated to what accuracy the data itself can be used for making future predictions. The FS methods take the procedure a stage further by using several trial physics-based models and investigating the prediction-decay curves for data generated by the models themselves (this data is sometimes known as "surrogate" data). The prediction-decay curves from

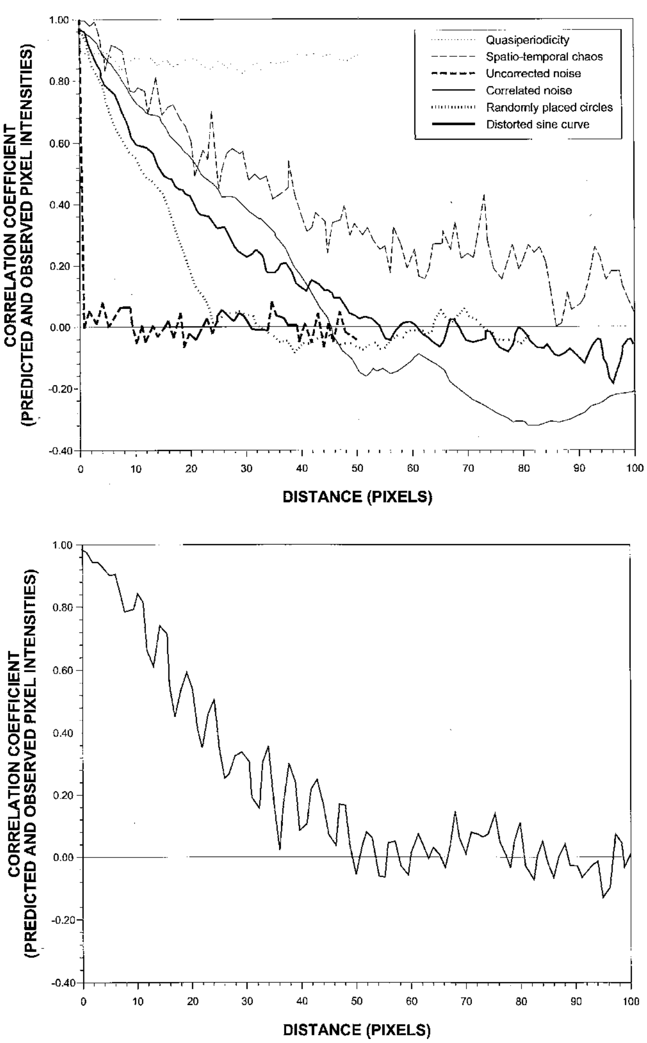

Figure 5. Forecasting Signatures: Prediction-decay curves for (Upper) sand ripple data and (Lower) six trial models (from Rubin, 1992. Reproduced with permission of the American Institute of Physics).

the trial models are then compared with that from the original data. The curves that most closely resemble the curve from the original data indicate the best of the trial models in describing the system dynamics. Note that the main purpose of this technique is not to make short-term predictions (although this is a by-product), but to understand generic dynamical behavior.

The trial models are usually chosen as simple models that are representative of different generic types of system behavior, such as linear vs non-linear, random vs chaotic *etc.* SUGIHARA and MAY (1990) have applied FS techniques to analyzing time series data for biological and ecological applications. RUBIN (1992) has extended the approach to analyze two-dimensional spatial patterns, and applied the technique to digitized photographs of wind-generated sand ripples. Rubin's application is particularly relevant for analyzing digitized images of coastal morphology, as this type of information becomes increasingly common from remote sensing sources such as the ARGUS video monitoring system (LIPPMANN and HOLMAN, 1989). Rubin used six trial models and concluded that the best predictions were derived from a non-linear model generating chaotic data, and by a linear model involving sine waves with correlated noise. Of these two, the non-linear model was favored by the separate consideration that branchings in the ripple patterns could not be simulated by a linear model. This further illustrates the general theme that some knowledge of the underlying physical processes is essential in deducing non-linear characteristics of the data. Figure 5 shows the prediction-decay curves for Rubin's original ripple data and the prediction-decay curves for the six trial models.

FS methods are a potentially powerful method for investigating generic system properties with relatively modest amounts of time series data (several hundreds upwards), although these quantities of data are uncommon in coastal morphology data sets from in-situ measurements. The most useful application to coastal morphological data may be the analysis of spatial and temporal data from digitized images from remote sensing sources. The method is restricted by the need to identify trial models at the outset. However, although the method will not authoritatively identify the actual system dynamics, it will identify the poor candidates, which can then be discarded. One point to beware is that for relatively small numbers of data (smaller than the low thousands), a linear sine-wave model with tailor-made correlated noise can do as well as any non-linear model (SUGIHARA and MAY, 1990). However, such a linear model would usually need to be constructed very artificially. Additional knowledge about the physical processes may rule this out as a serious candidate (as with branchings in the ripple patterns in RUBIN, 1992).

Fractal Analysis

The non-linear techniques discussed so far have their origins in systems with small numbers of dimensions. However, coastal morphology is a spatially extended system, and therefore inherently high dimensional. Each point in space represents a separate dimension, although points close to each other will be highly correlated, and the effective number of dimensions will be less. Nevertheless, the effective number of dimensions may well be sufficiently large that the application of the previous techniques can be impossible because of insufficient data, or even be theoretically unsound.

One technique that can be used for spatially extended, high-dimensional systems is fractal analysis. It has been found empirically that these types of complex natural systems often display fractal statistics of their dynamical variables, particularly when the systems are subjected to forcing that is only slowly varying (in comparison with the timescales of interest). Some examples include solar radiation, earthquakes, river discharges and turbulent fluid flows. The reason for the existence of fractal statistics, however, is still not properly understood. The most popular explanation is in terms of "self-organized criticality", an idea introduced by BAK *et al.* (1987). These authors suggested that complex, high-dimensional systems typically evolve to a critical state, analogous to a phase transition in matter, and are maintained there by self-organizational processes. In contrast to phase transitions in matter, however, this critical state is robust with respect to variability in the external forcing conditions, within certain limits. This hypothesis should be distinguished from a low-dimensional system evolving towards a chaotic attractor.

Introductions to fractal geometry and time series can be found in many textbooks such as TURCOTTE (1992) and HASTINGS and SUGIHARA (1993). We give here a brief background. A fractal shape is one that is self-similar, so that the pattern appears the same no matter how much it is magnified or contracted. An important use of fractal geometry is to provide a concise quantitative measure of the degree of irregularity of natural objects, described by the fractal dimension. The length of a fractal object L, measured in segments of length δ, is determined by a power law given by $L = k\delta^{1-D}$, where D is the fractal dimension and k a proportionality constant.

To connect the idea of a spatial fractal patterns with self-similar generating processes in time, the definition of a "random walk" or "Brownian process" is generalized to define a fractal process. Imagine we have a time series y(t) sampled at intervals Δt_1. The time increment variable, Δt, can therefore take values Δt_1, $2\Delta t_1$, $3\Delta t_1$ *etc.* Similarly the increments in y, defined as the variable $\Delta y = y(t+\Delta t) - y(t)$, can take values $y(t+\Delta t_1)-y(t)$, $y(t+2\Delta t_1)-y(t)$, $y(t+3\Delta t_1)-y(t)$ *etc.* The main feature of a fractal process is that the variance of the increments of y depends on the size of the time increment in the following way:

$$Var(\Delta y) = c(\Delta t)^{2H}$$

in which *Var* denotes the variance of Δy calculated for all possible pairs of values of y in the data series for a given Δt. H and c are constants, and H is known as the Hurst exponent. A fractal process is indicated by H being constant with respect to Δt. The value of H indicates the amount of persistence in the data (*i.e.* the extent to which each Δy is correlated to previous values of Δy). $H=0.5$ corresponds to a random walk, and there is no persistence in the data. For $0.5<H<1$, the process shows persistence (positive correlation to previous Δy), increasing for larger values of H. For $0<H<0.5$, the process shows antipersistence (negative correlation).

A fractal process can be demonstrated from the time series data in several ways. The most obvious method is to plot $Var(\Delta y)$ against Δt on a log-log graph. A fractal process will be indicated by the data values lying on a straight line, and the slope of the line will be two times the Hurst exponent. Other methods of fractal analysis are described in HASTINGS and SUGIHARA (1993), and SOUTHGATE and MÖLLER (2000).

Fractal analysis requires relatively few data values in comparison with other non-linear analysis techniques. Numbers in the low hundreds may be sufficient, but accuracy increases with more data. More confidence can be had in the results if several data sets are used that, on physical grounds, are believed to represent similar processes. Fractal analysis needs to be done carefully, and a number of sources of error and bias need to be considered. These difficulties, and methods to address them, are described in SOUTHGATE and MÖLLER (2000), and NORTH and HALLIWELL (1994).

Fractal analysis has been applied to beach level data at Linconshire, UK (SOUTHGATE and BELTRAN, 1996) and at Duck, North Carolina, USA (SOUTHGATE and MÖLLER, 2000). In the latter case, bathymetric data was measured along four profile lines, extending from the upper beach to about 7m water depth, over a ten-year period at intervals of about two weeks. This data set is particularly valuable because:

- It provides sufficient time series values at individual locations for fractal analysis
- Sufficient of the cross-shore region is covered, in which different generic dynamical behavior can be expected (*e.g.* LEE *et al.,* 1995 have classified four cross-shore zones with different general behavior at Duck, based on separate data analysis)
- Data exists for several locations within each cross-shore zone. Similar physical processes can be expected at these locations, thereby enhancing the reliability of analysis.

Results are described in detail in SOUTHGATE and MÖLLER (2000). The main conclusions are:

- *In zones where wave breaking is uncommon* (on the upper part of the beach, and seawards of about 5m water depth). Self-organizational processes appear dominant for periods up to about 12–24 months. At longer periods (up to about 3–4 years), forced responses dominate. This conforms to the observation that groups of severe storms, with a return period of about 2 years, have a major effect in these regions. Events that are more moderate are much less significant.
- *In zones where wave breaking is common* (between the waterline and 5m depth, in the region of breaker bars). Forced responses dominate for periods up to 12–24 months, and self-organizational processes are dominant at longer timespans. This conforms to the observation that waves break during moderate and common wave conditions, and result in strong variations of forcing over short timespans. Over longer times, the effects of severe events are proportionately less significant than the other zones, and self-organizational processes are stronger.

Figures 6 and 7 show some examples of fractal analysis. Figure 6 shows the Hurst exponents calculated from bathymetric time series at 10 m intervals along a cross-shore profile line (Profile 188). Figure 7 shows the timespans in different profile zones over which fractal behavior is most evident.

Neural Networks

Neural Networks (NN) are an entirely different approach to the non-linear analysis of data sets. The purpose of NN is short-term forecasting, rather than system characterization. Furthermore, the technique involves a black-box manipulation of the data rather than using a pre-specified method.

The architecture of a NN consists of a set of inputs and outputs connected by one or more hidden layers of nodes (or neurons). This is shown in Figure 8. Typically, each input and

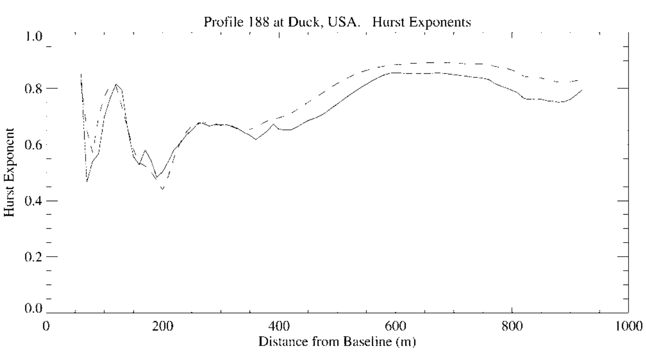

Figure 6. Fractal Analysis: Hurst exponent vs profile distance (Profile 188 at Duck, USA) calculated by two methods. Full line: Range method. Dashed line: Variance method.

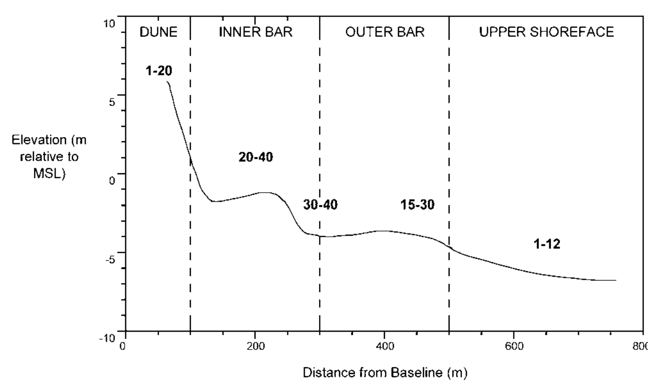

Figure 7. Fractal Analysis: Approximate timespans (months) showing fractal behavior in four different profile zones (Duck, USA).

output is connected to all the nodes in the adjacent layer. However, usually there are no connections between nodes in the same layer, or feedback to earlier layers. This type of NN is called a feedforward NN and is the most common type in practical applications.

Values at each node on each layer (including the output layer) are determined from "weights" that pertain to each connection. The values on each node are calculated as a linear sum of nodes values in the previous layer and the connection weights (symbols are shown in Figure 8; q_j is an intermediate variable in the calculation of p_j):

$$q_j = \left(\sum_i w_{ij} p_i\right) + b_j$$

in which b_j denotes a "bias" term. A further transformation to the node values is made by what is called the "activation function". This is usually a non-linear transformation such as:

$$p_j = 1/(1 + e^{-q_j}) \quad \text{or} \quad p_j = \tanh(q_j)$$

Another possibility is a step function in which p_j is zero if q_j is below a threshold value, and one if q_j is above the threshold.

Most NNs need to be "trained" first using pre-specified input and output data. This training requires a set of rules for adjusting the weights according to how the output compares with the target output data. The trained NN is then expected to exhibit a forecasting capability using different input data. Setting up a NN is a skilled task involving issues such as the number of layers, the number of nodes within each layer and the choice of activation function. However, once a NN is set up, it can be used in forecasting mode in an automated manner without requiring much human expertise. A drawback with this black box approach is that it provides little or no insight into the underlying mechanisms. NNs are therefore of little use for system characterization unless it is possible to unravel and understand the black box in some way (*e.g.* the strength of the connection weights may indicate some system properties).

Experiences with forecasting using NNs have been somewhat variable. The best results outperform those based on traditional statistical methods, but others have been much worse than these methods. An example is provided by the Santa Fe competition (WEIGEND, 1994), in which six sets of time series data (each of length between 1000 and 100,000 data values) from a variety of sources were provided to competitors. The competitors were required to make forecasts using this data and analysis methods of their choice, some of which were NNs. The data relating to the forecast period were not revealed to the competitors. The results showed that for some data sets the best, but also the worst, forecasts were from NNs. Furthermore there was a general failure of simplistic black box approaches using NNs. The successful NN applications involved some exploratory data analysis before the application of the NN. One conclusion that can be drawn is that human expertise in setting up NNs and pre- and post-assessment of data is a necessary condition for successful predictions. Naïve applications of NNs run a risk of going badly wrong.

There is little experience of using NNs for forecasting in coastal morphology or related areas. Possible reasons are the data requirements (upwards of several hundred data values per time series) and general lack of familiarity of the tech-

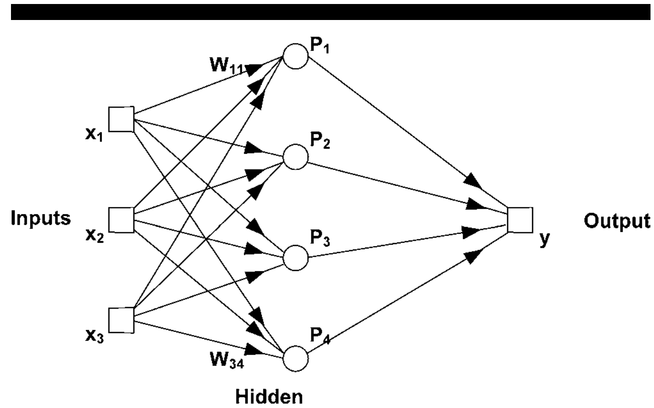

Figure 8. Neural Networks: Structure of a very simple neural network.

nique among coastal morphologists. However, KINGSTON and DAVIDSON (1999) have applied NNs to sand bar movement using remote sensed data from an ARGUS station at Perranporth in Cornwall, UK. NNs have also been applied in some related areas that typically have more abundant data, for example in river flood forecasting (CORNE *et al.,* 1998).

CONCLUSIONS

An important task in understanding, modeling and predicting coastal morphology over yearly and decadal timescales is to analyze field data to help us understand the general types of morphological behavior. However, this is a difficult task to undertake, for several reasons:

- Coastal morphological systems are non-linear, spatially extended and high dimensional. Data analysis techniques for identifying the generic properties of these systems are at an early stage of development.
- Coastal morphological data is often lacking in overall quantity, and temporal and spatial resolution, for proper use of these techniques.
- Because many of the relevant techniques have only recently been developed, information about them is scattered across a range of disciplines. Assessment of their usefulness for coastal morphology is hampered by lack of awareness of technical literature in different disciplines, differences in terminology and purpose, and need for interpretation to coastal morphology.

In this paper, and in the companion paper (LARSON *et al.,* 2003), we have attempted to identify and summarize the most useful techniques. To understand the relative merits of the different techniques it is important to categorize them in some way. This, itself, has not been easy, since there are no clear-cut distinctions. However, we have identified the following three modes of classification, recognizing that the dividing lines within each classification are not sharp ones.

- Purpose of data analysis: Data reduction, Phenomenology, Forecasting, System Characterization
- Statistician's approach vs Physical scientist's approach
- Linear vs Non-linear techniques

In LARSON *et al.* (2003), the emphasis is on relatively well-established, linear techniques, which are mainly applied for the purposes of data reduction, phenomenology and forecasting. In the present paper, we focus on recently developed non-linear techniques. These are mainly useful for system characterization and employ a physical scientist's approach.

In system characterization, we are trying to understand generic types of morphological behavior, such as the importance of forcing (that might result from random wave sequencing, strong events, human interference *etc*) vs internal dynamical behavior (*e.g.* equilibrium, periodic or chaotic behavior). This aim is to be contrasted with the aim of understanding constituent physical processes.

A common theme in the techniques described here is the use of time-lagged variables. This is mainly a consequence of a powerful theorem (Takens' theorem) in characterizing non-linear systems which states that a single time series, provided it is sufficiently long, contains all the information necessary to characterize the whole system. However, most of the successful applications of this approach have been to low-dimensional systems. The relevance to high-dimensional systems, such as coastal morphology, is less clear, and is the subject of on-going research. In parallel, some approaches specifically relevant for high-dimensional systems have been devised, such as fractal analysis. These approaches also have areas of uncertainty, for example in how fractal patterns are related to inherent dynamical behavior. These fundamental problems are being addressed in other related scientific fields, such as fluid turbulence, in which theoretical knowledge is more advanced and data more abundant than in coastal morphology.

A common difficulty with many data analysis techniques, and particularly recently developed ones, is that there usually are a number of sources of error or bias that can affect the results. This is too detailed an issue to cover in this paper, and readers are referred to the relevant technical papers.

Analysis of data, to establish generic system properties, is an essential component of the overall task of understanding, modeling and predicting coastal morphology on yearly and decadal timescales. At present, this type of activity is in its infancy. However, it will become increasingly feasible as theoretical developments are made, especially in related environmental sciences, and as more coastal morphological data becomes available, particularly from remote sensing sources.

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