

The effects of outside opportunity profits on the implementation of revenue-sharing contracts*

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Abstract

The effects of outside opportunity profits on the implementation of revenue-sharing contracts in distribution chains are studied. It is shown that these profits result in a smaller range of contracts being offered by the supplier to a retailer. In case of multiple competing retailers the outside opportunity profits may prevent the implementation of revenue-sharing contracts.

Key Words: outside opportunity profits, revenue-sharing contracts, distribution chain.

1 Introduction

Revenue-sharing contracts provide a mechanism to coordinate a distribution chain, which is a supply chain consisting of a supplier and one or more retailers. The contracts are such that the supplier charges the retailer a lower wholesale price per unit while the retailer transfers a part of his revenues to the supplier. As a result, the retailers will order the chain-optimal quantity from the supplier, that is, the distribution chain is coordinated. These contracts work particularly well in systems with a single selling season, like the video rental industry. This application and its underlying basic model have been studied in various forms by [1, 2, 4, 5] where it is shown to align the incentives of the retailers and coordinate the chain. Extensions of these models are discussed in [3, 7].

In this paper we take the outside opportunity profits, which are the profits that the firms may receive from some opportunity outside the contract, into account. These profits are minimal requirements for the firms to engage in a contract. We study the effects of these outside opportunity profits on the implementation of revenue-sharing contracts in distribution chains. It is shown that they imply a smaller range of revenue-sharing contracts offered by

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the supplier to a retailer. In case of multiple competing retailers, such a reduced range of contracts may even be empty, that is, the supplier and a retailer are not able to agree upon any revenue-sharing contract. We present two examples that show this remarkable effect. In these examples, for any revenue-sharing contract there is at least one firm that will not participate in the contract because it results in a profit lower than the outside opportunity profit. Thus, the outside opportunity profits may prevent the implementation of revenue-sharing contracts.

The outline of this paper is as follows. In Section 2 the effects of the outside opportunity profits on the implementation of revenue-sharing contracts in distribution chains with a single retailer are studied. Thereafter the effects for distribution chains with multiple retailers are investigated in Section 3. It is shown that the range of contracts offered by the supplier to a retailer are reduced and that such a range may even be empty in case of multiple retailers. Section 4 discusses our results.

2 Distribution chains with a single retailer

In this section we study the effects of outside opportunity profits on the implementation of revenue-sharing contracts in distribution chains with a single retailer. In the next section distribution chains with multiple competing retailers are analysed.

Consider a distribution chain with a supplier and a retailer. Both firms are risk neutral. The retailer orders q units of a good with a single selling season from the supplier. The supplier produces these goods at a cost price c per unit and delivers them to the retailer at a wholesale price w per unit. The retailer receives an expected revenue $R(q)$ from these goods, where $R(q)$ is assumed to be strictly concave and differentiable. This results in profit functions

$$\pi_r(q) = R(q) - wq$$

for the retailer and

$$\pi_s(q) = (w - c)q$$

for the supplier. The total profit in this distribution chain equals $\Pi(q) = \pi_r(q) + \pi_s(q) = R(q) - cq$ and it is optimal if $q = q^o$.

In this setting the firms may want to participate in a revenue-sharing contract. This is a contract $\{\phi, w\}$ offered by the supplier to the retailer with the property that it coordinates the distribution chain, that is, the total profit achieved under this contract equals the chain-optimal profit $\Pi(q^o)$. The idea underlying a revenue-sharing contract is to induce the retailer to buy many units of the good by offering a low wholesale price. In exchange the retailer should pay the supplier a part of his revenue. The first parameter ϕ , $0 \leq \phi \leq 1$, in the contract is the fraction of retail revenue that the retailer keeps while transferring the remainder, $(1 - \phi)R(q)$, to the supplier. The second parameter w is the wholesale price offered to the retailer. To

achieve coordination the supplier will set $w = \phi c$ because then the retailer will order the chain-optimal quantity q^o as it maximizes his profit $\phi R(q) - wq = \phi(R(q) - c)$. Under this revenue-sharing contract $\{\phi, \phi c\}$ the profit of the retailer equals

$$\pi_r^{RS}(q) = \phi R(q) - wq = \phi \Pi(q),$$

where the superscript RS denotes that this profit is achieved under the revenue-sharing contract. The supplier receives

$$\pi_s^{RS}(q) = (w - c)q + (1 - \phi)R(q) = (1 - \phi)\Pi(q).$$

Maximizing the retailer's profit π_r^{RS} is equivalent to maximizing the chain profit. Thus, the retailer will order q^o , resulting in his optimal profit $\pi_r^{RS}(q^o) = \phi \Pi(q^o)$. The remainder of the chain profit goes to the supplier, $\pi_s^{RS}(q^o) = (1 - \phi)\Pi(q^o)$.

Whether this contract will actually be implemented depends on the amounts of profit the firms receive from it. These amounts should be at least as large as the profits they may receive from some outside opportunity. In other words, if $\underline{\pi}_r$ and $\underline{\pi}_s$ denote the outside opportunity profits of the retailer and the supplier, respectively, (arising from a contract that need not coordinate the chain, e.g. a wholesale-price contract) then these firms are only willing to participate in the revenue-sharing contract if it results in profits not below $\underline{\pi}_r$ and $\underline{\pi}_s$, respectively. These constraints on the profits are the so-called participation constraints (sometimes also called win-win conditions): upon changing from one contract to another no firm may lose profits. Otherwise, at least one firm is not willing to participate and then the contract is not implemented.

In the presence of outside opportunity profits the supplier may only offer revenue-sharing contracts $\{\phi, \phi c\}$ that result in profits not lower than $\underline{\pi}_r$ for the retailer and not lower than $\underline{\pi}_s$ for himself. This induces the following conditions on the fraction ϕ : $\pi_r^{RS}(q^o) = \phi \Pi(q^o) \geq \underline{\pi}_r$ and $\pi_s^{RS}(q^o) = (1 - \phi)\Pi(q^o) \geq \underline{\pi}_s$. These conditions reduce to

$$\frac{\underline{\pi}_r}{\Pi(q^o)} \leq \phi \leq 1 - \frac{\underline{\pi}_s}{\Pi(q^o)}. \quad (1)$$

Notice that this is a nonempty range because the outside opportunity profits arise from a contract that need not coordinate the chain, $\underline{\pi}_r + \underline{\pi}_s \leq \Pi(q^o)$. Further, the range is included in the interval $[0, 1]$. Thus, the outside opportunity profits result in a smaller range of contracts being offered by the supplier. This proves the following result.

Theorem 2.1 *Consider a distribution chain with a supplier and a retailer. In the presence of outside opportunity profits the supplier will only offer revenue-sharing contracts $\{\phi, w\}$ where $w = \phi c$ and $\phi \in [0, 1]$ satisfies (1).*

Hence, the outside opportunity profits reduce the range of revenue-sharing contracts upon which the supplier and the retailer are able to agree from $[0, 1]$ to the range in (1).

3 Distribution chains with multiple competing retailers

Consider now a distribution chain with a supplier and n competing retailers. In this section we analyse the effects of outside opportunity profits on these chains.

Let q_i denote the order quantity of retailer i and let $\bar{q} = (q_1, \dots, q_n)$ be the orders placed at the supplier. The expected revenue of retailer i is denoted by $R_i(\bar{q})$ and w_i is the wholesale price per unit product that he pays to the supplier. Assume that R_i is a differentiable function that is unimodal in q_i . The retailers compete because products of retailers i and j are assumed to be substitutes: $\partial R_i^i / \partial q_j \leq 0$ for all $i \neq j$, where $R_i^j(\bar{q}) = \partial R_i(\bar{q}) / \partial q_j$. Hence, an increase of q_j has a negative effect on the marginal revenue of retailer i . The unit cost of the supplier is c . Thus, the profits of the firms are

$$\pi_{r_i}(\bar{q}) = R_i(\bar{q}) - w_i q_i \quad (2)$$

for retailer i and

$$\pi_s(\bar{q}) = \sum_{i=1}^n (w_i - c) q_i$$

for the supplier. The sum of these profits is the total profit $\Pi(\bar{q}) = \sum_{i=1}^n (R_i(\bar{q}) - c q_i)$ of this distribution chain. This joint profit is optimal if $\bar{q} = \bar{q}^o$, which satisfies the equalities

$$R_i^i(\bar{q}) = c - \sum_{j \neq i} R_j^i(\bar{q}) \quad (3)$$

for all retailers i .

Suppose the supplier offers retailer i a revenue-sharing contract $\{\phi_i, w_i\}$, $0 \leq \phi_i \leq 1$. Under this contract, the retailer transfers a fraction $(1 - \phi_i)$ of his revenues to the supplier and the supplier offers a relatively low wholesale price w_i . Hence, under this contract the profit of the retailer becomes $\phi_i R_i(\bar{q}) - w_i q_i$. As follows from (2) and (3), retailer i will order the chain-optimal quantity q_i^o if the wholesale price equals $w_i = \phi_i w_i^{RS}$ where $w_i^{RS} = c - \sum_{j \neq i} R_j^i(\bar{q}^o)$. As a result this contract coordinates the distribution chain.

The revenue-sharing contracts $\{\phi_i, \phi_i w_i^{RS}\}$ offered to all retailers i result in order quantities $\bar{q} = \bar{q}^o$, as argued above, and corresponding profits

$$\pi_{r_i}^{RS}(\bar{q}^o) = \phi_i (R_i(\bar{q}^o) - w_i^{RS} q_i^o) = \phi_i \pi_{r_i}(\bar{q}^o | w_i^{RS})$$

for retailer i , where by slight abuse of notation $\pi_{r_i}(\bar{q}^o | w_i^{RS})$ denotes the profit π_{r_i} of retailer i if all order quantities equal \bar{q}^o and his wholesale price is w_i^{RS} . The profit of the supplier,

$$\pi_s^{RS}(\bar{q}^o) = \sum_{i=1}^n (\phi_i w_i^{RS} - c) q_i^o + \sum_{i=1}^n (1 - \phi_i) R_i(\bar{q}^o) = \Pi(\bar{q}^o) - \sum_{i=1}^n \phi_i \pi_{r_i}(\bar{q}^o | w_i^{RS}),$$

is all that remains of the optimal chain profit.

Whether all firms are willing to participate in this contract, depends on the outside opportunity profits. According to the participation constraints, the profits resulting from the

revenue-sharing contract should satisfy $\pi_{r_i}^{RS}(\bar{q}^o) \geq \underline{\pi}_{r_i}$ for all retailers and $\pi_s^{RS}(\bar{q}^o) \geq \underline{\pi}_s$ for the supplier. These inequalities boil down to the inequalities

$$\phi_i \pi_{r_i}(\bar{q}^o | w_i^{RS}) \geq \underline{\pi}_{r_i} \quad (4)$$

for all retailers i and

$$\sum_{i=1}^n \phi_i \pi_{r_i}(\bar{q}^o | w_i^{RS}) \leq \Pi(\bar{q}^o) - \underline{\pi}_s. \quad (5)$$

We have proved the following Theorem.

Theorem 3.1 *Consider a distribution chain with a supplier and n competing retailers. In the presence of outside opportunity profits the supplier will only offer revenue-sharing contracts $\{\phi_i, w_i\}$ where $w_i = \phi_i w_i^{RS}$ and the $\phi_i \in [0, 1]$ satisfy (4) and (5).*

Again the outside opportunity profits reduce the range of contracts offered by the supplier to a retailer. But now such a range may be empty, as stated in the Remark below.

Remark 3.2 *There exist situations in which the outside opportunity profits prevent the implementation of revenue-sharing contracts because these would result in lower profits for at least one of the firms. This is caused by the range of feasible values of the ϕ_i in (4) and (5), which may be outside the range $[0, 1]$ or may even be empty.*

This remark is opposed to the result in Theorem 2.1 for distribution chains with a single retailer where the reduced range of contracts is always nonempty.

The examples below illustrate the effects in Remark 3.2 for a distribution chain with two retailers who are engaged in Cournot competition, see [6]. In the first example retailer 1 will not agree with any revenue-sharing contract because any such contract makes him worse off since he would receive $\pi_{r_1}^{RS}(\bar{q}^o) = 0$. In this situation any positive outside opportunity profit for retailer 1 will prevent his revenue-sharing contract from being implemented.

Example 3.3 Consider a distribution chain with a supplier and two competing retailers. The retailers are engaged in Cournot competition, that is, they compete through quantities. If retailer 1 has q_1 units for sale and retailer 2 q_2 units then the expected revenue for retailer 1 is given by $R_1(\bar{q}) = (7 - q_1 - 2q_2/3)q_1$ while retailer 2 earns an expected amount of $R_2(\bar{q}) = (8 - 2q_1/3 - q_2)q_2$. The unit cost of production equals $c = 5$.

Assume the supplier offers retailer i a revenue-sharing contract $\{\phi_i, w_i\}$ where $w_i = \phi_i w_i^{RS}$. The chain-optimal order quantities are $\bar{q}^o = (0, 3/2)$, resulting in $w_1^{RS} = 6$, $w_2^{RS} = 5$, $\pi_{r_1}(\bar{q}^o | w_1^{RS}) = 0$, $\pi_{r_2}(\bar{q}^o | w_2^{RS}) = 9/4$ and $\Pi(\bar{q}^o) = 9/4$.

According to (4) and (5), the revenue-sharing contracts $\{\phi_i, \phi_i w_i^{RS}\}$ satisfy the participation constraints if

$$\begin{cases} 0 \geq \underline{\pi}_{r_1}, \\ 9\phi_2/4 \geq \underline{\pi}_{r_2}, \text{ and} \\ 9\phi_2/4 \leq 9/4 - \underline{\pi}_s. \end{cases}$$

Contradiction if $\underline{\pi}_{r_1} > 0$, which is not unlikely. (For example, $\underline{\pi}_{r_1} > 0$ if this profit arises from a wholesale-price contract.) Hence, any positive outside opportunity profit for retailer 1 will prevent his revenue-sharing contract from being implemented. \square

In the next example the outside opportunity profits arise from a wholesale-price contract. Since $\underline{\pi}_{r_1} > \pi_{r_1}(\bar{q}^o | w_1^{RS}) > 0$ the lower bound of ϕ_1 arising from (4), $\underline{\pi}_{r_1} / \pi_{r_1}(\bar{q}^o | w_1^{RS})$, is larger than 1. Hence, the supplier will not agree upon such a revenue-sharing contract for retailer 1 because then he is subsidising the retailer's revenue instead of sharing it. Once again the outside opportunity profits prevent the revenue-sharing contract from being implemented.

Example 3.4 Consider the same situation as described in the previous example, but now the production cost per unit is a little lower: $c = 9/2$.

The outside opportunity profits of the firms follow from a wholesale price contract. Using such a contract, the supplier will set wholesale prices $w_i = R_i^i(\bar{q})$ resulting in quantity q_i being the order quantity for retailer i that maximizes his profit $R_i(\bar{q}) - w_i q_i$. Thus $w_1 = 7 - 2q_1 - 2q_2/3$ and $w_2 = 8 - 2q_1/3 - 2q_2$. Now the supplier's profit equals $\pi_s(\bar{q}) = (5/2 - 2q_1 - 2q_2/3)q_1 + (7/2 - 2q_1/3 - 2q_2)q_2$. This profit is optimal if $\bar{q} = \bar{q}^w = (3/8, 3/4)$. Hence, the supplier will charge wholesale prices $w_1 = 23/4$ and $w_2 = 25/4$. Given these wholesale prices the retailers will order $\bar{q} = \bar{q}^w$, as argued above. Hence, the outside opportunity profits equal $\underline{\pi}_{r_1} = 9/64$, $\underline{\pi}_{r_2} = 9/16$ and $\underline{\pi}_s = 57/32$.

Following a similar analysis as in the previous example, the chain-optimal order quantities are $\bar{q}^o = (3/20, 33/20)$, resulting in $w_1^{RS} = 28/5$, $w_2^{RS} = 23/5$, $\pi_{r_1}(\bar{q}^o | w_1^{RS}) = 9/400$, $\pi_{r_2}(\bar{q}^o | w_2^{RS}) = 1089/400$ and $\Pi(\bar{q}^o) = 123/40$. Now the revenue-sharing contracts $\{\phi_i, \phi_i w_i^{RS}\}$ satisfy the participation constraints if

$$\begin{cases} 9\phi_1/400 \geq 9/64, \\ 1089\phi_2/400 \geq 9/16, \text{ and} \\ 9\phi_1/400 + 1089\phi_2/400 \leq 123/40 - 57/32. \end{cases}$$

Notice that the lower bound of ϕ_1 equals $25/4$, which is larger than 1. This contradicts $0 \leq \phi_1 \leq 1$. \square

4 Discussion

We have shown that outside opportunity profits do have effect on the implementation of revenue-sharing contracts in distribution chains. In case of a distribution chain with a single retailer, the outside opportunity profits reduce the range of revenue-sharing contracts upon which the supplier and the retailer may agree. For a distribution chain with multiple retailers the same result holds, but now the reduction may result in an empty range of contracts. That is, the outside opportunity profits may prevent revenue-sharing contracts from being implemented.

These results show that the effect of outside opportunity profits on the implementation of revenue-sharing contracts may not be neglected. It would be interesting to investigate whether the outside opportunity profits have similar effects on the implementations of other contracts.

References

- [1] Cachon, G.P., Lariviere, M.A. (2002) Supply chain coordination with revenue-sharing contracts: strengths and limitations. *Working paper*, The Wharton School, University of Pennsylvania.
- [2] Dana, J., Spier, K. (2001) Revenue sharing and vertical control in the video rental industry. *Journal of Industrial Economics*, Vol. 49, pp. 223–245.
- [3] Giannoccaro, I., Pontrandolfo, P. (2004) Supply chain coordination by revenue sharing contracts. *International Journal of Production Economics*, Vol. 89, pp. 131–139.
- [4] Mortimer, J.H. (2004) Vertical contracts in the video rental industry. *Working paper*, Economics Department, Harvard University.
- [5] Pasternack, B.A. (2002) Using revenue sharing to achieve channel coordination for a newsboy type inventory model. In: Geunes, J., Pardalos, P.M., Romeijn, H.E. *Supply chain management: models, applications, and research directions*, Kluwer Academic Publishers, Dordrecht, chapter 6.
- [6] Tirole, J. (1988) *The theory of industrial organization*. MIT Press, Cambridge, MA.
- [7] Wang, Y., Jiang, L., Shen, Z-J. (2004) Channel performance under consignment contract with revenue sharing. *Management Science*, Vol. 50, pp. 34–47.