POOL MET ZORG

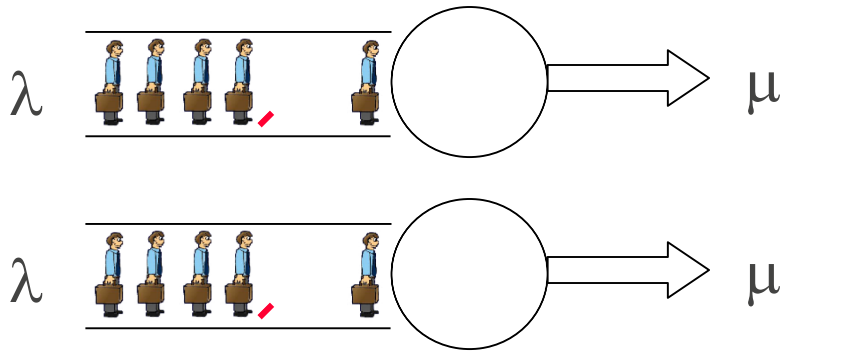
Nico van Dijk
CHOIR group /
Sanquin bloedvoorziening

University of Twente
Enschede, The Netherlands

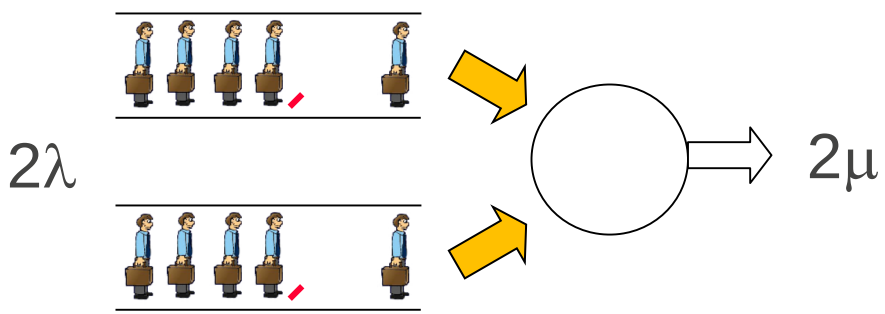
Streven

1.	Intro	(‘95)	(Postkantoor)
		(’05)	(CC) (E.vd Sluis)
		(’07)	(MRI-AMC, P.Joustra)
2.	Motivation	(’05)	(OK-IC, N.Kortbeek)
3.	Overflow	(’00,‘09)	(CC-SBR) (E.vd Sluis)
		(’00,’09)	(Mobile networks, R.J. Boucherie)
		(’18,’20, 21)	(Generic, B. Schilstra)
4.	Serial	(’18 /’20)	(SDU,B.Schilstra) (MC: Y.Cui)
5.	Opt	(’20/’21)	(MC/SDU, Y. Cui, B.Schilstra)

TWO PARALLEL SERVERS

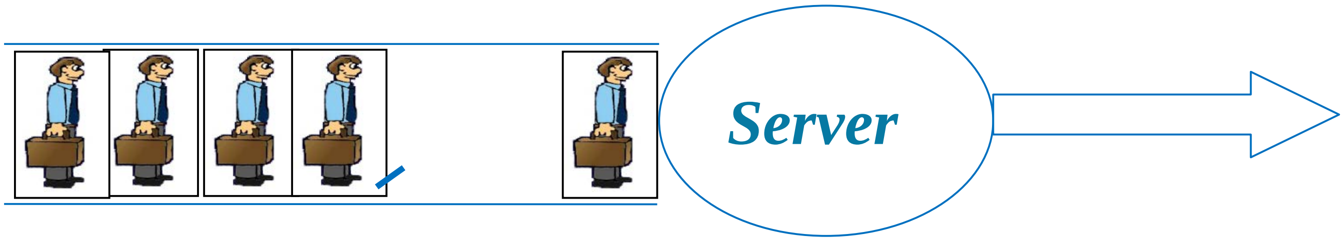


$$D = \left[\frac{1}{\mu - \lambda}\right]$$



$$D = \left[\frac{1}{2\mu - 2\lambda}\right]$$

50%



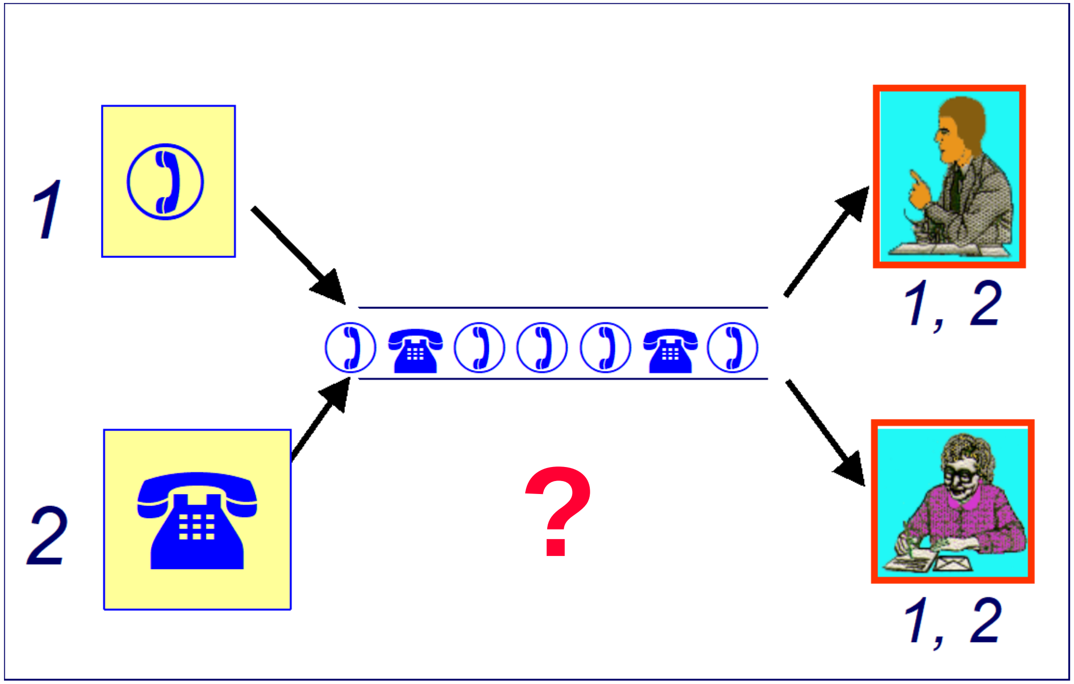
MEAN DELAY (SOJOURN TIME) ("DOORLOOPTIJD")

$$D = \left[\frac{1}{C - A} \right]$$

Service capacity μ (Max output)

Arrival intensity/rate (Throughput/input) λ

POOLING :



And yet?

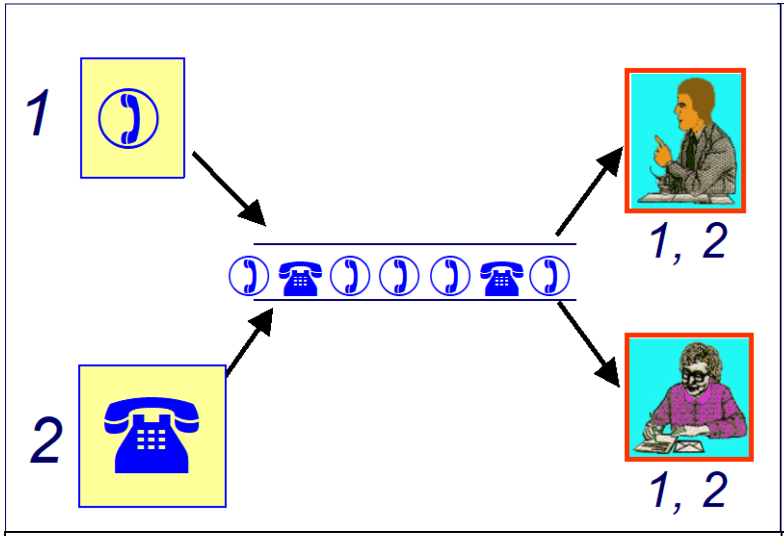
EXAMPLE

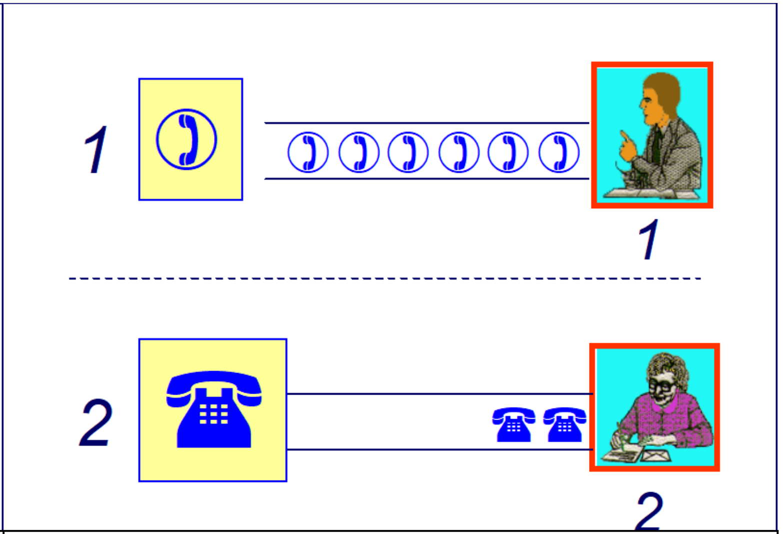
Type	Traffic	Service
1	50 / hr	1 min
2	5 / hr	10 min

$\rho = 0.83$

Deterministic

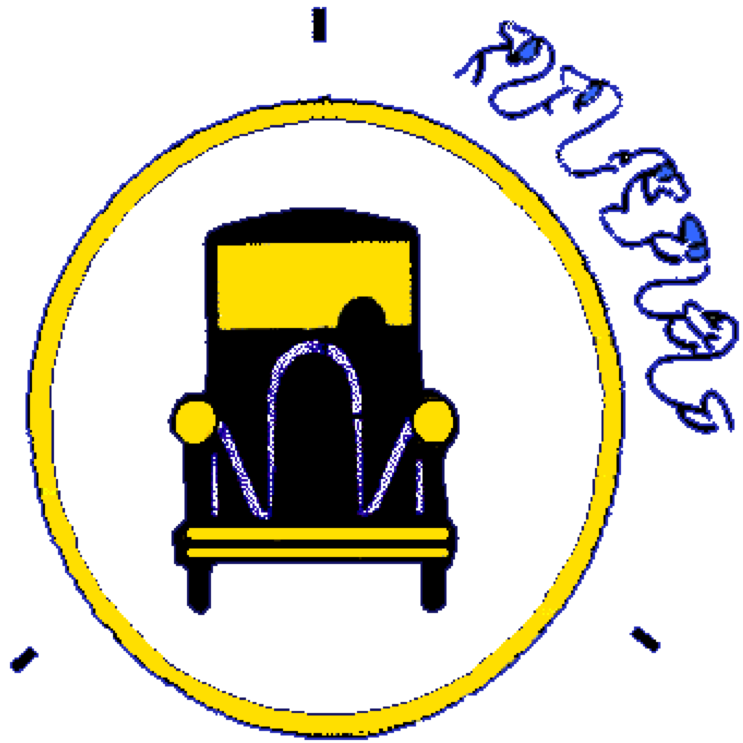
POOLING OR SEPARATE QUEUE





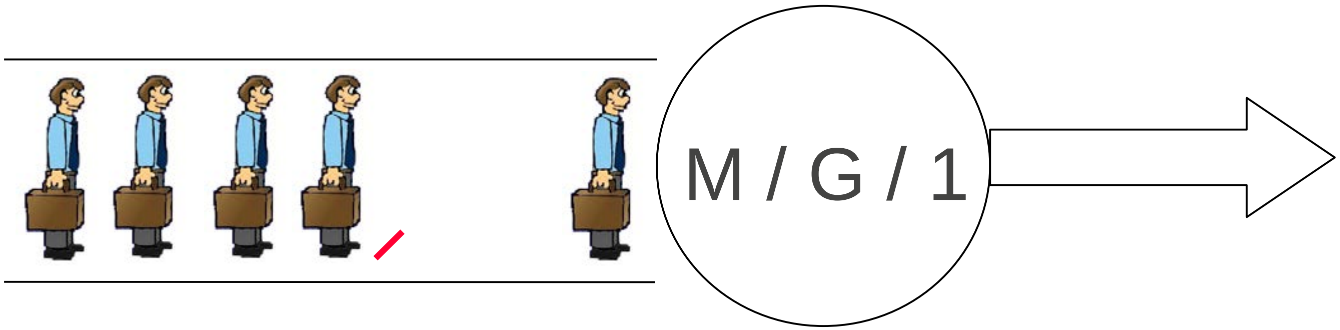
$W_1 = 6.15$ $W_2 = 6.15$	$W_1 = 2.50$ $W_2 = 25.0$
$W_A = 6.15$	$W_A = 4.55$

BUSSTOP



Busstop with 3 buses in the hour

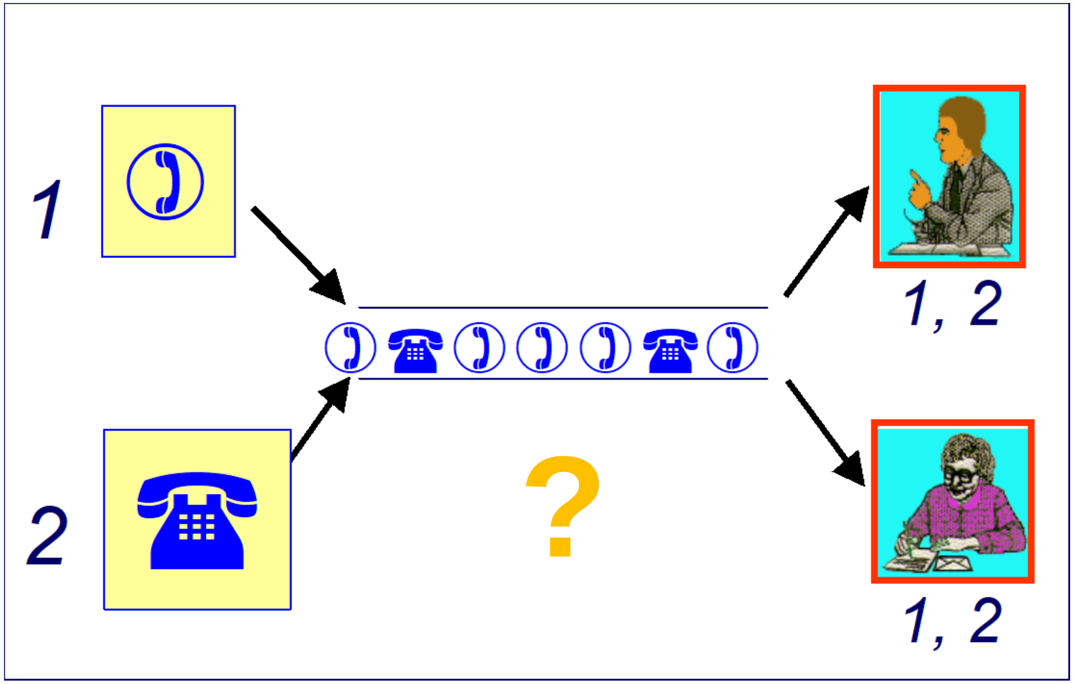
POLLACZEK-KHINTCHINE'S FORMULA



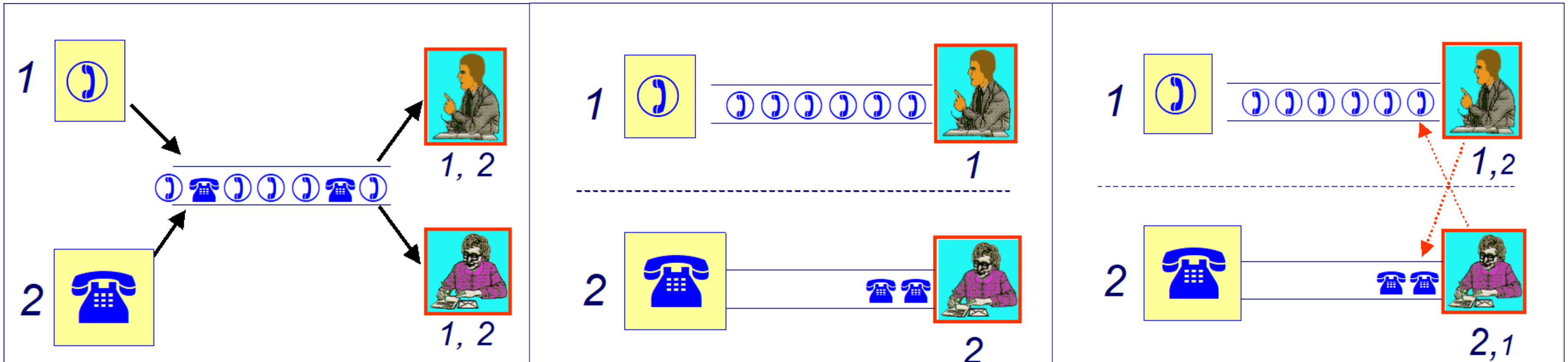
$$W_G = \tfrac{1}{2}(1 + c^2)W_{exp}$$

Coefficient of Variation

CAN WE DO BETTER ?

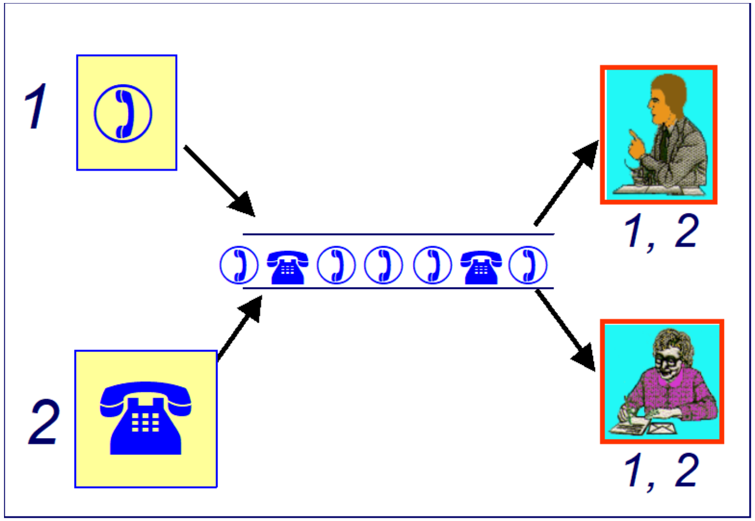


OVERFLOW RESULT

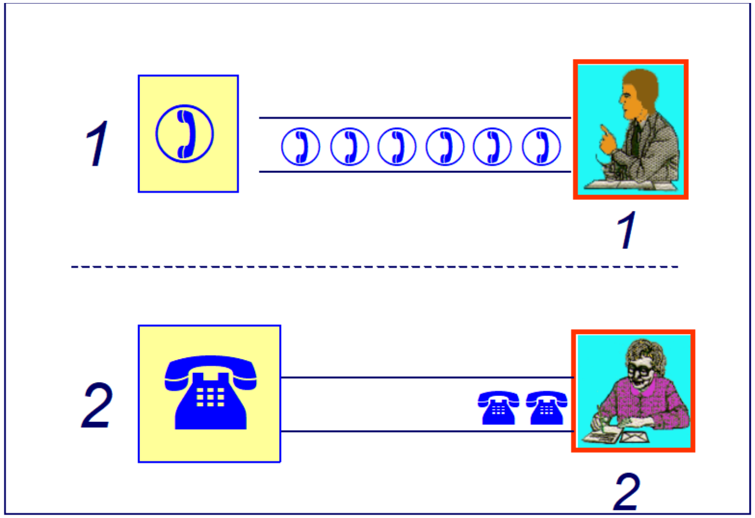


$W_1 = 6.15$ $W_2 = 6.15$	$W_1 = 2.50$ $W_2 = 25.0$	$W_1 = 3.66$ $W_2 = 8.58$
$W_A = 6.15$	$W_A = 4.55$	$W_A = 4.11$

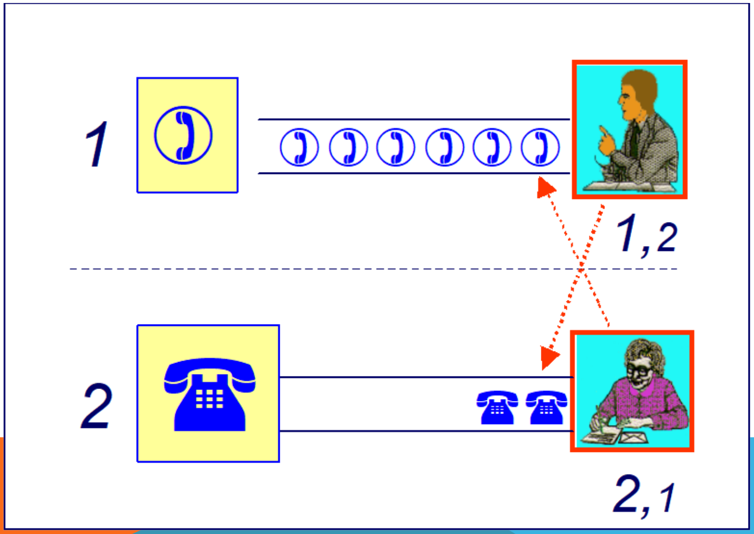
CAN WE DO BETTER?



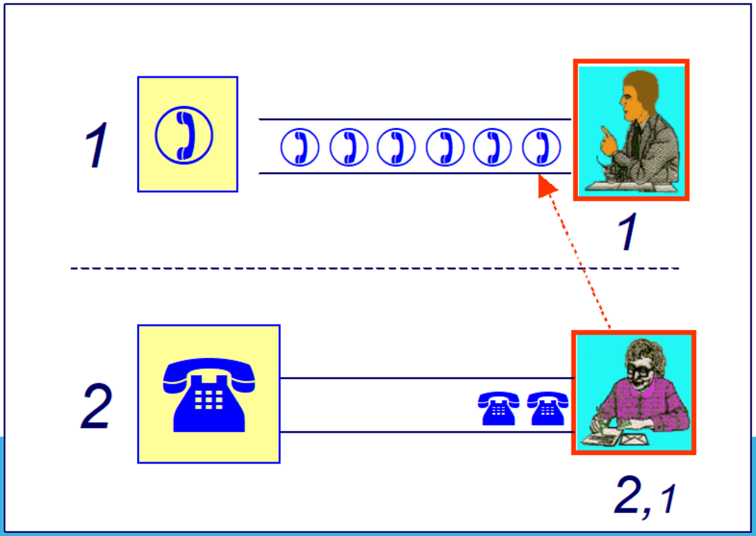
6.15



2.50
4.55
25.0

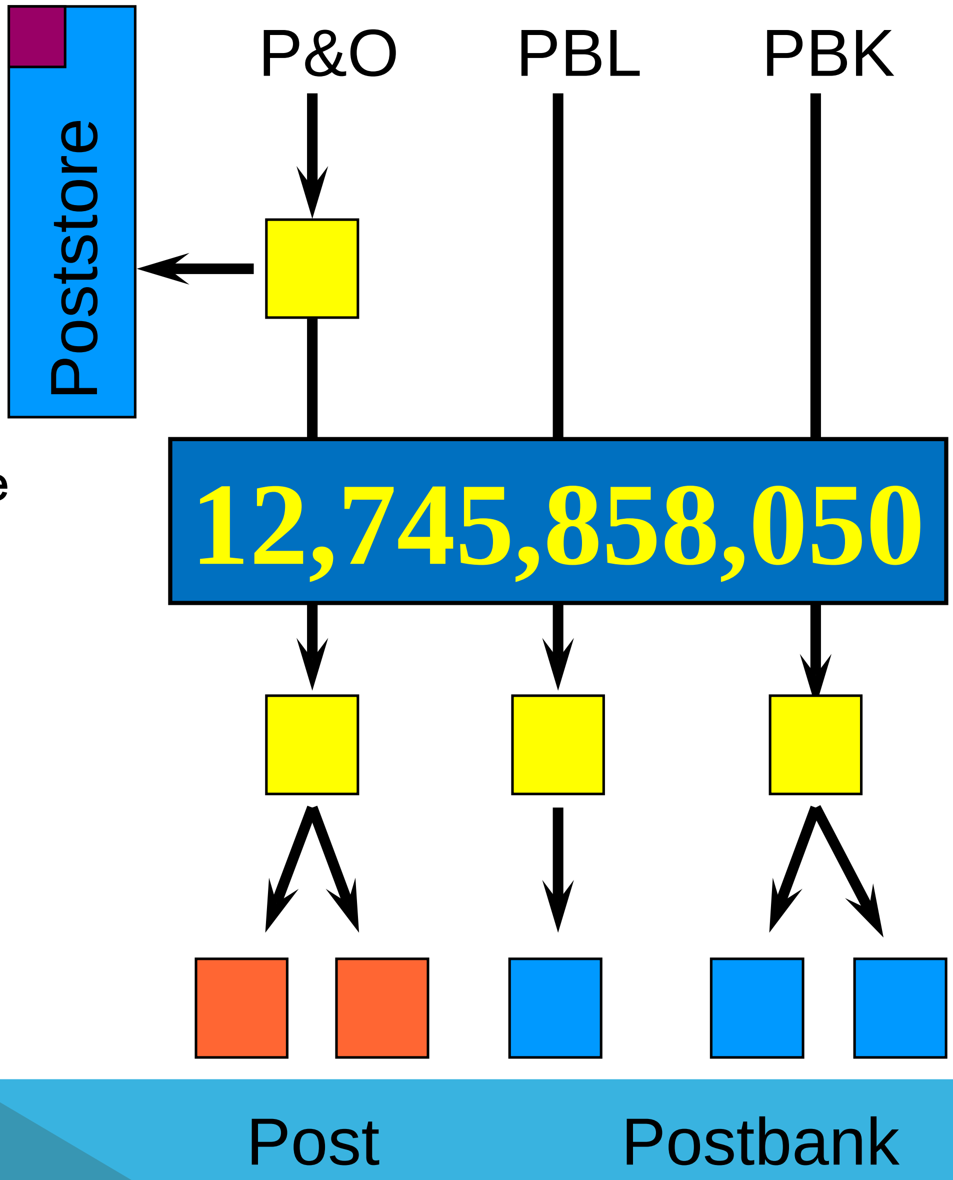


3.66
4.11
8.58



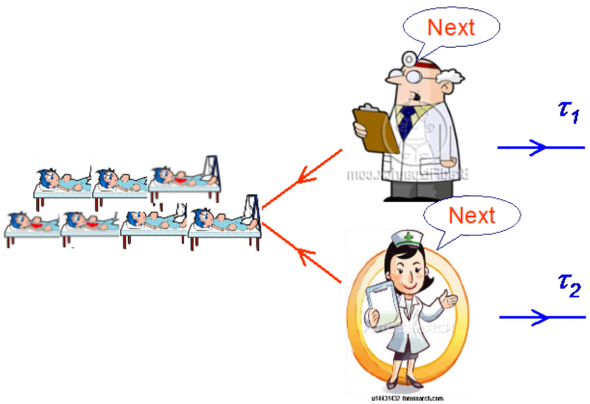
1.80
3.92
25.2

DUTCH (POSTKANTOREN B.V.) POSTAL OFFICE CASE ('95)



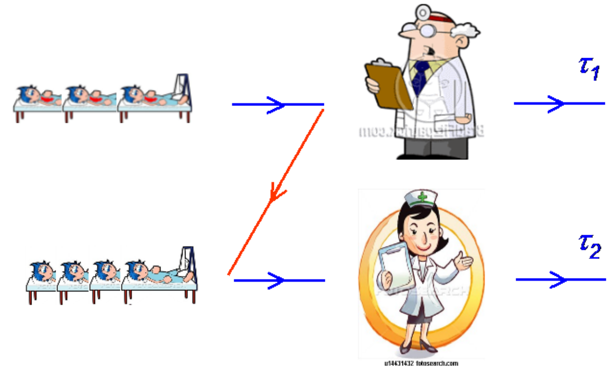
"Niet achter de komma:"

COMPARISON



$\tau_1 = 1$

$\tau_2 = 1.4$

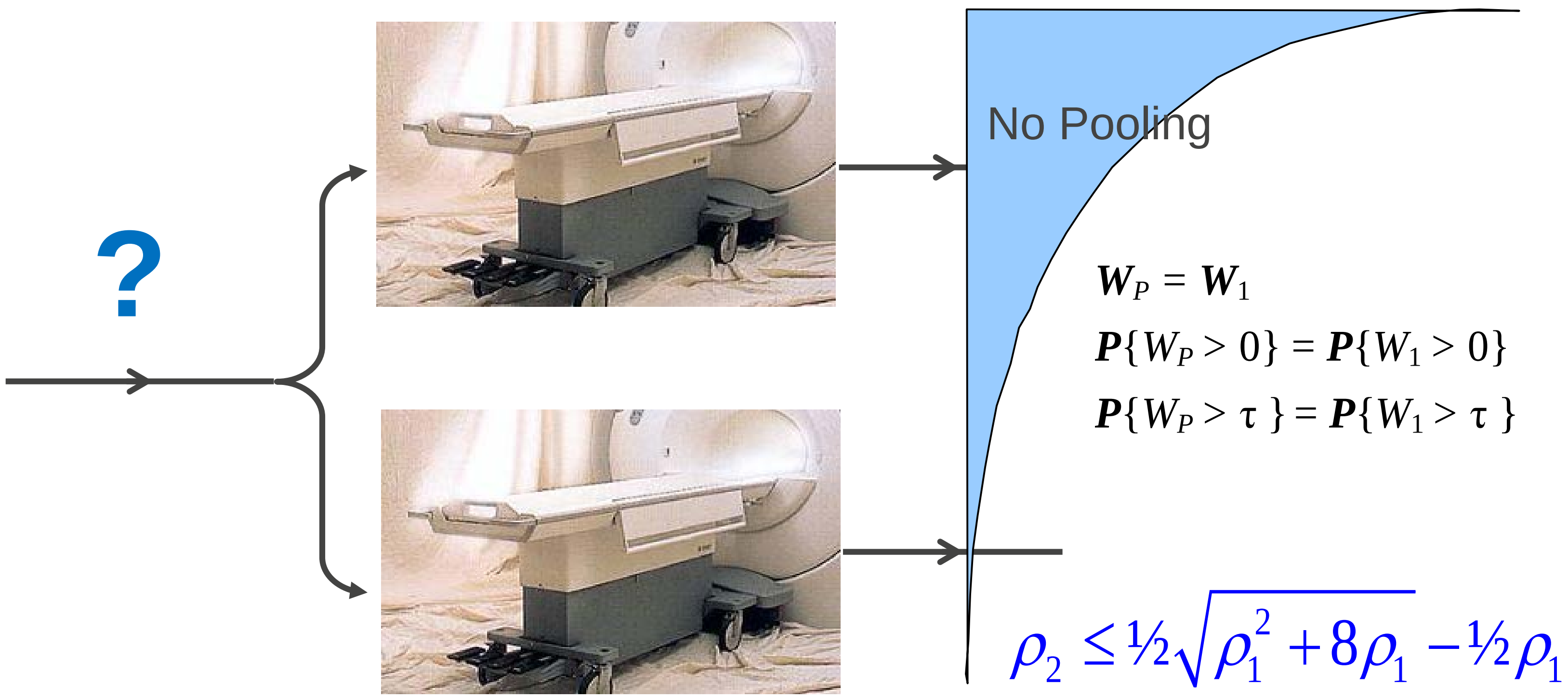


Pooled	Overflow
$W_A = 2.59$	$W_A = 2.37$

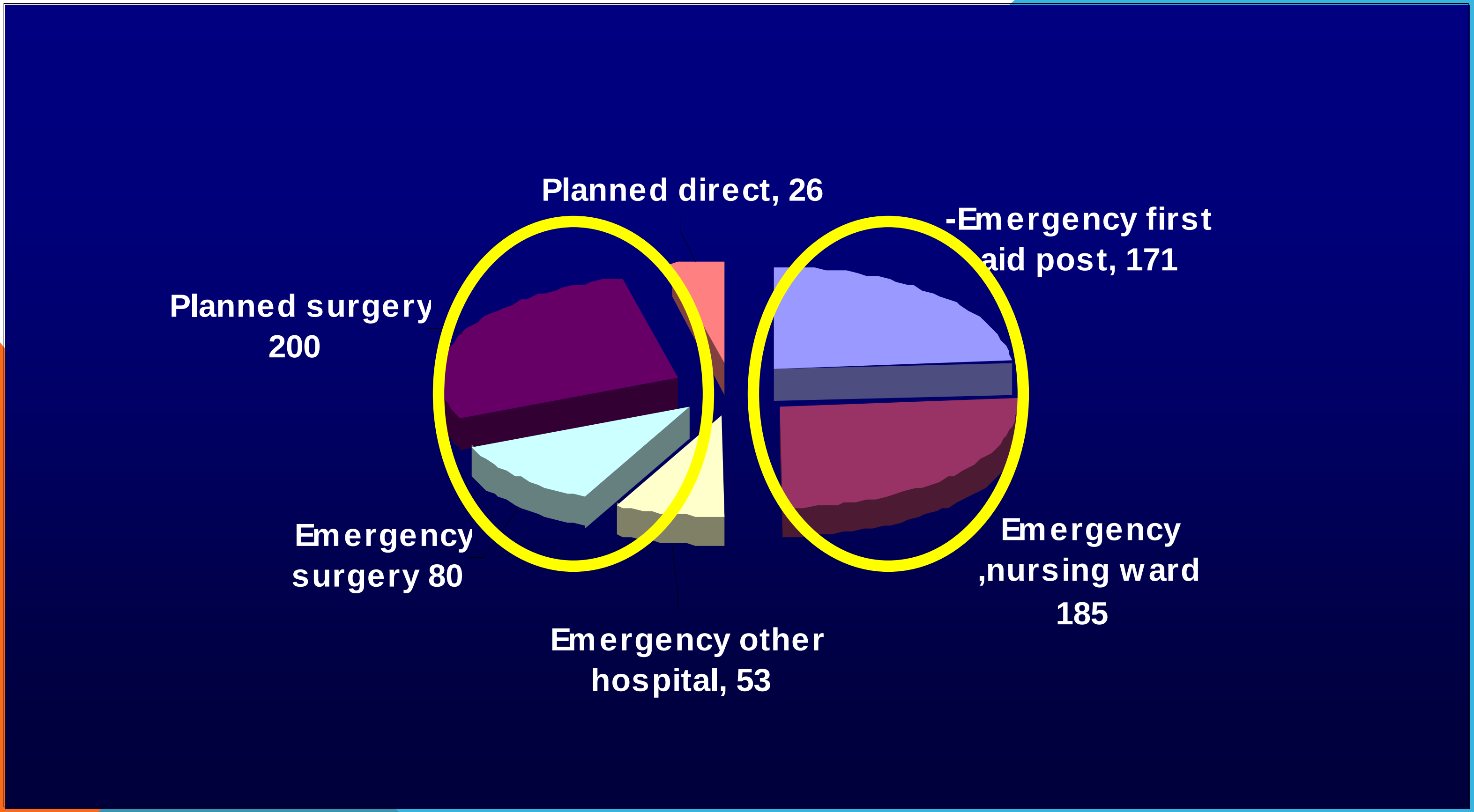
‘DIFFERENT NORMS (SUB) ACUTE PATIENTS

SHOULD WE POOL OR NOT?

AMC : P. JOUSTRA

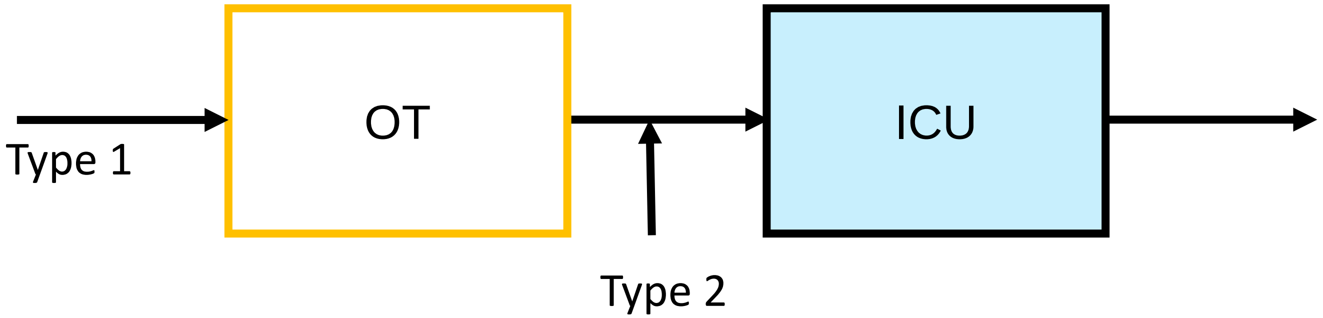


II MOTIVATION (IC)



Source Jeroen Bosch Hospital

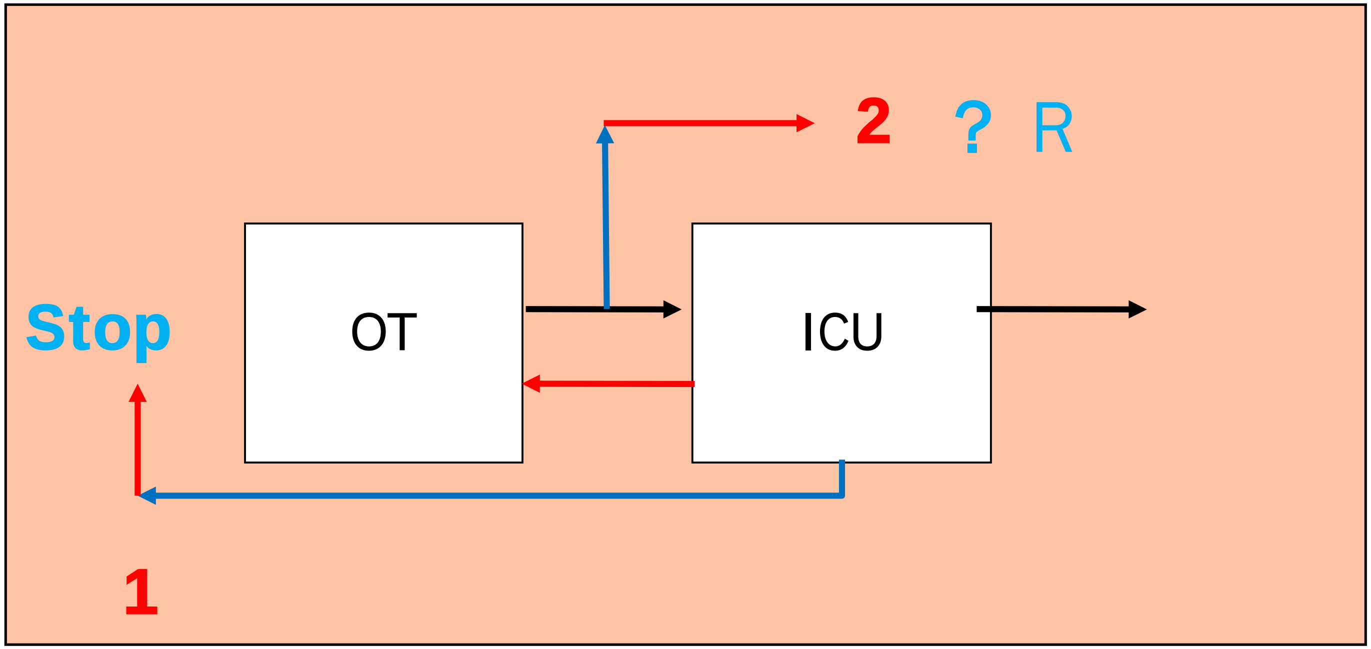
- Congestion of ICU
- Known % ? / visible ? / lives



- "No" (highly limited) measurements
- System is analytically "un"solvable (no pdrfm)
- ICU congestion/rejection probability ≈ 5 -15 % *

IIA

Motivation



Analytically solvable

$$\underbrace{\left\{\begin{array}{l}\pi(n_1,n_2)\mu_1 1_{(n_1>0)} \\ \quad + \\ \pi(n_1,n_2)\mu_2 1_{(n_2>0)} \\ \quad + \\ \pi(n_1,n_2)\lambda 1_{(n_1+n_2<N_1+N_2)}\end{array}\right\}}_{Outstream} = \underbrace{\left\{\begin{array}{l}\pi(n_1-1,n_2)\lambda 1_{(n_1>0)} \\ \quad + \\ \pi(n_1+1,n_2-1)\mu_1 \\ \quad + \\ \pi(n_1,n_2+1)\mu_2 1_{(n_1+n_2<N_1+N_2)}\end{array}\right\}}_{Instream}$$

$$\pi(n_1,n_2) = c\left(\frac{1}{\mu_1}\right)^{n_1}\left(\frac{1}{\mu_2}\right)^{n_2} \quad with\ c\ at\ R_L$$

RESULT

With
PF modifications

Erlang Loss probabilty

$$B\,(M/G/c/c) \leq R \leq B\,(M/G/c\text{-}1/c\text{-}1)$$

Original OT-IC model

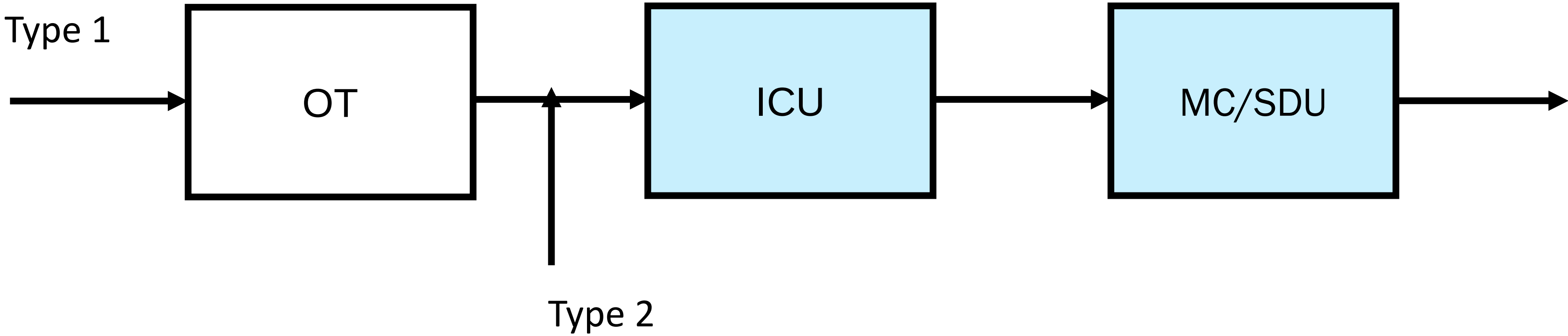
Upper bound for R

Erlang loss bounds for OT–ICU systems
(Queueing Systems, Special Issue
Erlang's centennial, 2009)
with N. Kortbeek,

IIB STEP DOWN UNIT

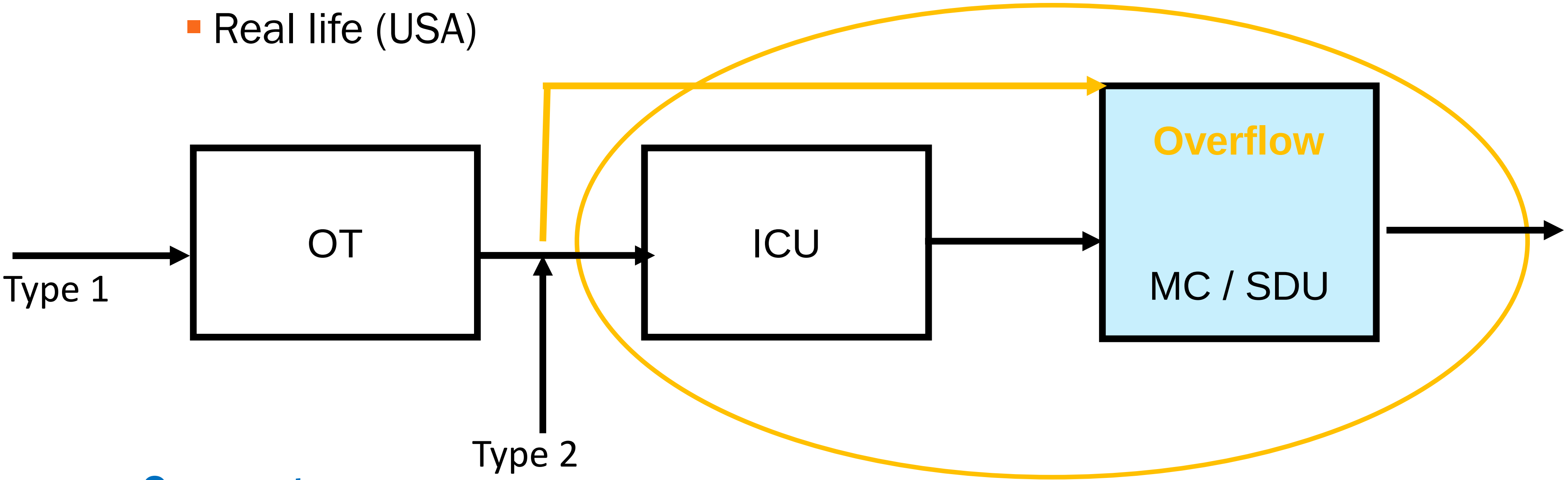
Motivation

- SDU (Intermediate care)
- (Old problem questioned)
- (Linda Green '09) : Real life (USA) : "also with SDU ?"



STEP DOWN UNIT (ALSO AS OVERFLOW)

- Intermediate care
- Real life (USA)



2 aspects

- SDU also as serial overflow
- Still different processes

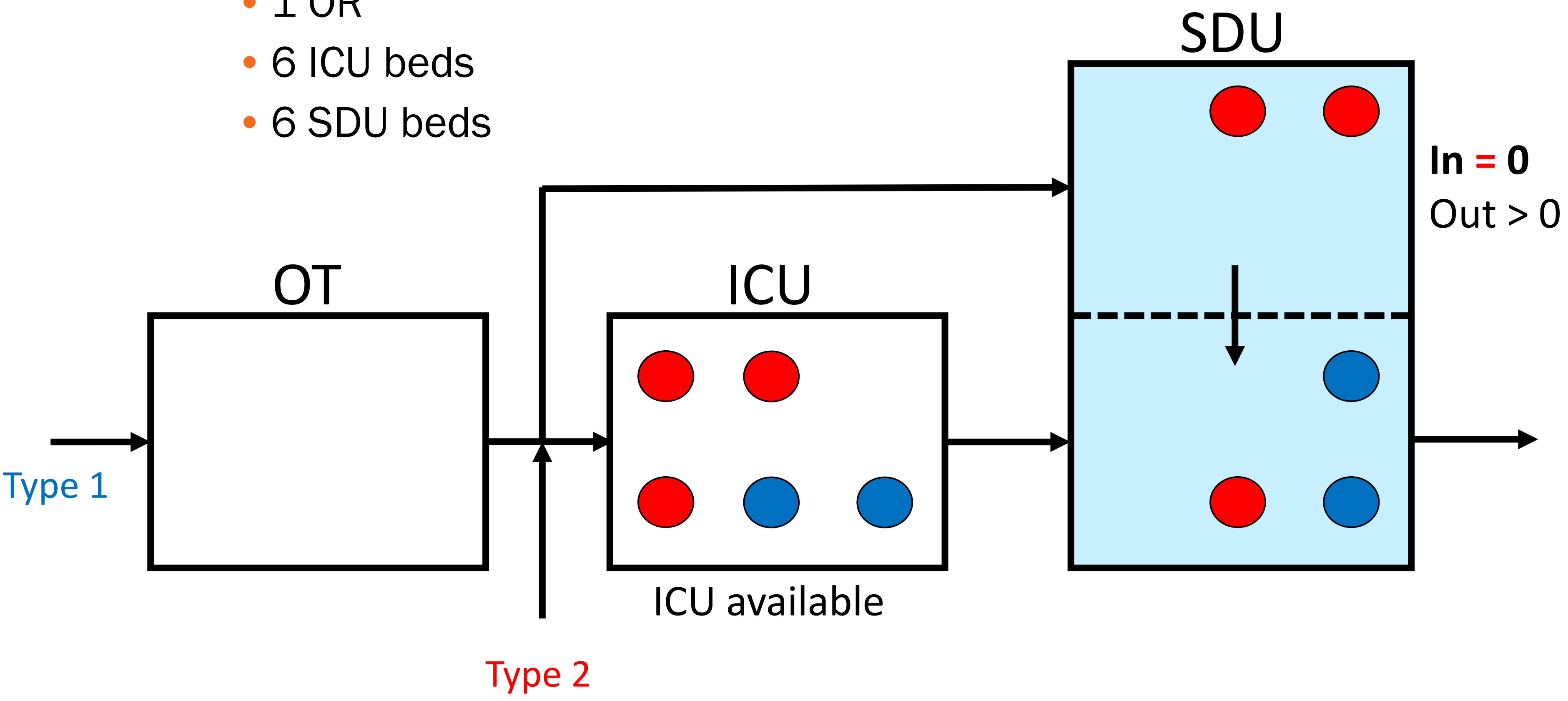
III A IC /MC

With
Yufan Cui /
Barteld Schilstra


Checklist technisch lezen – onderwijs en leesmethodes

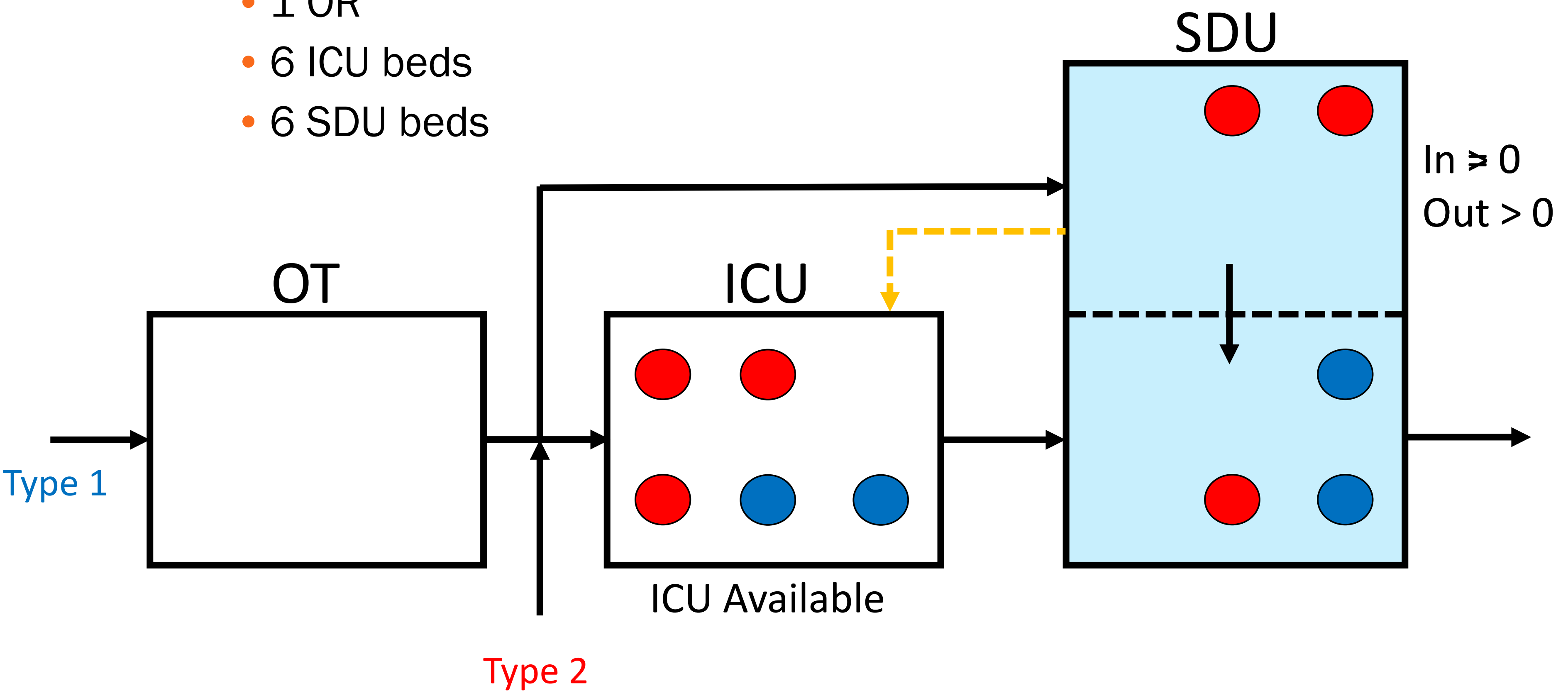
PRODUCT FORM FAILURE

- Example
  - 1 OR
  - 6 ICU beds
  - 6 SDU beds

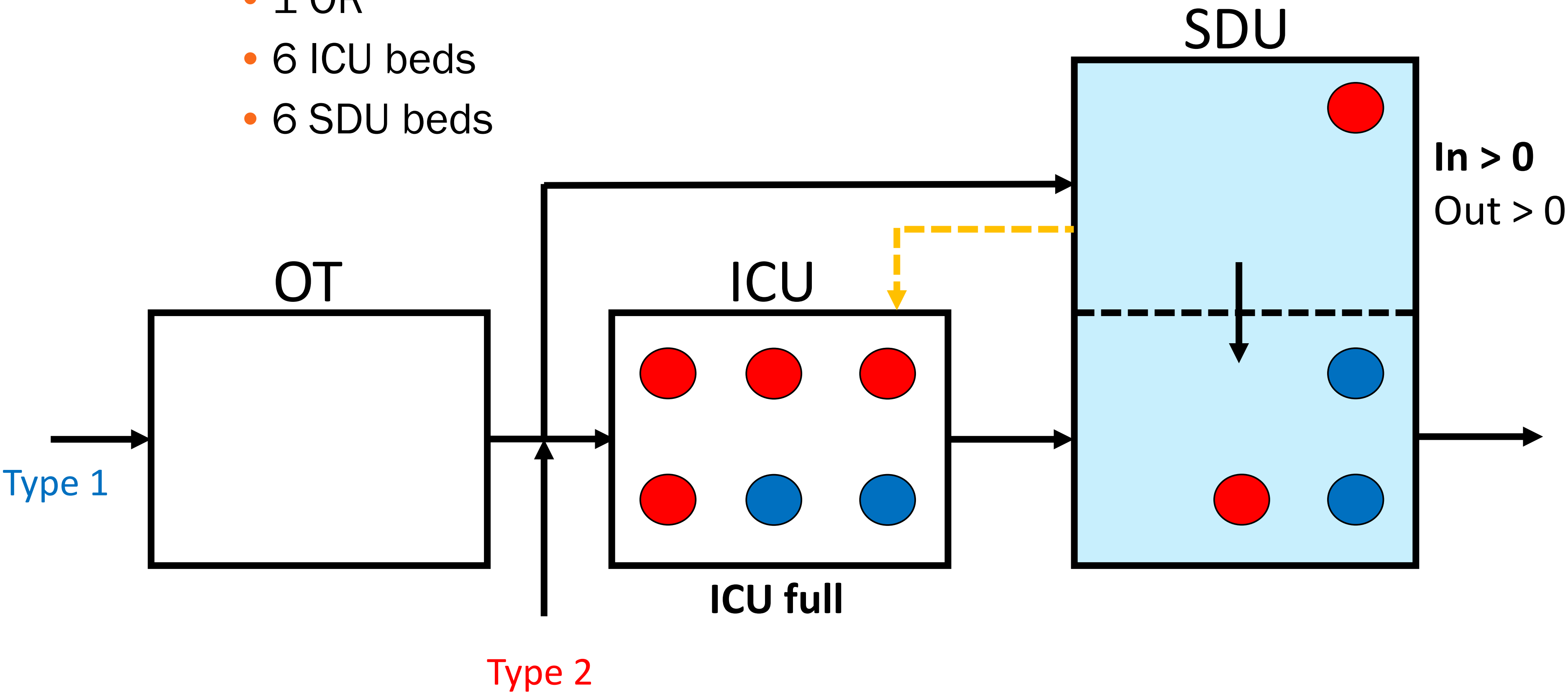


MODIFICATION
(CALL PACKING : ARTIFICIAL OR NOT)

- Example
  - 1 OR
  - 6 ICU beds
  - 6 SDU beds



- **Example**
  - 1 OR
  - 6 ICU beds
  - 6 SDU beds



3. PRECISE FORMULATION

- States: $(\mathbf{n}, \mathbf{m}) = \left(n_1\ , n_2^1\ , n_2^2, n_3^1, n_3^2, m^1, m^2\right)$

- Poisson arrivals with rate λ^i for type $i, i = 1{,}2$

- Service rates:
  - μ_1 for patients of type 1 at OT
  - μ_2^i for patients of type i at ICU, $i = 1{,}2$
  - μ_3^i for regular patients of type i at SDU, $i = 1{,}2$
  - γ^i for overflowed patients of type i at SDU, $i = 1{,}2$

GLOBAL BALANCE EQUATION

$$\left\{\begin{array}{l}\pi(n_1,n_2,m)n_1\mu_1 1_{\{n_1>0\}}+ \\ \pi(n_1,n_2,m)m\gamma 1_{\{m>0\}}+ \\ \pi(n_1,n_2,m)n_2\mu_2 1_{\{n_2>0\}}+ \\ \pi(n_1,n_2,m)\lambda_1 1_{\{n_1<N_1\}}+ \\ \pi(n_1,n_2,m)\lambda_1 1_{\{n_1=N_1\}}1_{\{(n_2,m+1)\in C\}}+ \\ \pi(n_1,n_2,m)\lambda_2 1_{\{(n_2+1,m)\in C\}}\end{array}\right\}\begin{array}{r}(1)\\(2)\\(3)\\(4)\\(5)\\(6)\end{array}$$

$$=$$

$$\left\{\begin{array}{l}\pi(n_1-1,n_2,m)\lambda_1 1_{\{n_1>0\}}1_{\{m=0\}}+ \\ \pi(n_1,n_2,m-1)\lambda_1 1_{\{n_1=N_1\}}1_{\{m>0\}}+ \\ \pi(n_1,n_2-1,m)\lambda_2 1_{\{n_2>0\}}+ \\ \pi(n_1+1,n_2,m)(n_1+1)\mu_1 1_{\{n_1<N_1\}}+ \\ \pi(n_1,n_2,m+1)N_1\mu_1 1_{\{n_1=N_1\}}1_{\{(n_2,m+1)\in C\}}+ \\ \pi(n_1,n_2,m+1)(m+1)\gamma 1_{\{n_1=N_1\}}1_{\{(n_2,m+1)\in C\}}+ \\ \pi(n_1,n_2+1,m)(n_2+1)\mu_2 1_{\{(n_2+1,m)\in C\}}\end{array}\right\}\begin{array}{r}(7)\\(8)\\(9)\\(10)\\(11)\\(12)\\(13)\end{array}$$

For $n_1 < N_1$	For $n_1 = N_1, m = 0$	For $n_1 = N_1, m > 0$
(1) = (7)	(1) = (7)	(1) + (2) = (8)
(3) = (9)	(3) = (9)	(3) = (9)
(4) = (10)	(5) = (11) + (12)	(5) = (11) + (12)
(6) = (13)	(6) = (13)	(6) = (13)

PRODUCT FORM RESULTS

- ### Modification 1 (Stop)

  With C normalizing constant

$$\pi(\mathbf{n},\mathbf{m}) = C\left(\frac{\lambda^1}{\mu_1}\right)^{n_1}\left\{\prod_{i=2,3}\prod_{t=1,2}\frac{1}{n_i^t!}\left(\frac{\lambda^t}{\mu_i^t}\right)^{n_i^t}\right\}\left\{\prod_{t=1,2}\frac{1}{m^t\,!}\left(\frac{\lambda^t}{\gamma^t}\right)^{m^t}\right\}$$

- ### Modification 2 (Call packing)

- With C normalizing constant and assuming $\mu_2^i = \gamma^i$, $i = 1,2$

$$\pi(\mathbf{n},\mathbf{m}) = C\left(\frac{\lambda^1}{\mu_1}\right)^{n_1}\left\{\prod_{t=1,2}\frac{1}{(n_2^t+m^t\,)!}\left(\frac{\lambda^t}{\mu_2^t}\right)^{(n_2^t+m^t\,)}\right\}\left\{\prod_{t=1,2}\frac{1}{n_3^t!}\left(\frac{\lambda^t}{\mu_3^t}\right)^{n_3^t}\right\}$$

VALUE

Proof

- Literature known ?? (Non-reversible / Serial overflow)

Bound (?)

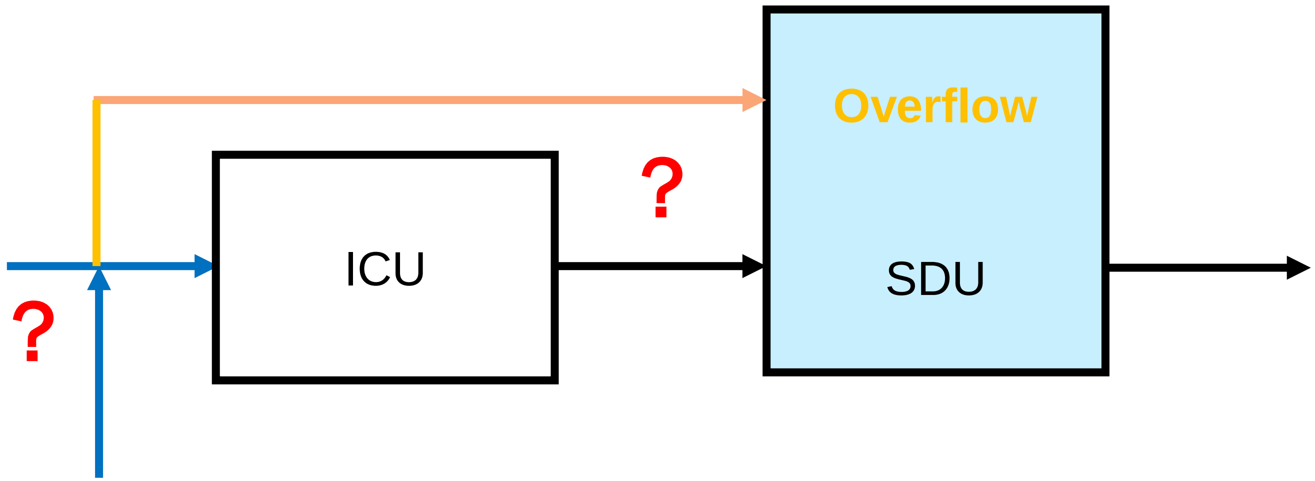
- Expect Upper bound for congestion probability of both ICU and SDU (overflow)

  - Realistic !

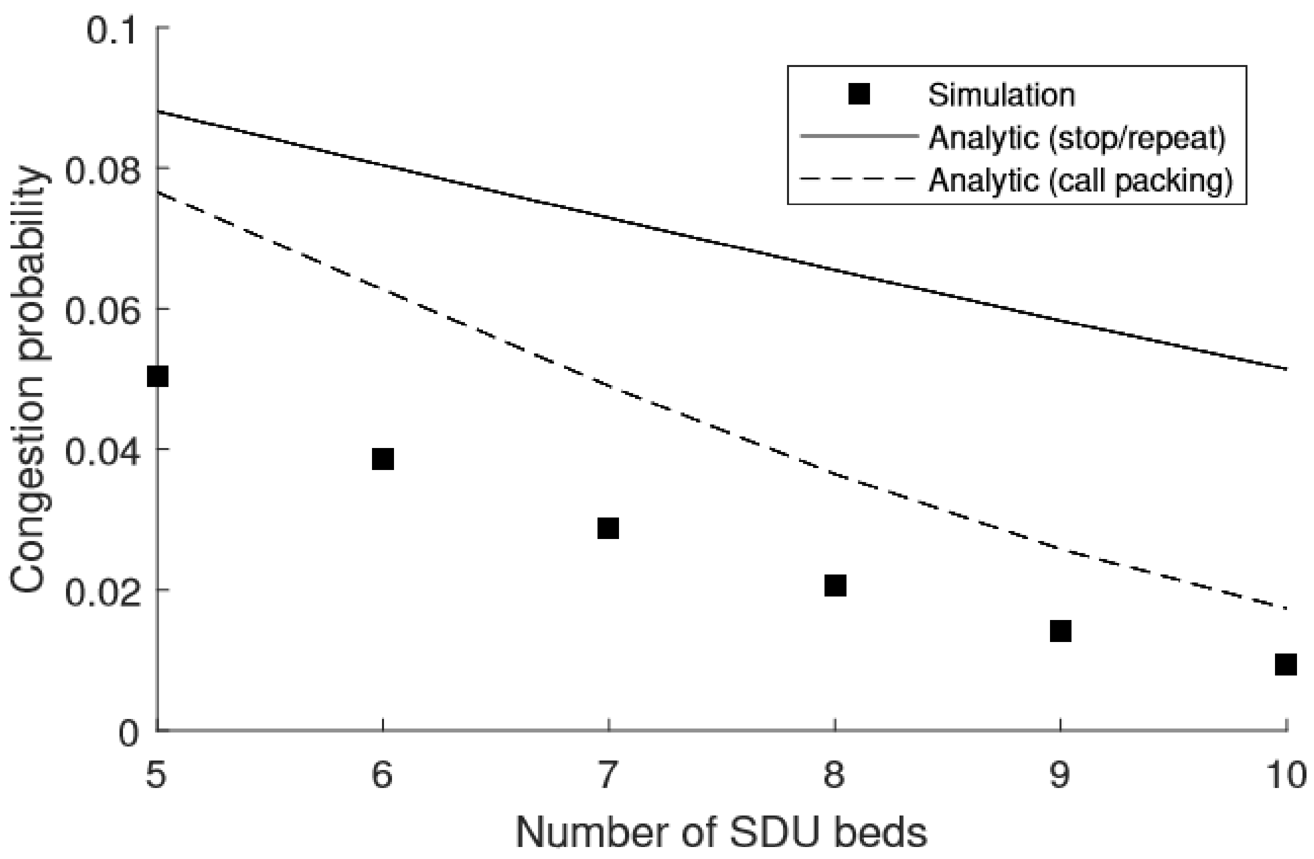
  This CP feature also appears partially realistic ?? !!! in hospitals

NUMERICAL RESULTS

- Can obtain congestion/rejection probabilities from the steady-state distributions
- Upper bound for congestion probability of ICU-SDU complex



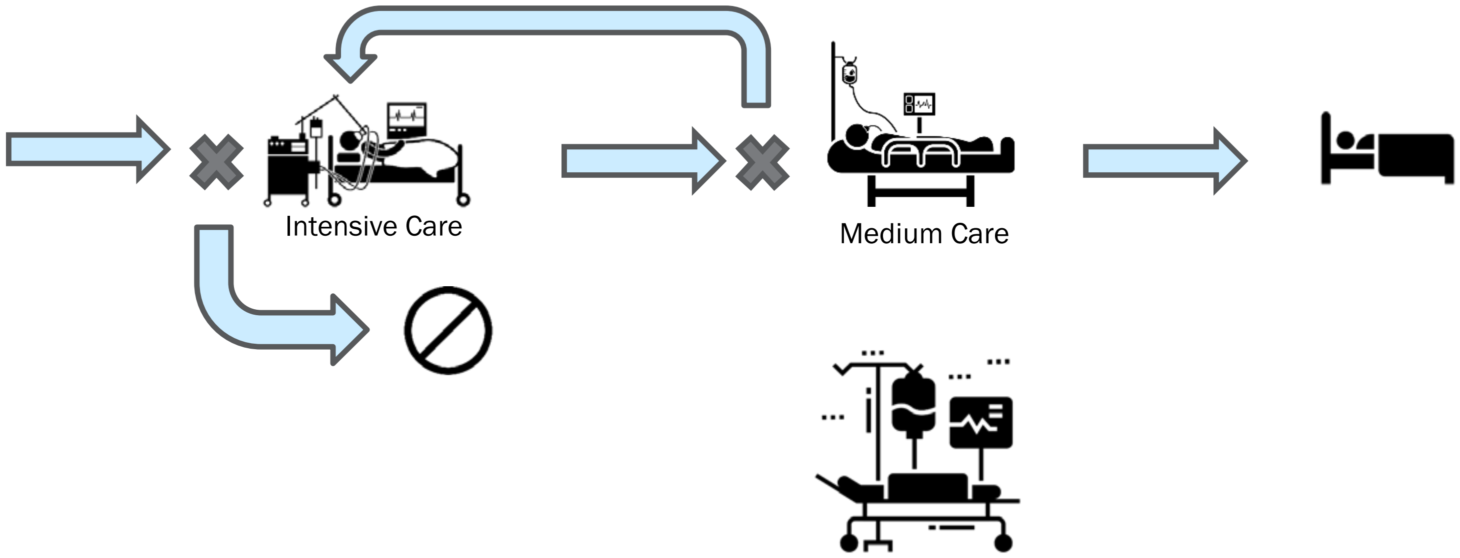
ICU-SDU CONGESTION EXAMPLE



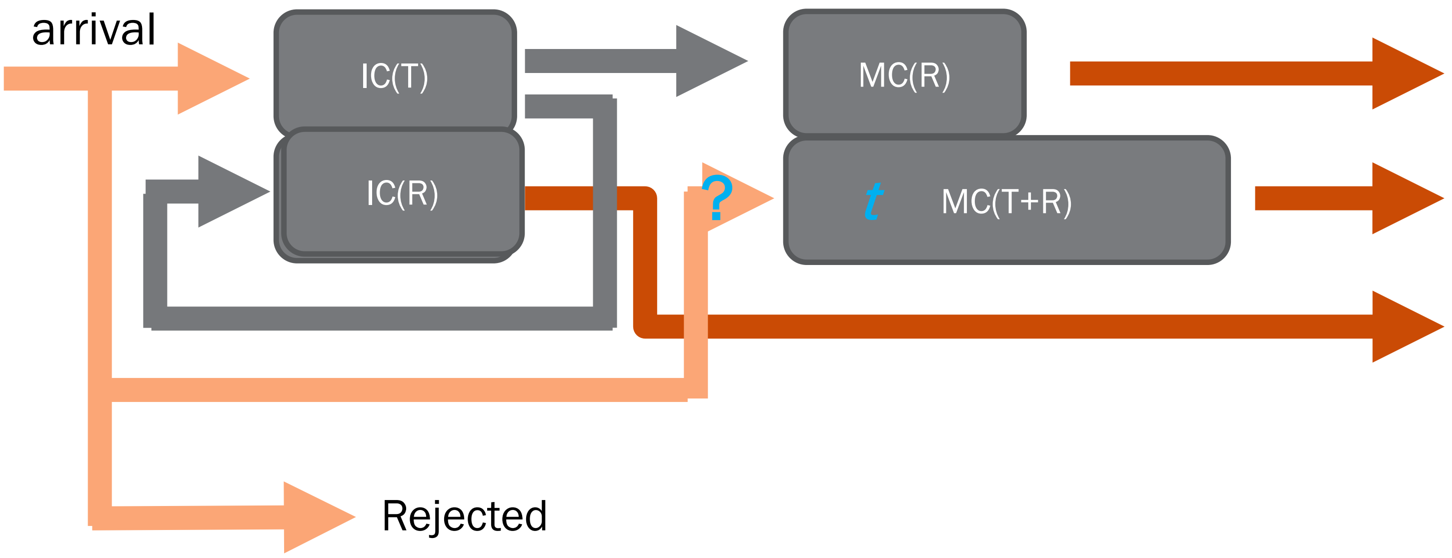
III B: FLEXIBLE MC

With
Yufan Cui

IC/MC ?



FLEXIBLE MODEL REAL



COSTS

- **Staffing**

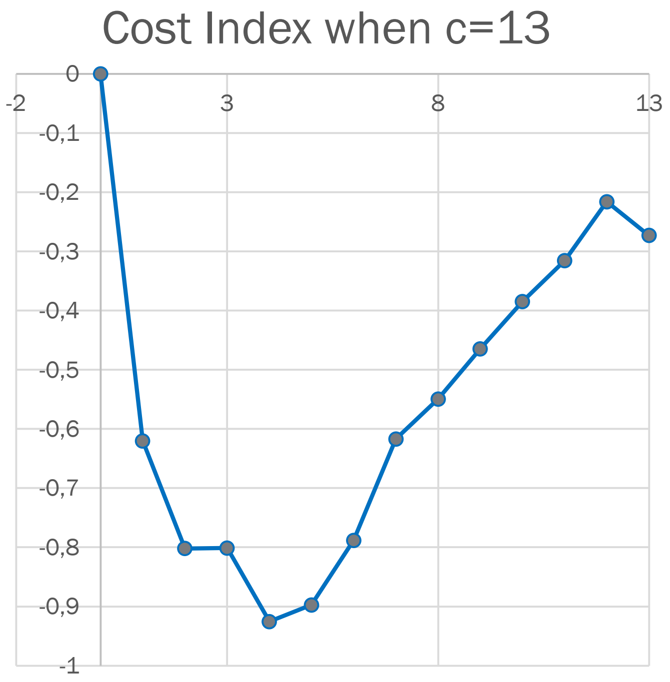
  (1;2 / 1:4)
- **Equipment**
  - Monitoring
  - Syringe pumps
  - Infusion
  - Ventilation
  - ABG machines

Fictitious write-offs

Cost Function					
IC	MC	IC2	TEMP	Reject	Refit
20	5	25	50	100	10000

NUMBER OF FLEXIBLE MC BEDS (M=N=13)

t	B	Cost Index
0	0.099464	0
1	0.08082	-0.6201
2	0.068356	-0.80207
3	0.059776	-0.80121
4	0.050585	-0.92541
5	0.045161	-0.8974
6	0.041528	-0.78826
7	0.039918	-0.61708
8	0.039097	-0.5495
9	0.038533	-0.46461
10	0.038306	-0.38448
11	0.038192	-0.31534
12	0.038162	-0.21592
13	0.037897	-0.27302



DATA

LUMC
2018
pat 1656

ICU : 3.46 days
MC : 2.08 days

Both ICU and MC
times : Lognormal

No exact costs

The new sheet for the simulation is as below,

Just simulated Opt

IC Congestion B Flexible MC beds	M=20, N=13 lognormal	M=20, N=13 Exp
0	7.6 %	7.7 %
3	6.3 %	6.5 %
6	7.6 %	7.4 %
10	9.8 %	9.3 %
13	10.1 %	8.5 %

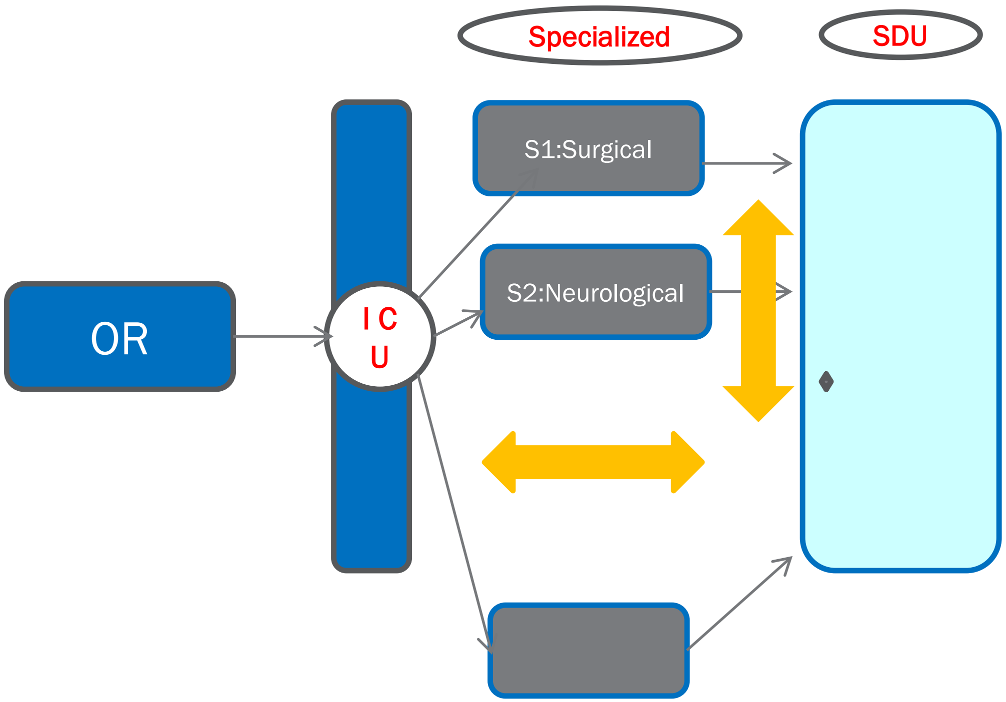
Optimum involved

(Gain ~ 25 ~ 100 p 4 year

Worth investing)

CHALLENGES

REAL COSTS
E.G. HIERARCHICAL AND SPECIALIZED ICU'S



CONCLUSIONS

- IC/MC Pooling :

  At least
  theoretical interest.
  (Product form modeling) if not practical ?

- Health care applications

  More than analytical
  More than (OT) planning

  How it works ? (standardized ?)
  Often multiple phased /resources
  Diverse stochasticity at different levels

- Questions / Comments : Welcome

Thanks for your attention