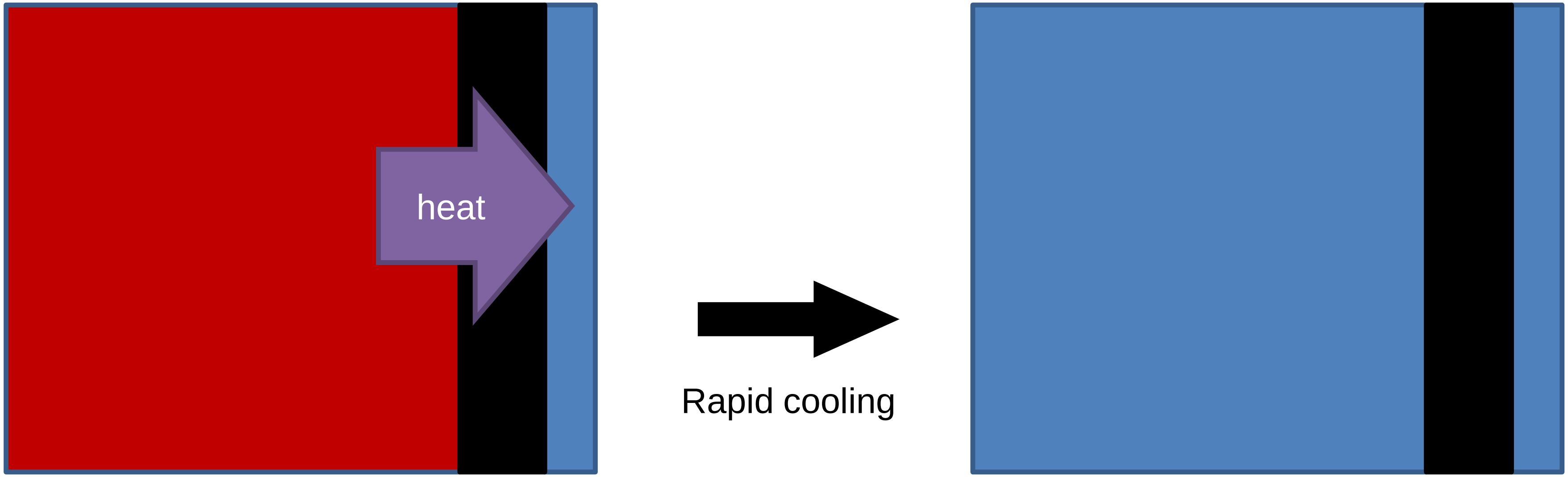
Thermal conductance of interleaving fins

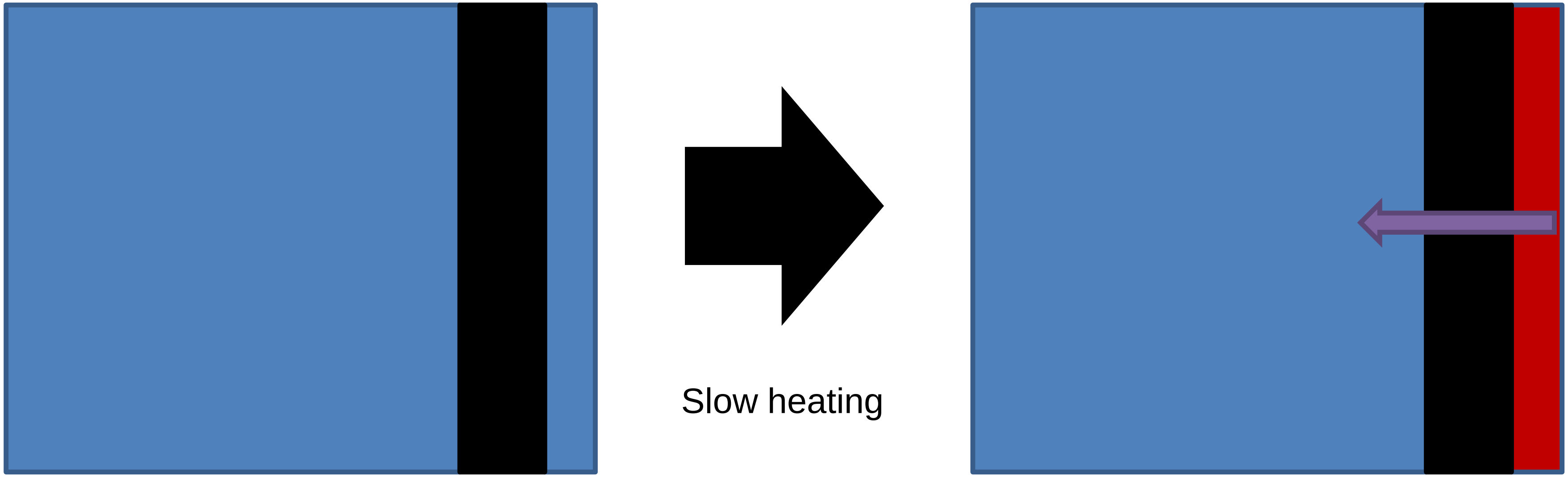
Michiel AJ van Limbeek,
Srinivas Vanapalli

UNIVERSITY OF TWENTE.

Heat exchange



Heat switch (cryo battery)



flux?

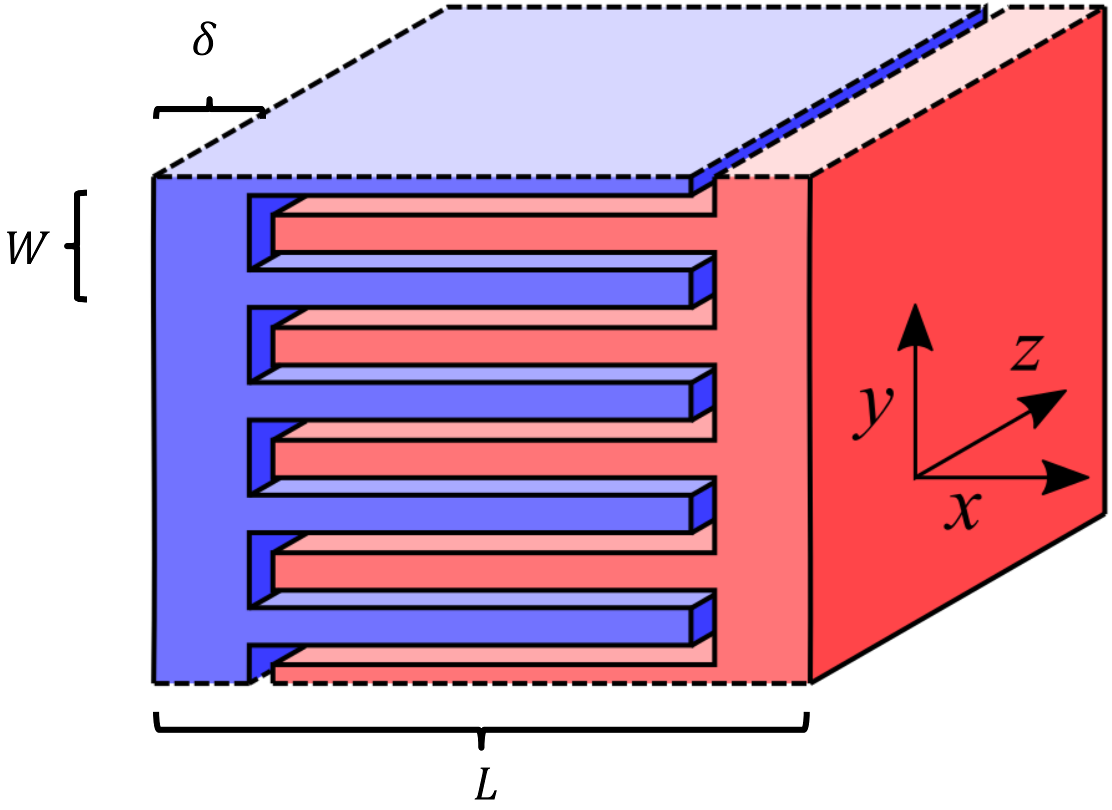
Problems

How to ‘switch’?

Solution: Interleaving fins

No fin: $q'' = \frac{k_g\left(T_h - T_c\right)}{D}$

Fins: $q'' = -\frac{k_g(T_h - T_c)}{D}\frac{(W - L - 2\delta - D)}{W}$



Working principle Heat switch

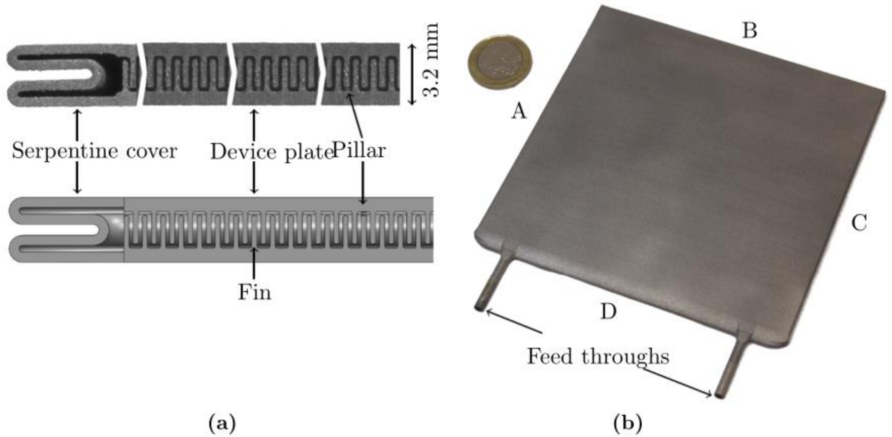
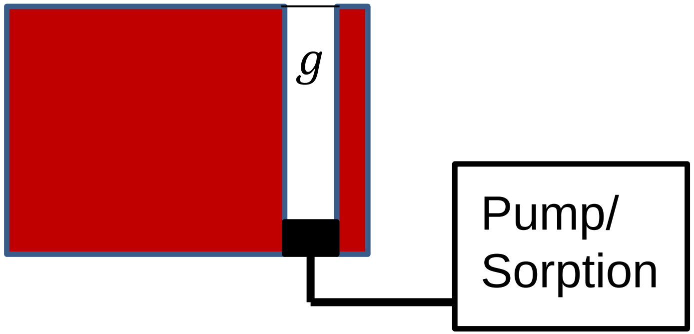
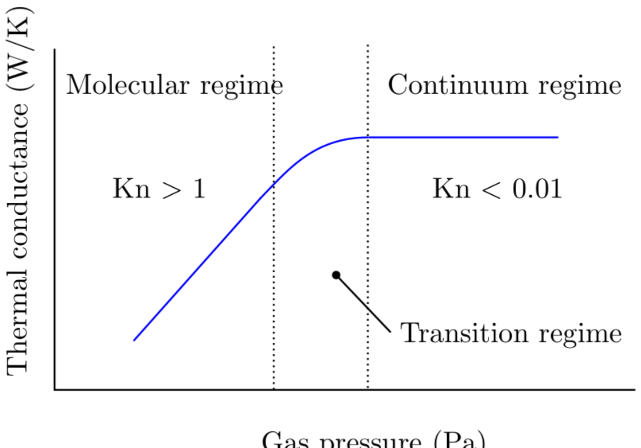
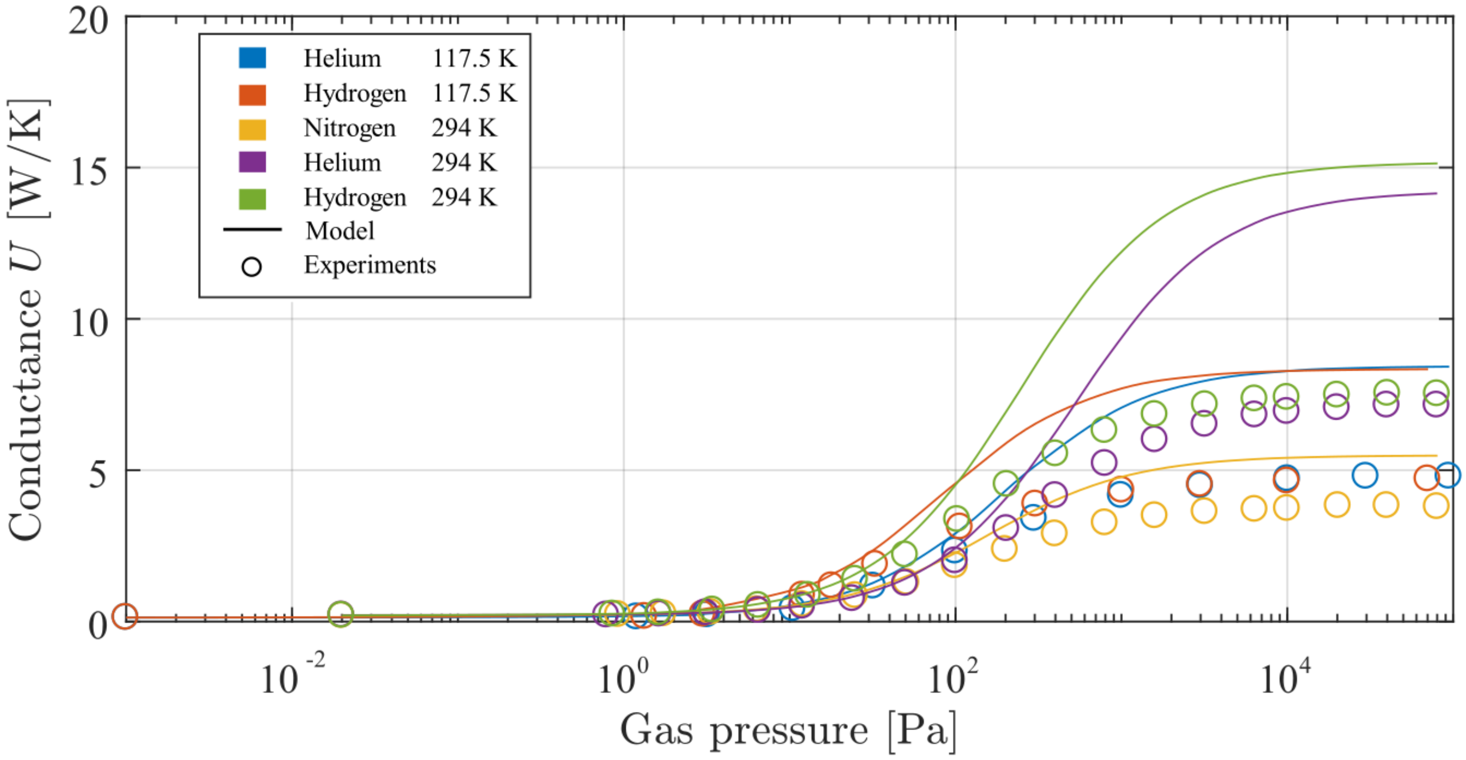


Figure 2.1: The flat-panel gas-gap heatswitch. (a) A cross-section of the heat switch. (b) A photograph of the heat switch. Image by M.A.R. Krielaart[2]



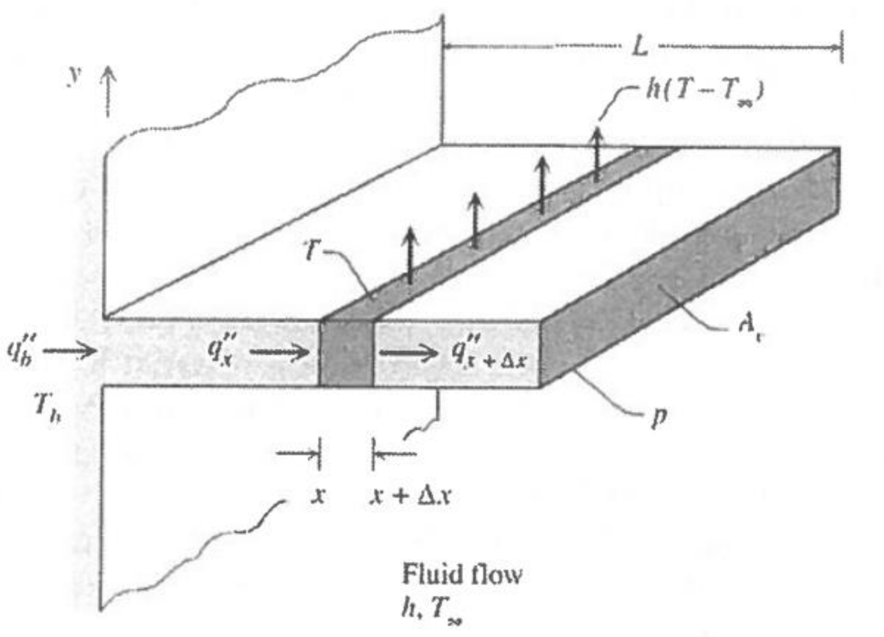


Testing…



Helium 117.5 K
Hydrogen 117.5 K
Nitrogen 294 K
Helium 294 K
Hydrogen 294 K
Model
Experiments
20
15
10
5
0
Conductance U [W/K]
10^{-2}
10^{0}
10^{2}
10^{4}
Gas pressure [Pa]

Fin in infinite medium: Bi



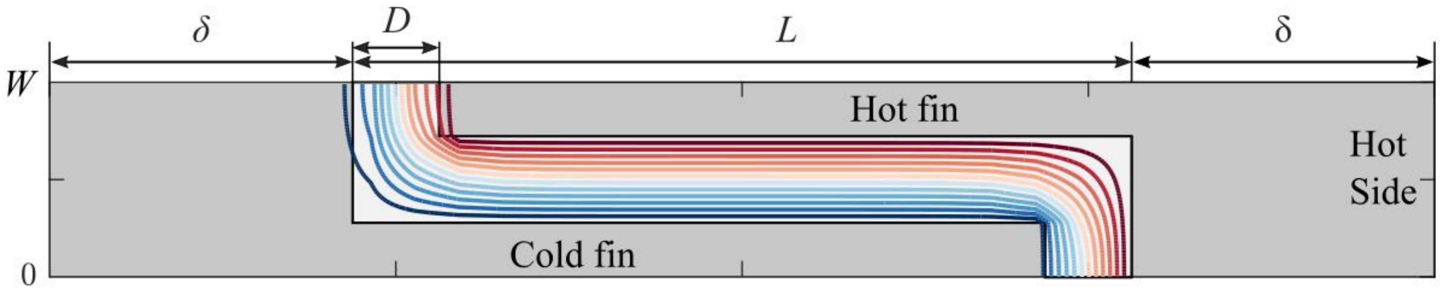
$$T \sim \exp(-Bi^{1/2}x)$$

$$Bi = \frac{hL}{k_s}$$

A. Bejan, **Heat Transfer** – *J.Wiley&Sons* 1993

Cooling effect

$$\frac{k_g}{k_s} = 10^{-5}$$



$$\Theta_0 = -1$$

$$T(0,0) - T(0,W) = \Theta_0(T_h - T_c)$$

$$\Theta_0 = -0.03$$

Model

- Rescale $\hat{x} = \bar{x}\hat{L}$ and $\hat{y} = \bar{y}(\widehat{W} - \widehat{D})$

$$\hat{k}_s \left(\partial_{\hat{x}\hat{x}}\hat{T} + \partial_{\hat{y}\hat{y}}\hat{T}\right) = \hat{k}_s \left(\partial_{\bar{x}\bar{x}}\hat{T} + \frac{\hat{L}^2}{(\hat{W} - \hat{D})^2}\partial_{\bar{y}\bar{y}}\hat{T}\right) = 0$$

- For large $\frac{\hat{L}^2}{(\widehat{W} - \widehat{D})^2}$:

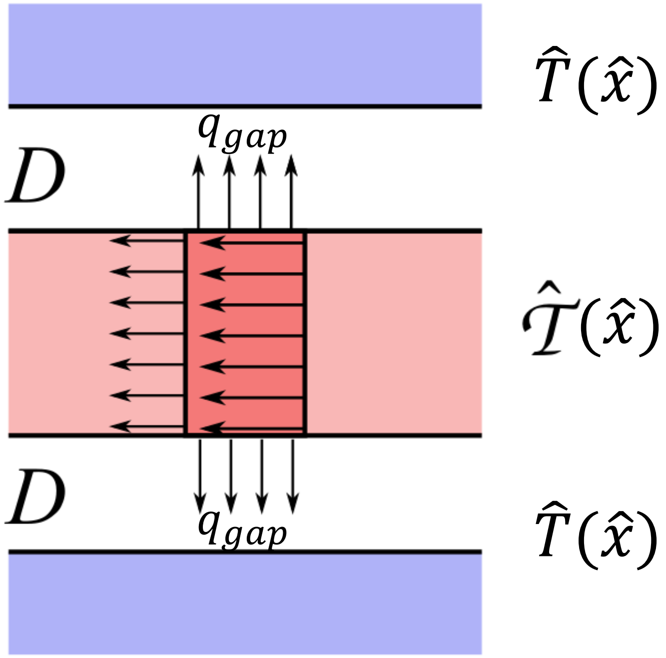
$$\hat{T}(\hat{x}, \hat{y}) \approx T(\hat{x})$$ → 1D-model

1D-model

Two profiles:
$T(x)$ for cold fin and $\mathcal{T}(x)$in the hot fin

Gap: Fourier's law:

$$q_{gap} = -k_g \nabla T \approx -\frac{k_g\left(T(x) - \mathcal{T}(x)\right)}{D}$$



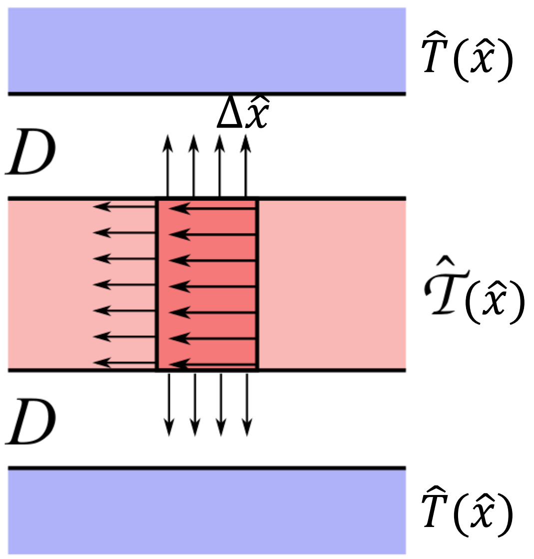
1D-model

$$\int_v \nabla \cdot \vec{j}\, \mathrm{d}V = \oint \vec{j} \cdot \vec{n}\, \mathrm{d}\ell = 0$$

$$\hat{k}_s(\hat{W} - \hat{D}) \left(\frac{\mathrm{d}\hat{T}(\hat{x} + \Delta\hat{x})}{\mathrm{d}\hat{x}} - \frac{\mathrm{d}\hat{T}(\hat{x})}{\mathrm{d}\hat{x}} \right) + 2\Delta\hat{x}\hat{q}_{\mathrm{gap}} = 0.$$

$$\hat{k}_s(\hat{W} - \hat{D}) \frac{\mathrm{d}^2\hat{T}}{\mathrm{d}\hat{x}^2} - \frac{2\hat{k}_g}{\hat{D}}(\hat{T} - \hat{\mathcal{T}}) = 0 \quad \text{and}$$

$$\hat{k}_s(\hat{W} - \hat{D}) \frac{\mathrm{d}^2\hat{\mathcal{T}}}{\mathrm{d}\hat{x}^2} - \frac{2\hat{k}_g}{\hat{D}}(\hat{\mathcal{T}} - \hat{T}) = 0 \quad .$$



Solving model: Θ

Rescaled lengths with $\hat{L}$

$$\Theta = T(x) - \mathcal{T}(x)$$

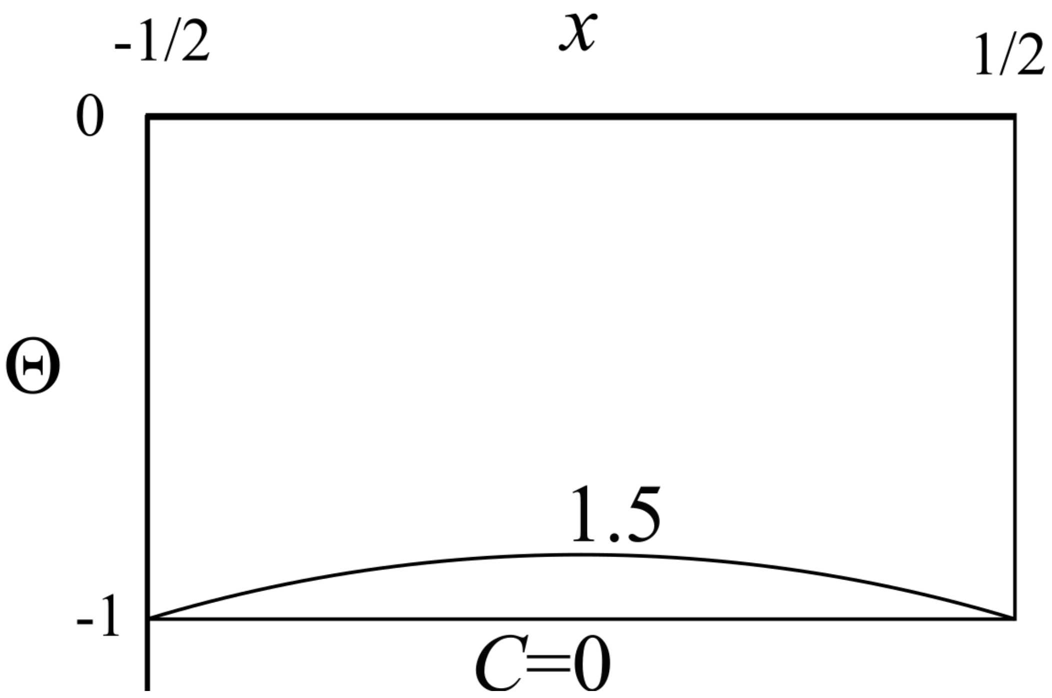
$$\frac{\mathrm{d}^2\Theta}{\mathrm{d}x^2} - C^2\Theta = 0$$

with $C^2 = \frac{4\hat{k}_g\hat{L}^2}{\hat{k}_s\hat{D}(\hat{W}-\hat{D})}$

$$\Theta = \Theta_0 \cosh\left(C^{\frac{1}{2}}x\right)$$

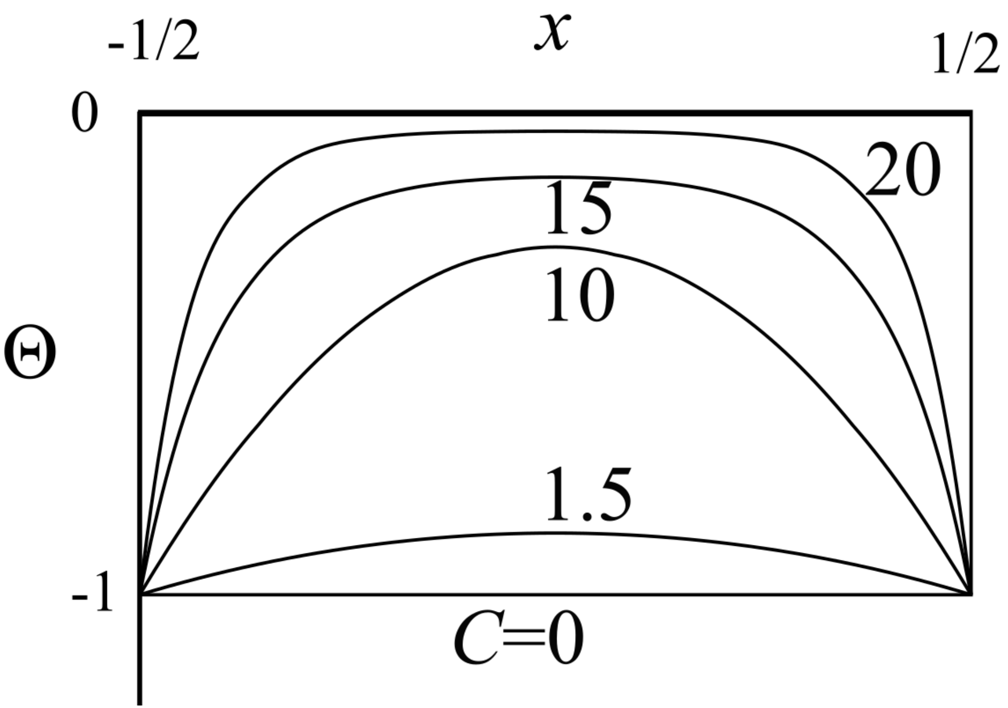
$$T'' - \frac{1}{2}C^2\Theta = 0, \quad \mathcal{T}'' + \frac{1}{2}C^2\Theta = 0,$$

Intermezzo:
no base plate $\delta = 0$



$$\Theta = \Theta_0 \cosh\left(C^{\frac{1}{2}} x\right)$$

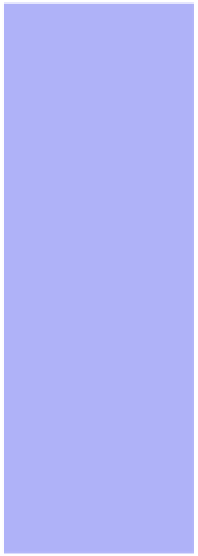
Intermezzo:
no base plate $\delta = 0$



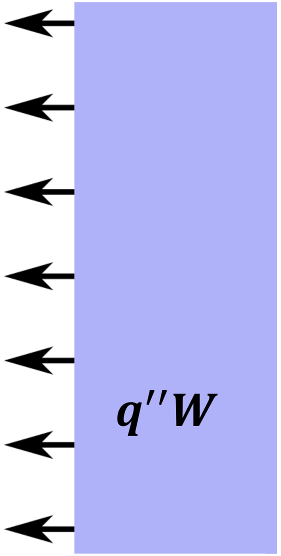
$$\Theta = \Theta_0 \cosh\left(C^{\frac{1}{2}}\, x\right)$$

BC and more…

1) $\mathcal{T}(-1/2) = T(-1/2) = T_{cold}$



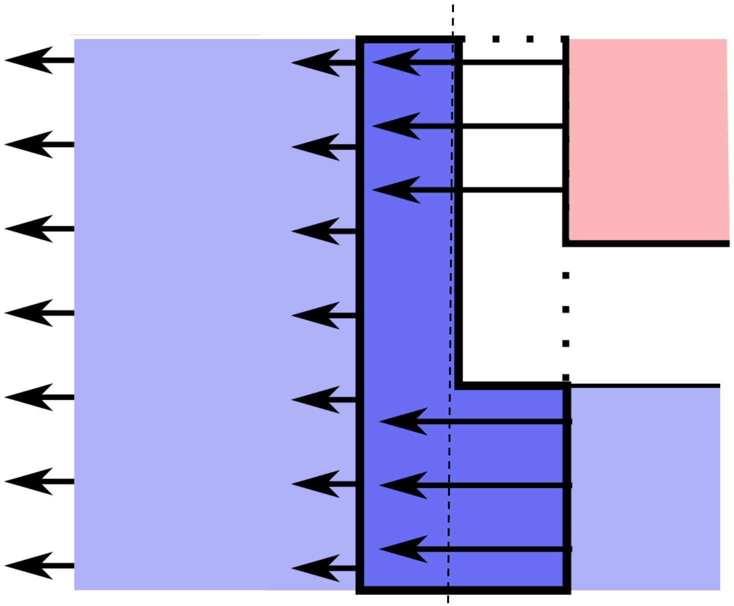
BC and more…



1) $\mathcal{T}(-1/2) = T(-1/2) = T_{cold}$
2) Flux continuity

$$q''W = -k_s W \frac{dT}{dx}$$

BC and more…



1) $\mathcal{T}(-1/2) = T(-1/2) = T_{cold}$

2) Flux continuity

$$q''W = -k_s W \frac{dT}{dx}$$

$$= -k_s(W - D)\frac{dT}{dx} + k_g(W - D)\frac{\Theta}{D}$$

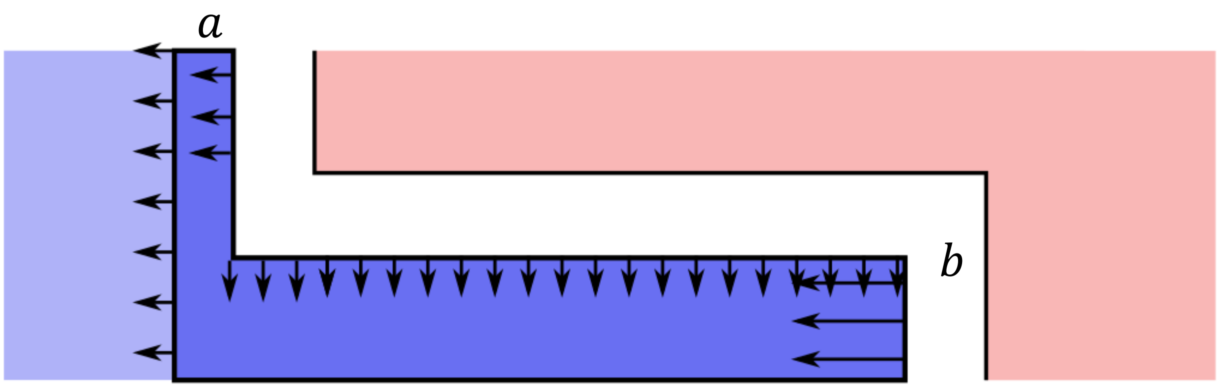
BC and more…

1) $\mathcal{T}(-1/2) = T(-1/2) = T_{cold}$

2) Flux continuity

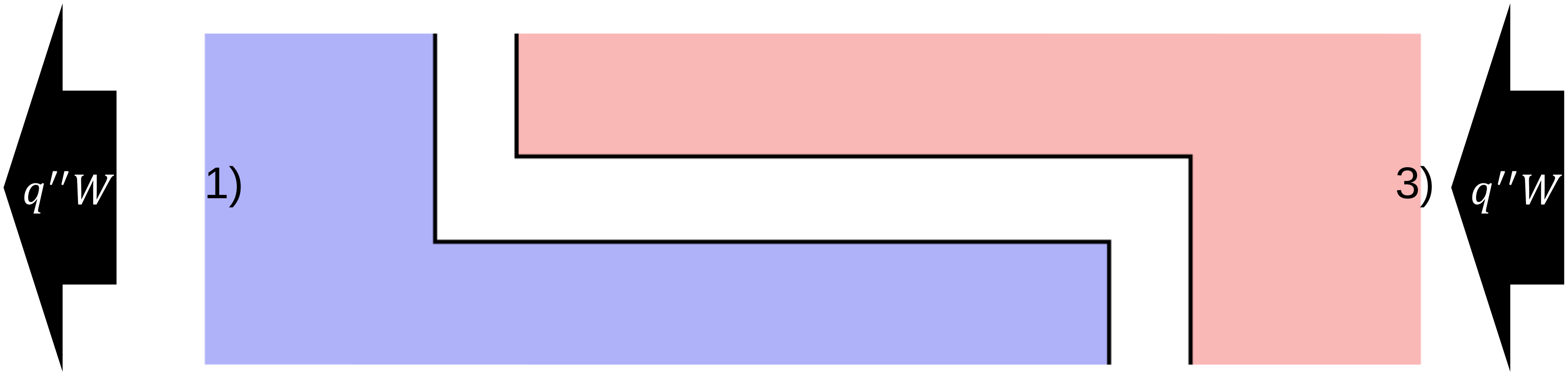
$$q''W = -k_s W \frac{dT}{dx}$$

$$= -k_s(W - D)\frac{dT}{dx}|_a + 2k_g \frac{1}{D}\int_a^b \Theta \, \mathrm{d}x - k_s(W - D)\frac{dT}{dx}|_b$$



BC and more…

1) $\mathcal{T}(-1/2) = T(-1/2) = T_{cold}$
2) Flux continuity: $q''W$
3) $\mathcal{T}(1/2) = T(1/2) = T_{hot}$

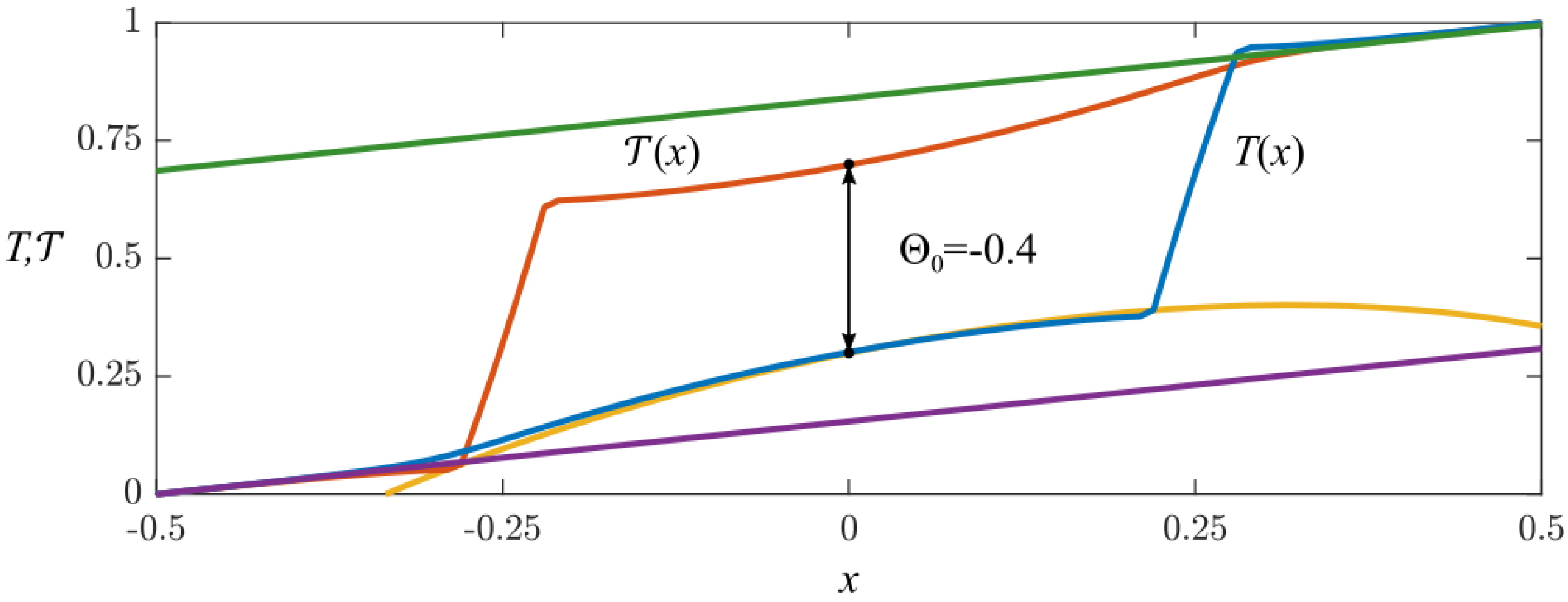


Flux, $\mathcal{J}(x)$ (red) and $I'(x)$ (blue) are extracted

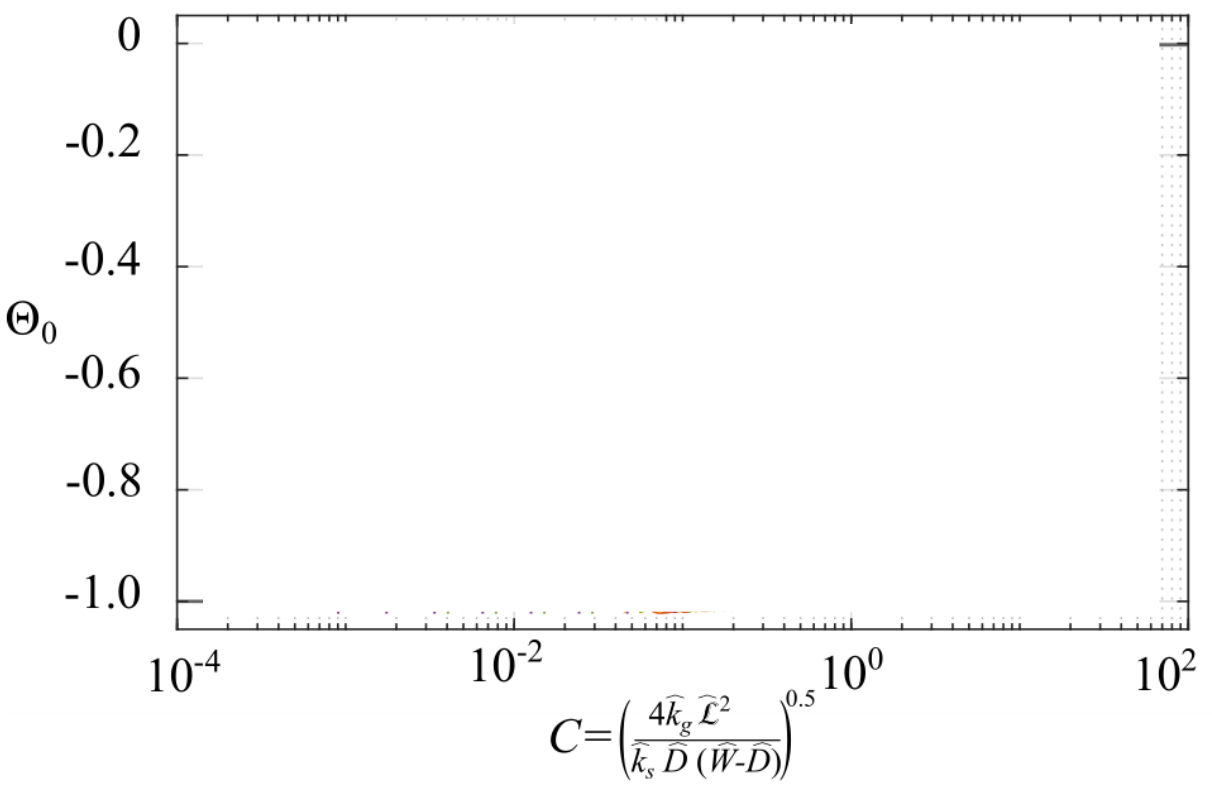
Numerical benchmark

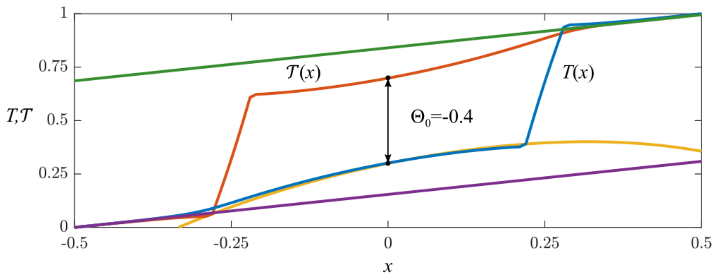
Analytical model is Weak Solution: three pieces

Weak solution 3 pieces

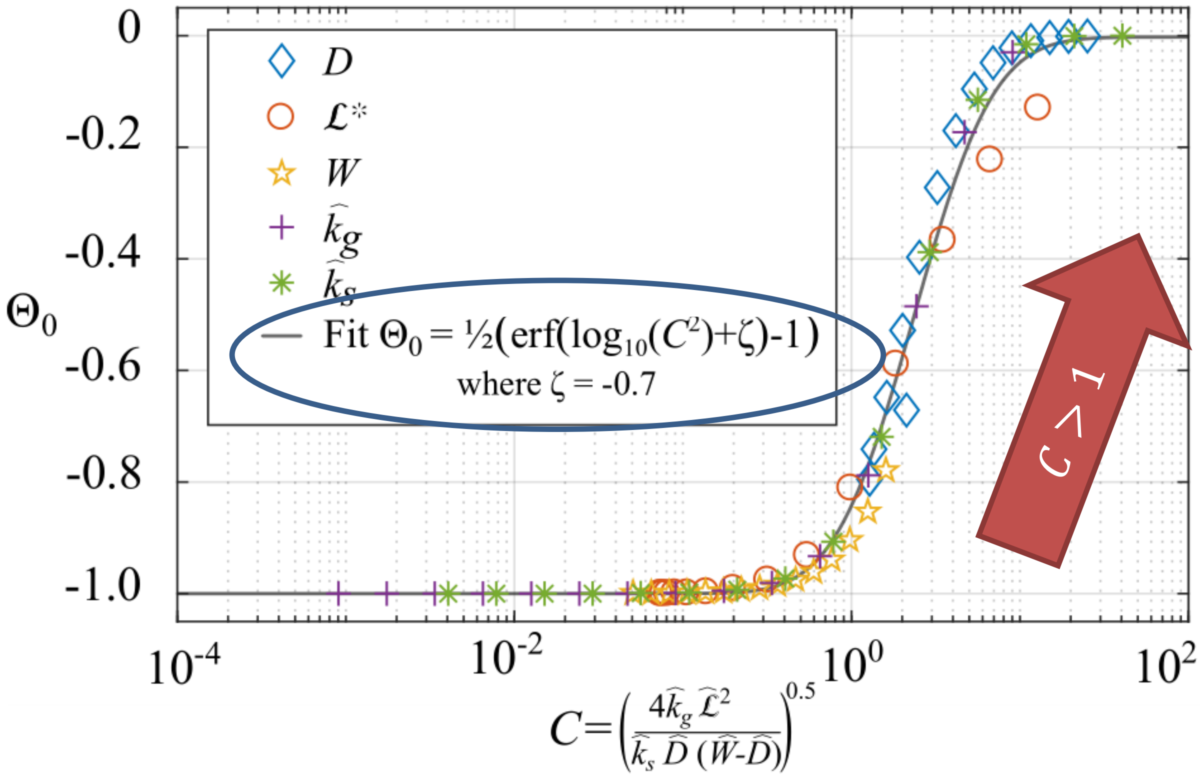


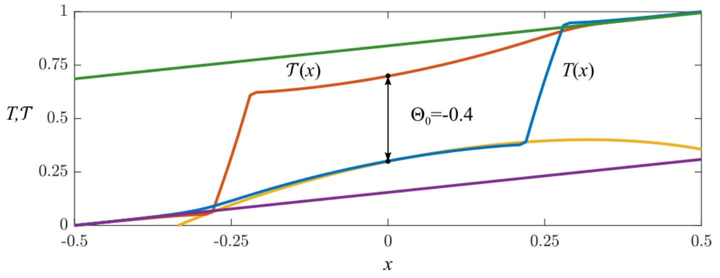
Cooling strength: C and Θ_0





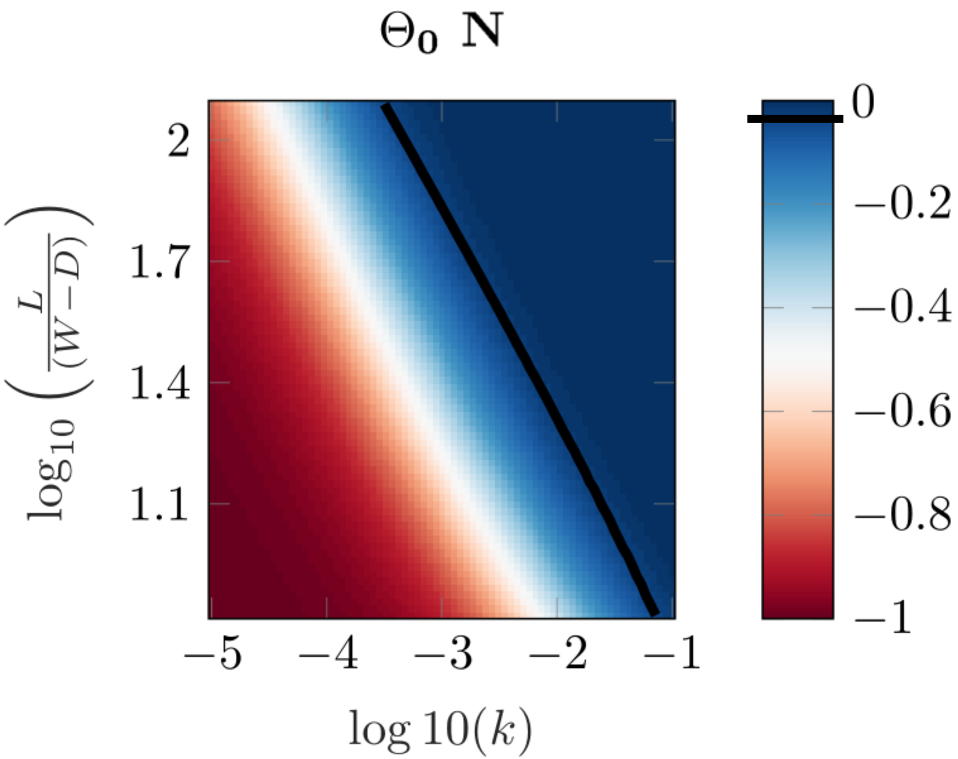
Cooling strength: C and Θ_0

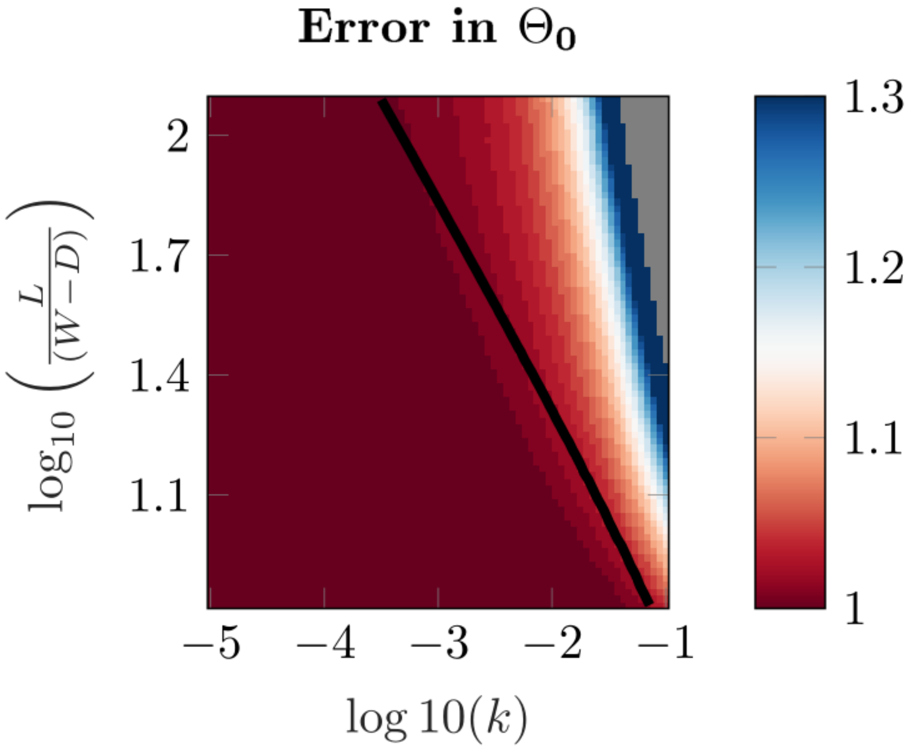




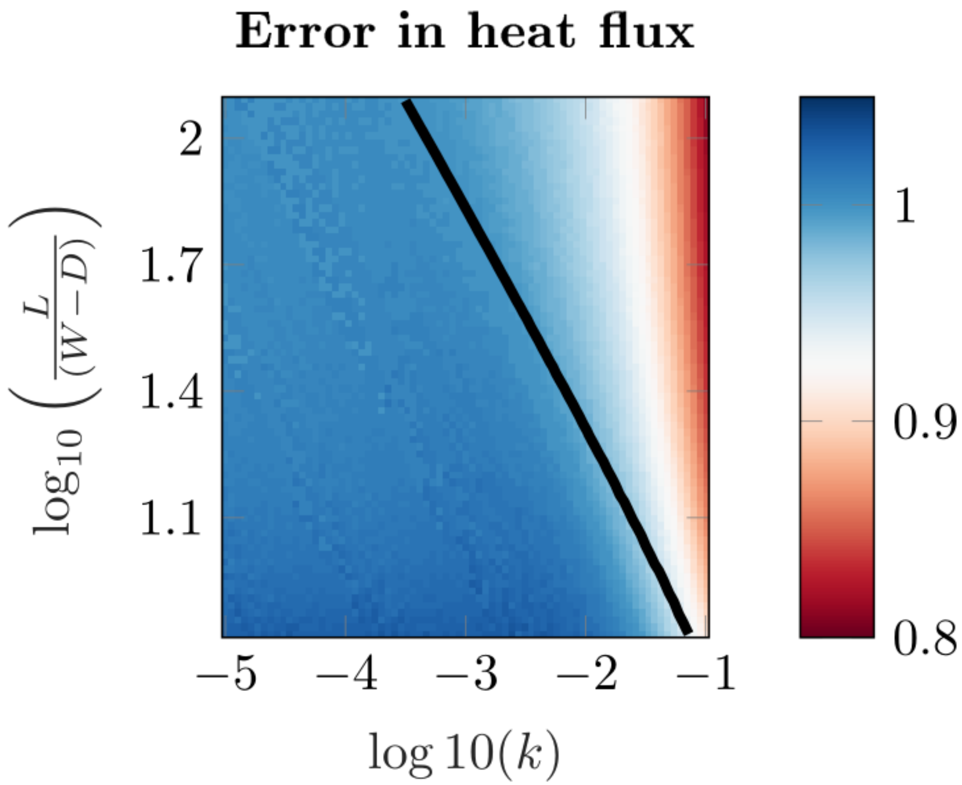
Benchmark against numerics

$k = \hat{k}_g / \hat{k}_s$



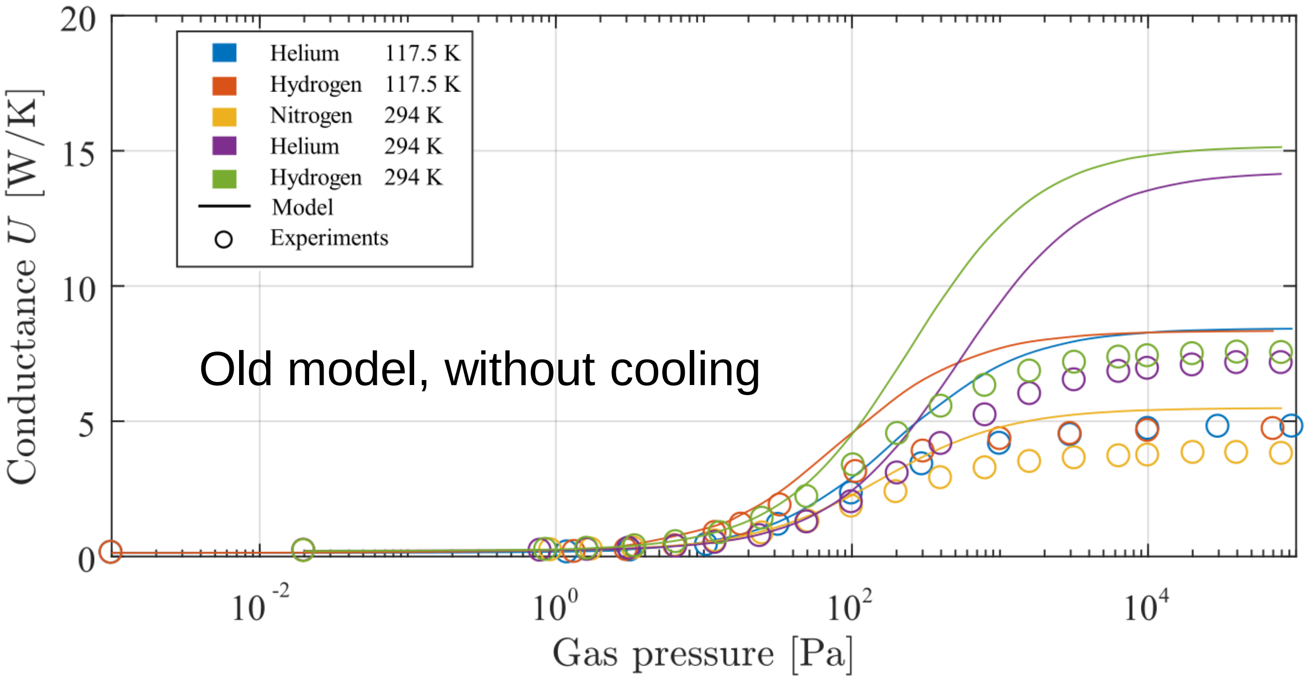


Heat flux

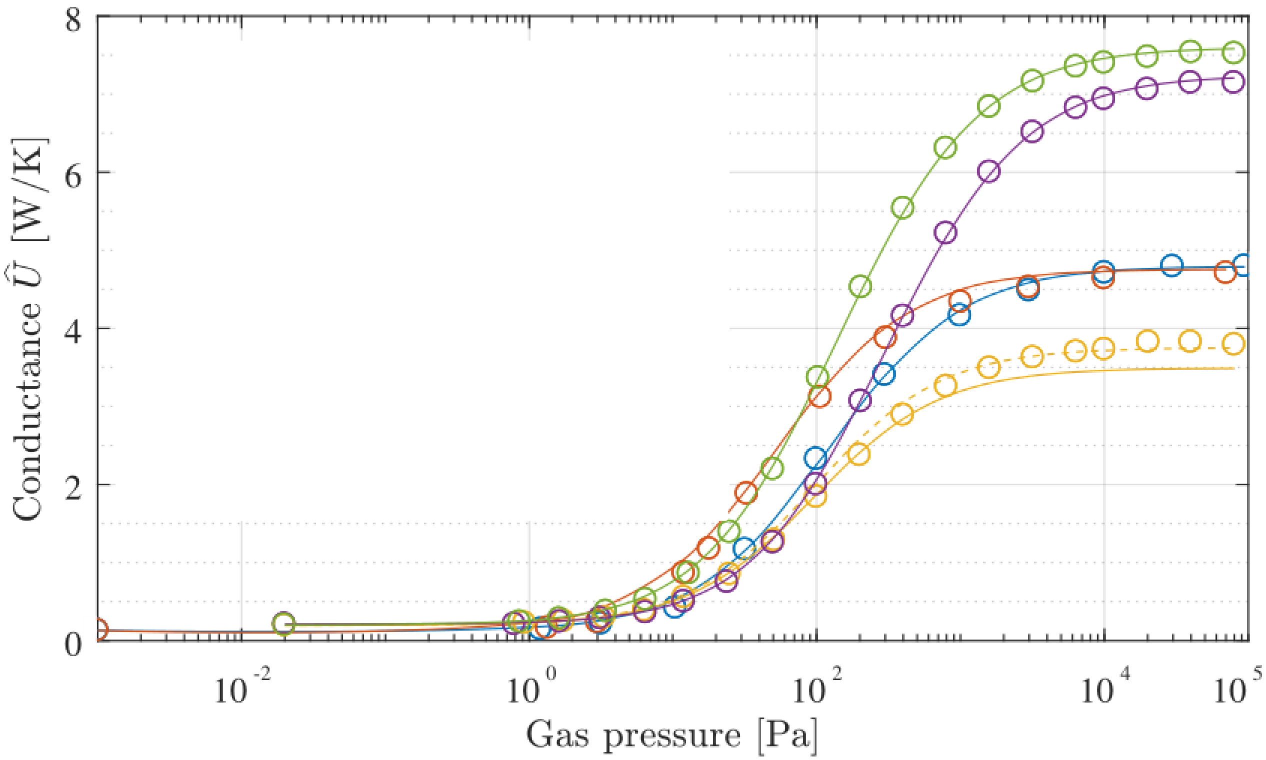

Error in heat flux
2
1.7
1.4
1.1
$\log_{10}\left(\frac{L}{(W-D)}\right)$
−5
−4
−3
−2
−1
$\log 10(k)$
1
0.9
0.8

Analytical prediction for the conductance

Does the model work?



Finally, success!



Summary

Cooling can occur in interleaving heat fins

$$C^2 = \frac{4\hat{k}_g \hat{L}^2}{\hat{k}_s \widehat{D}\left(\widehat{W} - \widehat{D}\right)} > 1$$

Quick-and-dirty fit for Θ_0

Analytic model predicts performance well

Allows for easy optimalization/implementation studies

Limitations

Constant gap spacing

Constant width of fin

Symmetric geometry