Mutual control structures

Hans Peters, Maastricht University

For a player set $N = \{1, \ldots, n\}$, a *mutual control structure* is a map $C : 2^N \to 2^N$ which is monotonic, i.e., $S \subseteq T \Rightarrow C(S) \subseteq C(T)$ for all $S, T \in 2^N$, and satisfies $C(\emptyset) = \emptyset$. If $T = C(S)$, then this is interpreted as every player in T being controlled by the coalition S.

An important inspiration for the concept of a mutual control structure derives from situations in which firms and investors own shares of each other. An important question in such situations is how the implicit power structure looks like. If firm 1 owns shares of firm 2 and firm 2 owns shares of firm 3, then indirectly firm 1 may have power over firm 3, and so on.

An *invariant* mutual control structure C is one satisfying the following condition: for all $S, T, R \in 2^N$, if $T \subseteq C(S)$ and $R \subseteq C(S \cup T)$, then $R \subseteq C(S)$. Thus, an invariant mutual control structure also incorporates the indirect control relations. We show that any mutual control structure can be made invariant, indeed, by incorporating indirect control relations in a specific way. Alternatively, a mutual control structure can be represented as an n-tuple of simple games, expressing the control relations, and then we show that we reach the same result, namely an invariant mutual control structure, by performing specific substitutions in the simple games.

The second part of the paper is concerned with establishing a power index for mutual control structures. We propose a specific index, and present an axiomatic characterization of this index. In particular, the index reflects the indirect control relations between the players.

(The presentation is based on joint work in progress with Dominik Karos, University of Saarbrücken, Germany.)