Assignment classes 8 and 9 of MQSN (LNMB, 2016): Fluid queues

Note that this assignment has changed slightly w.r.t. the earlier published version: see questions (8) and (9) where no eigenvalues and -vectors need to be determined. NB. assignments for classes 6 and 7 remain as published earlier.

Deadline for this and all other assignments of classes 6–9: **see website.**
Handing in is by email, per group of two students (same groups as for part Boucherie).
Please attach pdf-files only; scans of handwritten solutions are fine.
Motivate all your answers.

Main references (for this assignment; see also slides):

V. Kulkarni. Fluid models for single buffer systems. In J.H. Dshalalow, editor, Frontiers in queueing. Models and Applications in Science and Engineering, pp. 321–338, Boca Raton, Florida, 1997. CRC Press.

S. Aalto and W.R.W. Scheinhardt (2000), Tandem fluid queues fed by homogeneous on-off sources, *Operations Research Letters* 27, pp. 73–82.

M. Mandjes, D. Mitra and W. Scheinhardt (2003), Models of network access using feedback fluid queues. *Queueing Systems* 44, pp. 365–398.

Steady state distribution

Consider a single fluid on-off source. The on-time is exponential with mean μ^{-1}, the off-time exponential with mean λ^{-1}. While on, traffic is generated at rate r. The source feeds into an infinite buffer with link speed C. To avoid trivialities, assume $r > C$ and that the system is stable.

(1) What is the stability condition? Write down the Kolmogorov forward equations for the queue fed by this single source. What differential equation can be obtained for the joint steady state distribution of the buffer content and the state of the fluid source?

(2) Give the boundary conditions for the infinite-buffer case.

(3) Solve the differential equation. What are the eigenvalues and eigenvectors? What is the probability of exceeding level y?

(4) Give the solution when there is a finite buffer B. What is the loss probability? Is the stability condition still needed in this case?

(P.T.O.)

Performance measures (finite buffer case)

From now on take the mean on-time 1 second and the mean off time 2 seconds. Let $r = 4$ kbit/s and $C = 2$ kbit/s.

(5) How large should the buffer be (in kbits) to ensure a packet loss ratio of less than 10^{-4}?

(6) Give a formula for the mean delay. Compute the value of the mean delay.

(7) Give the throughput and the utilization of the system.

(8) Indicate how the analysis changes when the on-time is Erlang(2) distributed (each of the phases has mean length $(2\mu)^{-1}$). Can you give the boundary conditions?

(9) Indicate how the analysis changes when two exponential on-off sources (as described above) feed into the buffer, rather than just one. Let $C = 3$. (No need for computations)

Feedback

Consider the original finite buffer model fed by a single on-off source as in (4), but now assume that the mean (remaining) on-time is changed to 1/10 second as long as the buffer is full, due to some feedback mechanism; the distribution is still exponential.

(10) Find the packet loss ratio for the buffer size found in (5).

Network (infinite buffers)

Consider the original infinite buffer of (1)-(3) with input as before output capacity 2, and suppose the outflowing fluid is fed into another fluid buffer ('buffer 2') which has output capacity 5/3. The output of this buffer 2 is deterministically split in two equal flows, which are being fed into separate buffers (buffers 3 and 4, respectively), each of which has an output capacity of 3/4.

(11) Give the (steady-state) mean buffer contents and the utilizations for buffers 1–4.

(12) Determine the correlation coefficient between the buffer contents of buffers 1 and 2.

(13) Can you also determine the correlation coefficient between the buffer contents of buffers 2 and 3?

(14) What about the correlation coefficient between the buffer contents of buffers 3 and 4?