Asymptotic behavior of second order linear autonomous neutral delay differential equations

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Introduction

Delay (Neutral) delay differential equations arise from a variety of applications including control systems, electrodynamics, mixing liquids, neutron transportation, population models etc. Among these applications, the stability and asymptotic behaviour of solutions of these model equations is especially essential to many investigators.

Objectives

The objective is to study the asymptotic behaviour of the second order linear autonomous neutral delay differential equation by two approaches:

- ***Spectral approach***
- ***ODE approach***

The advantages and disadvantages can be reflected by applying these two approaches to some examples

Methods

Spectral approach:

The spectral approach gives emphasis on the explicit computation of the large time behaviour by using spectral projections.

ODE approach:

The main idea of ODE approach is that of transforming the second order delay differential equation into first order delay differential equation, by using of a real root of the corresponding characteristic equation. Meanwhile, by using this asymptotic behaviour , the stability criterion of the trivial solutions can be obtained.

Results

The asymptotic behaviour of the following second order linear autonomous neutral delay differential equation (1.1) with initial value (1.2) are obtained by spectral approach and ODE approach respectively.

$$x''(t)+cx''(t-\alpha)=ax(t)+bx(t-\beta) \qquad (1.1)$$

where a, b, c are real numbers, α, β are positive real numbers.

The initial condition of (1.1) is the form

$$x(t)=\varphi(t) \quad \text{for } -r \le t \le 0, \qquad (1.2)$$

φ(t) is a given continuously differentiable real-valued function on the initial interval [-r,0].

Furthermore, the conditions given by the ODE approach are further studied and some examples are shown to illustrate the main results.

Examples:

1. $x''(t)+x''(t-1)=x(t)+x(t-1)$ with initial condition (1.2)
2. $x''(t)+x''(t-1)=x(t)-x(t-1)$ with initial condition (1.2)

Conclusions

The asymptotic behaviour of IVP (1.1) and (1.2) is obtained by these two approaches. The advantages and disadvantages of these two approaches are illustrated through their applications to some examples.

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