

TO BE ANNOUNCED

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Symposium on Stochastic Processes
Enschede, 26 September 2014

REMINISCENCES AND AFTERTHOUGHTS

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**Springer Series
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Approach**



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**An Introduction
to Orthogonal
Polynomials**

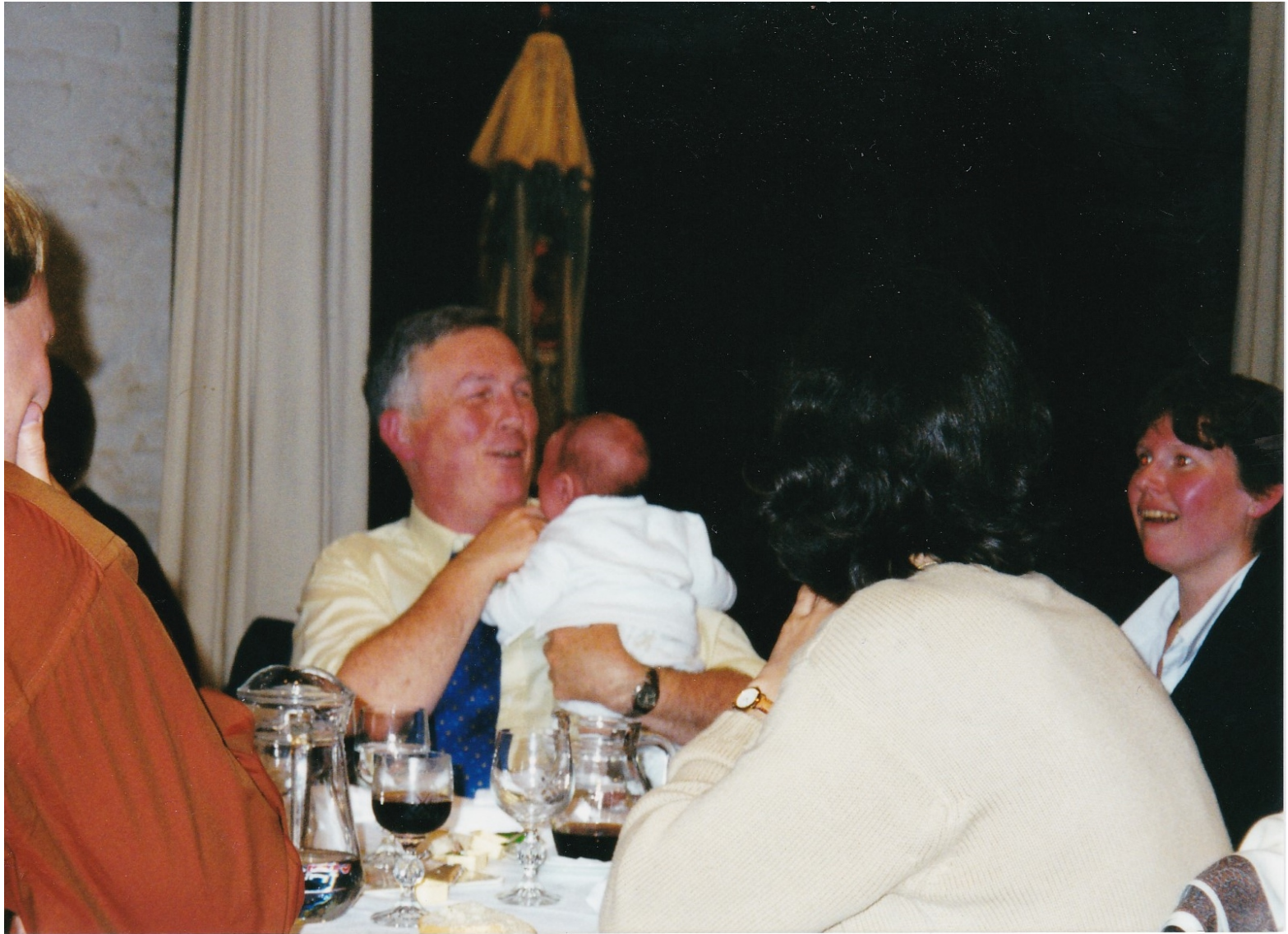
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MATHEMATICS AND ITS APPLICATIONS
A Series of Monographs and Texts VOLUME 13





















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A REVIEW OF BEAUTIFUL FORMULAS

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$$P_{ij}(t) = \pi_j \int_0^{\infty} e^{-xt} Q_i(x) Q_j(x) \psi(dx)$$

$$\pi_j \int_0^{\infty} Q_i(x) Q_j(x) \psi(dx) = \delta_{ij}$$

AE Stochastic Processes and Their Applications:

... The paper is difficult to read because hardly anybody knows orthogonal polynomials anymore.

but

... I do not feel it is appropriate to complain about proofs just because they use tools that we lack.

definition: $\{P_n(x), n = 0, 1, \dots\}$ (monic, $\deg(P_n) = n$) is *orthogonal polynomial sequence (OPS)* if there exists (Borel) measure ψ (of total mass 1) such that

$$\int_{-\infty}^{\infty} P_n(x)P_m(x)\psi(dx) = k_n\delta_{nm}$$

with $k_n > 0$ (ψ is not necessarily unique)

Favard's theorem:

$\{P_n(x), n = 0, 1, \dots\}$ is OPS \iff there exist $c_n \in \mathbb{R}, \lambda_n > 0$

such that

$$\begin{aligned} P_n(x) &= (x - c_n)P_{n-1}(x) - \lambda_n P_{n-2}(x) \\ P_0(x) &= 1, \quad P_1(x) = x - c_1 \end{aligned}$$

OPS $\{P_n(x), n = 0, 1, \dots\}$ satisfies

$$P_n(x) = (x - c_n)P_{n-1}(x) - \lambda_n P_{n-2}(x)$$
$$P_0(x) = 1, \quad P_1(x) = x - c_1$$

theorem: the following are equivalent:

(i) $\text{supp}(\psi) \subset [0, \infty)$

(ii) there exist numbers $\alpha_n > 0$, $\beta_{n+1} > 0$ and $\gamma_n \geq 0$ such that $c_1 = \alpha_1 + \gamma_1$ and for $n > 1$,

$$c_n = \alpha_n + \beta_n + \gamma_n$$
$$\lambda_n = \alpha_{n-1}\beta_n$$

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$$\xi_1 = \inf \text{supp}\{\psi\}$$

questions:

$$\xi_1 = ?$$

$$\sum_{n=1}^{\infty} \pi_n Q_n(\xi_1) < \infty?$$

$$\lim_{n \rightarrow \infty} Q_n(\xi_1) < \infty?$$

$$z = 1 + M' - M$$

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$GI/GI/\infty$ with $F = H$ and mean 1

$$z = M' = 2 \int_0^{\infty} H(t)(1 - H(t))dt$$

$$z = 2 \text{ Gini}(H) = \int_0^{\infty} \int_0^{\infty} |t_1 - t_2| dH(t_1) dH(t_2)$$

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A RETROSPECT AND A LOOK FORWARD

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