Insensitivity of the Mean Field Solution for Randomized Routing in Erlang Loss Systems

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ABSTRACT

Many cloud systems provide dedicated resources for clients to use. When resources are finite, not all requests can be admitted and this leads to blocking.

Systems like AZURE guarantee an admitted job a certain amount of resources VM (Virtual Machines) that abstract processor speed, memory, etc. This results in an Erlang type of loss system. In recent work we studied the properties of using randomized routing where the jobs are routed to least busy resources randomly chosen amongst the many resources. In particular using mean field techniques we established that such schemes are very close to the optimal blocking that can be obtained from such systems. This analysis assumed that the holding times were i.i.d exponential. Moreover we showed that the stationary point of the mean field satisfies a balance condition that suggests insensitivity that was confirmed through simulation.The case of general i.i.d service times remained open although the possibility of such a result holding was suggested in work by Bramson.

In this talk I will show that the result holds when the service times are generally distributed with a Coxian distribution. Coxian distributions are known to be dense in the class of general distributions with rational Laplace transforms. Indeed we show that the equilibrium points of the mean field equations are identical thus establishing insensitivity of the stationary distribution corresponding to the global asymptotic stability (GAS) of the mean field. We then prove a continuity result associated with these models that shows that the queueing process is continuous in the Skorohod topology with respect to the weak convergence of Coxian distributions to general distributions on the positive reals.

I will also talk about an extension to the Krasovskii-Lyapunov theorem on stability of nonlinear differential equations in compact spaces that is of relevance to establishing GAS of the mean field equation an essential step in showing the interchange arguments when monotonicity is not present as in multirate loss systems.

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