Solution concepts for TU games with directed cooperation structures

Anna Khmelnitskaya (St. Petersburg State University, Russia)
coauthored with Dolf Talman and Őzer Selçuk

Abstract:
In classical cooperative game theory it is assumed that any coalition of players may form. However, in many practical situations the collection of feasible coalitions is restricted by some social, economical, hierarchical, communicational, or technical structure. The study of TU games with limited cooperation introduced by means of undirected communication graphs was initiated by Myerson (1977).

In our study we consider directed graph games in which possible cooperation is prescribed by a directed graph (digraph). The approach to the solution for digraph games depends on the interpretation of a directed cooperation structure. If it is assumed that a digraph represents a flow situation when some links may merge while others split into several separate ones, we restrict our consideration to the class of cycle-free digraph games (without directed cycles, undirected cycles are not excluded). We introduce web-type values, in particular the tree value, for cycle-free digraph games axiomatically, each one with respect to a chosen coalition of players that is assumed to be an anti-chain in the digraph and is considered as a management team. We provide the explicit formula representation and simple recursive algorithms to calculate these values and we study their efficiency and stability. We also define the average web value as the average of the web values over all management teams in the digraph. In general these values are not component efficient. As application we consider the water distribution problem of a river with multiple sources, a delta and possibly islands.

Another possible interpretation of a directed cooperation structure is the assumption that a digraph represents the dominance structure or subordination of players such that after each player any of his subordinates may follow as long as this does not hurt the total subordination among the players prescribed by the digraph. In this case we define a covering tree for a digraph as a tree that keeps the subordination of players prescribed by the digraph but in general is not a spanning tree. We introduce the average covering tree value as the average of the tree values over all covering trees in the digraph. The average covering tree value appears to be component efficient. Under the same assumption concerning the digraph we consider also another solution concept, the so-called Shapley value for digraph games, defined as the average of the marginal contribution vectors corresponding to those permutation of players that do not violate the dominance of players induced by the digraph.