How to Solve an Allocation Problem?

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Summary

Game theory proposes several allocation solutions: we know **(a)** fairness properties, **(b)** how to develop **(c)** methods building on these properties, and **(d)** how to calculate **(e)** allocations. We also know how to influence the perceived fairness and realization of allocation solutions. However, we cannot explain properly that theoretically fair allocation methods are rarely used. To obtain more insight into these issues we solved an allocation problem in a purchasing cooperative case study by confronting theory with perceptions. We find large theoretical and perception differences and inconsistencies between and within the five steps from a to e. We note that theoretically fair methods tend to be more complex than theoretically unfair methods. In addition, the allocations of some simple methods are perceived fairer than the allocations of complex methods in our case study. To improve theoretical solutions the focus should be on a and c. To influence perceptions the focus should be on b, c, and d. Finally, all five steps are modeled into comparable fairness measures and a general model. Using this model implies that both theory and perceptions are considered in solving allocation problems.

Educator and practitioner summary

Allocation conflicts occur often; despite that we know in theory what fair allocation solutions are; despite that we know in practice how to positively influence the perceived fairness and realization of allocation solutions. In this paper we therefore solve an allocation problem in a purchasing cooperative by confronting theory with perceptions. We develop comparable fairness measures and develop a general model for solving allocation problems.

Keywords

Changing Perceptions; Allocation Solutions; Cooperative Purchasing

Introduction of the topic

Recently two next-door organizations independently decided to sell their organizations. The value of organization A was € 4 million; the value of organization B was € 2 million. Organization C showed interest in buying A and B together for € 8 million. Despite or because of the surplus of € 2 million dissension arose between A and B concerning the surplus allocation. Even so much dissension arose that the sale for € 8 million to C was eventually cancelled.

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The example above is perhaps exceptional, but allocation problems occur in all kinds of situations. In this paper we analyze and solve an allocation problem in a purchasing cooperative. Purchasing cooperatives consist of two or more organizations that share and/or bundle purchasing related, information, resources, and/or volumes in order to create symbiotic relationships (based on Schotanus and Telgen, 2005). Examples of symbiotic relationships are mutualism and parasitism (e.g. Johnson et al., 1997). In cooperative purchasing preferably mutualistic relationships are created in which all of the cooperating organizations benefit financially or in any other way by cooperating. As we illustrated in the example, even in mutualistic relationships allocation problems can occur. The same accounts to mutualistic purchasing cooperatives. Especially in cooperatives in which the cooperating organizations differ in terms like size. Due to these differences it is difficult to find fair allocation solutions in theory (e.g. Heijboer, 2003; Schotanus, 2004). We propose that to be able to tackle allocation problems one should look at least at the theoretical fairness of an allocation solution.

Theoretical fairness

Based on analytical research numerous authors proposed different solutions to theoretical unfairness in allocation problems. Some authors proposed solutions to allocation problems in cooperative purchasing (e.g. Heijboer, 2003; Schotanus, 2004; Singer, 1985). They proposed, and/or analyzed several allocation methods like (see appendix A for their descriptions):

- Average Cost Pricing, i.e. Proportional Amount (e.g. Heijboer, 2003);
- Compromise Price (Schotanus, 2004)
- Compromise Value (Borm et al., 1992);
- Equal Amount (e.g. Heijboer, 2003);
- Equal Price (e.g. Schotanus, 2005b);
- Nucleolus (e.g. Schmeidler 1969);
- Shapley Value (Shapley, 1953).

Heijboer (2003) and Schotanus (2004) analyzed the fairness of these allocation methods for purchasing cooperatives in terms of cooperative game theory. To this end they used general theoretical properties of fairness (e.g. Friedman, 2003; Moulin, 2001; Shapley, 1953) like (see appendix B for their descriptions):

- Additivity (ADD);
- Dummy (DUM);
- Efficiency (EFF);
- Fair Ranking (FR);
- Monotonicity (MON);
- Stability (STA);
- Symmetry (SYM).

The analytical body of knowledge about theoretical allocation properties and methods is large. See for instance journals as the International Journal of Game Theory, the Journal Games and Economic Behavior, the Journal of Economic Theory or the Journal of Applied Mathematics. Consequently, we already know **(a)** numerous theoretical properties of fairness. We know **(b)** how to develop **(c)** theoretically fair allocation methods building on these properties. And we know **(d)** how to calculate **(e)** theoretically fair allocations using these methods (see figure 1).

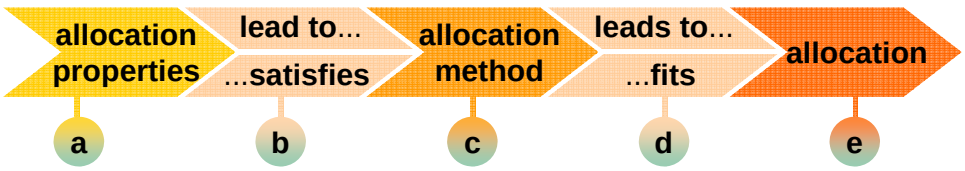

Figure 1. Generic Representation of the Allocation Process

The two most popular allocation methods in analytical literature are the Shapley Value and the Nucleolus (Meurer, 1999). Both methods are considered to be theoretically fair in various situations (e.g. Heijboer, 2003). Nevertheless, only a few real-life applications exist of the Nucleolus. In most cases, its use has been motivated as an alternative to the Shapley value (Serrano, 1999). The Shapley value has been applied in some real-life situations, like calculating political power (Shapley and Shubik, 1954); calculating landing and take-off fees for Birmingham airport (Littlechild and Thompson, 1977); and allocating railways infrastructure costs (Fragnelli et al., 2000). Still, the real-life application of the Shapley Value has been very limited. To our knowledge the method has never been used in for instance purchasing cooperatives. The Equal Price method is usually used in purchasing cooperatives (Schotanus, 2005b), despite the fact that it:

- Is considered to be theoretically unfair in terms of cooperative game theory (Heijboer, 2003);
- Sometimes leads to premature endings of cooperatives (Schotanus, 2004);
- Sometimes hinders the establishment of new cooperatives (Schotanus, 2005a).

In several situations practitioners know the issues above and they are aware of the existence of alternative allocation methods which are theoretically fair. Apparently, in allocation problems more aspects play a role than theoretical fairness. The perceived realization of an allocation method might play an important role as well. If a person is aware of the existence of an allocation method, then that does not have to mean that this person also understands how this method actually works. Therefore we propose that to be able to tackle allocation problems one should look at least at the interaction between the theoretical fairness and perceived realization of allocation solutions.

Perceived realization

Both the complexity of allocation problems and methods are linked to perceived realization, i.e. perceived understanding. With complexity we refer to computational complexity theory and measures as the number of steps that it takes to solve allocation problems with a certain method (e.g. Barton et al. 1987). In addition, we take into account cognitive psychology by looking at how complicated allocation problems and methods are from the perspectives of all those involved (e.g. Funke, 1998). Note that the outcomes of the allocation methods proposed in this paper can be calculated within minutes at a regular computer for small cooperatives (≤ nine organizations). So, we do not consider the computation time of allocation methods as a limiting condition.
We assume that the more complex an allocation problem and/or method is, the more difficulties usually arise concerning the perceived realization of allocation methods (based on Walker and Wooldridge, 1995). In this paper we simply assume that the perceived realization can be positively influenced by increased knowledge. Nevertheless, in practice it might be very difficult to positively influence the perceived realization of complex allocation problems and methods.
Perceived realization might influence perceived fairness as well. If it is not clearly understood how a method works, then the outcomes might be perceived as unfair. In addition, perceived fairness might differ between individuals as we illustrated in the example in the beginning of this paper. This could finally lead to allocation conflicts. Therefore we propose that to be able to tackle allocation problems one should look at the interaction between the theoretical fairness, perceived realization, and perceived fairness of allocation solutions.

Perceived fairness

Several decisions usually have to be taken within the steps of the allocation process. Examples of decisions are determining which allocation methods to compare and how to involve all those concerned. Based on empirical research numerous authors proposed many different solutions to perceived unfairness of decisions in allocation problems. They suggest that the perceived fairness of decisions consists of three aspects (e.g. Tax et al., 1998):

- Procedural fairness (e.g. Leventhal, 1980): how are the decisions made (Tang and Baldwin, 1996)?
- Distributive fairness (e.g. Alexander and Ruderman, 1987): what are the decisions (Tang and Baldwin, 1996)? How do the decisions affect me and my comparable others (Xia, 2004)?
- Interactional fairness (e.g. Tax et al., 1998): how are the decisions presented and implemented (Hoffman and Kelly, 2000)?

The empirical body of knowledge about how people perceive allocation methods and its outcomes is large (Alexander and Ruderman, 1987; Bolton et al., 2005; Kagel and Roth, 1995). See for instance journals as the Journal of Applied Psychology, Social Justice Research, the Journal of Economic Psychology or the Academy of Management Journal. Consequently, we know that the three aspects of perceived fairness interact in complex ways (Skarlicki and Folger, 1997). In addition, it is widely accepted that the perceived fairness of decisions can be positively influenced if (e.g. Folger and Konovsky, 1989; Johnson et al., 1997; Leventhal, 1980; Moorman, 1991; Tang and Baldwin, 1996; Tyler and Bies, 1989):

- There is two-way respectful communication between all those concerned;
- The procedures are applied consistently and accurately;
- The procedures are correctable, ethical, and bias is suppressed;
- All those concerned may express concerns and influence the procedures;
- All those concerned are familiar and well informed with the situation, with the procedures, and the procedure outcomes;
- And the final aspect is related to revenue management research results. These results suggest that the perceived fairness of pricing can be positively influenced by framing (Kimes, 2003; Kimes and Wirtz, 2004), i.e., the presentation of economic equivalent prices as a gain or a loss. We suggest that the same effect could apply to allocation problems. Thus, in mutualistic relationships the emphasis should be placed on the fact that all participating organizations gain by cooperating. An emphasis on some organizations receiving more or less gains than the others should be prevented.

Perceived unfairness by individuals in allocation solutions tends to lead to reactions of these individuals to restore fairness in terms of equity theory (Adams, 1963; Adams, 1965). Some individuals are willing to impose a cost – both on self and others – to resist perceived unfairness (Greenberg, 1990; Sheppard et al., 1992). This can even happen in mutualistic relationships like we illustrated in the example in the beginning of this paper.

Research relevance and objectives

Summarizing the previous sections, the body of knowledge about theoretically fair allocation solutions is large. The empirical body of knowledge about influencing perceptions is large as

well. However, we do not know very well how theory and perceptions interact in allocation problems. To our knowledge no well-established journals exist as a fictitious Journal of Applied Psychology and Applied Mathematics Research. Still, allocation conflicts occur often and we can hardly explain that despite frequent allocation conflicts due to perception differences, theoretically fair allocation methods are rarely used. It is therefore our main objective to obtain more insight into influencing and solving allocation problems by confronting theory with perceptions in the allocation process (see figure 2).

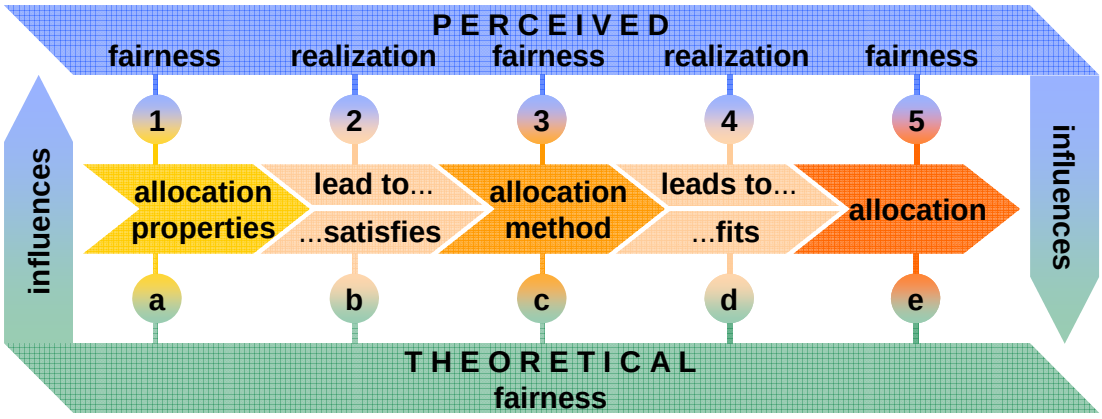

Figure 2. Theoretical versus Perceived Realization and Perceived Fairness in the Allocation Process

As mentioned before, in this paper we analyze and solve a typical allocation problem in a purchasing cooperative. For that reason, our more specific objectives are linked to cooperative purchasing. In cooperative purchasing it is currently not known:

Step **(1a)**: What the perceived importance of properties of fairness is among practitioners. It could be that an allocation method satisfies seven out of eight properties. On the surface of it this may sound fair, but if the eighth property that is not satisfied is perceived as very important the method is not fair after all. Furthermore, the importance of the properties may depend on the characteristics of practitioners and organizations. This may lead to conflicting perceptions of fairness.

Step **(2b)**: Whether or not it is perceived that an allocation method satisfies certain properties of fairness. For instance, it might be that a practitioner thinks that a method satisfies a property of fairness, but that it can be proven theoretically that this is false.

Step **(3c)**: Whether or not fair allocation methods in terms of cooperative game theory are perceived to be fair in practice as well.

Step **(4d)**: Whether or not it is perceived that the allocation of a method is perceived as fair as the corresponding method. For instance, it might be that a practitioner perceives a certain method as fair, but perceives the matching allocation in cold hard cash as unfair.

Step **(5e)**: Whether or not the actual allocation is perceived to be fair.

It is one of our specific objectives to shed more light on these steps and their interactions. Furthermore, we aim to build recommendations on which steps should be influenced if one would want to influence theory and/or perceptions. Our final specific objective is to develop a general model for solving allocation problems. With this model we aim to find a theoretically fair and perceived fair allocation method for the purchasing cooperative studied.

Organizational context

The heads of purchasing of eleven medical organizations – all members of one purchasing cooperative – are involved in our study. The cooperative is well-known in the public sector in its homeland and is considered as a classic example of successful cooperative purchasing. The cooperative booked plentiful purchasing savings, employs one full time purchasing manager, and has numerous cooperative contracts. Nevertheless, the cooperating organizations are not always like hand and glove. The added value of cooperating is sometimes a discussion point, as is the allocation of gains. The allocation is difficult for this cooperative as the organizations differ from each other (e.g. differences in purchasing volume, professional level of the purchasing function, and level of involvement). Recently, two organizations even left the cooperative leaving nine organizations. See table 1 for some quantitative properties of these organizations.

Size	Organization	Annual Procurement in €	Number of Beds	Annual Procurement in € per Bed
Large	1	55.000.000	881	62.429
	2	54.000.000	712	75.843
	3	40.600.000	617	65.802
Medium	4	28.800.000	600	48.000
	5	23.000.000	390	58.974
	6	21.500.000	359	59.889
Small	7	12.000.000	275	43.636
	8	11.500.000	187	61.497
	9	9.500.000	140	67.857
Total		255.900.000	4.161	61.500

Table 1. Quantitative Properties of the Cooperating Organizations

To prevent more organizations leaving the cooperative the gain allocation method was brought up for discussion. The gains were usually allocated with the Equal Price method, i.e. all organizations pay the same price per item (e.g. Schotanus, 2005b). This method is perceived as unfair by the cooperative. For this reason a new method called Differential Pricing was tested in one tender. This method implies that all organizations receive a small fixed discount percentage. On top of that, larger organizations receive a larger extra discount percentage. Differential Pricing is perceived as unfair as well by the cooperative. Therefore, the researchers were asked to assist the cooperative in finding a fairer method than Equal Price and Differential Pricing.

Empirical Methodology

We first carried out interviews with two key persons to better understand the allocation problem. Based on these interviews and secondary data we built a draft questionnaire with a mix of question types. The questionnaire was first sent to a focus group to test the questions. We paid special attention to the clear explanation of properties of fairness and allocation methods. The final questionnaire was filled in by all of the nine organizations and consisted of three parts:

Part **(1)**: The first part consisted of general questions.

Part **(2)**: The second part consisted of questions related to two cases based on actual contracts of the cooperative. For both cases we provided actual allocations, but we did not provide information on which allocation methods were used. For all allocations for the

cooperative the respondents were asked to indicate whether or not they perceived the allocation as fair.

Part **(3)**: The third part consisted of questions related to:

- The perceived importance of several properties of fairness;
- Whether or not several properties of fairness are perceived as realized for the purchasing cooperative for the well-known Equal Price method;
- Whether or not several allocation methods are perceived as fair.

The next phase of our study consisted of sending a brief report of a theoretical analysis of allocation problems to all of the respondents. We send this report in order to increase knowledge on theoretical allocation properties, methods, and realizations. The final phase consisted of a presentation and discussion of our findings at a workshop. All of the respondents attended this workshop and in the end of the meeting a preliminary decision was made concerning the allocation method. After some time to consider the matter the final decision was made.

Findings and discussion

We discuss our findings by using the five steps described in figure 2 as a connecting thread. We compare the outcomes of the five steps after we have discussed each of them individually.

Step (1a): Axiomatic fairness

For this first step we asked the respondents for the perceived importance of several properties of fairness. This concerns the axiomatic fairness. We state that an allocation method is axiomatically fairer than another method if it satisfies more properties of fairness that are perceived as important. As a fairly subjective measure we assign a 5 (very fair) to a method that satisfies all of the properties used. We assign a 1 (very unfair) to a method that satisfies no properties. For the methods k in between we calculate the axiomatic fairness AF_k by taking into account theory and the average perceived importance API_p of each allocation property p for M properties. The variable s_{kp} has got a value 1 or 0 if property p respectively is satisfied or is not satisfied in theory by method k for the purchasing situations of the cooperative. AF_k can then be formulated as:

$$AF_k = 1 + \frac{4\sum_{p=1}^{M}\left(s_{kp} \cdot API_p\right)}{\sum_{p=1}^{M} API_p} \qquad \textbf{(1)}$$

Note that one general mark of the axiomatic fairness of a method cannot exist as the perceived importance of properties of fairness may differ per situation. Note also that several properties conflict in theory and cannot both be satisfied at the same time in specific situations (e.g. Herrero, 1999). So, it is theoretically impossible to satisfy all of the properties in different situations while using one allocation method. One theoretically fairest method can therefore not exist.

In table 2 we show the average perceived importance API_p of several properties p for the cooperative. The table also shows if the properties of different methods are theoretically satisfied for the purchasing situations of the cooperative (proofs have been omitted here). Finally, the allocation methods are ranked by their complexity and their axiomatic fairness AF_k is given.

p	API_p of all Organi-zations	Simple				Moderately Complex		Complex	
		Equal Amount	Average Cost Pricing	Equal Price*	Differential Pricing*	Compromise Value	Compromise Price	Nucleolus	Shapley Value**
EFF	4,9	✓	✓	✓	✓	✓	✓	✓	✓
STA	4,4	×	×	✓	×	✓	✓	✓	✓
DUM	4,2	×	✓	✓	✓	✓	✓	✓	✓
FR	4,0	✓	✓	×	✓	✓	✓	✓	✓
SYM	3,6	✓	✓	✓	✓	✓	✓	✓	✓
ADD	3,4	✓	×	×	×	×	×	×	✓
MON	3,3	✓	✓	✓	✓	✓	✓	✓	✓
AF_k		3,8	3,9	3,9	3,9	4,5	4,5	4,5	5,0

Note: ✓, (×) means theoretically (not) satisfied in general for the purchasing situations of the cooperative
* is already perceived as unfair in previous tenders by the purchasing cooperative
** is considered to be too complex by the purchasing cooperative
The 2nd column is measured on a five point Likert scale from 1 (not important) to 5 (very important)

Table 2. Axiomatic Fairness of Allocation Methods

We found no remarkable differences between the individual scores of the respondents. Interesting methods can therefore be easily chosen given the preconditions, the allocation properties used, and their perceived importance. Here the Compromise Value is the fairest and relatively least complex method. The methods Average Cost Pricing and Equal Amount are simple alternatives.
The consideration between complexity and theoretical fairness concerning an allocation method is related to what we call the complexity dilemma. Some allocation methods satisfy the same properties of fairness as other methods and satisfy some extra properties as well. Those methods are theoretically fairer than the others. However, theoretically fairer allocation methods tend to be more complex. To our experience an allocation method as the Equal Price method is usually considered as simple and is theoretically unfair for purchasing cooperatives. A method as the Compromise Value is usually considered as moderately complex and is theoretically quite fair. A method as the Shapley Value is often considered as (too) complex and is theoretically fair.

Step (2b): Axiomatic realization

For this second step we asked the respondents which properties of fairness are satisfied by the Equal Price method that they know very well. This concerns the axiomatic realization. We state that the axiomatic realization from allocation properties to a method is higher if differences between theory and perception are lower. As a fairly subjective measure we assign a 5 (very high) if there are no differences between theory and perception (e.g. satisfied in theory and satisfied in perception). We assign a 1 (very low) if the differences are maximal. For the methods k in between we calculate the axiomatic realization AR_k by comparing theory with the average perceived realization APR_{kp} of each allocation property p for M properties. The variable s_{kp} has got a value 1 or 0 if property p respectively is satisfied or is not satisfied in theory for the purchasing situations of the cooperative by method k. AR_k can then be formulated as:

$$AR_k = 5 - \frac{4\sum_{p=1}^{M}\sqrt{\left(s_{kp} - APR_{kp}\right)^2}}{M} \qquad \textbf{(2)}$$

In table 3 we show the theoretical realization s_{kp}, the average perceived realization APR_{kp} and the axiomatic realization AR_k for different groups of organizations and k is the Equal Prize method. The allocation properties p are ranked by their average perceived importance.

p	s_{kp}	APR_{kp} of all Organizations	APR_{kp} of Large Organizations	APR_{kp} of Medium Organizations	APR_{kp} of Small Organizations
EFF	1	1,0	1,0	1,0	1,0
STA	1	0,4	0,3	0,7	0,3
DUM	1	0,3	0,3	0,3	0,2
FR	0	0,4	0,3	0,7	0,2
SYM	1	0,4	0,3	0,7	0,3
ADD	0	0,4	0,0	0,5	0,7
MON	1	0,9	0,8	1,0	1,0
AR_k		3,5	3,6	3,6	3,3

Note: The 2nd column is measured on a true (1) or false (0) scale with a do not know option (0,5)

Table 3. Axiomatic Realization of k = Equal Price

To be able to make a well-founded decision concerning an allocation method special attention needs to be given to important allocation properties with large average differences between perception and theory. The same applies if there are many do not know answers for a certain property. Finally, if there are allocation methods with low AR_k scores these need to be given special attention as well. If necessary, step 1b needs to be reconsidered.
Here attention needed to be given to STA, DUM, and FR as these are perceived as important properties but have low APR_k scores. For instance, sometimes the respondents thought that the Equal Price method satisfies FR, while it does not in theory.

Step (3c): Methodical fairness

For this third step we asked the respondents for the perceived fairness of several allocation methods. This concerns a direct measurement of the methodical fairness. In table 4 we show the average methodical fairness MF_k for different groups of organizations. The methods k are ranked by their complexity.

k	MF_k of all Organizations	MF_k of Large Organizations	MF_k of Medium Organizations	MF_k of Small Organizations
Equal Amount	2,1	2,0	1,0	3,3
Average Cost Pricing	3,8	4,0	4,7	2,7
Equal Price*	3,8	3,0	4,0	4,3
Differential Pricing*	2,3	3,0	2,7	1,3
Compromise Value	2,7	1,7	3,3	3,0
Compromise Price	3,1	2,0	4,3	3,0
Nucleolus	2,3	1,7	3,0	2,3

Note: * is already perceived as unfair in previous tenders by the purchasing cooperative
The columns are measured on a five point Likert scale from 1 (not fair) to 5 (very fair)

Table 4. Methodical Fairness of Allocation Methods

Remarkably, a theoretically fair method as the Nucleolus is perceived as unfair. The same accounts to a smaller extent to the Compromise Value and the Compromise Price. Possible

explanations could be related to the complexity of the methods. If the respondents are not sure what the exact outcomes of a method are, then they could perceive the method as unfair. We discuss another explanation in section 5e.
Furthermore, there seem to be perception differences between the different groups of organizations. Small organizations dislike the Average Cost Pricing and the Differential Pricing method compared to larger organizations. In theory these methods favor larger organizations (proofs have been omitted here). Larger organizations dislike the Equal Price and the Equal Amount method. In theory these methods favor smaller organizations (proofs have been omitted here). Organizational size plays apparently an important role in the perceived fairness of the allocation methods analyzed. On average, the method Average Cost Pricing is perceived as the fairest one. When considering the different groups the method scores a high minimum as well.

Step (4d): Methodical realization

For this fourth step we compared the outcomes of step 3c and 5e. We choose this indirect measure to reduce the number of questions in our questionnaire. We measured the perceived fairness directly in both step 3c and 5e. As we measured the perceived fairness indirectly in step 1a, we did not apply the same comparison in step 2b.
We state that the methodical realization from allocation methods to allocations is higher if perception differences are lower. As a fairly subjective measure we assign a 5 (very high) if there are no perception differences. We assign a 1 (very low) if the differences are maximal. For the methods k in between we calculate the methodical realization MR_k by comparing the perceived fairness of methods MF_{ik} with the perceived fairness of matching allocations for each organization i for N organizations. The perceived fairness of allocations DF_{ikl} is calculated over O cases for each allocation l that theoretically matches method k. MR_k can then be formulated as:

$$MR_k = 5 - \frac{\sum_{i=1}^{N} \sqrt{(MF_{ik} - \frac{\sum_{l=1}^{O} DF_{ikl}}{O})^2}}{N} \quad \textbf{(3)}$$

In table 5 we show the methodical realization MR_k of several methods k for different groups of organizations.

k	MR_k	MR_k of Large Organizations	MR_k of Medium Organizations	MR_k of Small Organizations
Equal Amount	4,4	4,5	5,0	3,5
Average Cost Pricing	4,3	4,5	4,7	3,3
Equal Price	4,1	4,2	4,2	3,8
Differential Pricing	3,3	2,5	3,7	4,0
Compromise Value	3,5	3,7	3,3	3,5
Compromise Price	3,4	3,7	3,2	3,5
Nucleolus	3,8	3,0	4,3	4,3

Note: The columns are measured on a five point Likert scale from 1 (not fair) to 5 (very fair)

Table 5. Methodical Realization of Allocation Methods

To be able to make a well-founded decision concerning an allocation method special attention needs to be given to methods with a low MR_k, especially if these methods are interesting

according to step 1a and 3c. If necessary, step 3c needs to be reconsidered. Note that the more complex a method is, the more difficulties could arise concerning the perceived realization.
Here attention needed to be given to the Differential Pricing method with a low minimum score for one of the groups. The methods Average Cost Pricing, and the Compromise Value needed attention as well as these are interesting according to step 1a and 3c, but do not have high scores for all of the groups.

Step (5e): Distributive fairness

For this final step we asked the respondents for the perceived fairness of actual allocations in two comparable cases, without providing information on which methods were used. In both cases the same amount of gains needed to be allocated among the organizations. The only difference between the cases concerned the used quantity discount price function, i.e. a derived function from quantity discount schedules.
The perceived fairness of allocations concerns a direct measurement of the distributive fairness. In table 6 we show the average distributive fairness DF_k for different groups of organizations. The methods k are ranked by the extent they favor large organizations in the two cases analyzed. The final column shows the similarities in perceptions PS_k between the first case PFC_{1i} and the second case PFC_{2i} for each organization i for N organizations. PS_k can then be formulated as:

$$PS_k = 5 - \frac{\sum_{i=1}^{N} \sqrt{\left(PFC_{1i} - PFC_{2i}\right)^2}}{N} \qquad \textbf{(4)}$$

For instance, a methods scores a value of 5,0 if there are no perception differences between both cases for all organizations.

k	DF_k of all Organizations	DF_k of Large Organizations	DF_k of Medium Organizations	DF_k of Small Organizations	PS_k of all Organizations
Differential Pricing*	3,4	3,5	4,0	2,5	4,6
Average Cost Pricing**	4,3	4,2	4,3	4,3	4,8
Equal Price*,***	3,2	3,2	3,2	3,3	1,9
Compromise Price	2,6	2,7	2,5	2,5	3,9
Compromise Value	2,4	2,3	2,3	2,5	4,3
Nucleolus	2,4	3,0	2,3	1,8	4,4
Equal Amount**	1,4	1,8	1,0	1,5	4,9

Note: * is already perceived as unfair in previous tenders by the purchasing cooperative
** has the exact same allocations in both cases
*** generally favors small organizations to a larger extent than in the two cases analyzed
The columns are measured on a five point Likert scale from 1 (not fair) to 5 (very fair)

Table 6. Distributive Fairness of Allocation Methods

It is remarkable that the actual allocations of theoretically fair methods as the Nucleolus are perceived as unfair. The sheer complexity of these methods cannot explain this as the respondents only had to asses the allocations. A somewhat far-fetched explanation is that the respondents could assess the fairness of allocations by (subconsciously or consciously) deriving the underlying method. However, the edge is taken off that explanation by step 4d. Therefore, we propose that the explanation relates to the size of the organizations. Here the Average Cost Pricing method is perceived as the fairest method. Except for Differential Pricing, the perceived

fairness of the other methods – including the complex ones – decreases steadily when small organizations are favored to a larger extent.
Another remarkable result concerns the fact that there are not many perception differences between the different groups. Only one clear difference is found in the Differential Pricing method that favors large organizations to a large extent. Perception differences in the Nucleolus method can be explained by an outlying score of one the respondents.
Finally, it is evident that the Equal Price method is perceived inconsistently with a low PS_k score in the final column of table 6. While using a common linear quantity discount price function (Schotanus, 2006) the method is perceived as (very) fair, but while using a common curved function (Schotanus, 2006) the method is perceived as unfair. This indicates the importance of considering several cases when choosing an allocation method for the long run.

How to solve an allocation problem?

To solve an allocation problem the perceived realization needs to be sufficient (step 2b and 4d) and the theoretical and perceived fairness need to be weighted up against each other (step 1a, 3c, and 5e). Influencing the perceived fairness of 1a is difficult as the perceived fairness of allocation properties is often considered as a limiting condition. The perceived fairness of 5e is an outcome of the method in 3c. Therefore, if necessary the perceived fairness of the method in 3c should preferably be influenced (based on Alexander and Ruderman, 1987).
Only the Average Cost Pricing method scored sufficiently in most of the steps in our case study (see table 7). In addition, the method lies in between the methods Equal Price and Differential Pricing when looking at favoring respectively small or large organizations in general. The main disadvantage of the Average Cost Pricing method is that it does not satisfy STA. The Compromise Price or the Compromise Value method would have been theoretically fairer alternatives.

p	API_p	← Favors Small Organization in General						Favors Large Organization in General →
		Equal Amount	Equal Price*	Nucleolus	Compromise Value	Compromise Price	Average Cost Pricing	Differential Pricing*
EFF	4,9	✓	✓	✓	✓	✓	✓	✓
STA	4,4	×	✓	✓	✓	✓	×	×
DUM	4,2	×	✓	✓	✓	✓	✓	✓
FR	4,0	✓	×	✓	✓	✓	✓	✓
AF_k		3,8	3,9	4,5	4,5	4,5	3,9	3,9
Minimum AF_k		3,6	3,8	4,5	4,5	4,5	3,9	3,9
MF_k		2,1	3,8	2,3	2,7	3,1	3,8	2,3
Minimum MF_k		1,0	3,0	1,7	1,7	2,0	2,7	1,3
DF_k		1,4	3,2	2,4	2,4	2,6	4,3	3,4
Minimum DF_k		1,0	3,2	1,8	2,3	2,5	4,2	2,5
PS_k		4,9	1,9	4,4	4,3	3,9	4,8	4,6
Minimum PS_k		4,7	1,3	4,3	4,3	3,7	4,7	4,3
Average**		2,8	3,0	3,2	3,3	3,4	4,0	3,3

Note: ✓, (×) means theoretically (not) satisfied in general for the purchasing situations of the cooperative
* is already perceived as unfair in previous tenders by the purchasing cooperative
** the averages are fairly subjective and are only indicated for easy reference
Perception is measured on a five point Likert scale from 1 (negative) to 5 (positive)

Table 7. Perceived Fairness of Allocation Methods Summarized

We propose a general model for solving allocation problems in figure 3 as a summary of the previous sections. The steps in this model imply among other things that all those concerned in the allocation problem are familiar with the situation, understand it as good as possible, and are involved in the complete transparent process. In addition, bias is suppressed and consistency is enforced to a large extent. Using the model took the respondents about three hours by filling in the questionnaire, reading relevant materials, and attending a workshop.

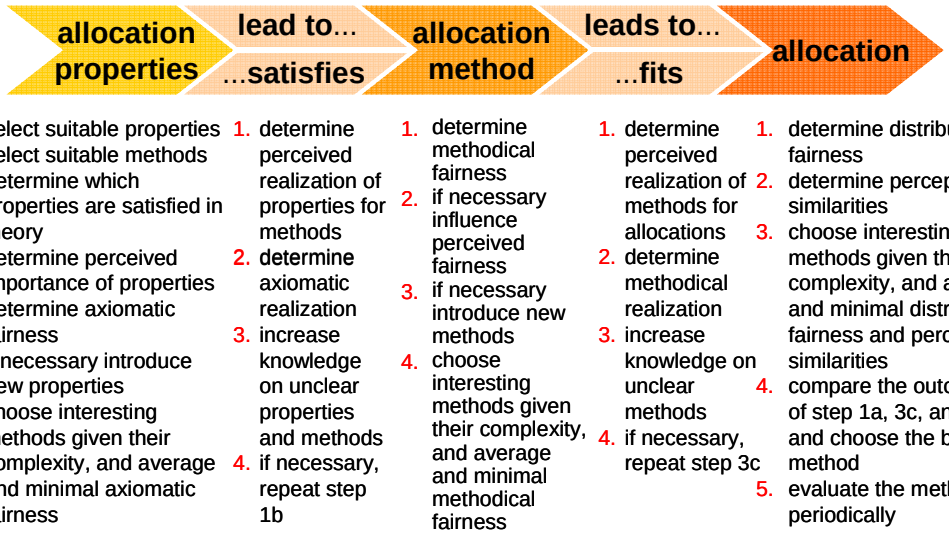

Figure 3. A General Model for Solving Allocation Problems

Limitations and further research

There are some limitations that should be kept in mind when interpreting our results. These could be interesting aspects for further research as well. First of all, to find significant relationships between different groups a large-scale survey will be necessary. More relationships could also be found when studying the five steps individually into more detail. Finding significant relationships is however not the objective of this paper, as is obtaining more insight into influencing and solving allocation problems by studying the broader picture.

Second, not all specific aspects of perceived fairness are taken into account in our study. To reduce the number of questions in our questionnaire we did not use a construct for perceived fairness. In addition, some aspects not defined as perceived fairness as e.g. applicability could have influenced the respondents' answers related to perceived fairness.

Third, the two cases we choose in step 5e differ only in one aspect. More aspects could be considered. In addition, more measures could be used if questionnaires are sent to the respondents at different points in time.

Fourth, allocation problems become even more complex when also considering cooperative costs and logistical gains. Finding a fair solution in these situations might become impossible without assuming that some aspects (e.g. allocation of costs) should be considered as limiting conditions.

Finally, our fairness properties could be defined more specifically. We used a broad definition for a property as FR. This makes it difficult to find differences between separate groups. Other properties and methods might be considered as well. Note that it will be difficult for most people to fully grasp more complex properties and methods. We already encountered difficulties with explaining the ones we used. Further research to new theoretical allocation methods to create a better fit with perception might therefore need a new rigorous way of thinking.

Conclusions

Our conclusions relate to our main objective: to obtain more insight into solving allocation problems by taking into account the perceived fairness and realization of theoretically fair allocation solutions. Our specific objectives are linked to cooperative purchasing. First, we conclude that both theoretical and perceived fairness have been studied intensively separately. However, more research is necessary on the interaction between theory and perceptions to be able to fully understand and solve allocation problems.

Second, the interaction between theory and perceptions can be modeled into a five step model (see figure 2). The steps deal with **(1a)** axiomatic fairness, **(2b)** axiomatic realization, **(3c)** methodical fairness, **(4d)** methodical realization, and **(5e)** distributive fairness. For each of the steps a comparable measure is developed which can be used to efficiently solve allocation problems. To improve theoretical allocation solutions the focus should be on 1a and 3c. To influence perceptions the focus should be on steps 2b, 3c, and 4d.

Third, the consideration between theory and complexity leads to a complexity dilemma: theoretically fair methods tend to be more complex than theoretically unfair methods. On top of that, complex methods are perceived as less fair than some simple methods in our case study.

Fourth, the perceived realization of allocation methods was initially not completely clear to the respondents. For instance, sometimes they thought that a well-known method satisfies a certain property, while it does not in theory. Sometimes it was the other way around.

Fifth, the perceived fairness of allocations of the Equal Price method is very inconsistent. In one tender its allocations are perceived as fair and in another tender as unfair. Remarkably, the allocations of several theoretically fair methods are also perceived as unfair. Our explanation is that organizational size plays an important role in the perceived fairness of a method.

Sixth, the Average Cost Pricing method is overall perceived as the fairest method for the purchasing cooperative studied. The Compromise Price or the Compromise Value method would have been theoretically fairer alternatives.

Finally, all of the steps discussed in this paper can be modeled into a general model for solving allocation problems (see figure 3). Using this model implies that both theory and perceptions are taken into account to solve allocation problems as good as possible.

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Appendix A: Descriptions of the allocation methods

(1) Average Cost Pricing: The gains are divided proportionally by the purchasing quantities.
(2) Compromise Price: The gains are divided proportionally by the added value of the organizations, while taking into account the minimal claim of an organization. The added value of an organization A equals the difference between the total savings of the cooperative and the savings of the cooperative without organization A. The minimal claim of an organization A equals the part of the savings that are created by and for organization A (Schotanus, 2004).
(3) Compromise Value: The gains are divided proportionally by the added value of the organizations (Borm et al., 1992) in purchasing cooperatives (Heijboer 2003).
(4) Differential Pricing: All organizations receive a same small fixed discount percentage. On top of that, larger organizations receive a larger extra discount percentage. Note that determining the percentages is the difficult part of this method.
(5) Equal Amount: The gains are divided equally.
(6) Equal Price: All organizations pay the same price per item (e.g. Schotanus, 2005b)
(7) Nucleolus: The Nucleolus minimizes the maximum dissatisfaction level of all possible coalitions. As a measure for dissatisfaction, an excess factor is used. (Schmeidler 1969). The Nucleolus implies among other things that for each player the pay-off of cooperation is higher than the pay-off of working alone (Heijboer, 2003). It finds a solution that minimizes total

dissatisfaction for a cooperative. However, this does not have to mean that all cooperating organizations are satisfied.

(8) Shapley Value: The Shapley Value looks at different sequences of organizations entering a cooperative for all possible coalitions. When organization A enters an empty coalition no savings are allocated to this organization. When organization B enters this coalition it receives all of the savings this organization creates for the cooperative. For all of the possible coalitions for each organization all of its allocations are added up and divided by the total number of coalitions. This amount equals the allocation of an organization (Shapley, 1953).

Appendix B: Descriptions of the allocation properties

(1) ADD: A cooperative could be used for multiple (types of) items at the same time. Each item could be treated as a separate game with a separate allocation. The gains from all items could also be added up and be allocated at once. It seems fair that when the same allocation method would be used for each item separately or for all items together the total amount allocated to each organization should be the same. This is another way of saying that ADD has to hold.

(2) DUM: Means that a non-contributing organization should not get anything.

(3) EFF: All gains are allocated back to the organizations in a cooperative.

(4) FR: Means that this property is satisfied if an organization with an equal or larger quantity of items to be purchased through a cooperative receives an equal or larger share of the gains.

(5) MON: Satisfying this property means that if the quantity of items to be purchased by one organization in a cooperative stays equal or becomes larger than in a former situation, this organization should receive an equal or larger amount of gains.

(6) STA: Means that for each organization the pay-off of cooperation in the grand coalition is equal or higher than the pay-off of working alone or in any other sub coalition.

(7) SYM: Means that equal organizations in a cooperative should get equal pay-offs.