

TISCA User day

# A Numerical-Experimental Study on chemo-mechanical degradation of COncrete sewer Pipes (NESCOP)

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prof. dr. ir. A.S.J. Suiker

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# Introduction



# Problem Statement

## Assessment Methods

- Age determination
- CCTV

Need for better measurements or better prediction methods/design rules



Image retrieved from:  
<https://sunburstev.com/drain-line-camera-inspection-cctv-technology/>



Image retrieved from:  
<https://www.wellingtondrainage.co.nz>

# Project Goals

## Chemo-mechanical model

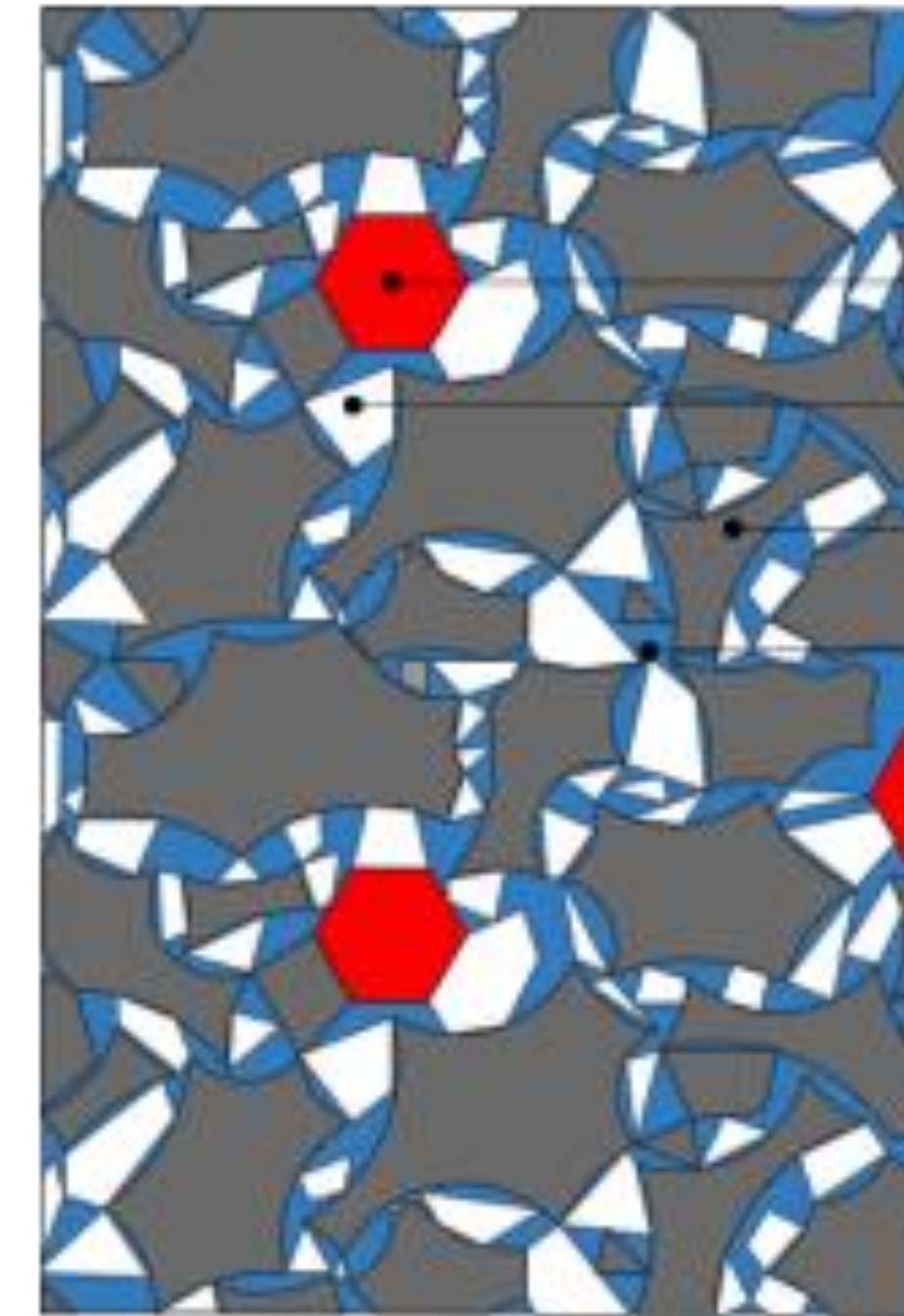
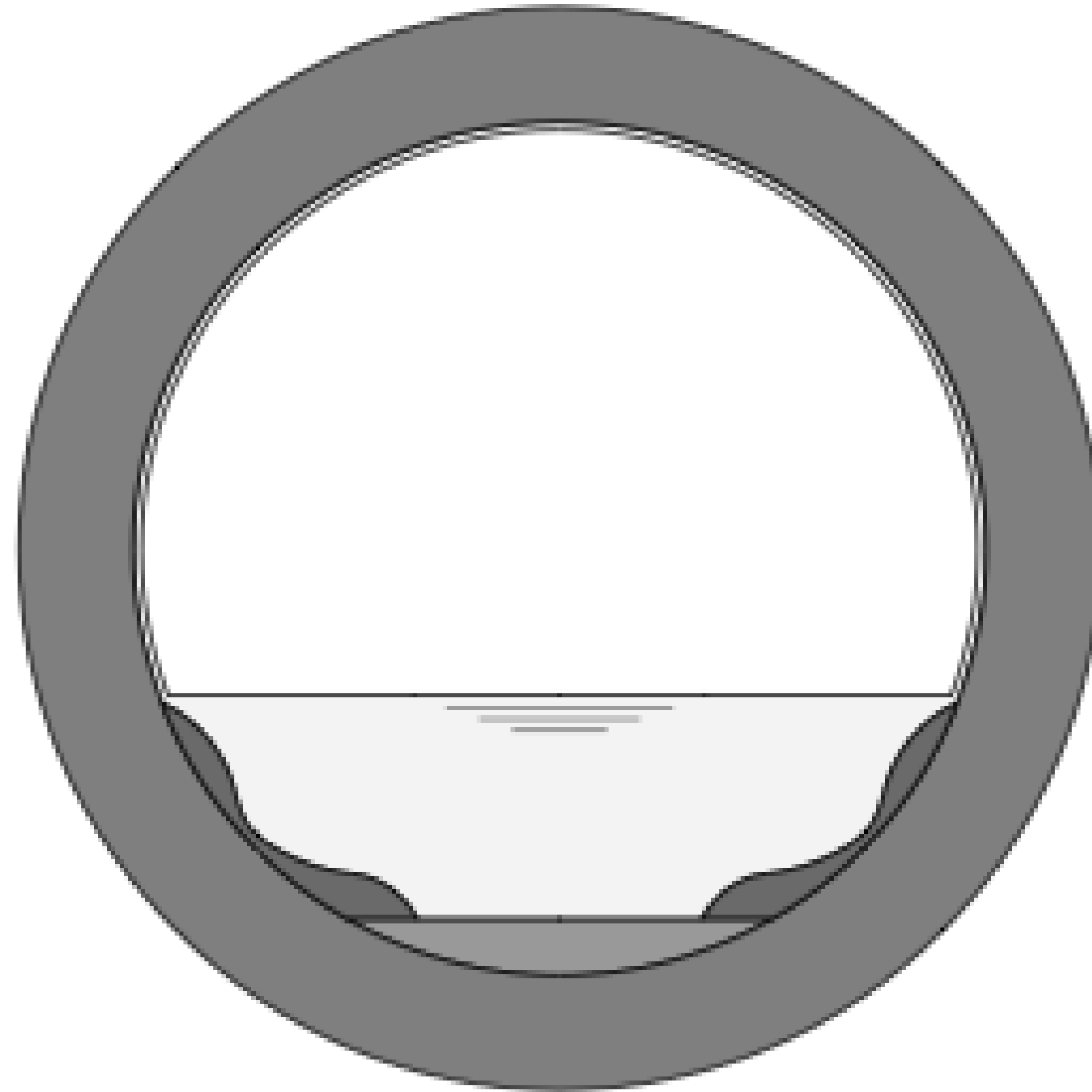
- Prediction of remaining load bearing capacity
- Focus on transferability instead of complexity

## Experiments

- Used to calibrate and validate the chemo-mechanical model



## Biogenic Sulfide Corrosion: process



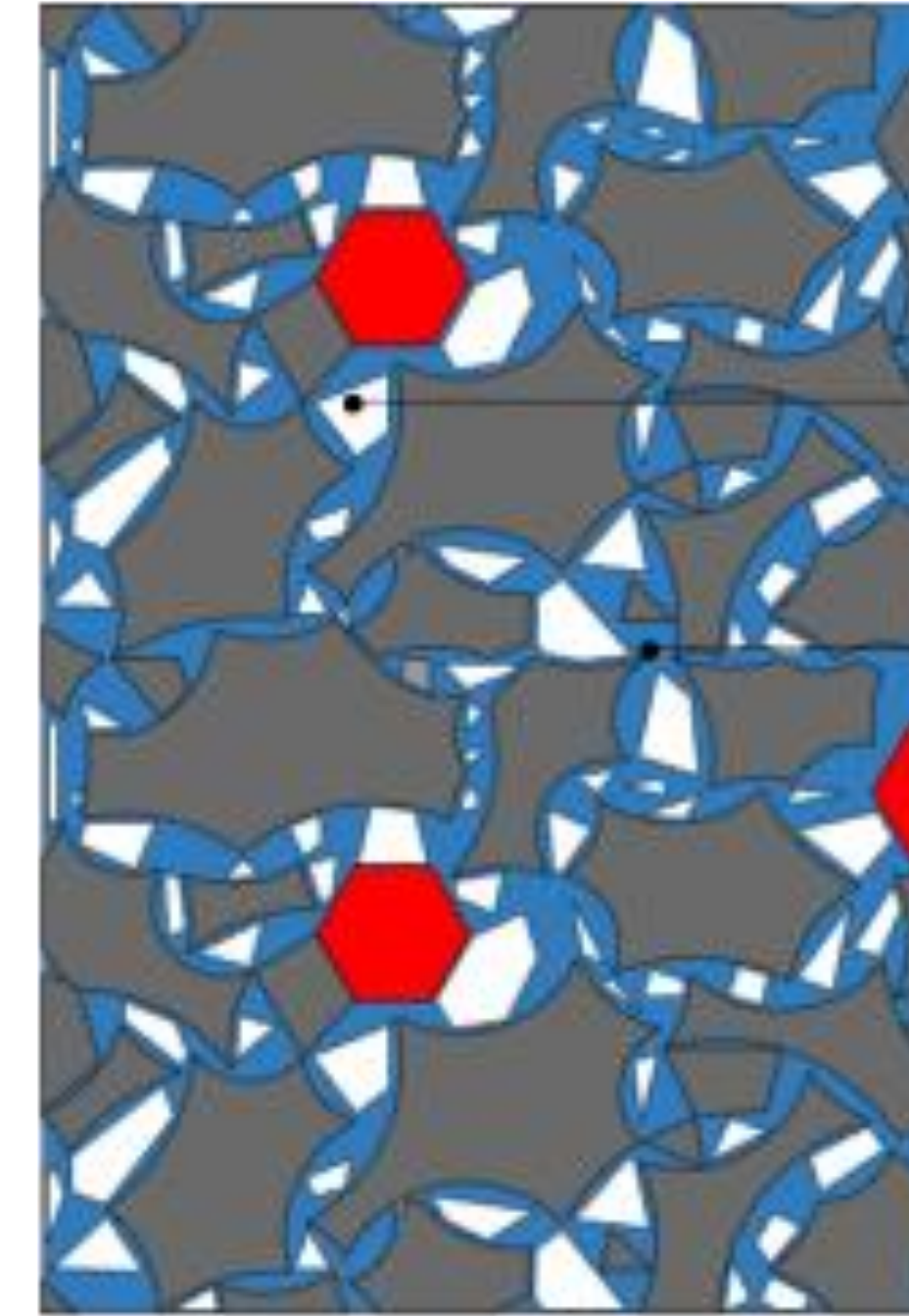
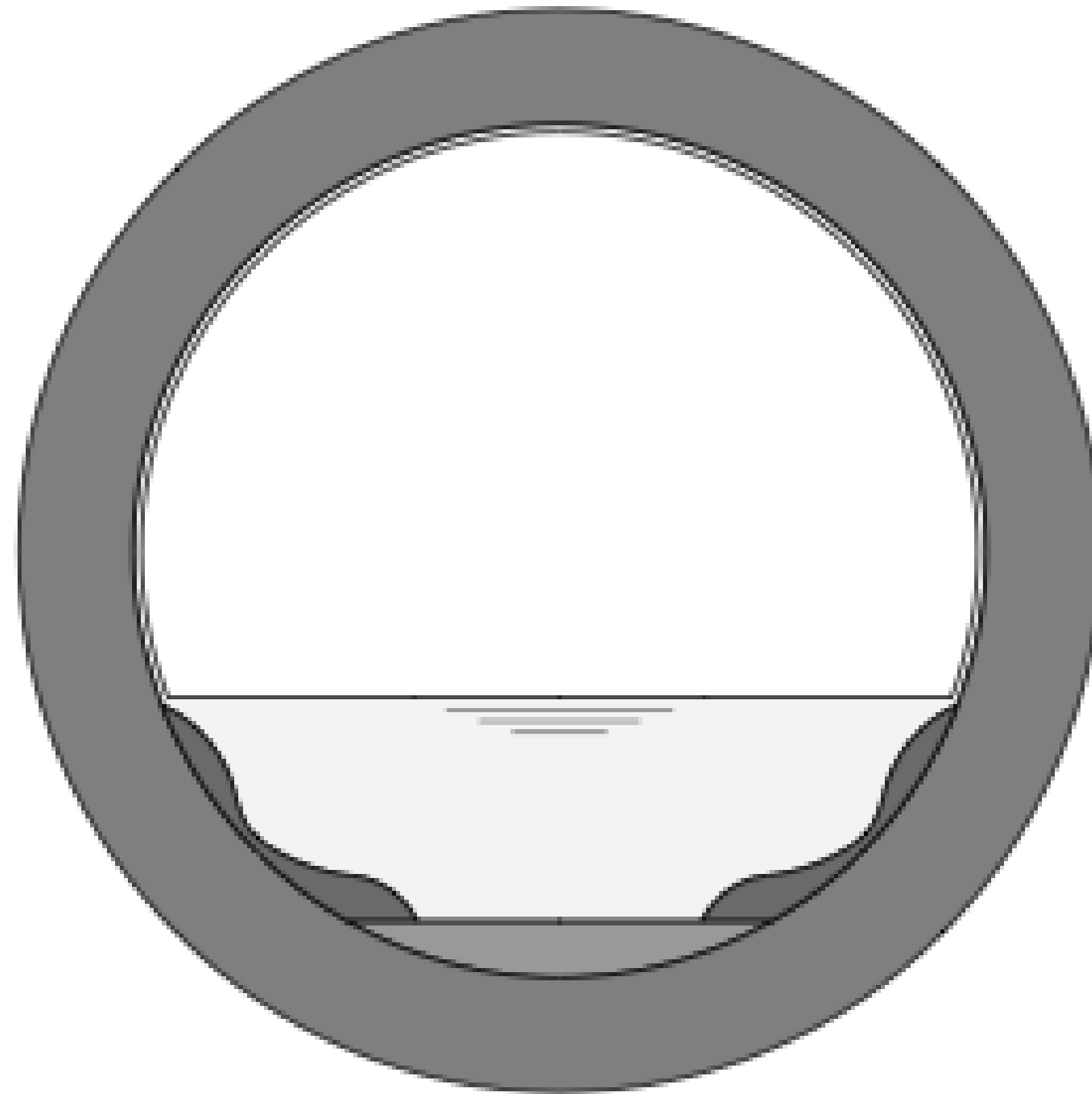
Aluminate crystal

Calcium hydroxide crystal

Calcium-silicate hydrate

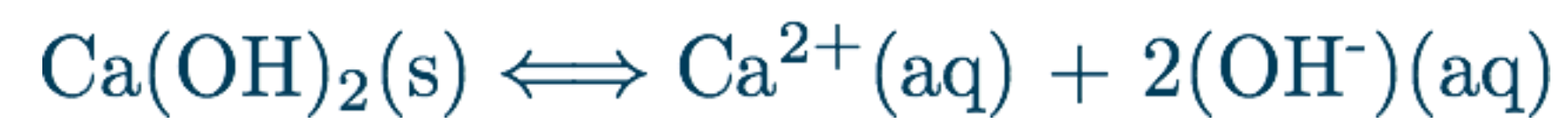
Hydrated pore

## Biogenic Sulfide Corrosion: process



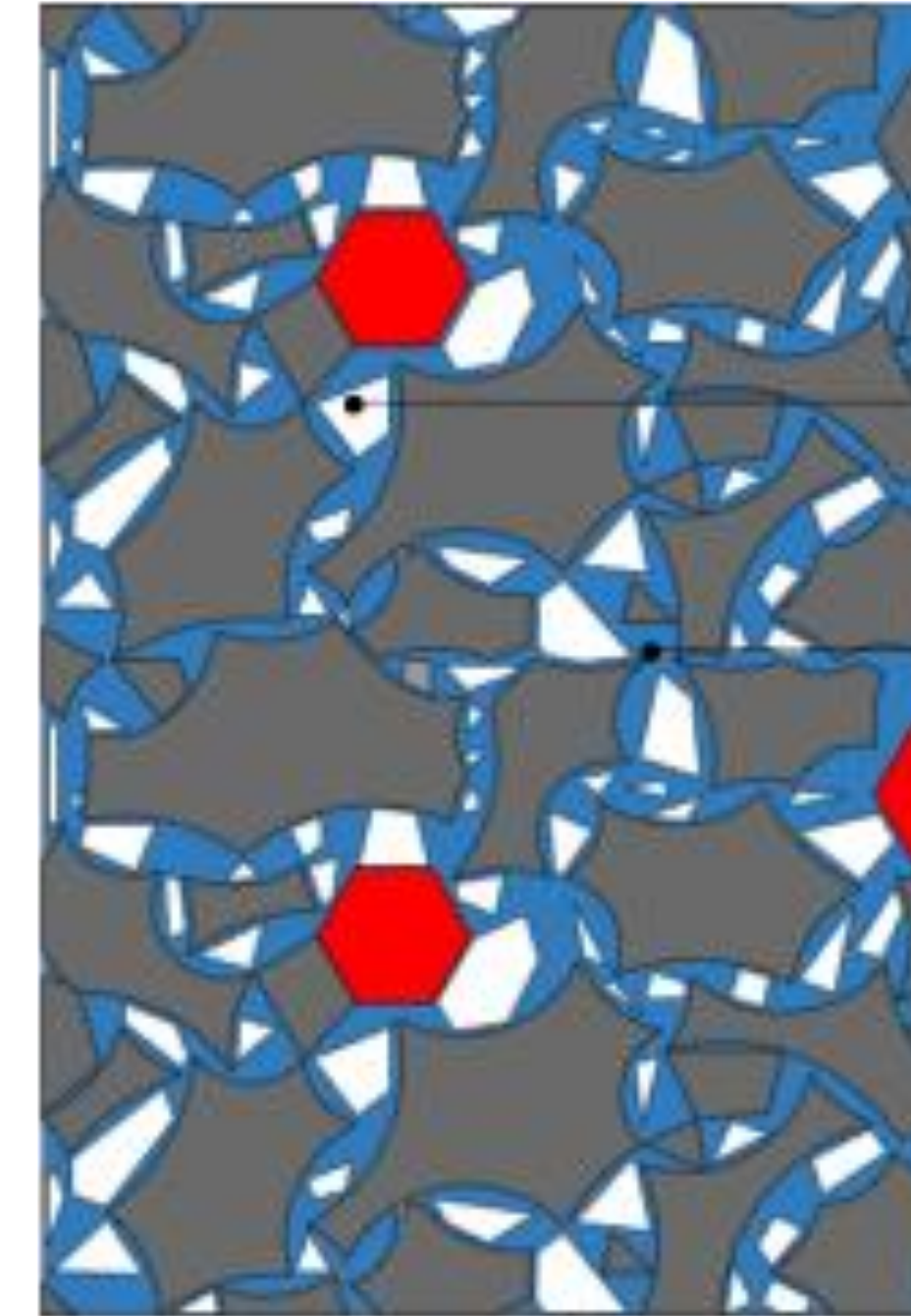
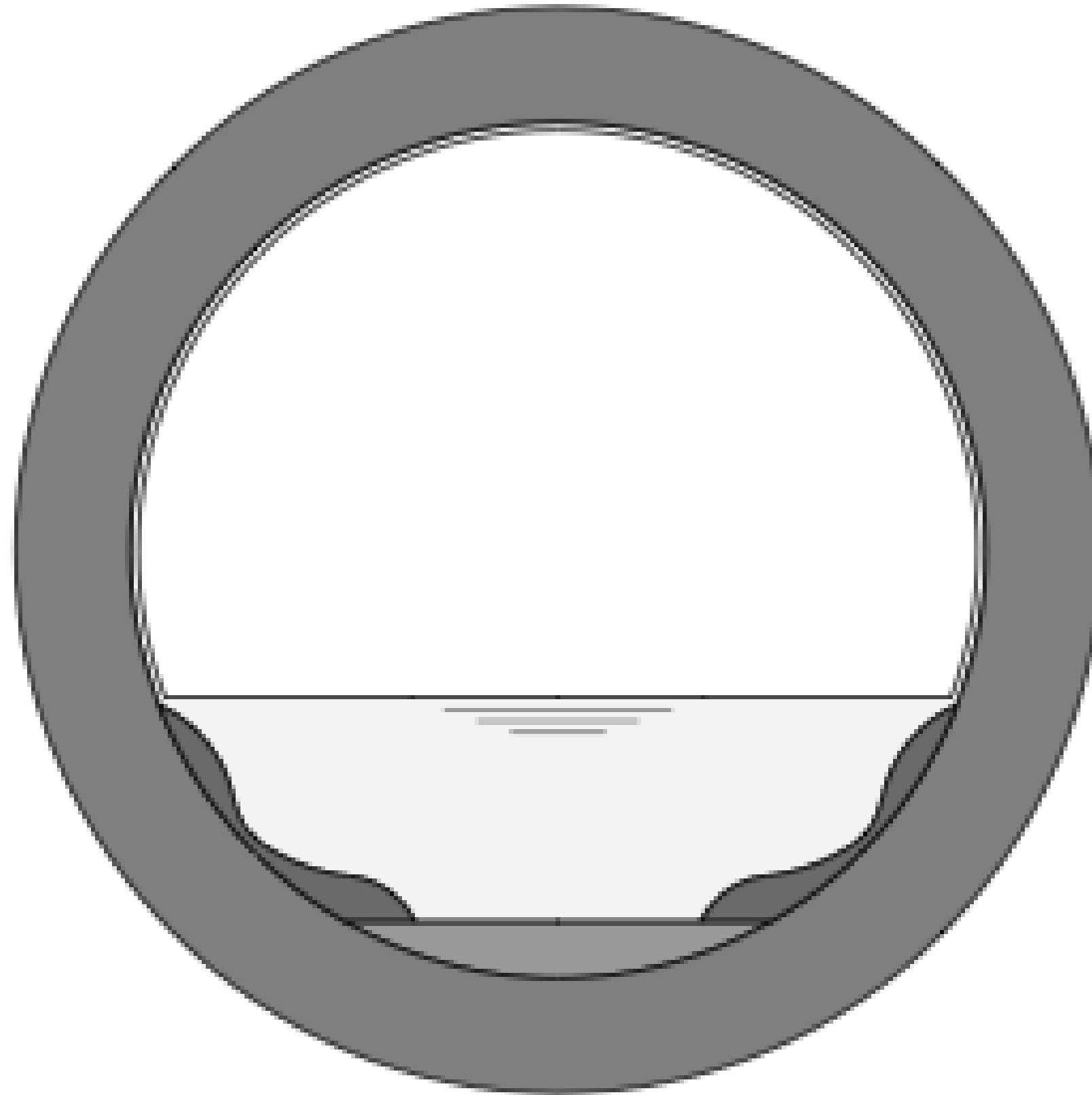
Partly dissolving calcium  
hydroxide crystal

Hydrated pore containing  
calcium hydroxide ions



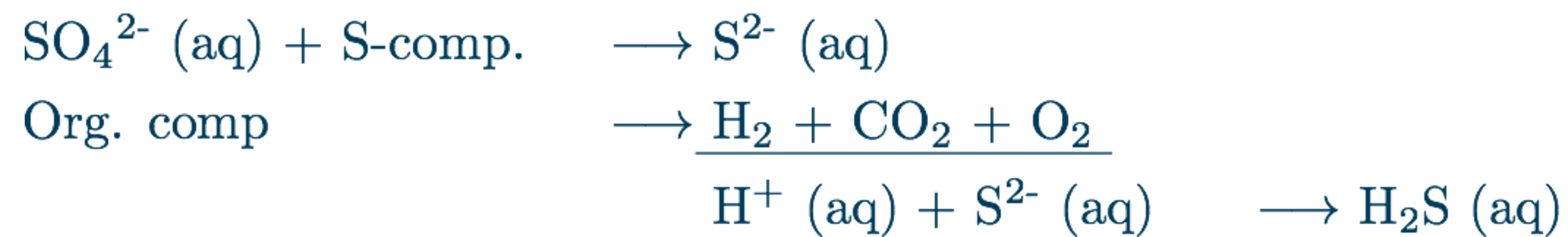


## Biogenic Sulfide Corrosion: process

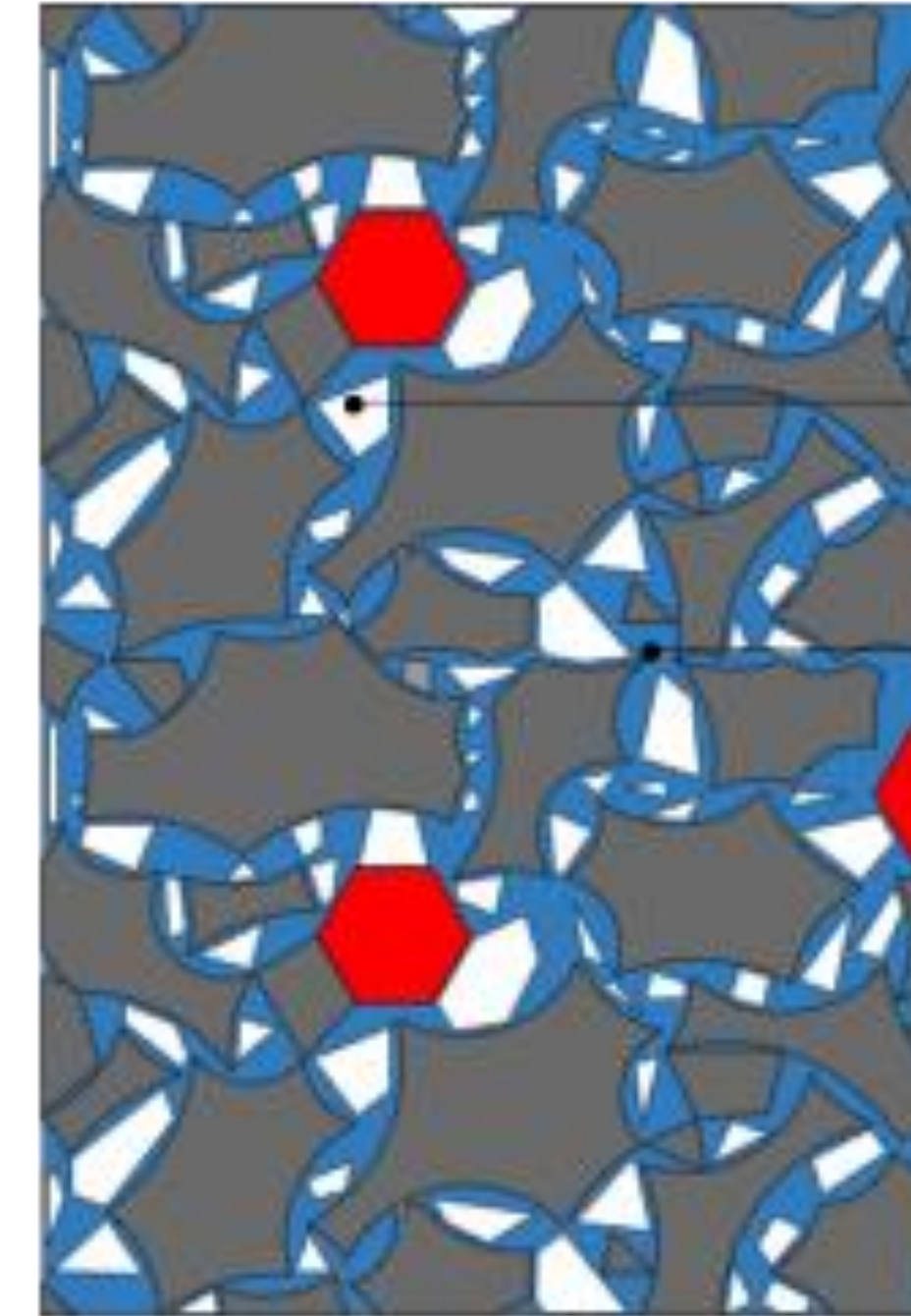
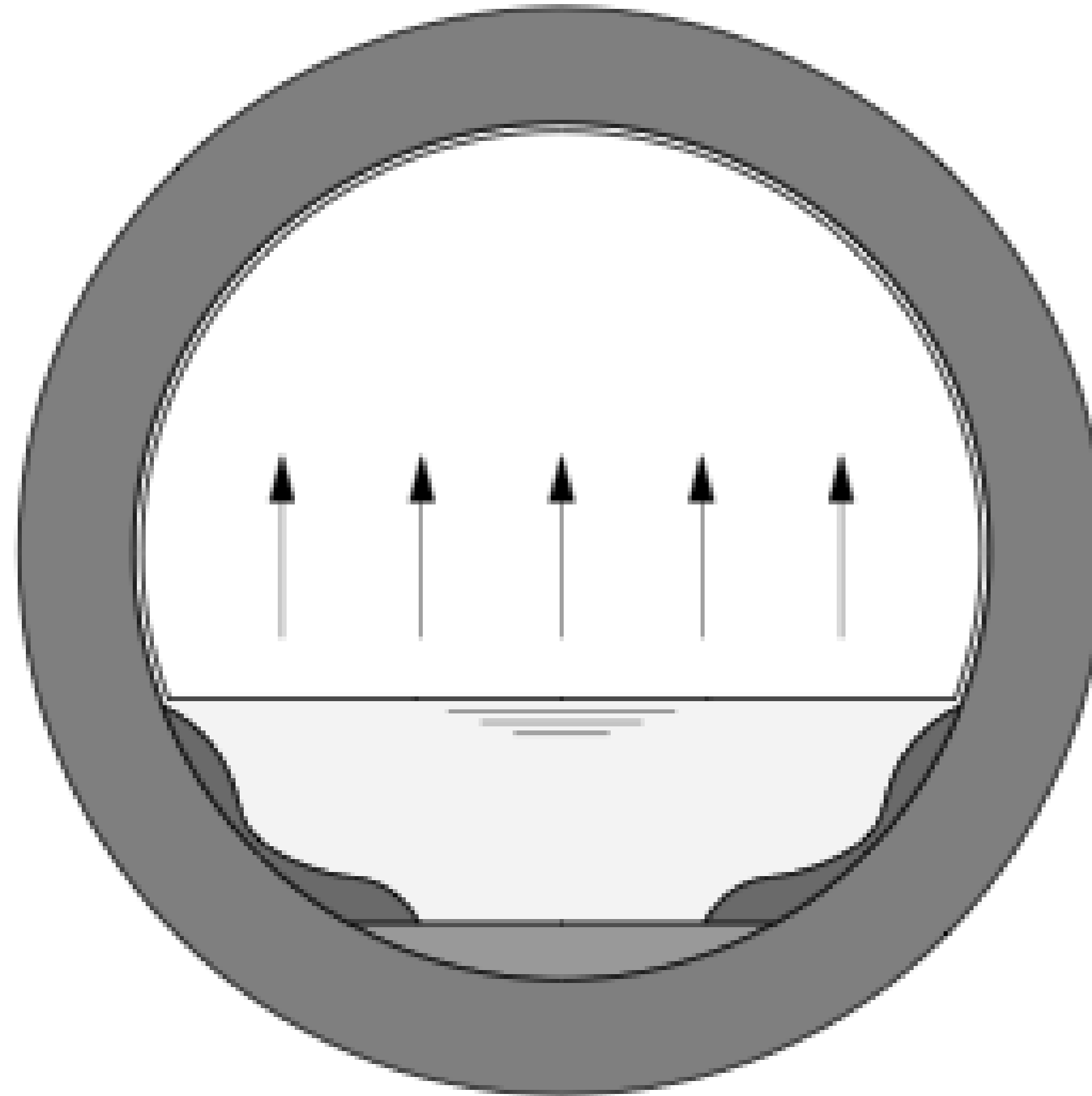


Partly dissolving calcium  
hydroxide crystal

Hydrated pore containing  
calium hydroxide ions



## Biogenic Sulfide Corrosion: process

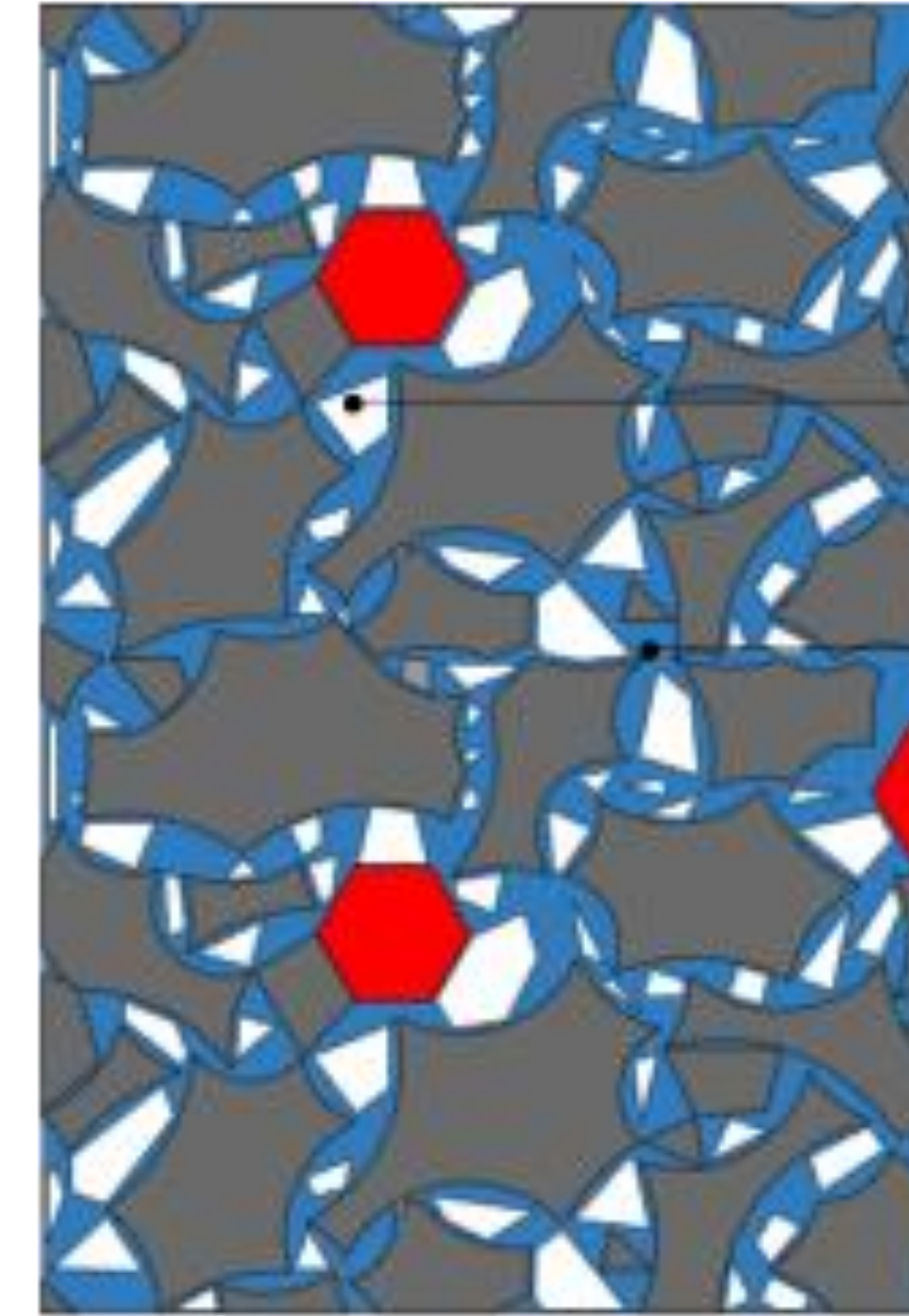
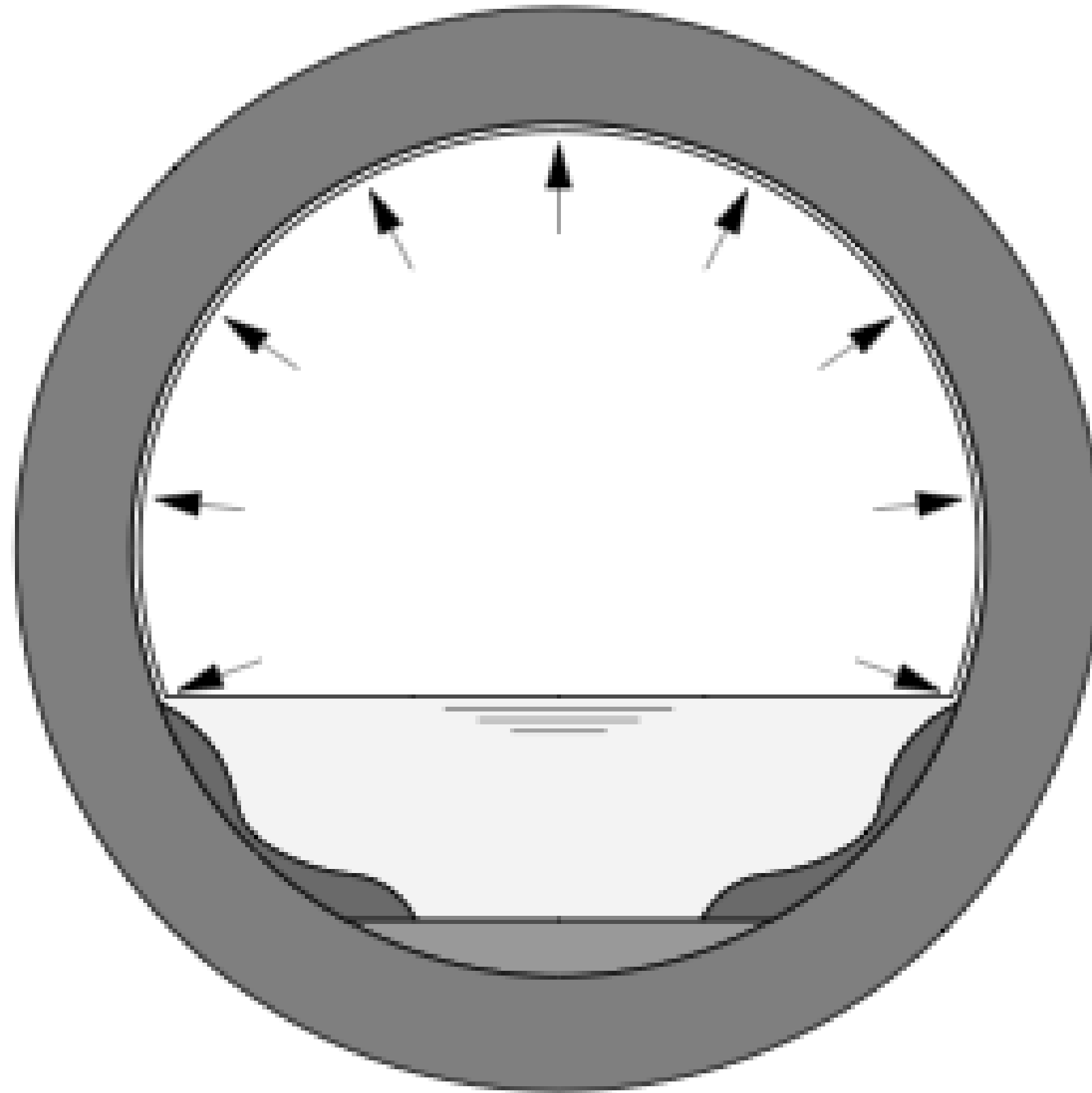


Partly dissolving calcium  
hydroxide crystal

Hydrated pore containing  
calcium hydroxide ions



## Biogenic Sulfide Corrosion: process

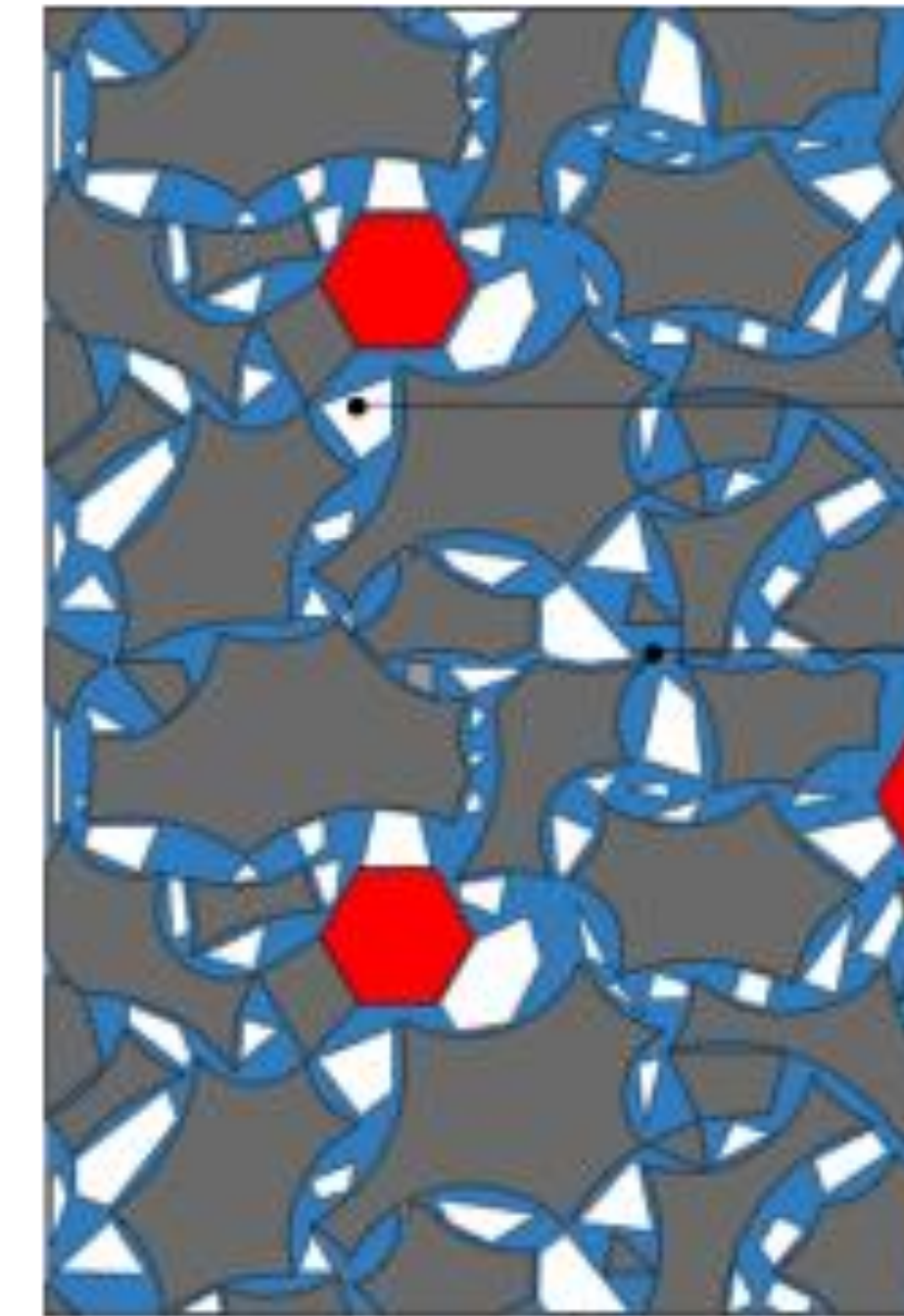
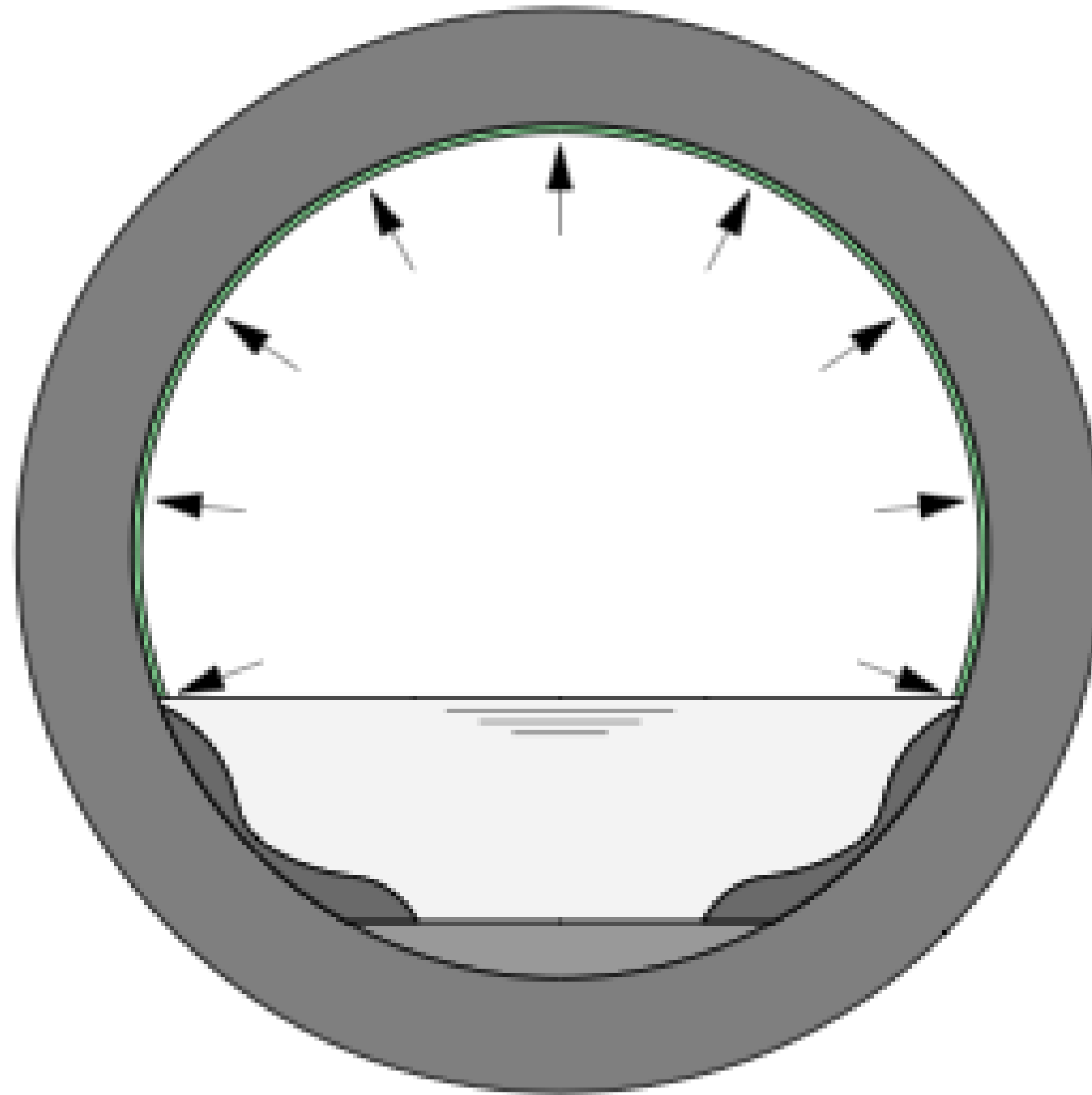


Partly dissolving calcium hydroxide crystal

Hydrated pore containing calcium hydroxide ions



## Biogenic Sulfide Corrosion: process



Partly dissolving calcium hydroxide crystal

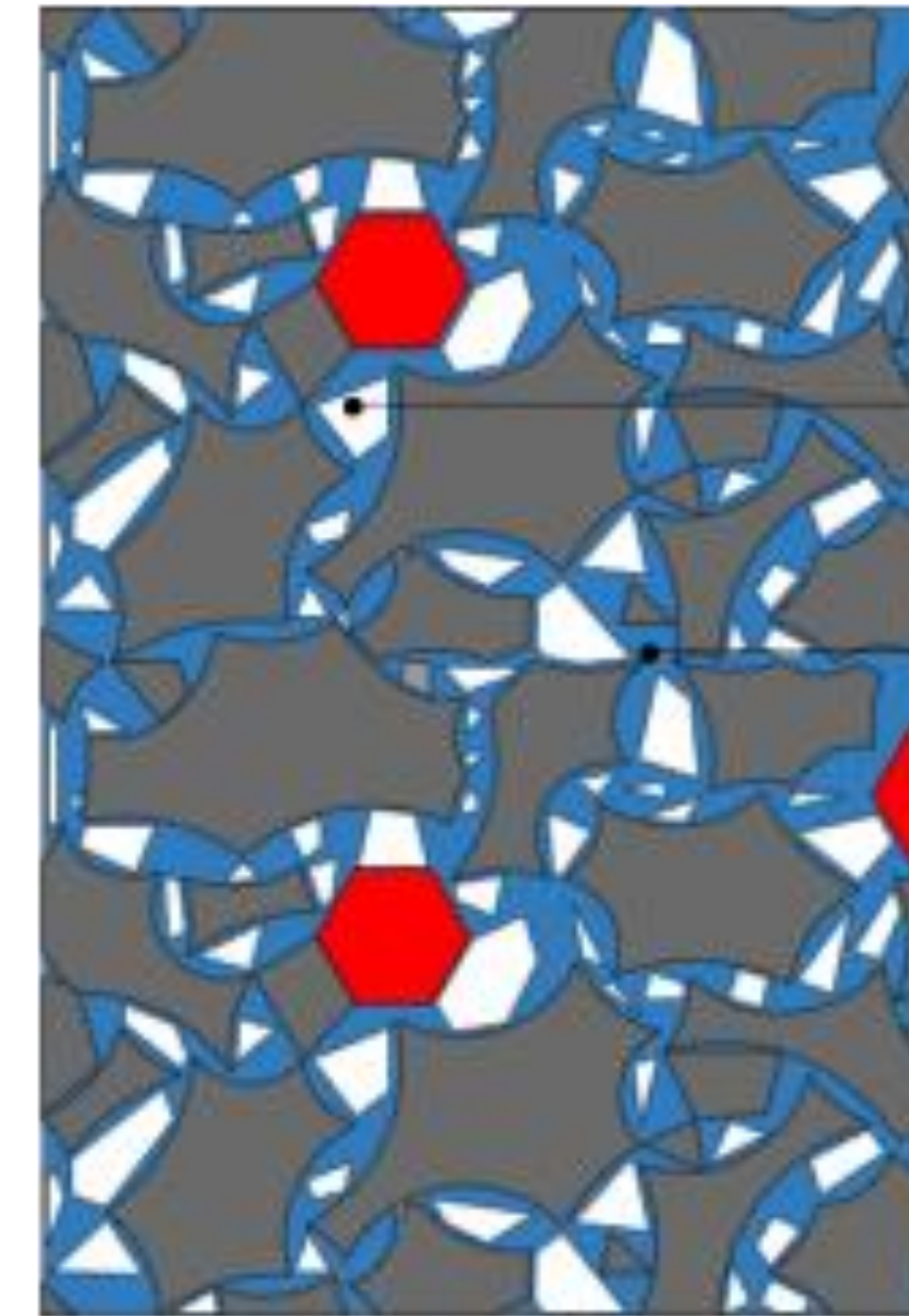
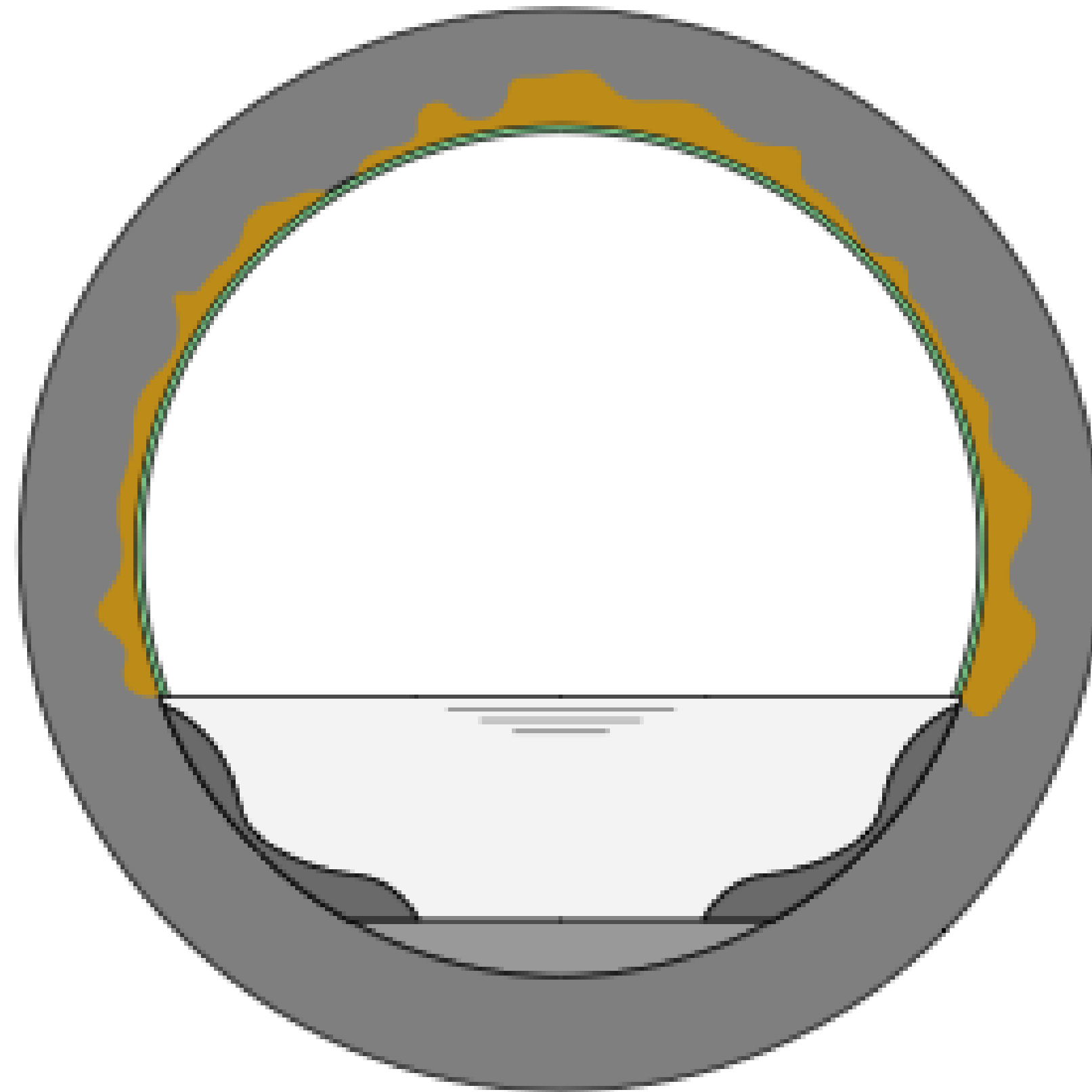
Hydrated pore containing calcium hydroxide ions

### Possible reasons for pH reduction:

- Carbonation
- $H_2S$  concentration
- Fungi



## Biogenic Sulfide Corrosion: process

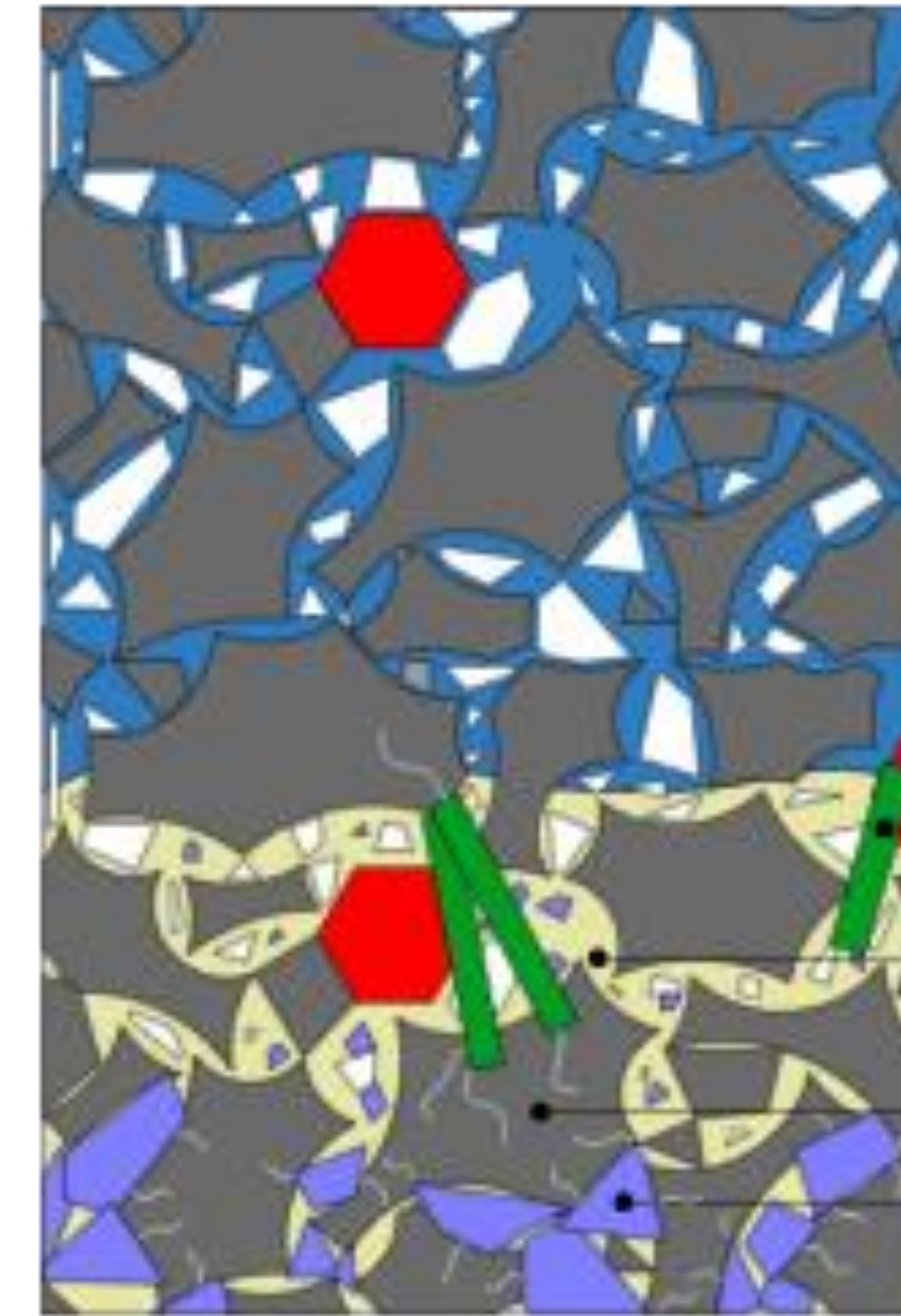
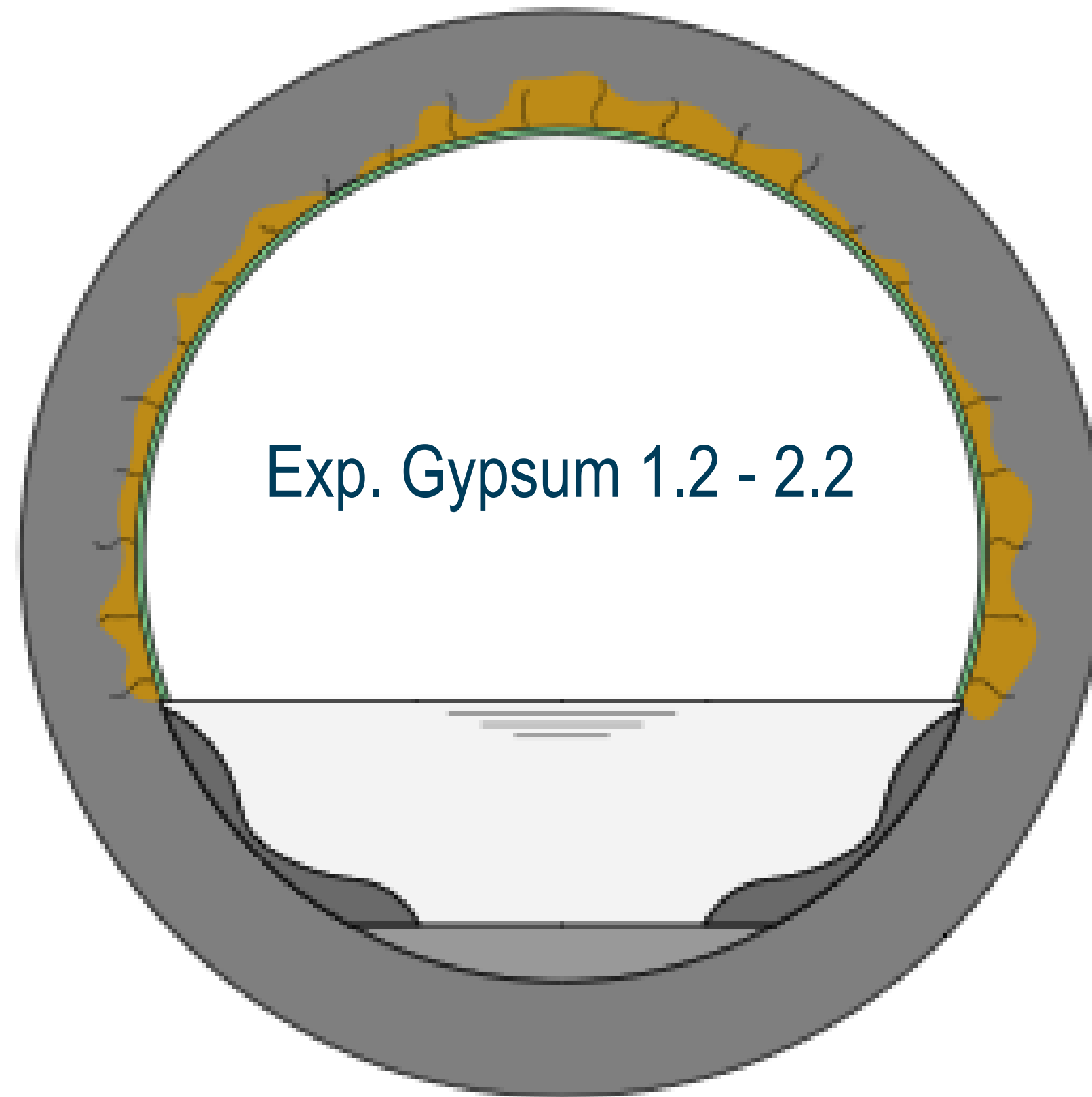


Partly dissolving calcium  
hydroxide crystal

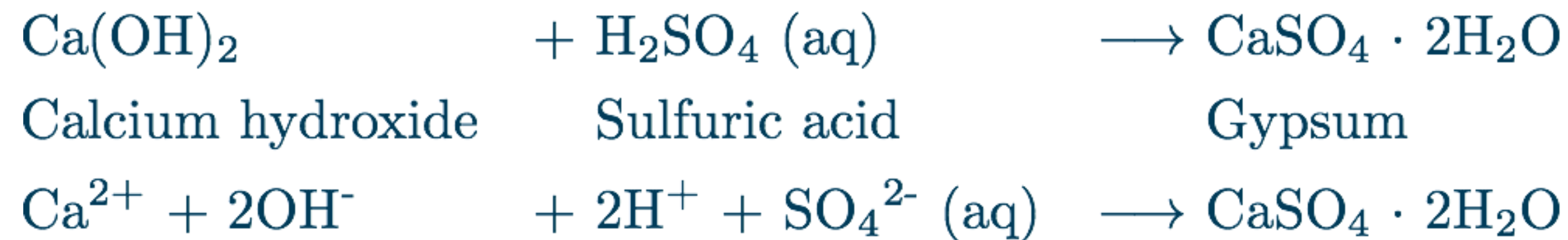
Hydrated pore containing  
calium hydroxide ions



## Biogenic Sulfide Corrosion: process

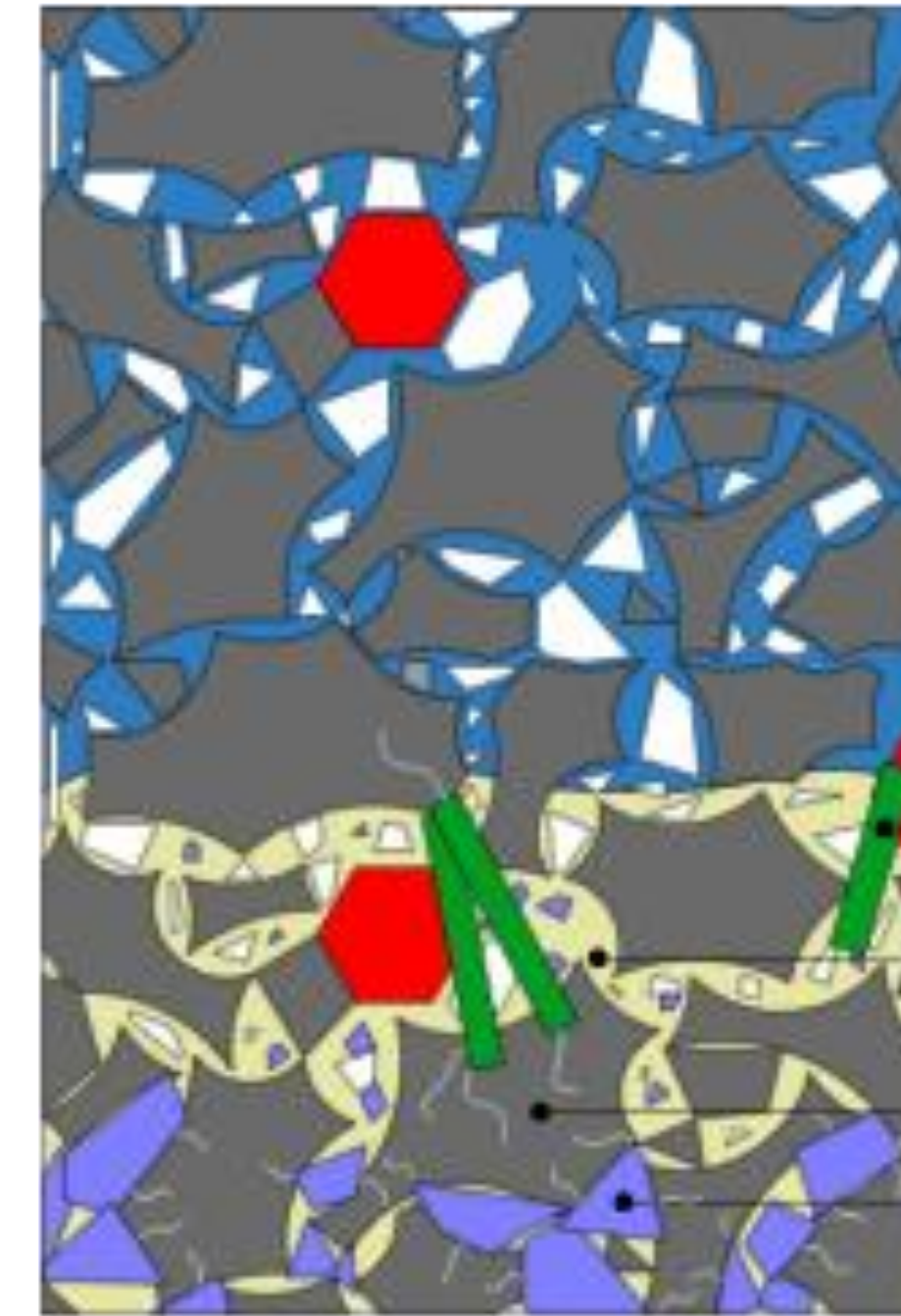
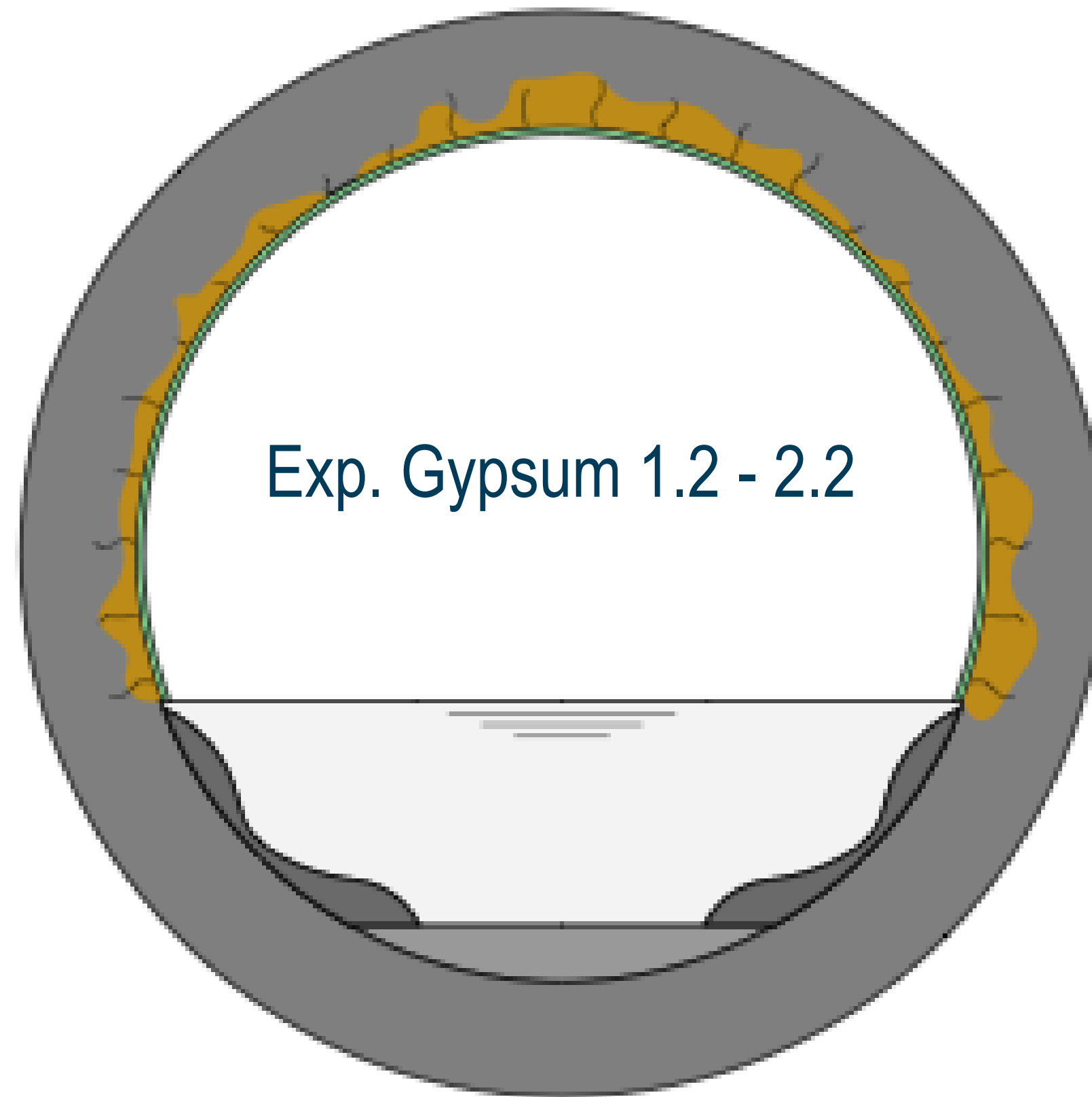


- Ettringite crystals
- Diffusing sulphuric acid
- Fracturing calcium-silicate hydrate
- Gypsum crystals

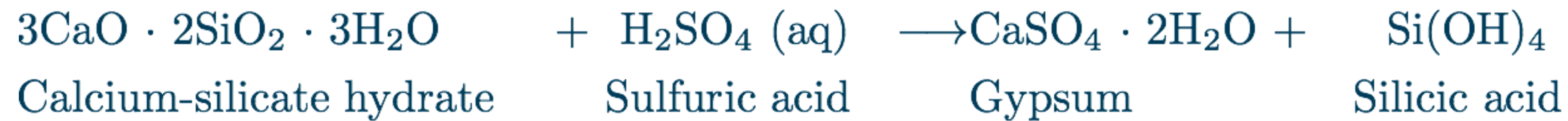




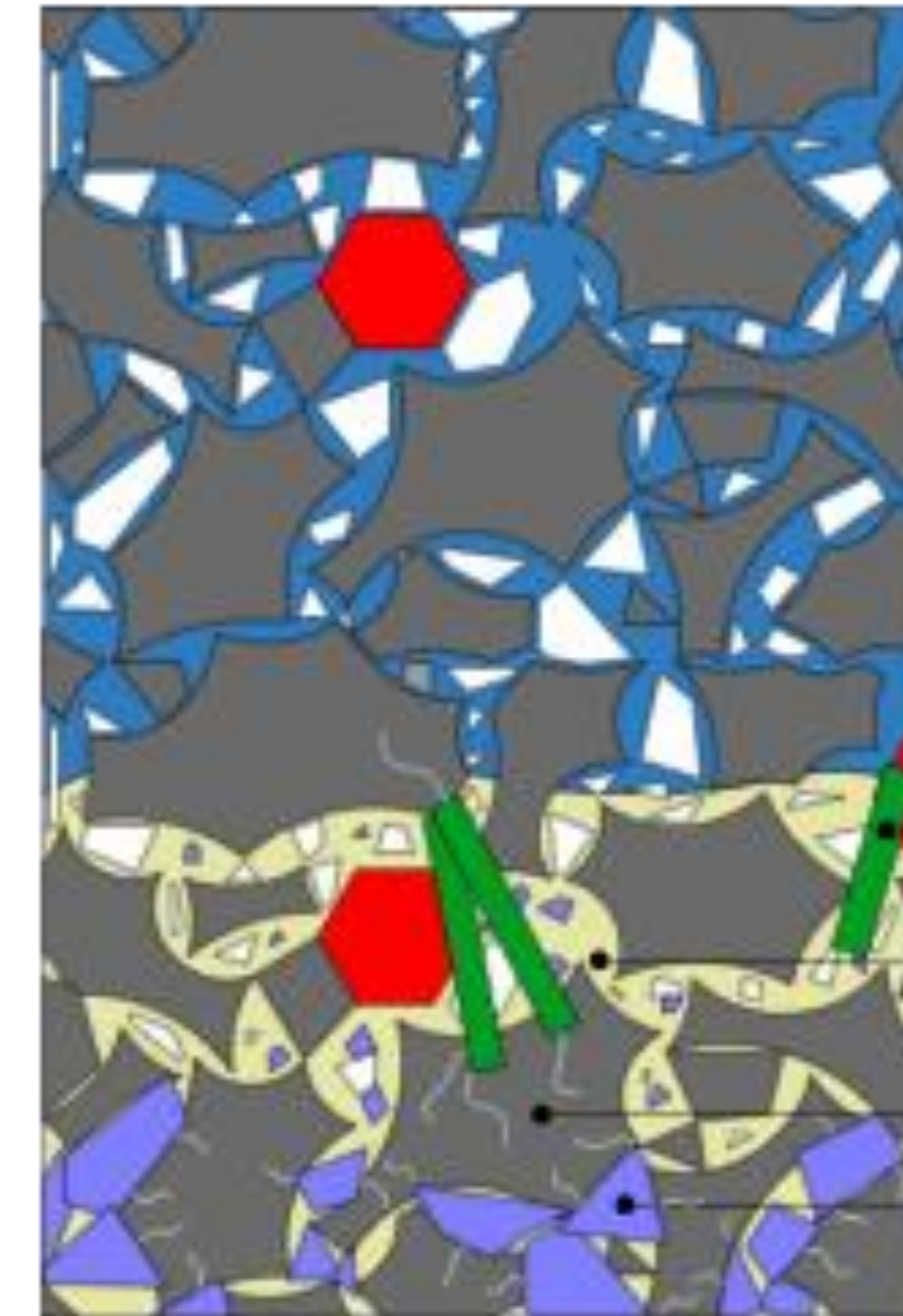
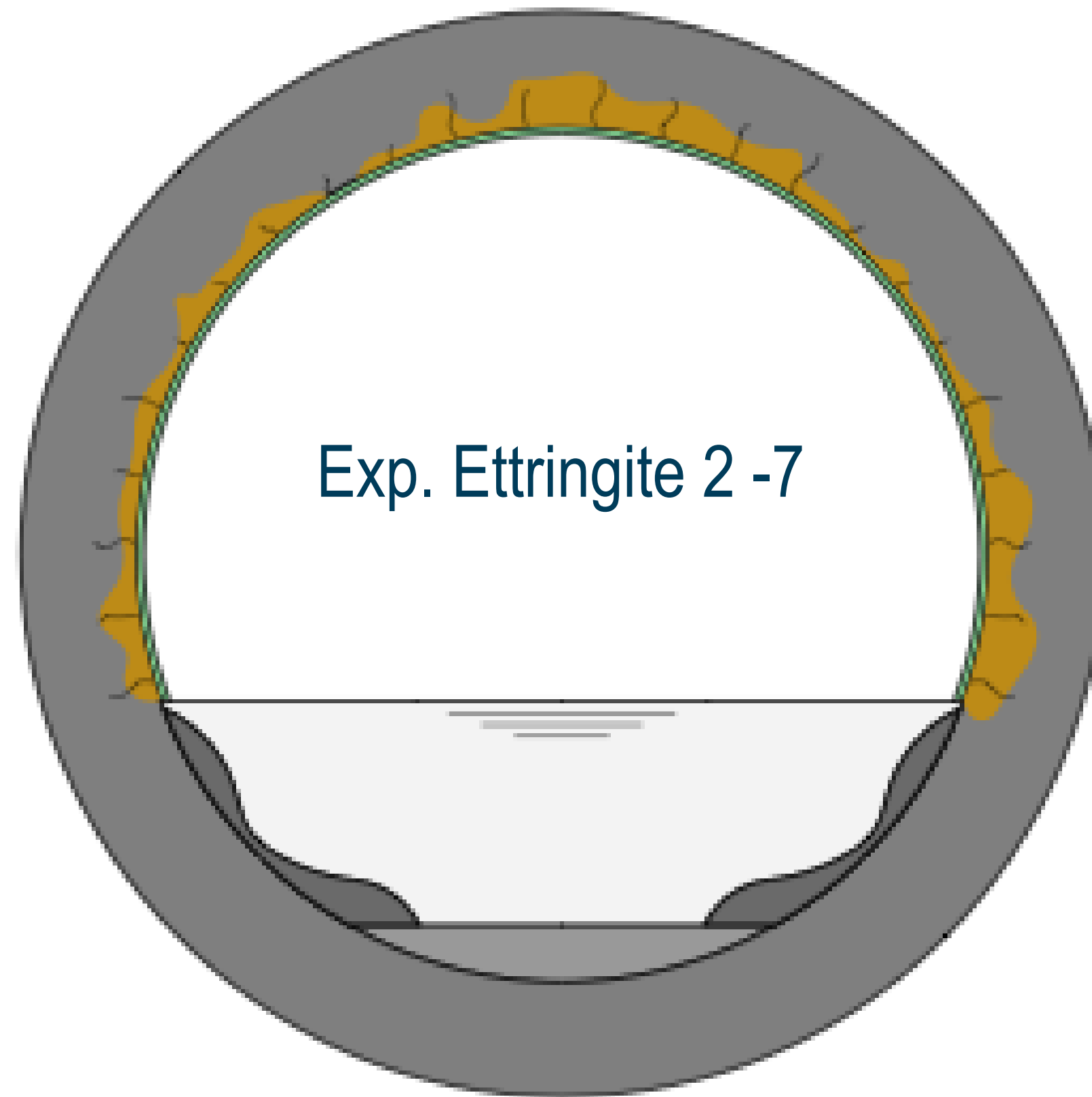
## Biogenic Sulfide Corrosion: process



- Ettringite crystals
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## Biogenic Sulfide Corrosion: process



Ettringite crystals

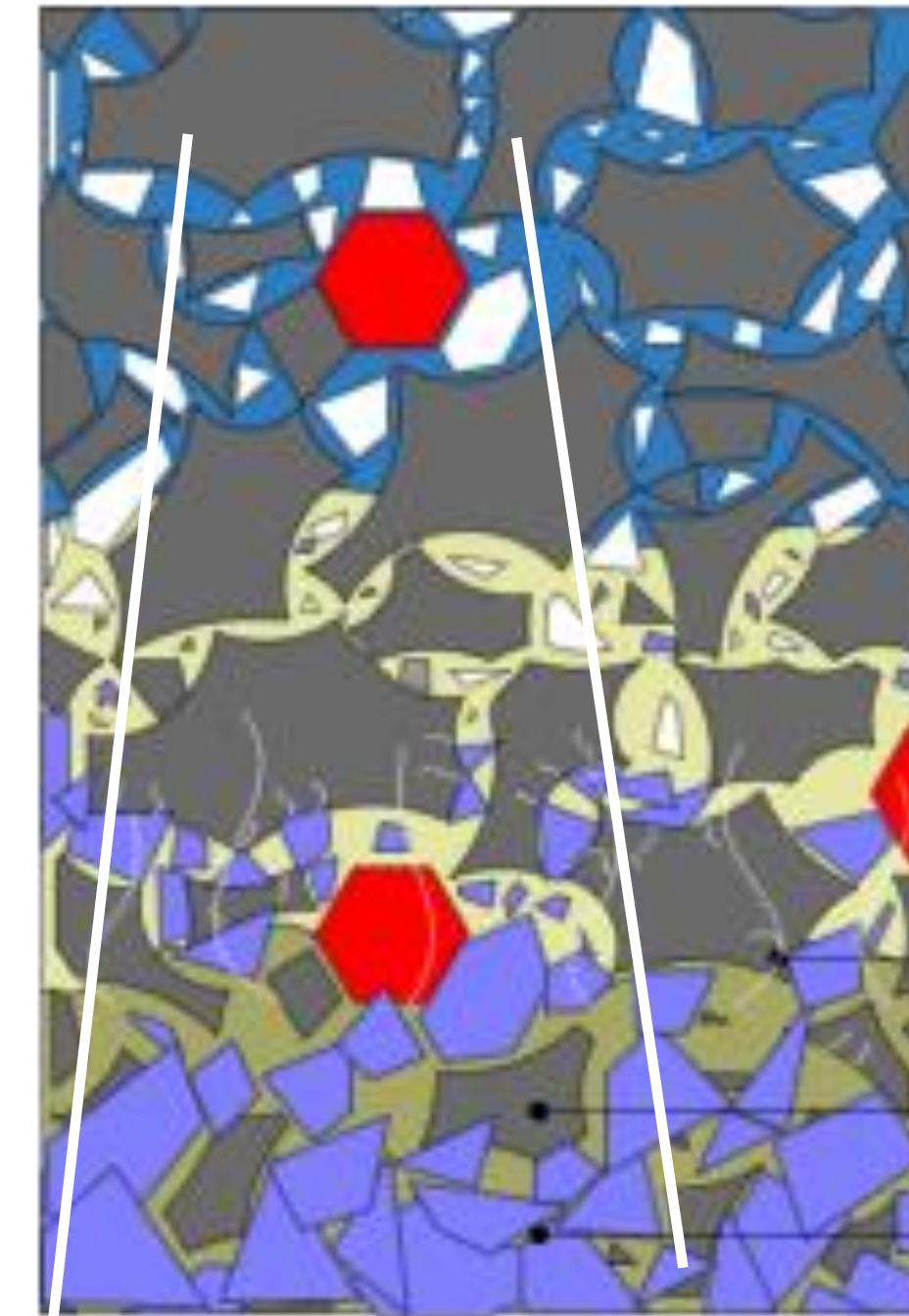
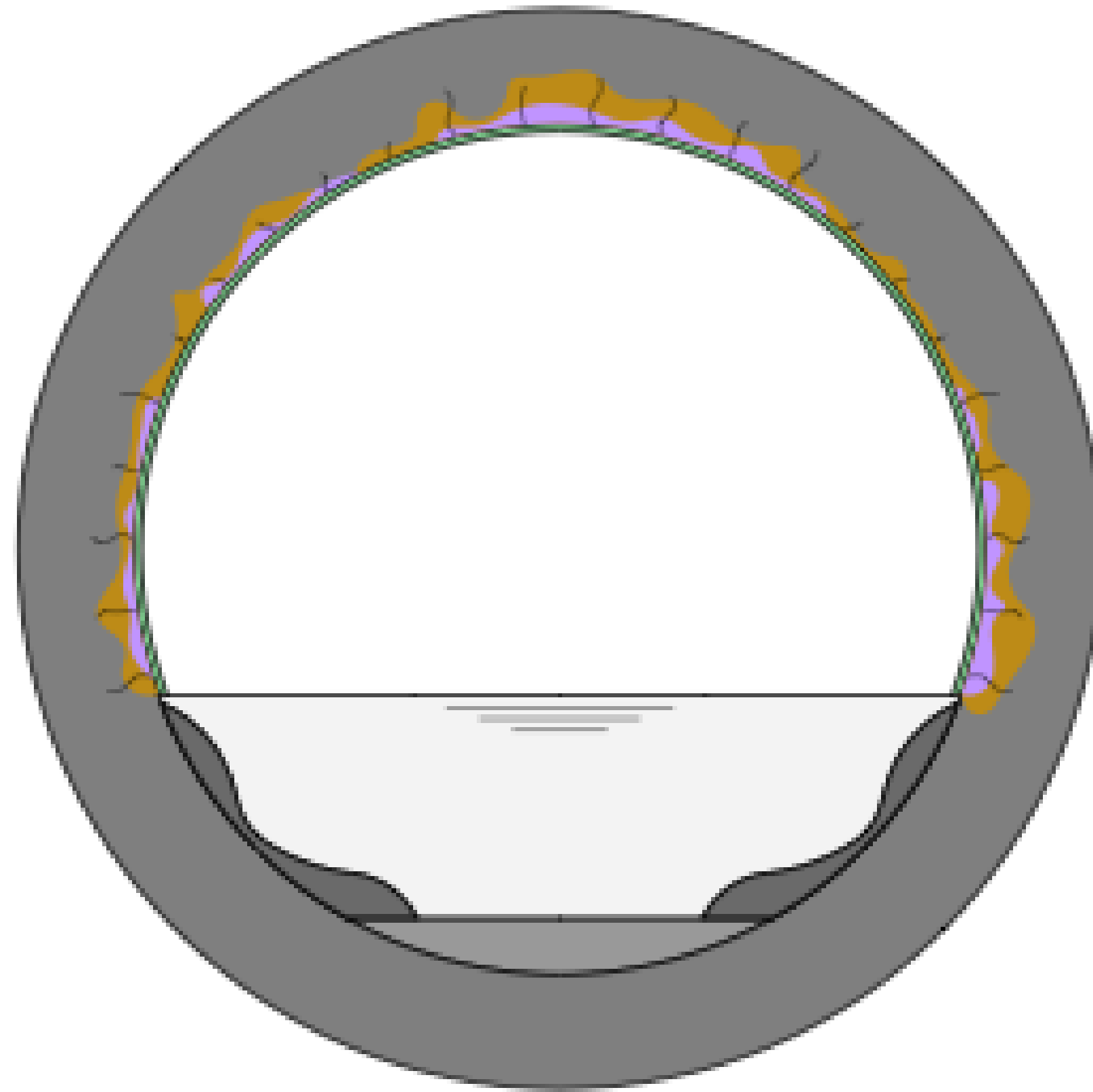
Diffusing sulphuric acid

Fracturing calcium-silicate  
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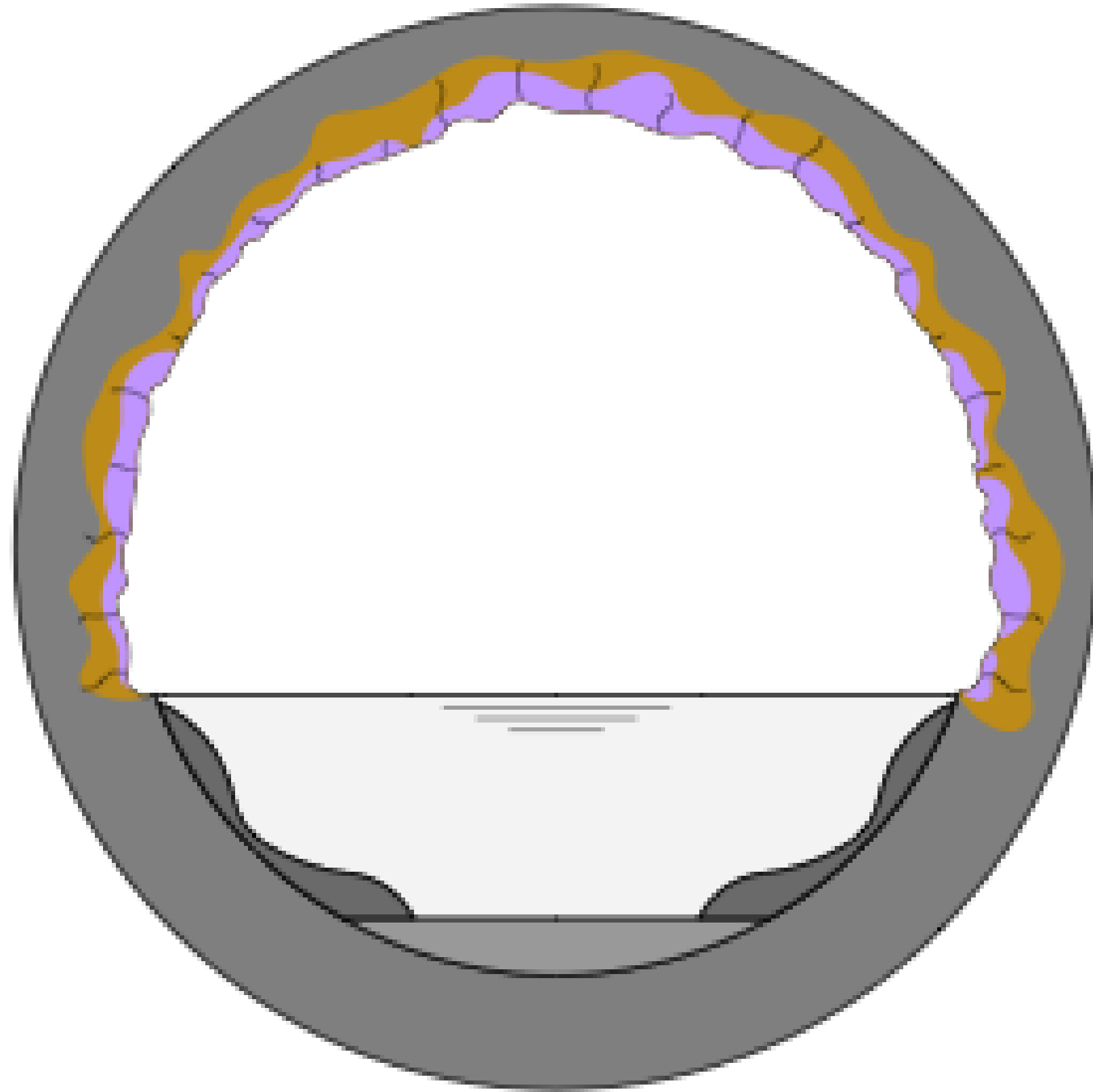


## Biogenic Sulfide Corrosion: process



- Macro cracks
- Dissolving calcium-silicate hydrate
- Gypsum layer

## Biogenic Sulfide Corrosion: process

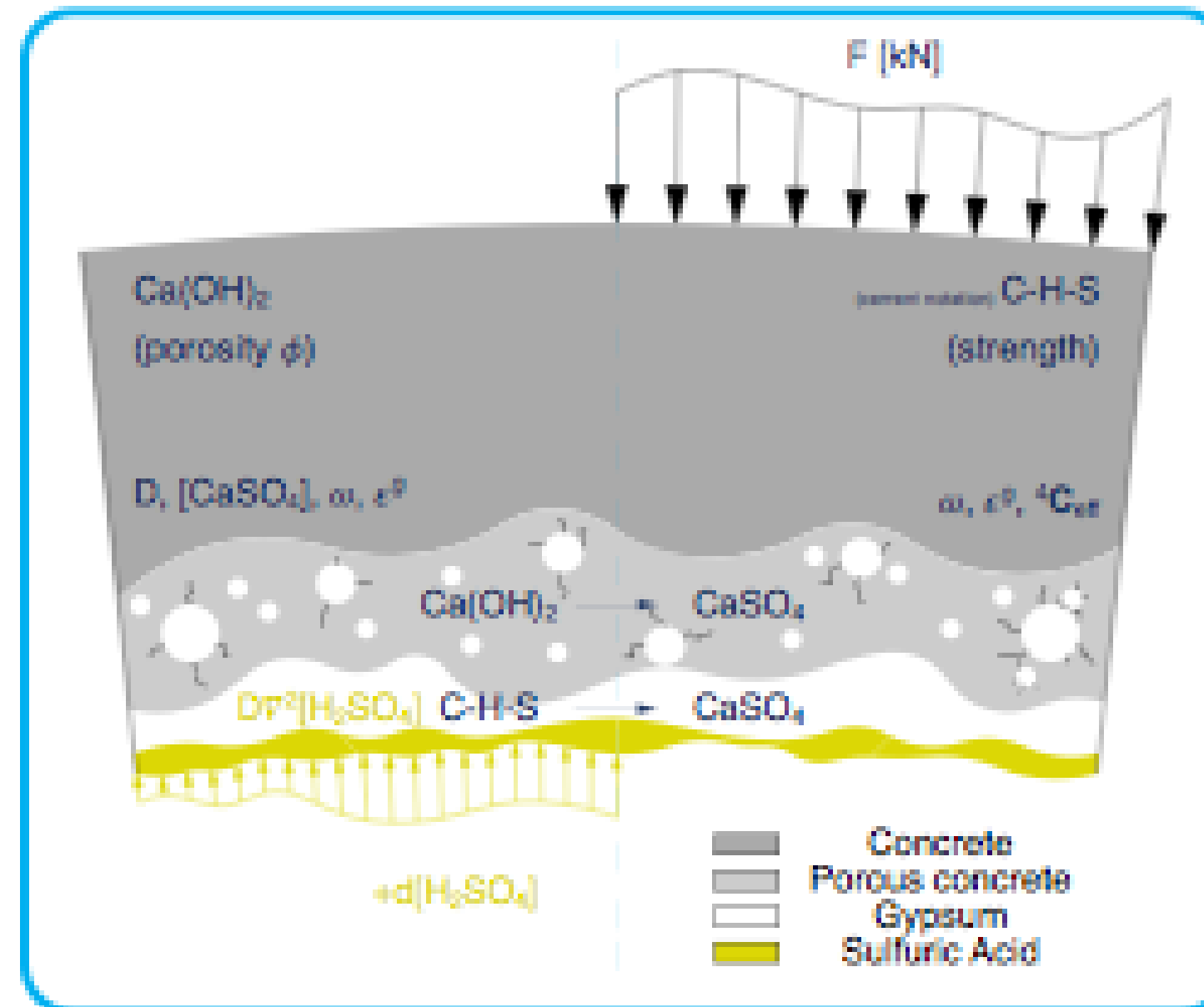




# Modelling Approach

Diffusion Processes

$$\frac{\partial [C_i]}{\partial t} = \nabla \cdot (D_i \nabla [C_i]) + \mathbf{R}_{1 \times m} \mathbf{Q}_{m \times 1}$$



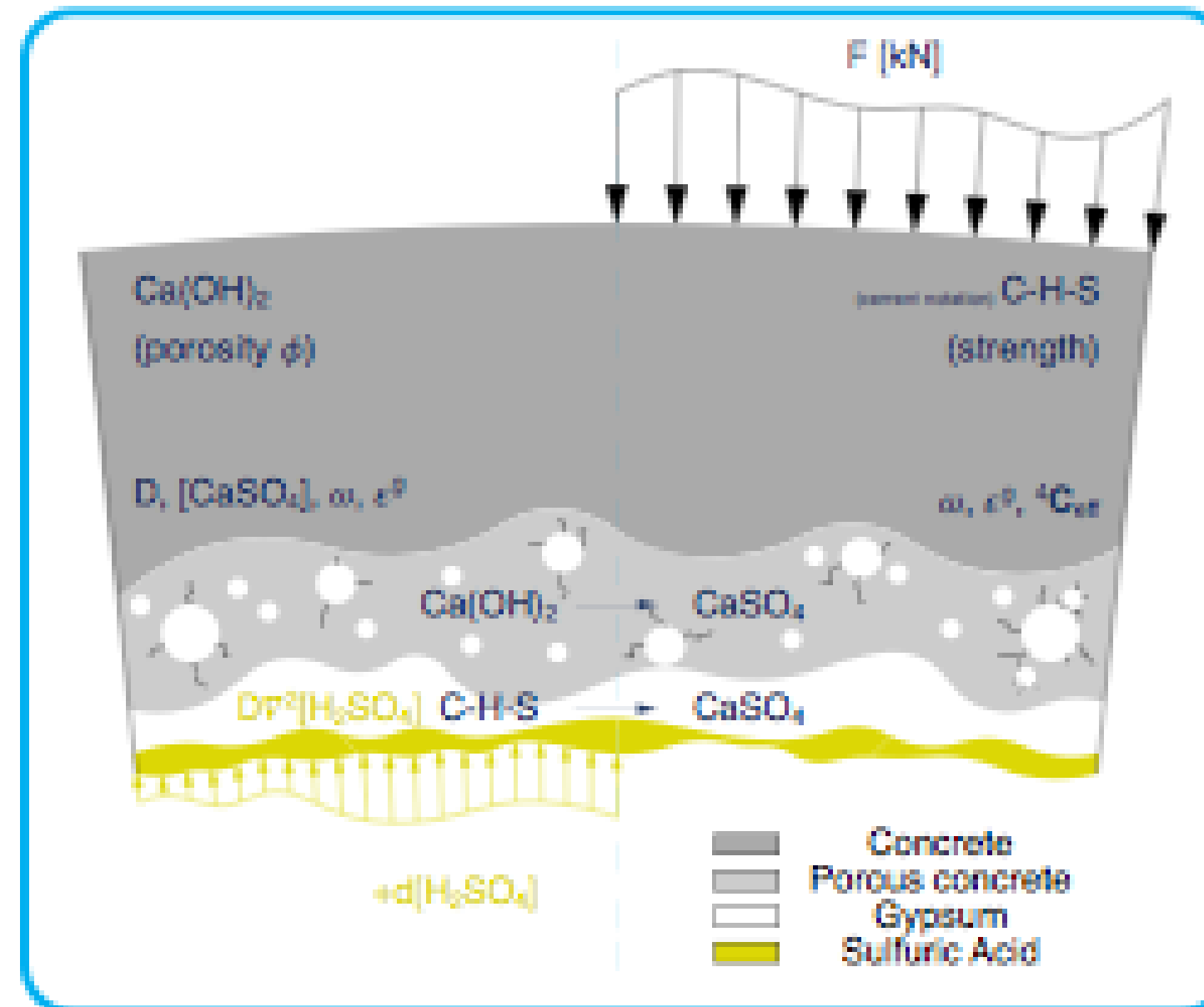
$$\sigma = (1 - \omega)^4 C_0 : \epsilon^e$$

Continuum Damage Mechanics



Diffusion Processes

$$\frac{\partial[C_i]}{\partial t} = \nabla \cdot (D_i \nabla[C_i]) + \mathbf{R}_{1 \times m} \mathbf{Q}_{m \times 1}$$



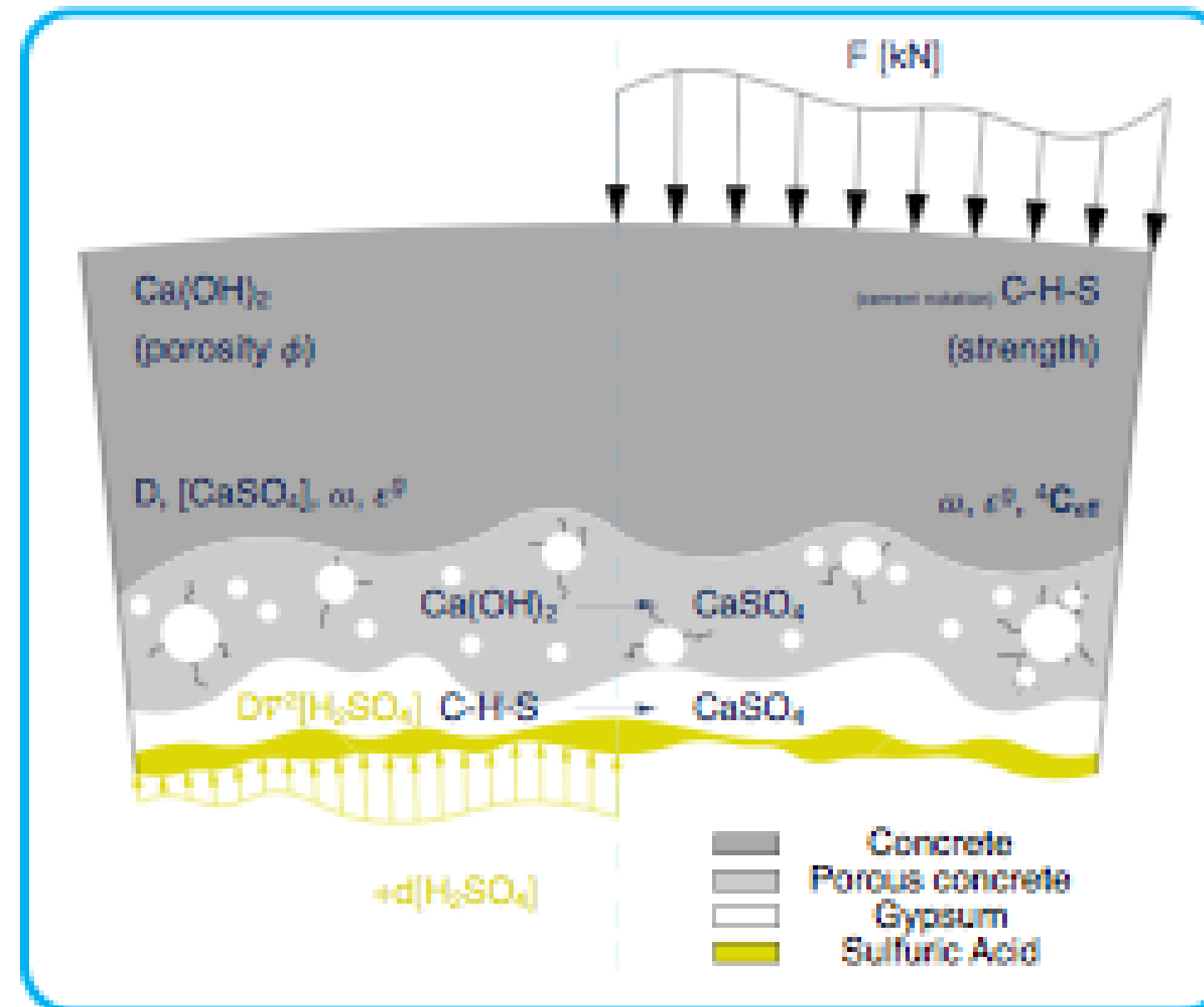
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Continuum Damage Mechanics

Diffusion Processes

$$\frac{\partial [C_i]}{\partial t} = \nabla \cdot (D_i \nabla [C_i]) + \mathbf{R}_{1 \times m} \mathbf{Q}_{m \times 1}$$

$$D = D(\omega)$$



$$\omega = \omega(\kappa)$$

$$\boldsymbol{\epsilon}^e = \boldsymbol{\epsilon} - \boldsymbol{\epsilon}^g$$

$$\varphi = \frac{[CSH]}{[CSH]_0}$$

$$\boldsymbol{\sigma} = (1 - \omega) \mathbf{C}_0 : \boldsymbol{\epsilon}^e$$

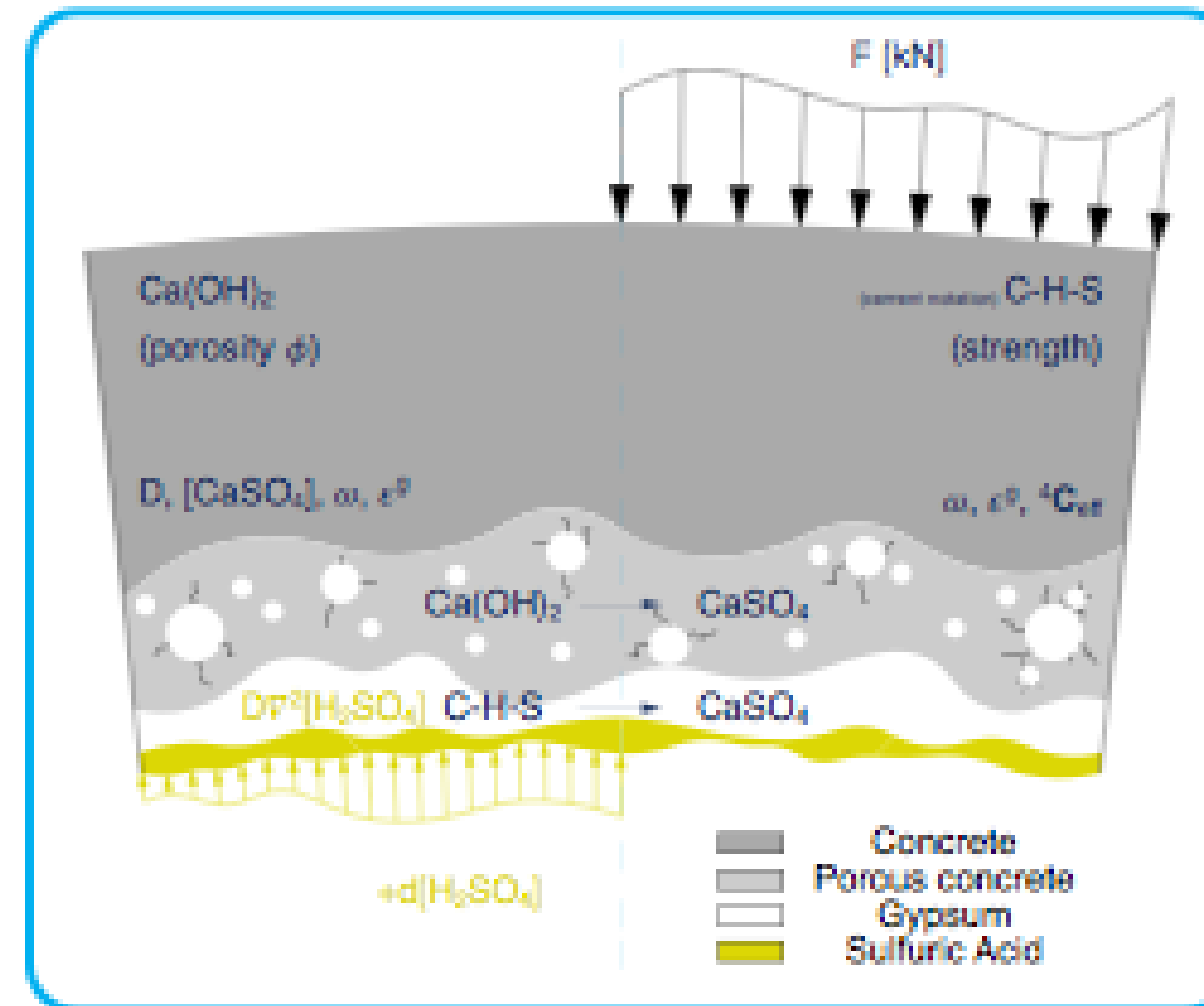
Continuum Damage Mechanics



## Diffusion Processes

For all relevant chemical species  $i$ , involved in  $m$  reactions:

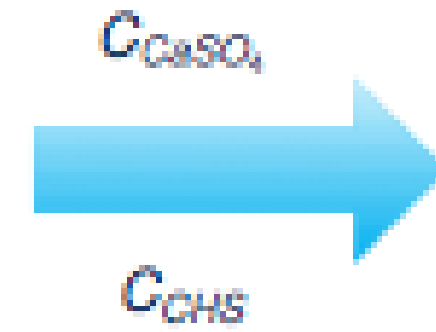
$$\frac{\partial C_i}{\partial t} = \nabla \cdot (D_i(\omega) \nabla C_i) + \mathbf{R}_{1 \times m} \mathbf{Q}_{m \times 1}$$



### Diffusion Processes

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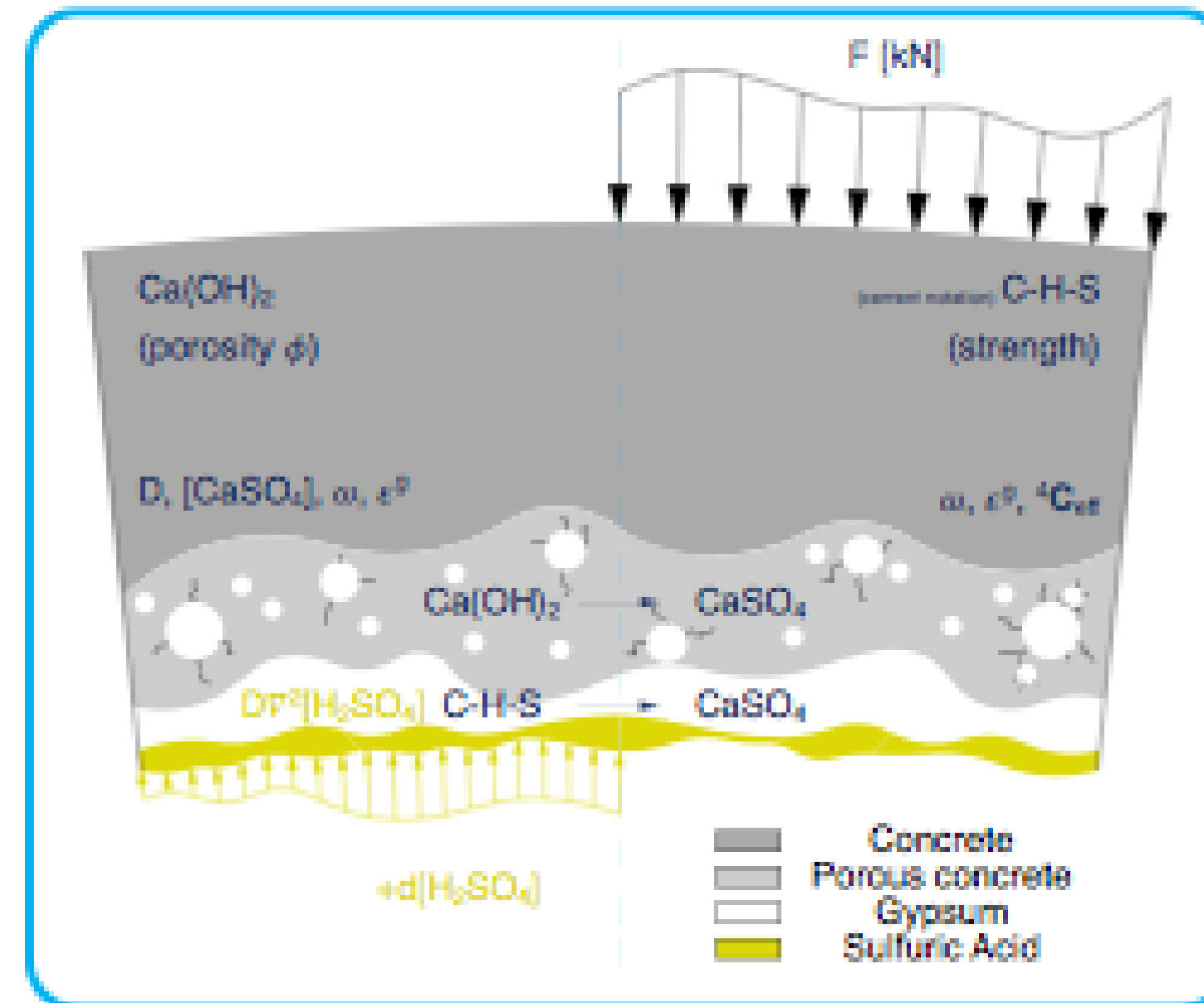


$$\varphi = \frac{C_{CHS}}{C_{CHS}|_0}$$

Volume fraction of C-H-S

$$\epsilon^{chem} = \epsilon^g C_{CaSO_4} I$$

Growth strain tensor

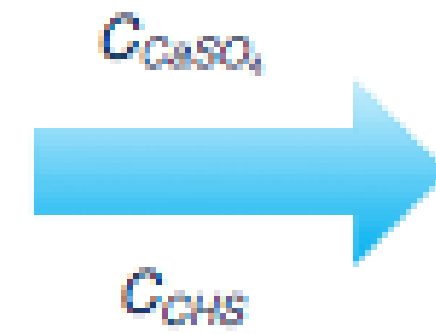




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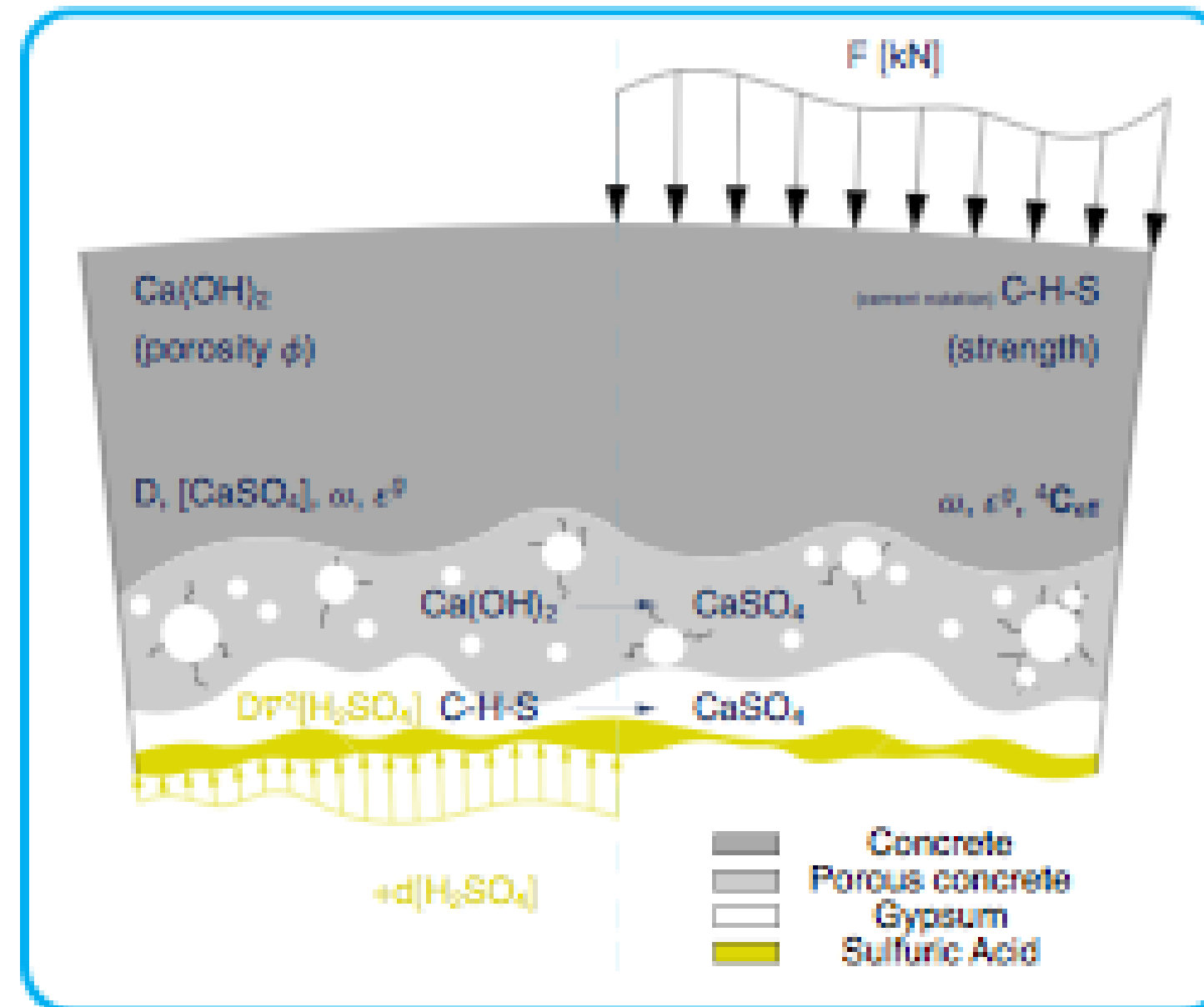


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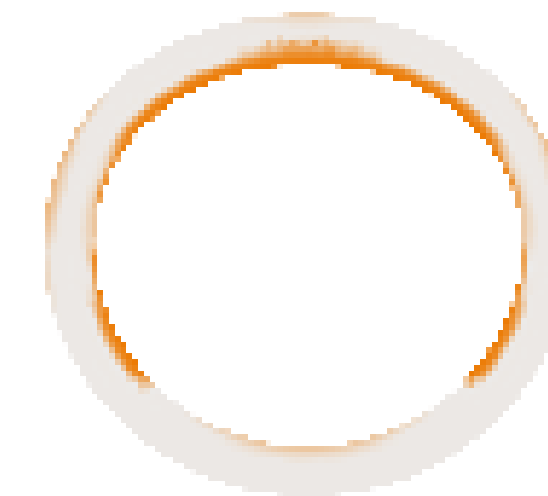
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Growth strain tensor

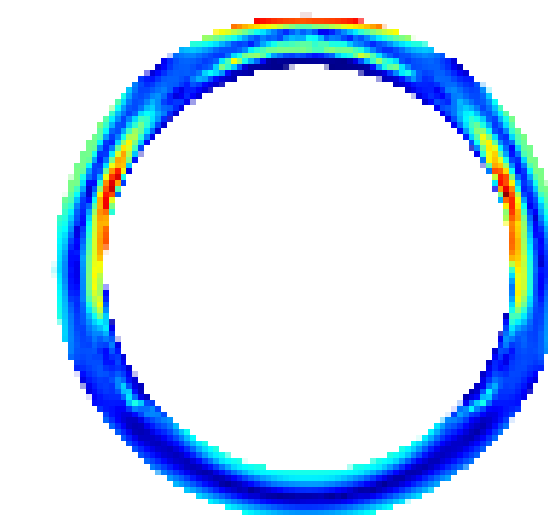


Total Damage:

$$\Omega = (1 - \omega_n(\kappa)) (1 - \varphi)$$



$$\boldsymbol{\sigma} = \Omega^4 \mathbf{C}_{CHS,0} : (\boldsymbol{\epsilon} - \boldsymbol{\epsilon}^{chem})$$

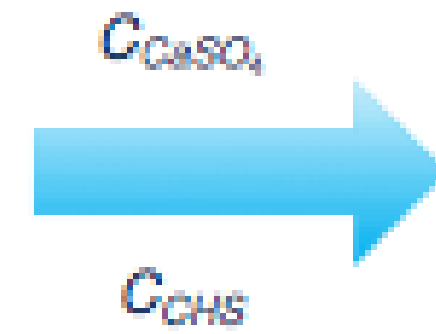


Continuum Damage Mechanics

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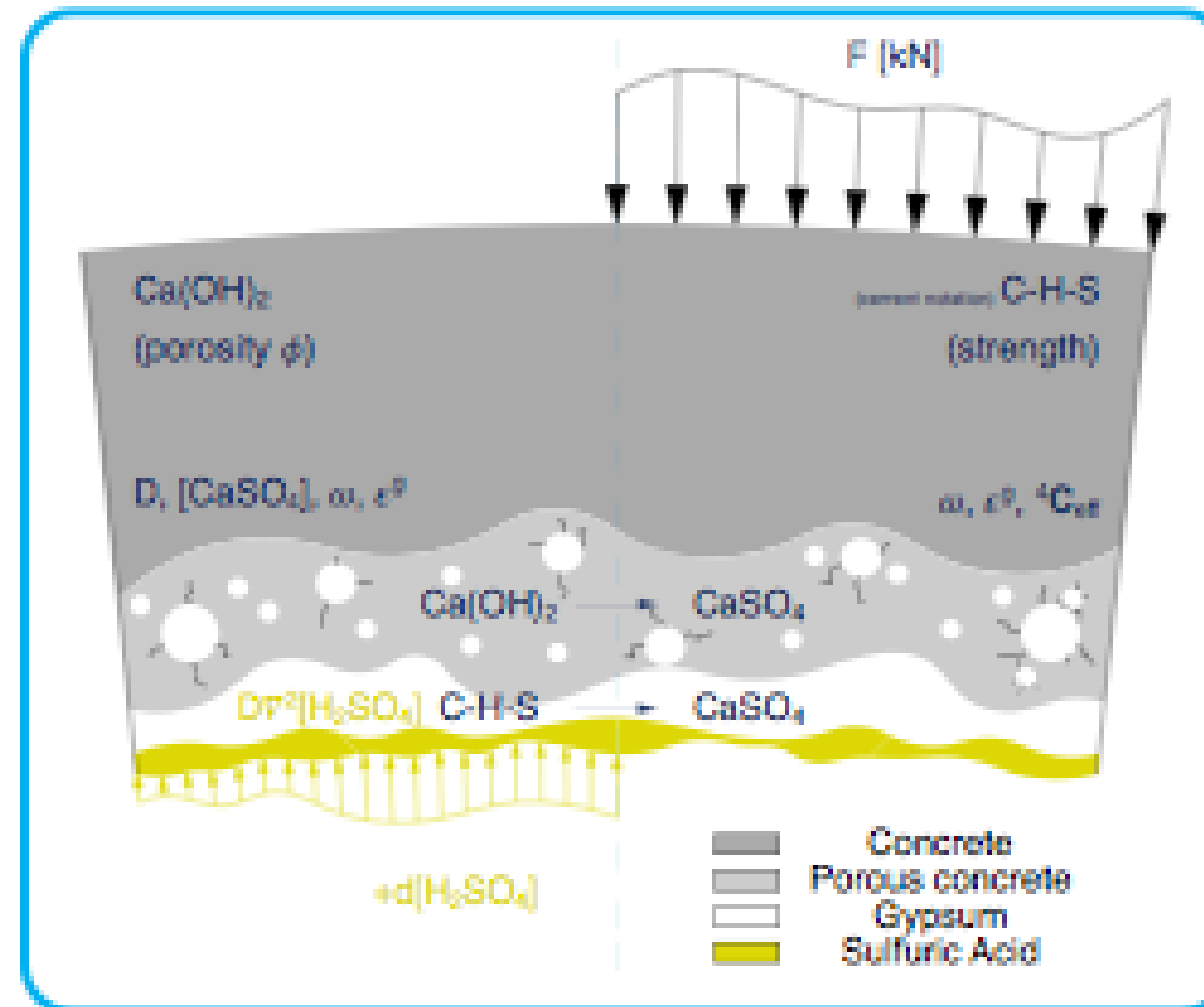


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Volume fraction of C-H-S

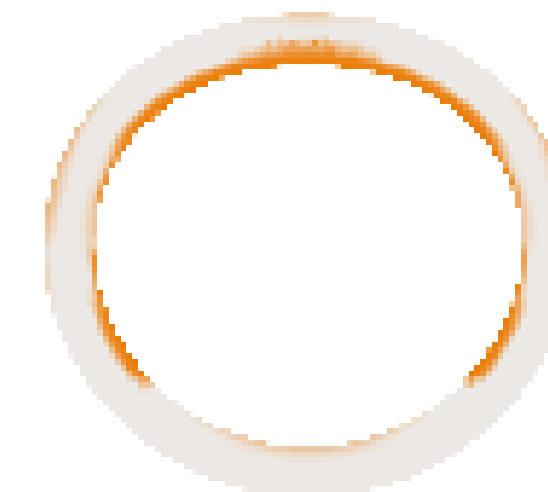
$$\boldsymbol{\epsilon}^{chem} = \epsilon^g C_{CaSO_4} \mathbf{I}$$

Growth strain tensor

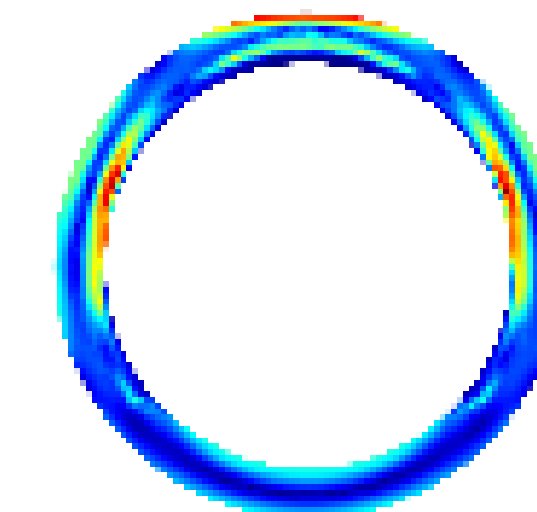


Total Damage:

$$\Omega = (1 - \omega_n(\kappa)) (1 - \varphi)$$



$$\boldsymbol{\sigma} = \Omega^4 C_{CHS,0} : (\boldsymbol{\epsilon} - \boldsymbol{\epsilon}^{chem})$$



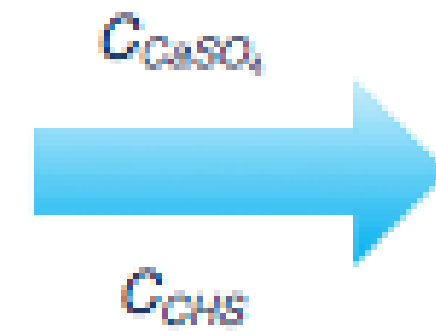
$$\omega_{n+1}(\kappa) = \frac{\kappa_U \kappa - \kappa_0}{\kappa \kappa_U - \kappa}$$

Continuum Damage Mechanics

### Diffusion Processes

For all relevant chemical species  $i$ , involved in  $m$  reactions:

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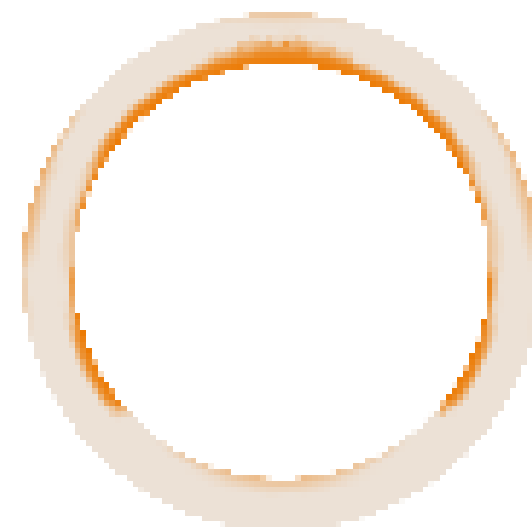
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Volume fraction of C-H-S

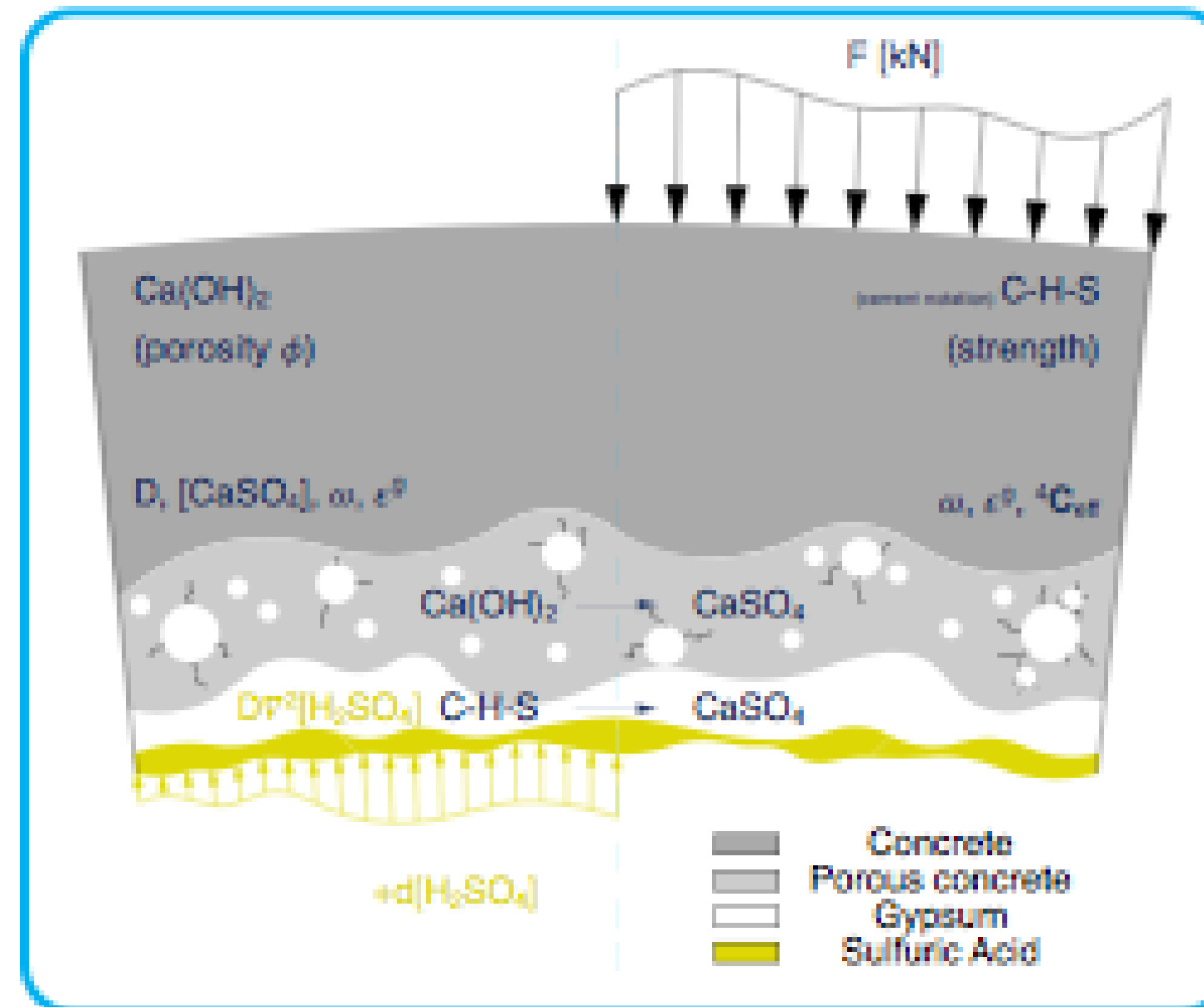
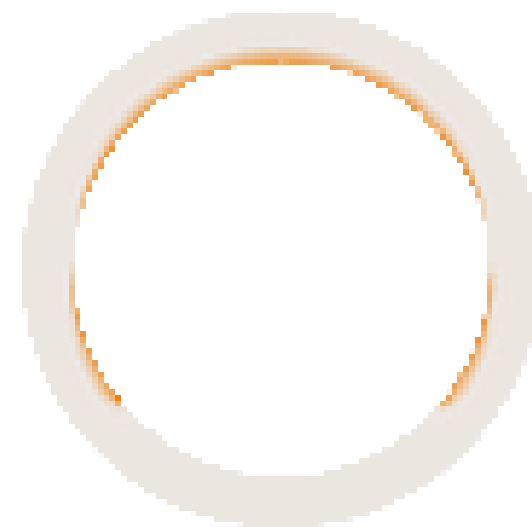
$$\boldsymbol{\epsilon}^{chem} = \epsilon^g C_{CaSO_4} \mathbf{I}$$

Growth strain tensor

Diffusion Coefficient:  
 $D = f(\omega_{n+1}, C_{CaSO_4}, \boldsymbol{\epsilon})$

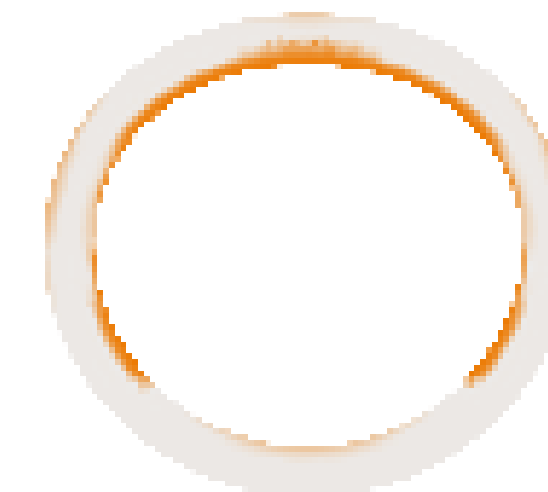


$C_{CaSO_4}$

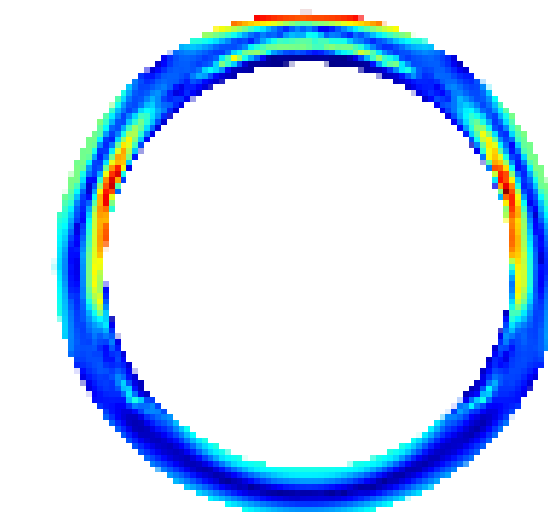


Total Damage:

$$\Omega = (1 - \omega_n(\kappa)) (1 - \varphi)$$



$$\boldsymbol{\sigma} = \Omega^4 \mathbf{C}_{CHS,0} : (\boldsymbol{\epsilon} - \boldsymbol{\epsilon}^{chem})$$



$$\omega_{n+1}(\kappa) = \frac{\kappa_U \kappa - \kappa_0}{\kappa \kappa_U - \kappa}$$

$\kappa$   
 $\epsilon$

Continuum Damage Mechanics

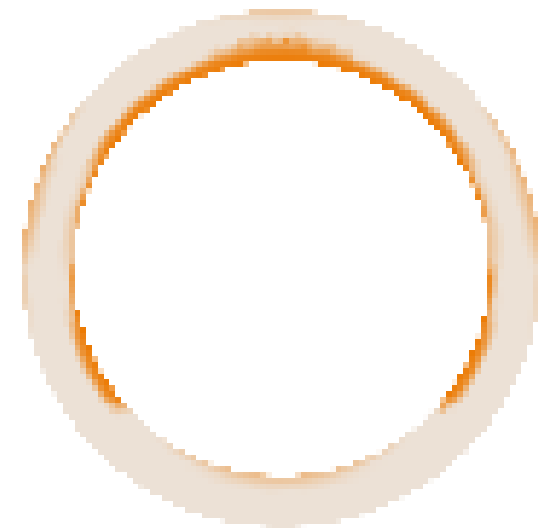


### Diffusion Processes

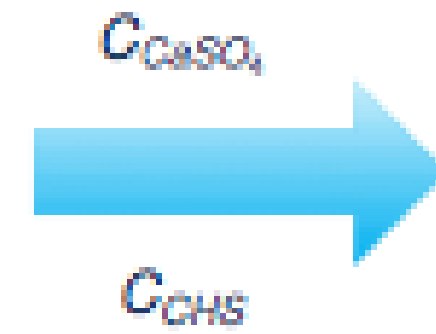
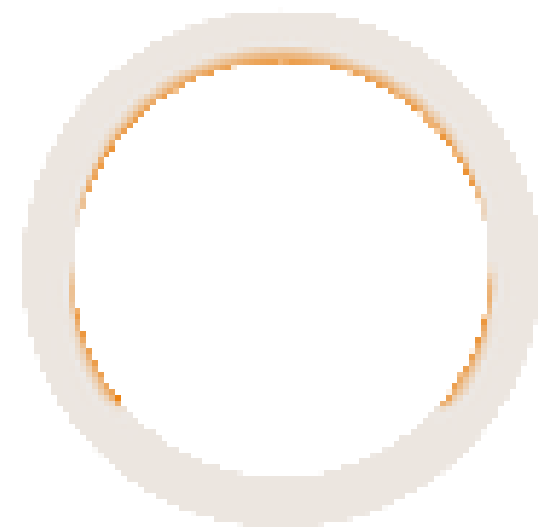
For all relevant chemical species  $i$ , involved in  $m$  reactions:

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Diffusion Coefficient:  
 $D = f(\omega_{n+1}, C_{CaSO_4}, \epsilon)$



$C_{CaSO_4}$



$$\varphi = \frac{C_{CHS}}{C_{CHS|0}}$$

Volume fraction of C-H-S

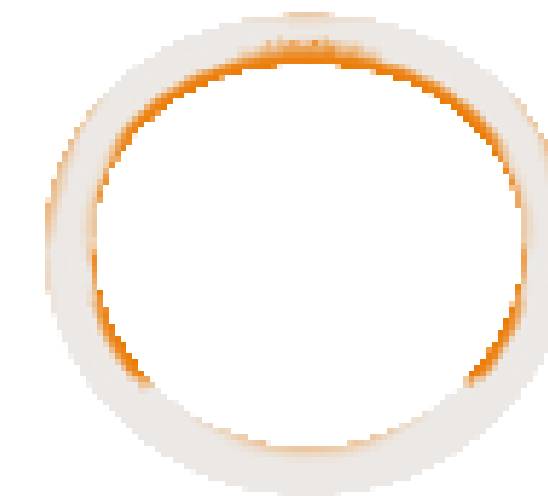
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Growth strain tensor

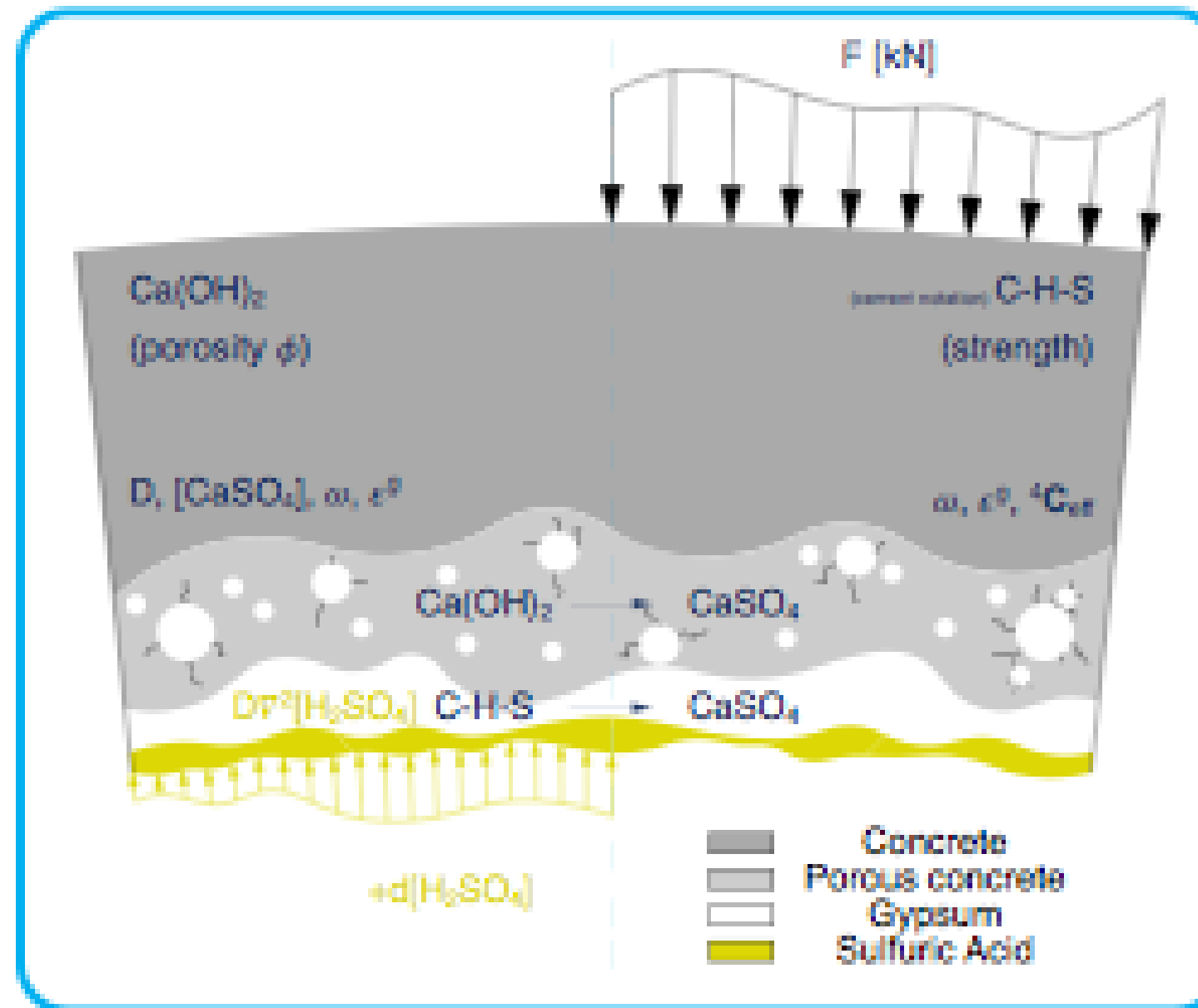
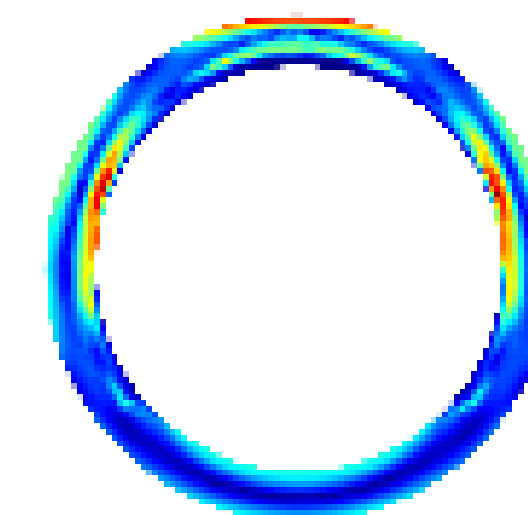


Total Damage:

$$\Omega = (1 - \omega_n(\kappa)) (1 - \varphi)$$



$$\sigma = \Omega^4 C_{CHS,0} : (\epsilon - \epsilon^{chem})$$

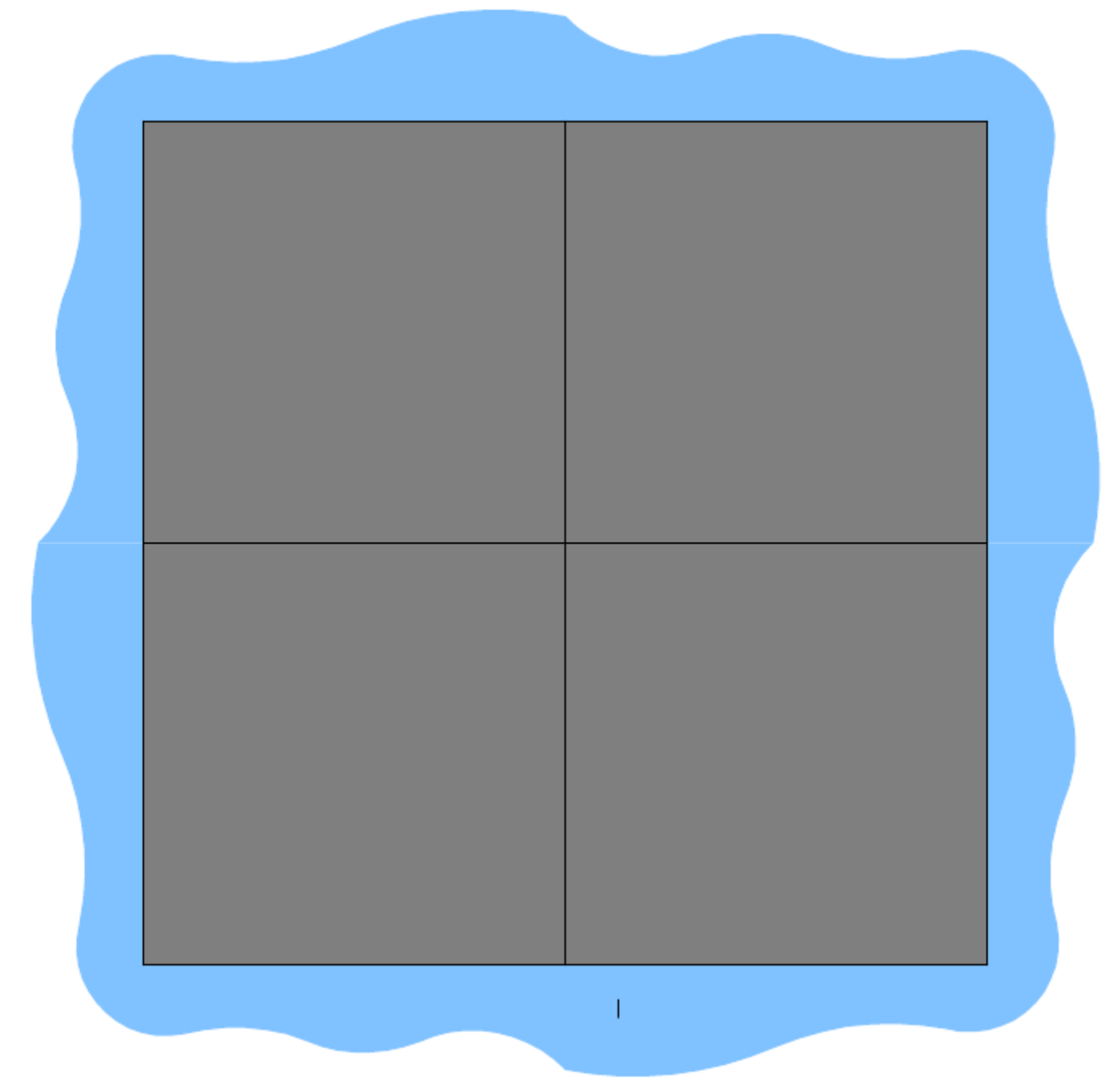


$$\omega_{n+1}(\kappa) = \frac{\kappa_U \kappa - \kappa_0}{\kappa \kappa_U - \kappa}$$

Continuum Damage Mechanics

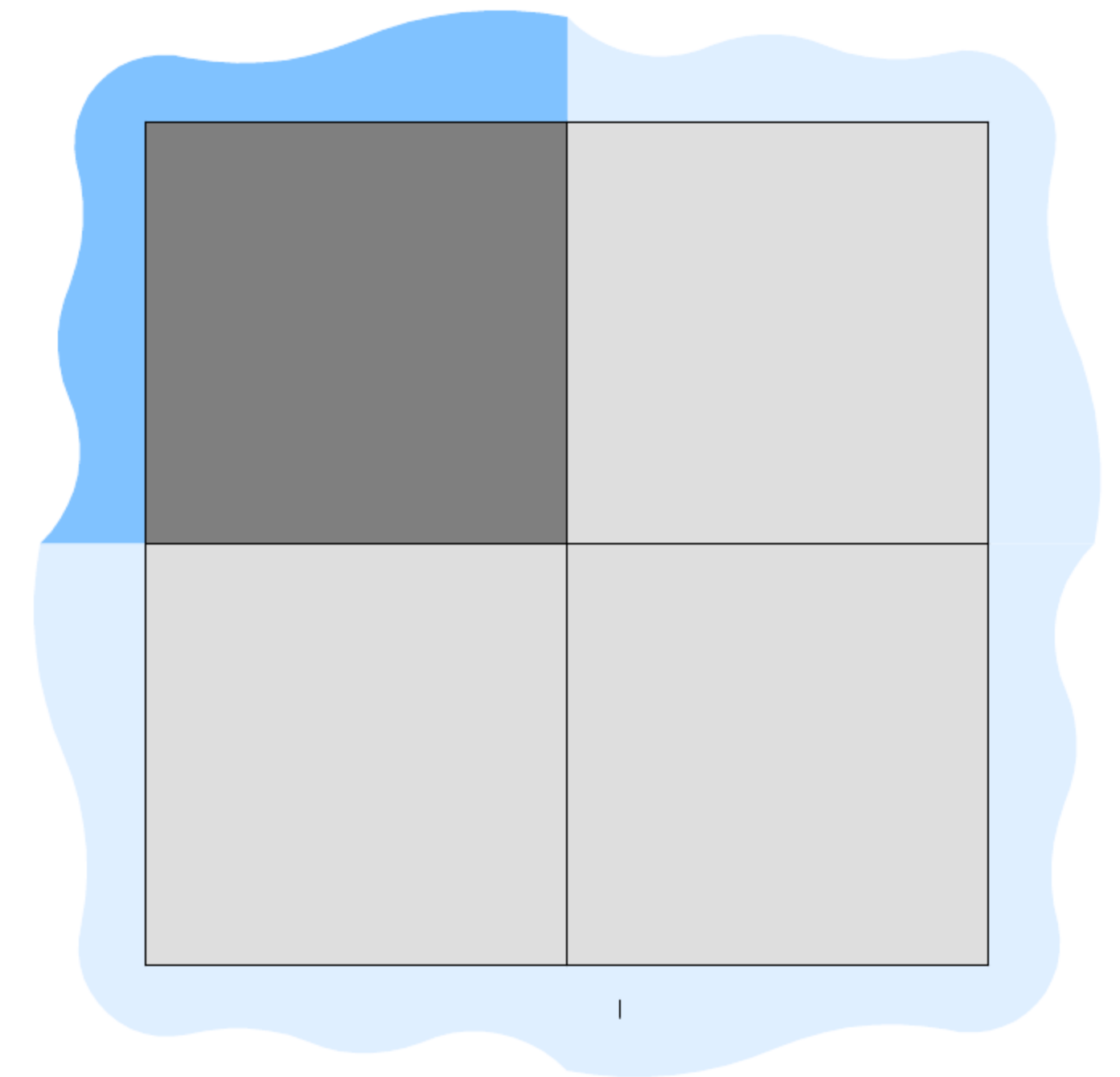
# Parameters

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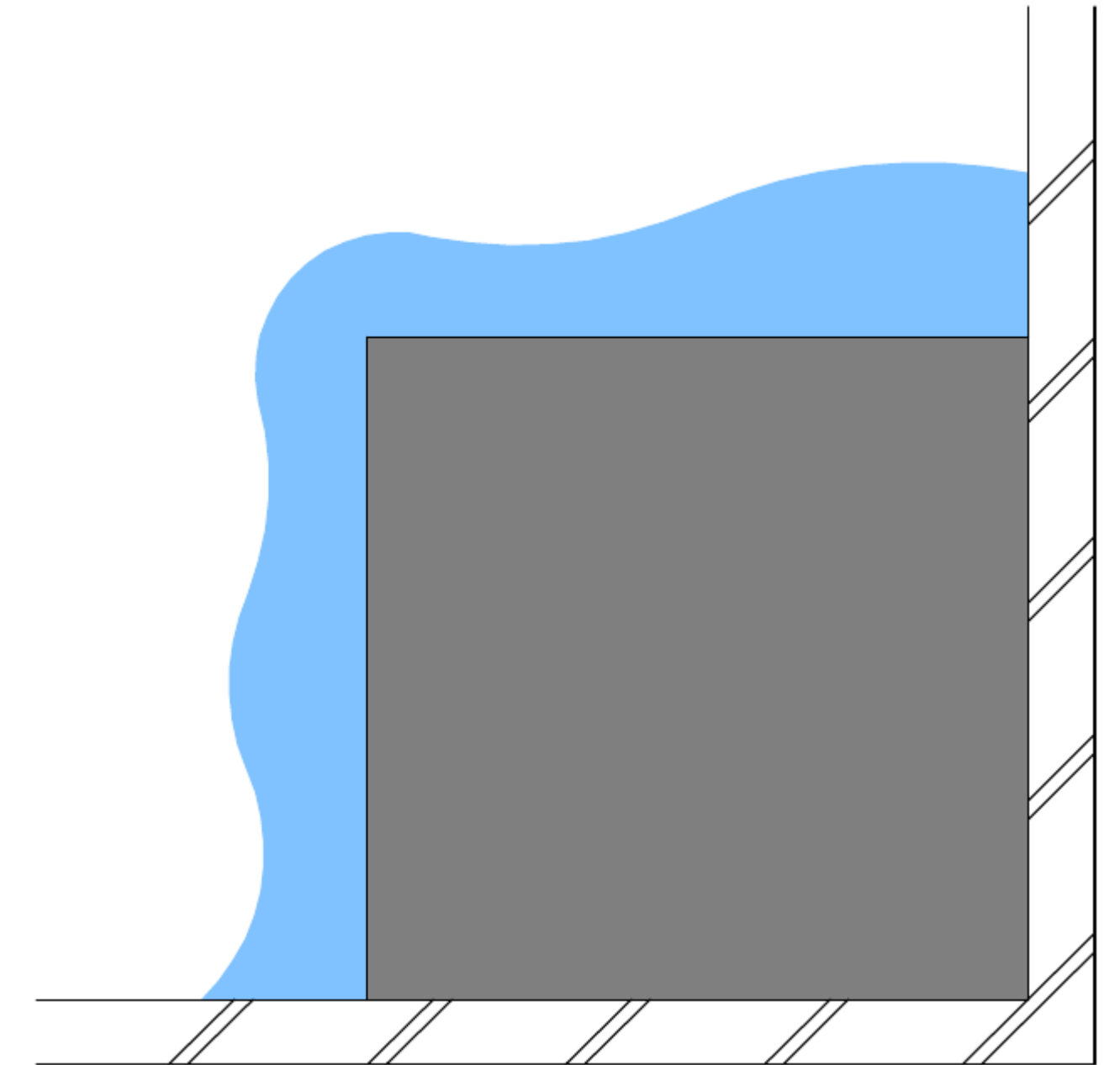




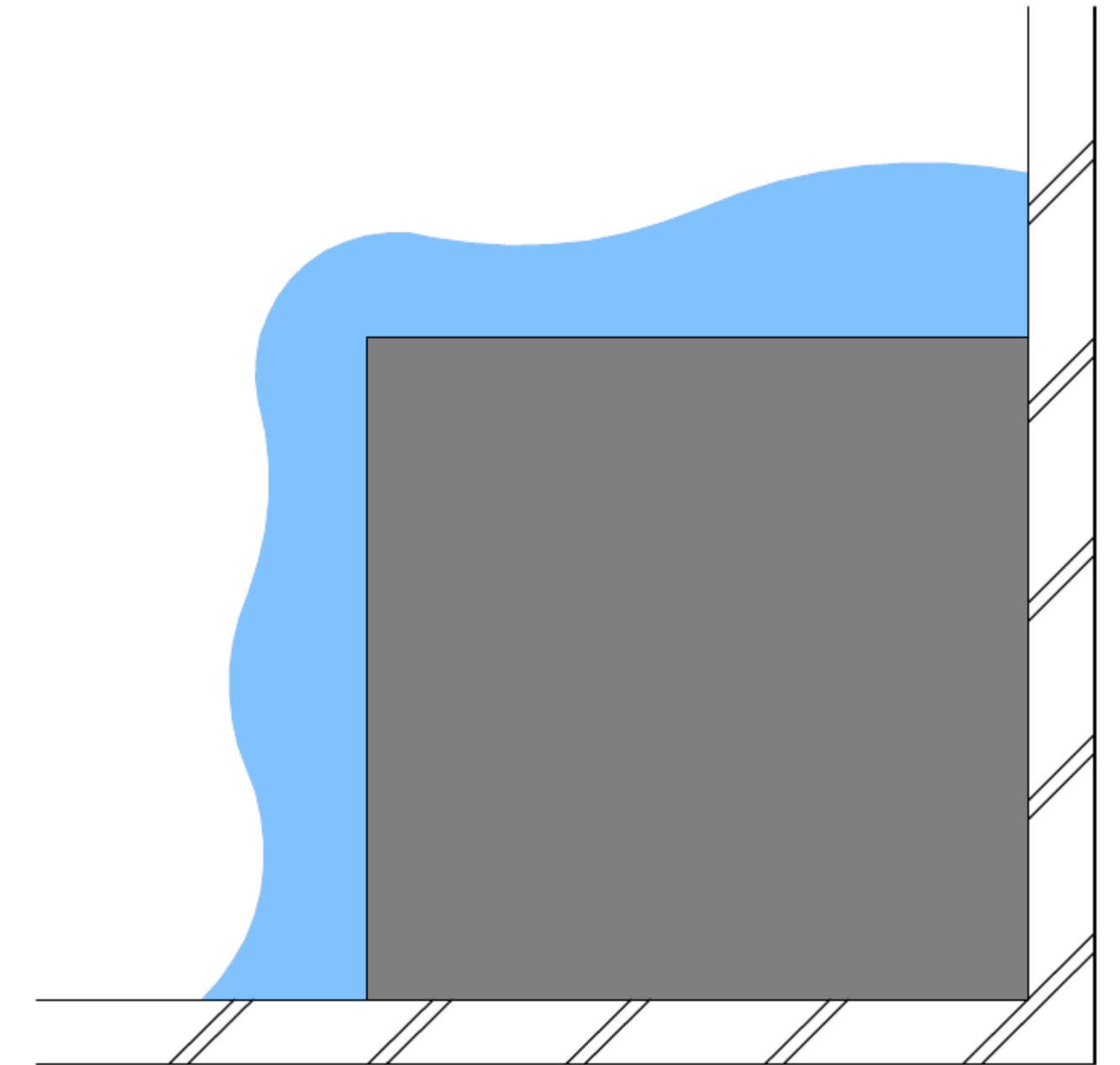
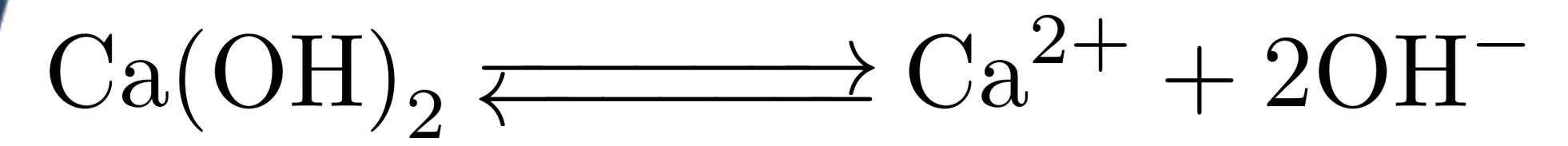
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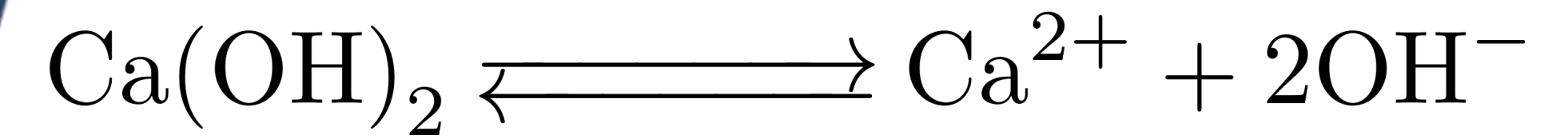


# CH dissolution

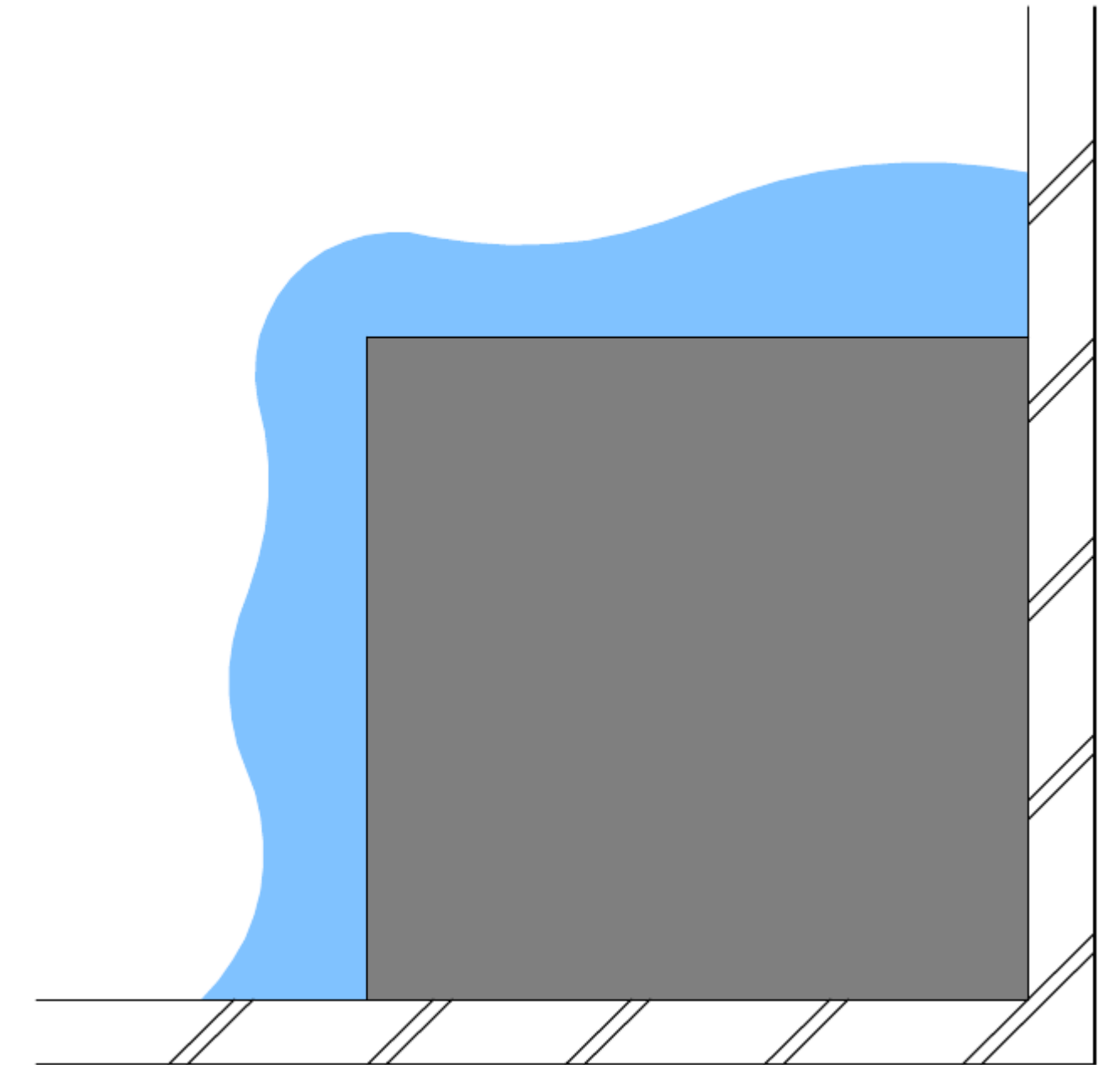




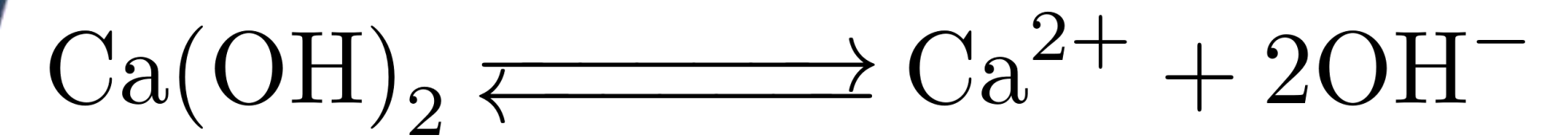
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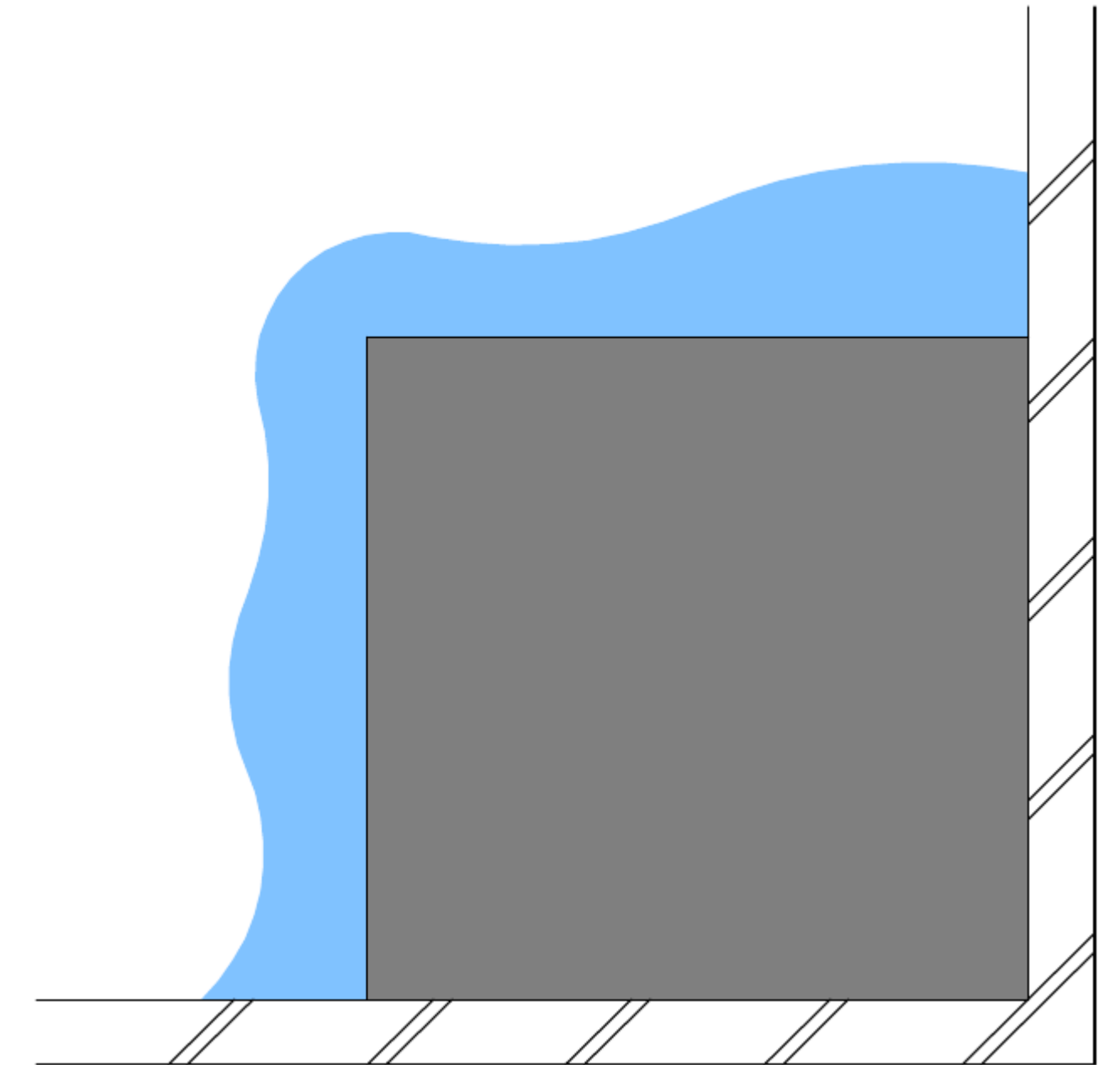
$$\begin{aligned} \frac{\partial[\text{CH}]}{\partial t} &= +k_{\text{CH}} ([\text{Ca}^{2+}][\text{OH}^-]^2 - K_{\text{CH}}) \\ \frac{\partial[\text{Ca}^{2+}]}{\partial t} &= D_0 \nabla^2 [\text{Ca}^{2+}] - k_{\text{CH}} ([\text{Ca}^{2+}][\text{OH}^-]^2 - K_{\text{CH}}) \\ \frac{\partial[\text{OH}^-]}{\partial t} &= D_0 \nabla^2 [\text{OH}^-] - 2k_{\text{CH}} ([\text{Ca}^{2+}][\text{OH}^-]^2 - K_{\text{CH}}) \end{aligned}$$

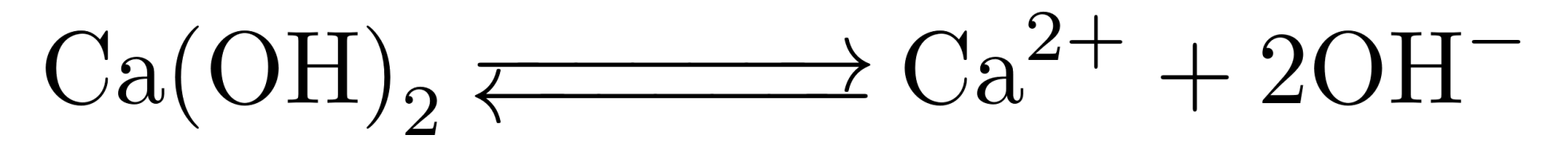


# CH dissolution

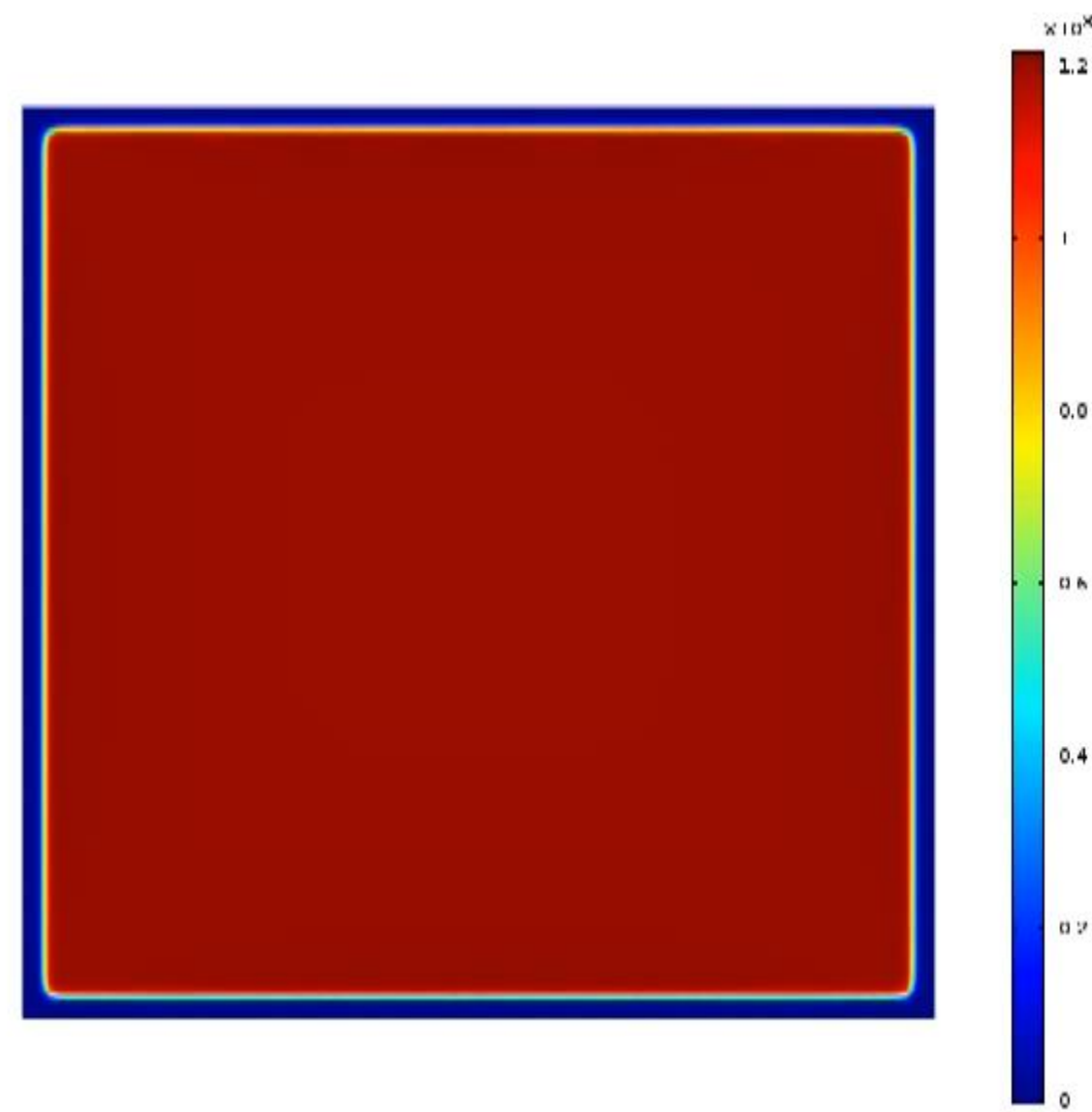


$$\begin{aligned} \frac{\partial[CH]}{\partial t} &= +k_{CH}Q_{CH} \\ \frac{\partial[Ca^{2+}]}{\partial t} &= D_0 \nabla^2 [Ca^{2+}] - k_{CH}Q_{CH} \\ \frac{\partial[OH^-]}{\partial t} &= D_0 \nabla^2 [OH^-] - 2k_{CH}Q_{CH} \end{aligned}$$

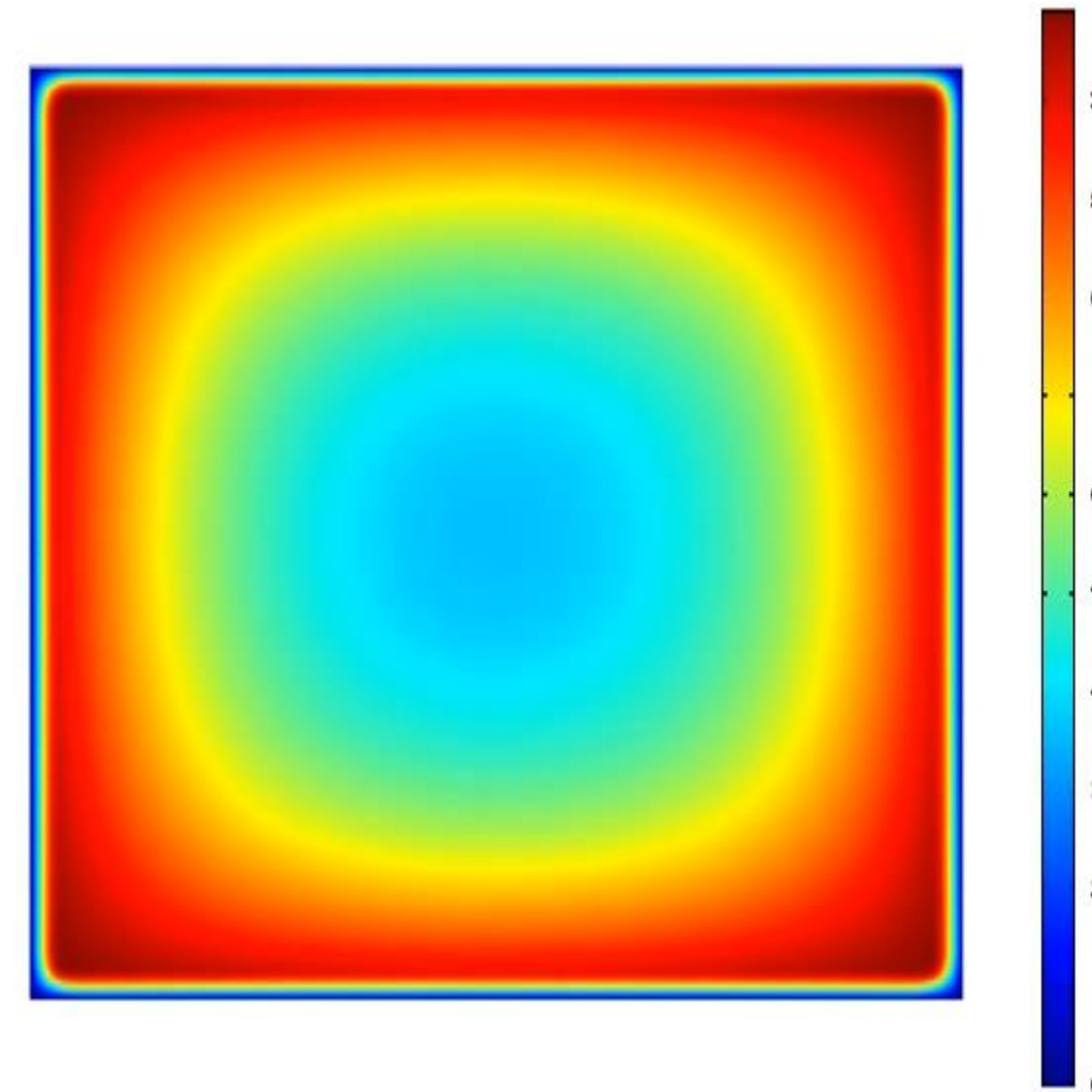




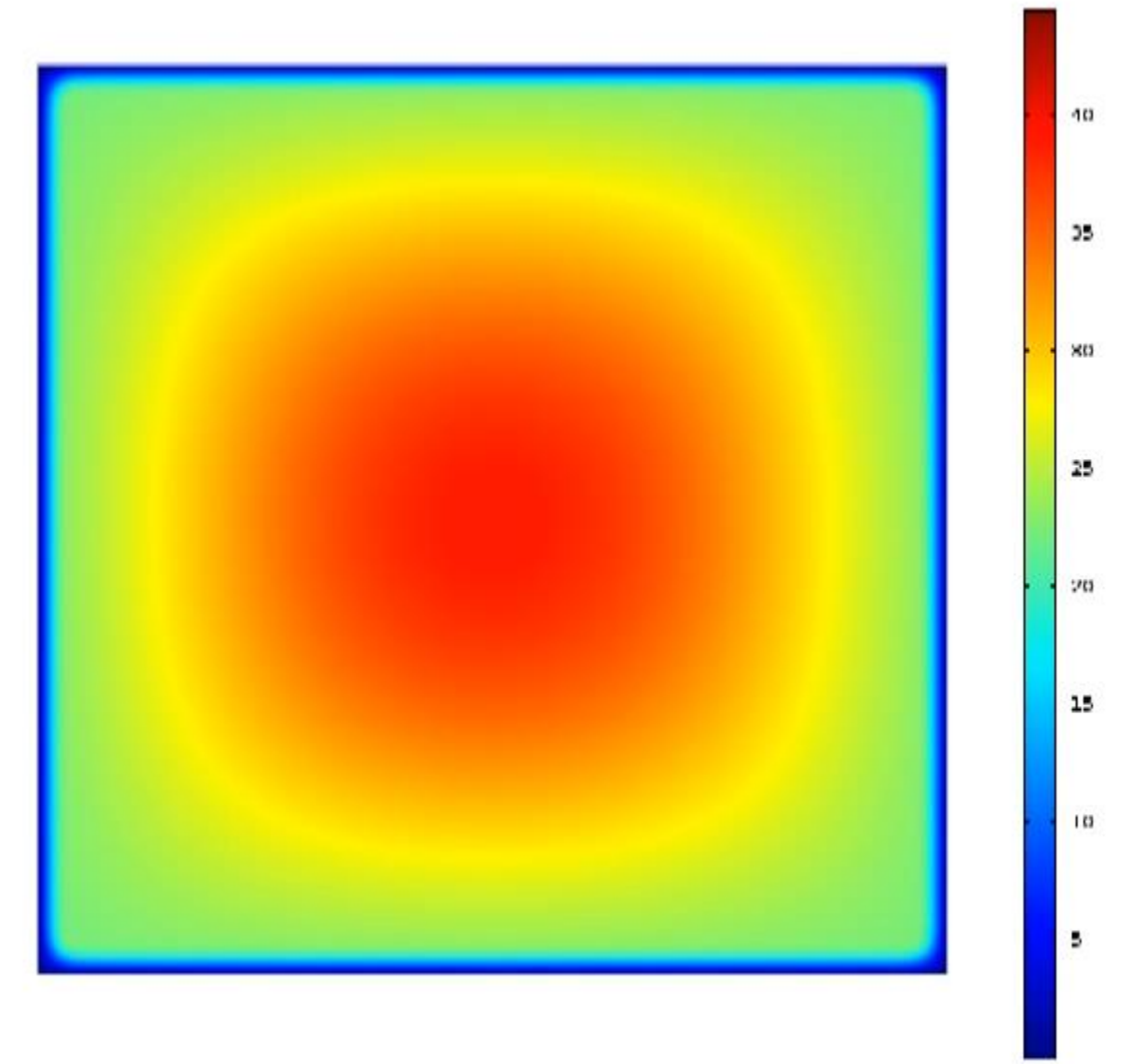
CH (Calcium Hydroxide)



Ca (Calcium)

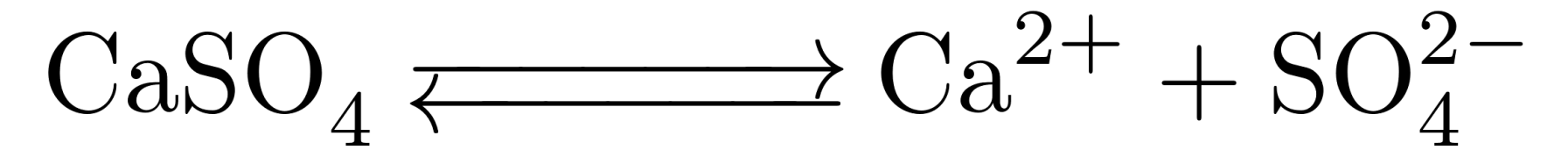
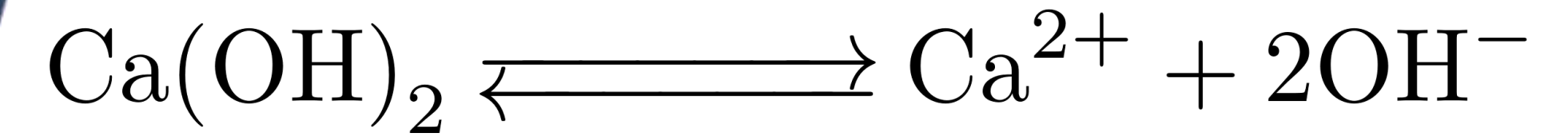


OH (Hydroxide)

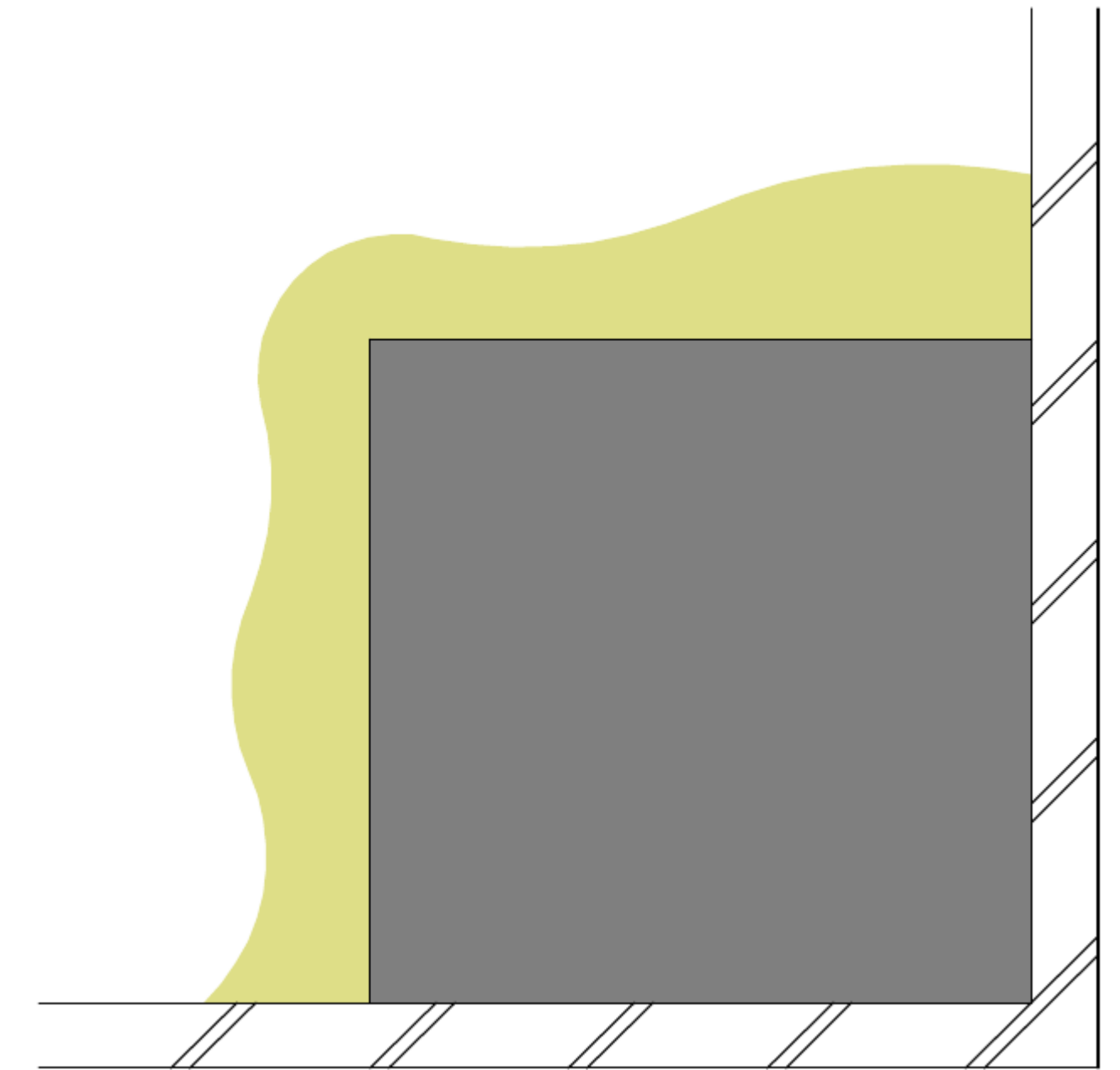




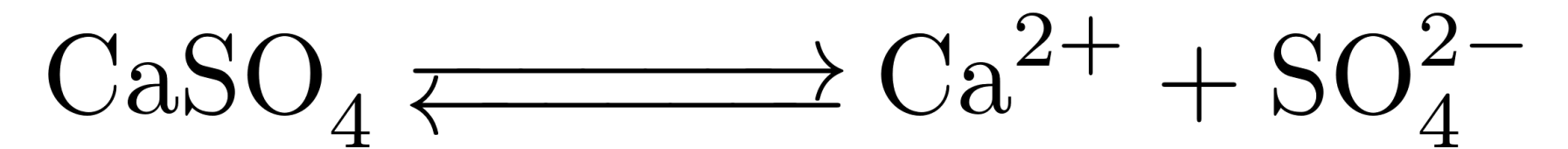
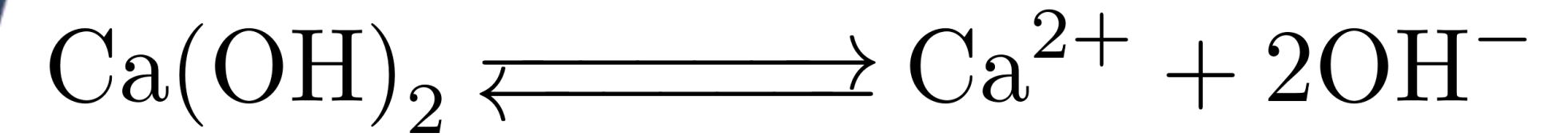
# Gypsum formation



$$\begin{aligned} \frac{\partial[CH]}{\partial t} &= +k_{CH}Q_{CH} \\ \frac{\partial[CaSO_4]}{\partial t} &= +k_{CaSO_4}Q_{CaSO_4} \\ \frac{\partial[Ca^{2+}]}{\partial t} &= D_0\nabla^2[Ca^{2+}] - k_{CH}Q_{CH} - k_{CaSO_4}Q_{CaSO_4} \\ \frac{\partial[OH^-]}{\partial t} &= D_0\nabla^2[OH^-] - 2k_{CH}Q_{CH} \\ \frac{\partial[SO_4^{2-}]}{\partial t} &= D_0\nabla^2[SO_4^{2-}] - k_{CaSO_4}Q_{CaSO_4} \end{aligned}$$



# Gypsum formation



$$\frac{\partial [CH]}{\partial t} = + k_{CH} ([Ca^{2+}][OH^-]^2 - K_{CH})$$

$$\frac{\partial [CaSO_4]}{\partial t} =$$

$$\frac{\partial [Ca^{2+}]}{\partial t} = D_0 \nabla^2 [Ca^{2+}] - k_{CH} ([Ca^{2+}][OH^-]^2 - K_{CH})$$

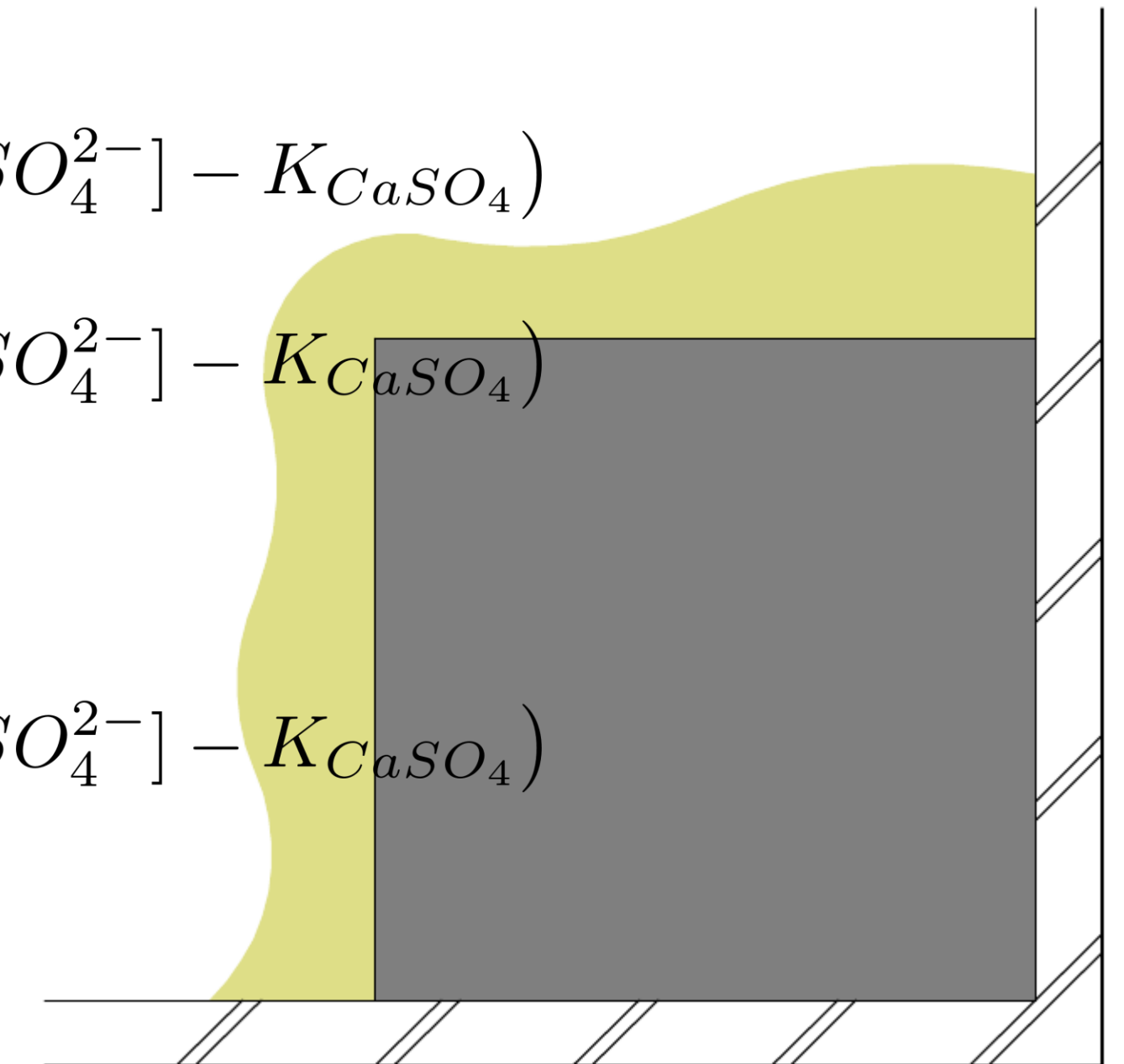
$$\frac{\partial [OH^-]}{\partial t} = D_0 \nabla^2 [OH^-] - 2k_{CH} ([Ca^{2+}][OH^-]^2 - K_{CH})$$

$$\frac{\partial [SO_4^{2-}]}{\partial t} = D_0 \nabla^2 [SO_4^{2-}]$$

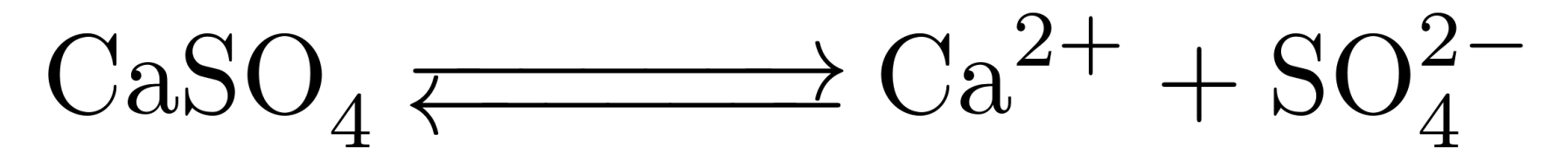
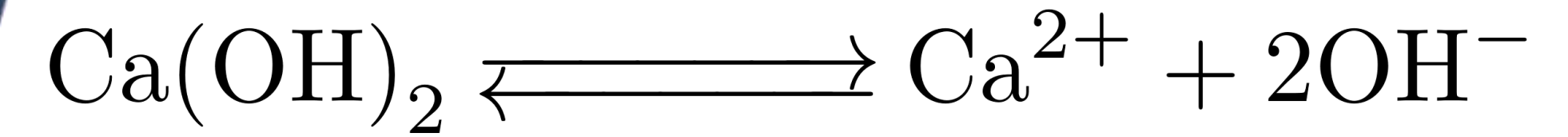
$$+ k_{CaSO_4} ([Ca^{2+}][SO_4^{2-}] - K_{CaSO_4})$$

$$- k_{CaSO_4} ([Ca^{2+}][SO_4^{2-}] - K_{CaSO_4})$$

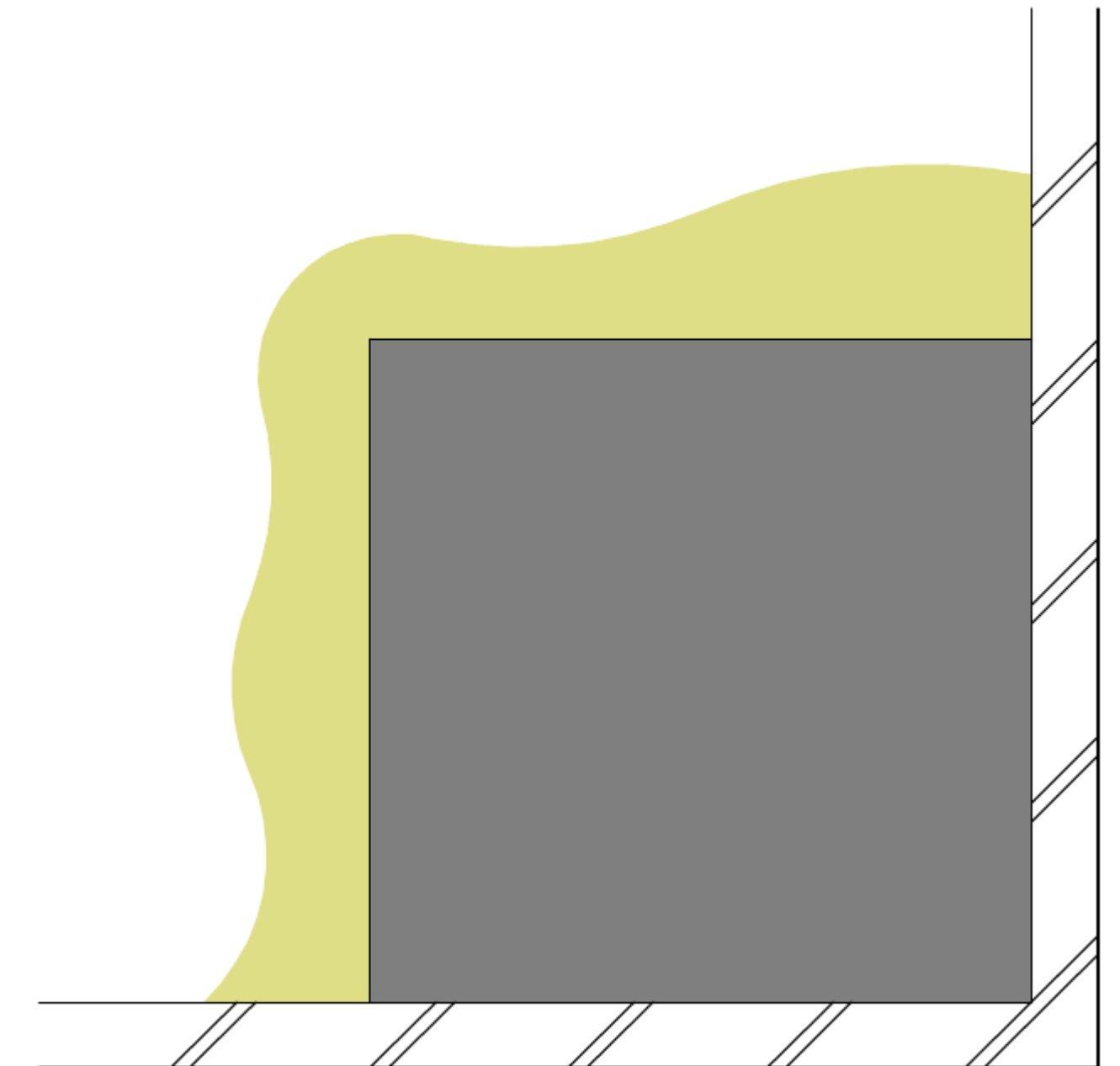
$$- k_{CaSO_4} ([Ca^{2+}][SO_4^{2-}] - K_{CaSO_4})$$



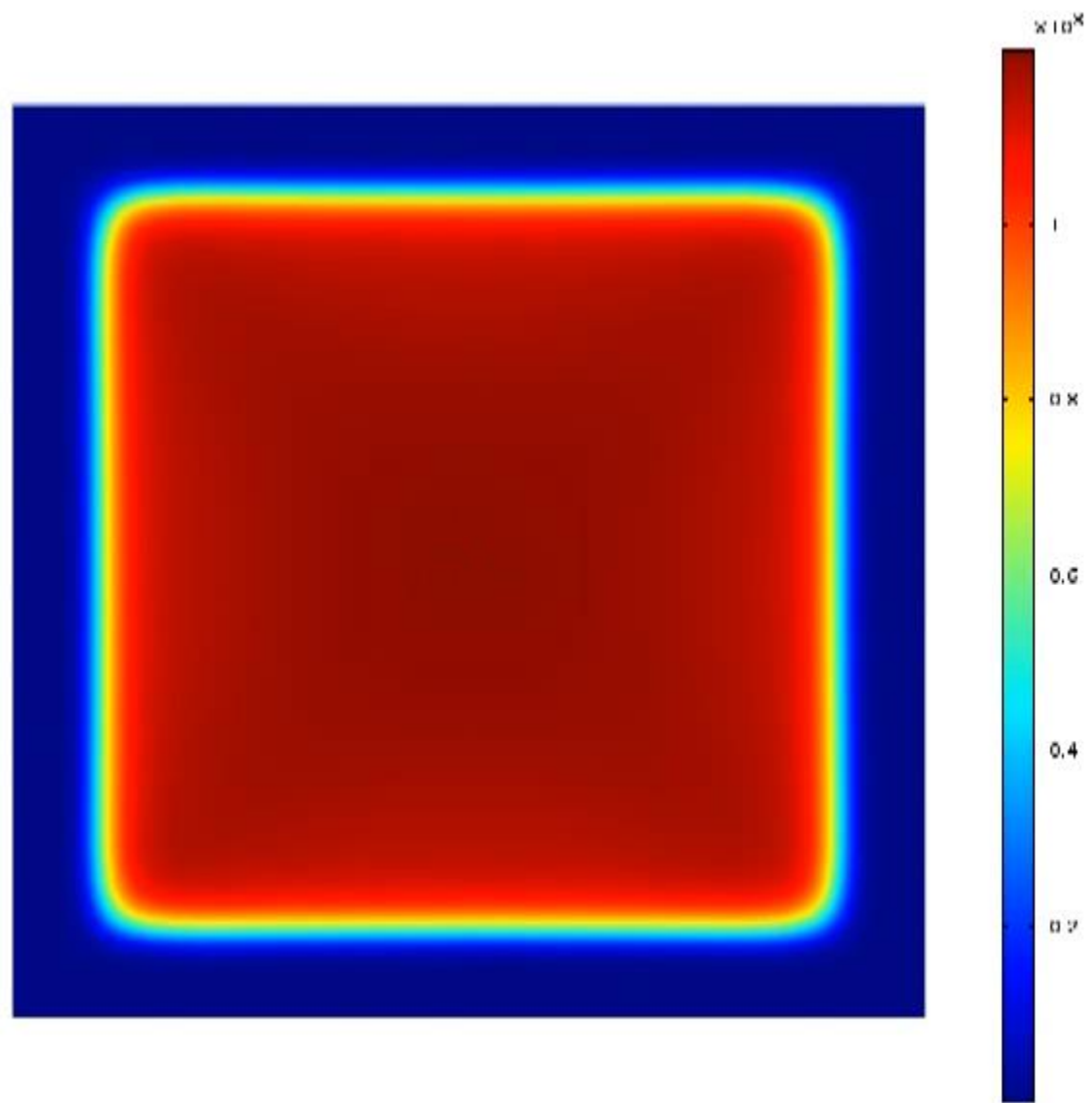
# Gypsum formation



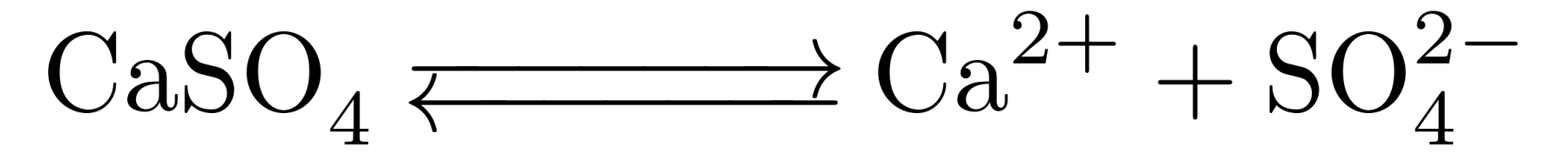
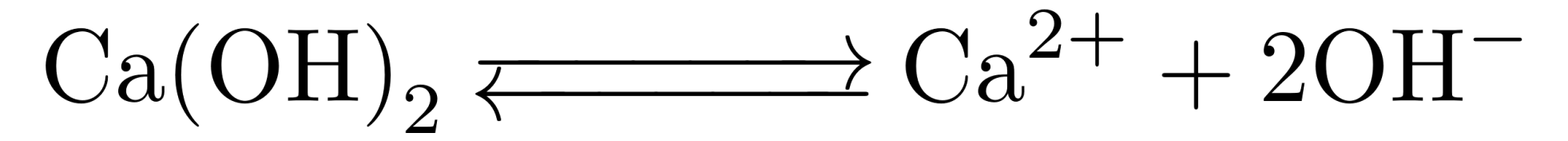
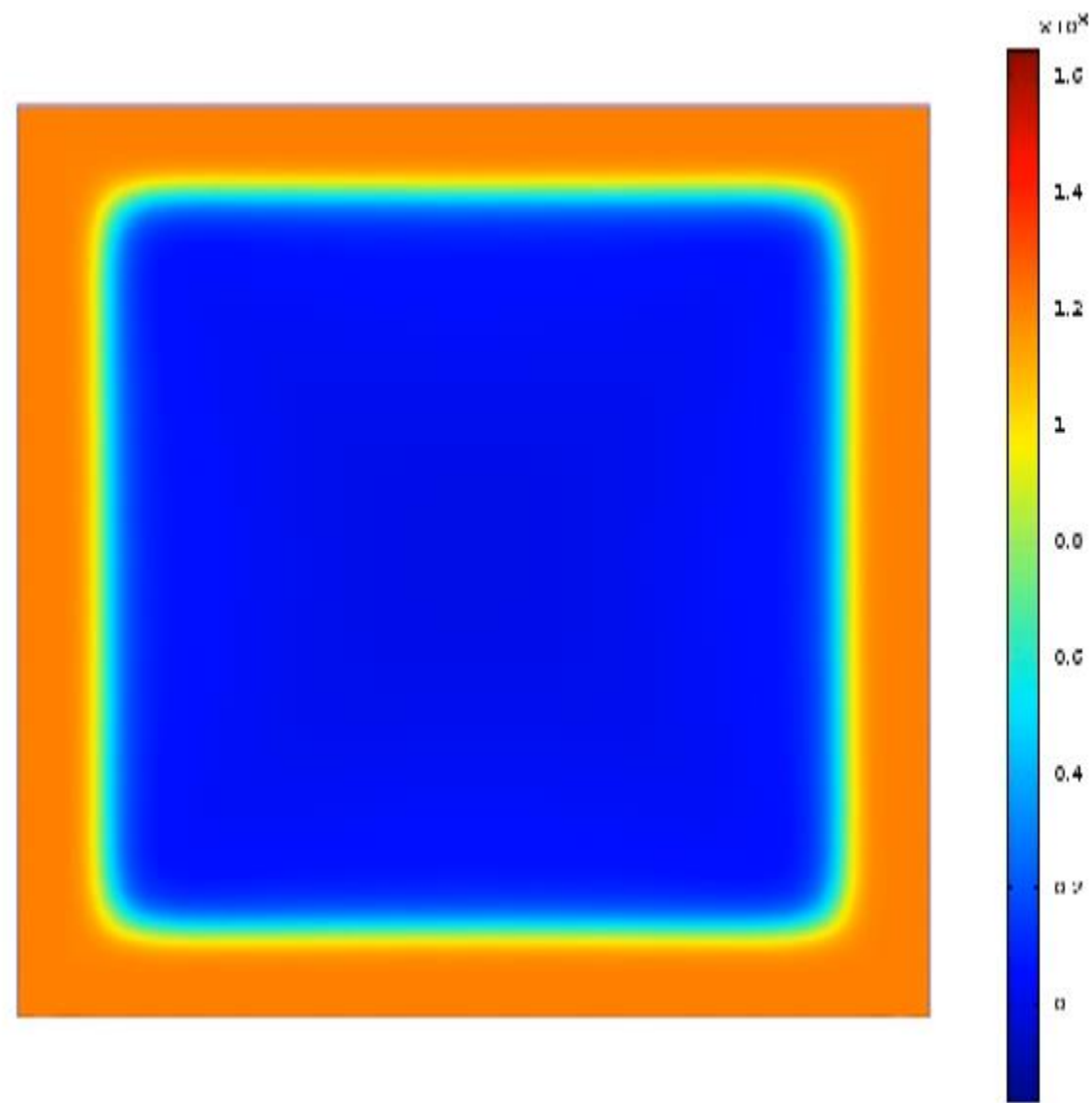
$$\begin{aligned} \frac{\partial[CH]}{\partial t} &= + k_{CH}Q_{CH} \\ \frac{\partial[CaSO_4]}{\partial t} &= + k_{CaSO_4}Q_{CaSO_4} \\ \frac{\partial[Ca^{2+}]}{\partial t} &= D_0 \nabla^2 [Ca^{2+}] - k_{CH}Q_{CH} - k_{CaSO_4}Q_{CaSO_4} \\ \frac{\partial[OH^-]}{\partial t} &= D_0 \nabla^2 [OH^-] - 2k_{CH}Q_{CH} \\ \frac{\partial[SO_4^{2-}]}{\partial t} &= D_0 \nabla^2 [SO_4^{2-}] - k_{CaSO_4}Q_{CaSO_4} \end{aligned}$$



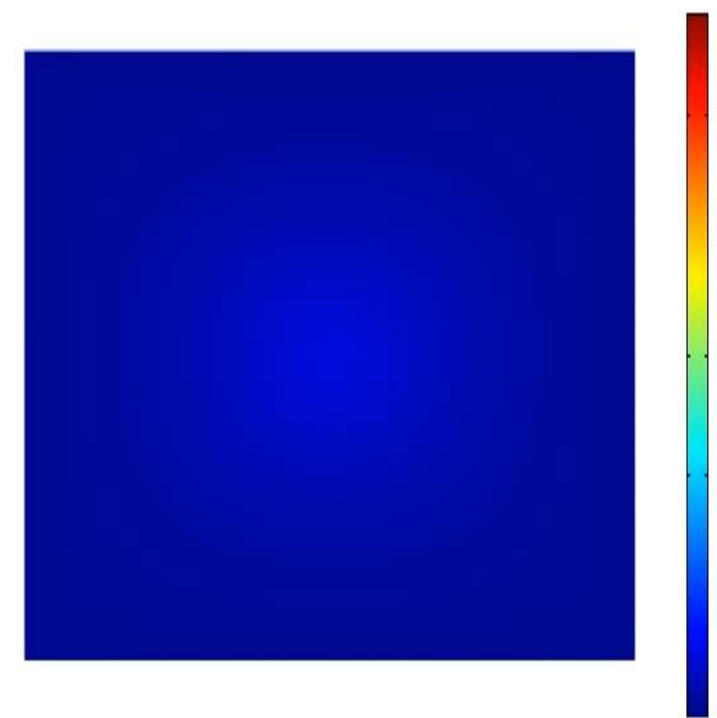
CH (Calcium Hydroxide)



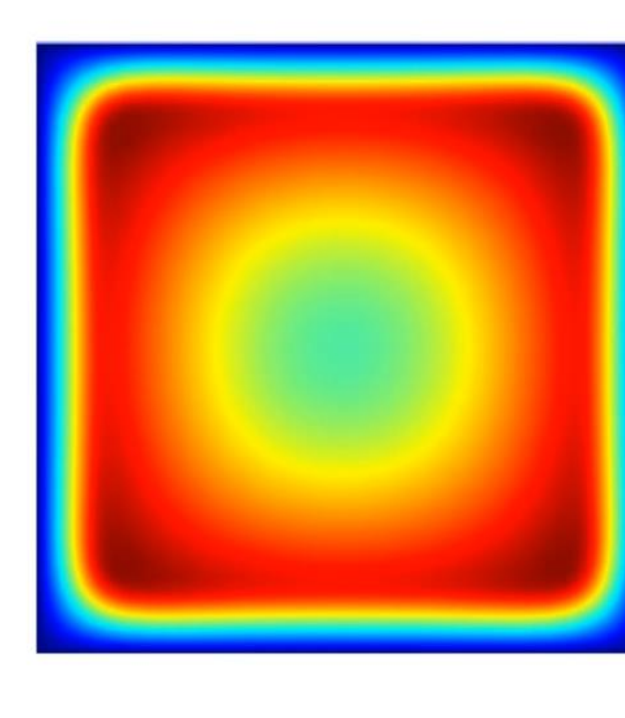
CaSO4 (Gypsum)



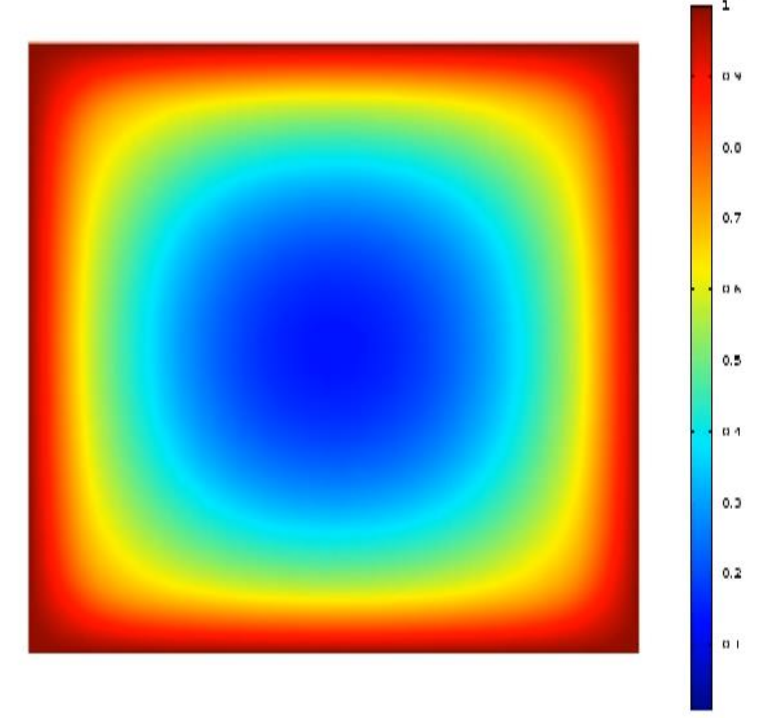
Ca



OH

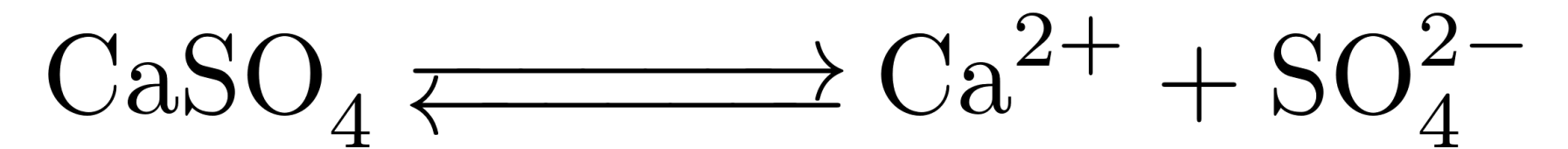
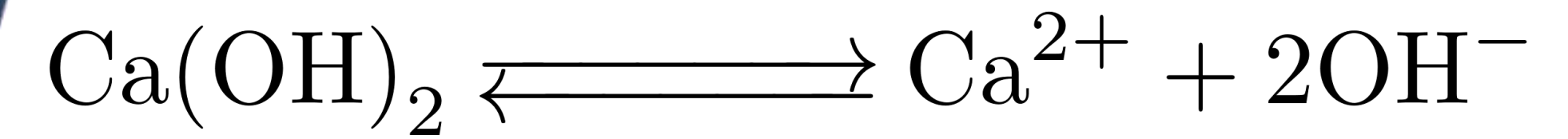


SO4



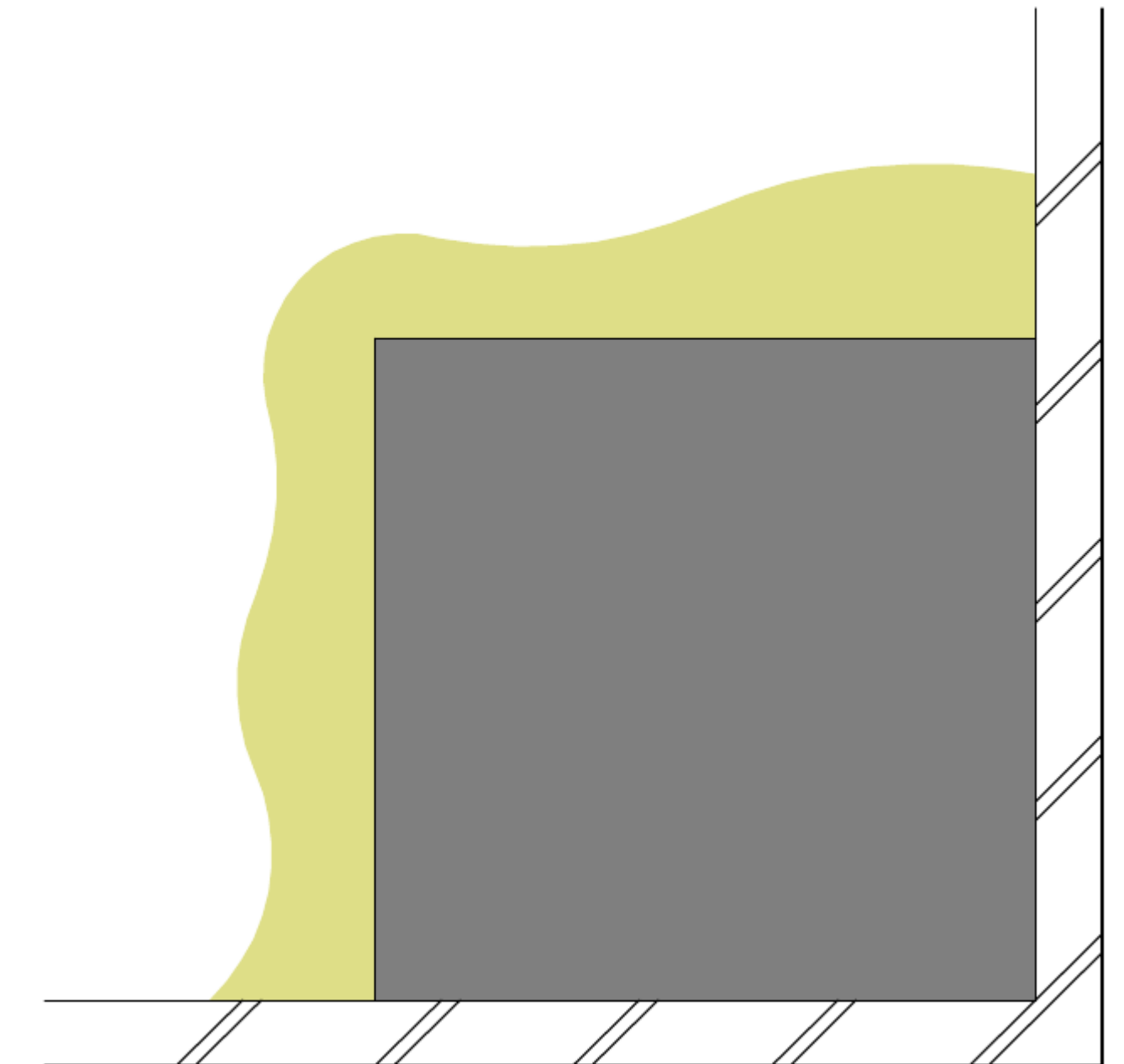


# Linear Elastic Mechanics

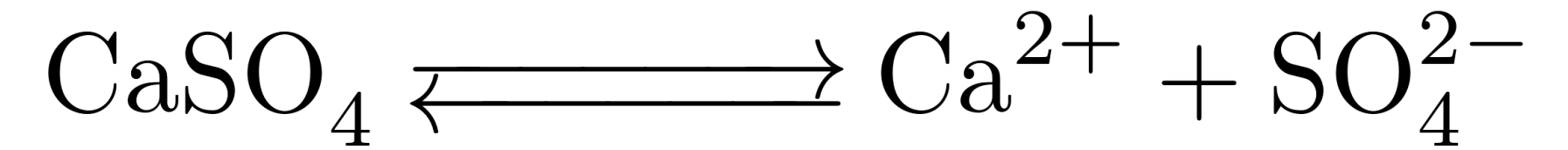
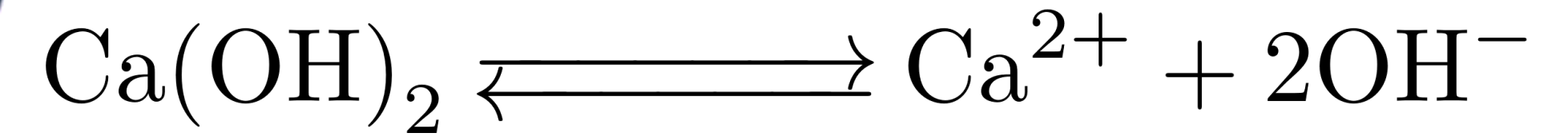


$$\begin{aligned} \frac{\partial[CH]}{\partial t} &= +k_{CH}Q_{CH} \\ \frac{\partial[CaSO_4]}{\partial t} &= +k_{CaSO_4}Q_{CaSO_4} \\ \frac{\partial[Ca^{2+}]}{\partial t} &= D_0 \nabla^2 [Ca^{2+}] - k_{CH}Q_{CH} - k_{CaSO_4}Q_{CaSO_4} \\ \frac{\partial[OH^-]}{\partial t} &= D_0 \nabla^2 [OH^-] - 2k_{CH}Q_{CH} \\ \frac{\partial[SO_4^{2-}]}{\partial t} &= D_0 \nabla^2 [SO_4^{2-}] - k_{CaSO_4}Q_{CaSO_4} \end{aligned}$$

$$\begin{aligned} \boldsymbol{\sigma} &= {}^4\mathbb{C}_0 \boldsymbol{\varepsilon}^e \\ \text{where: } \boldsymbol{\varepsilon}^e &= \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^g \\ &= \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^g [CaSO_4] \mathbf{I} \end{aligned}$$

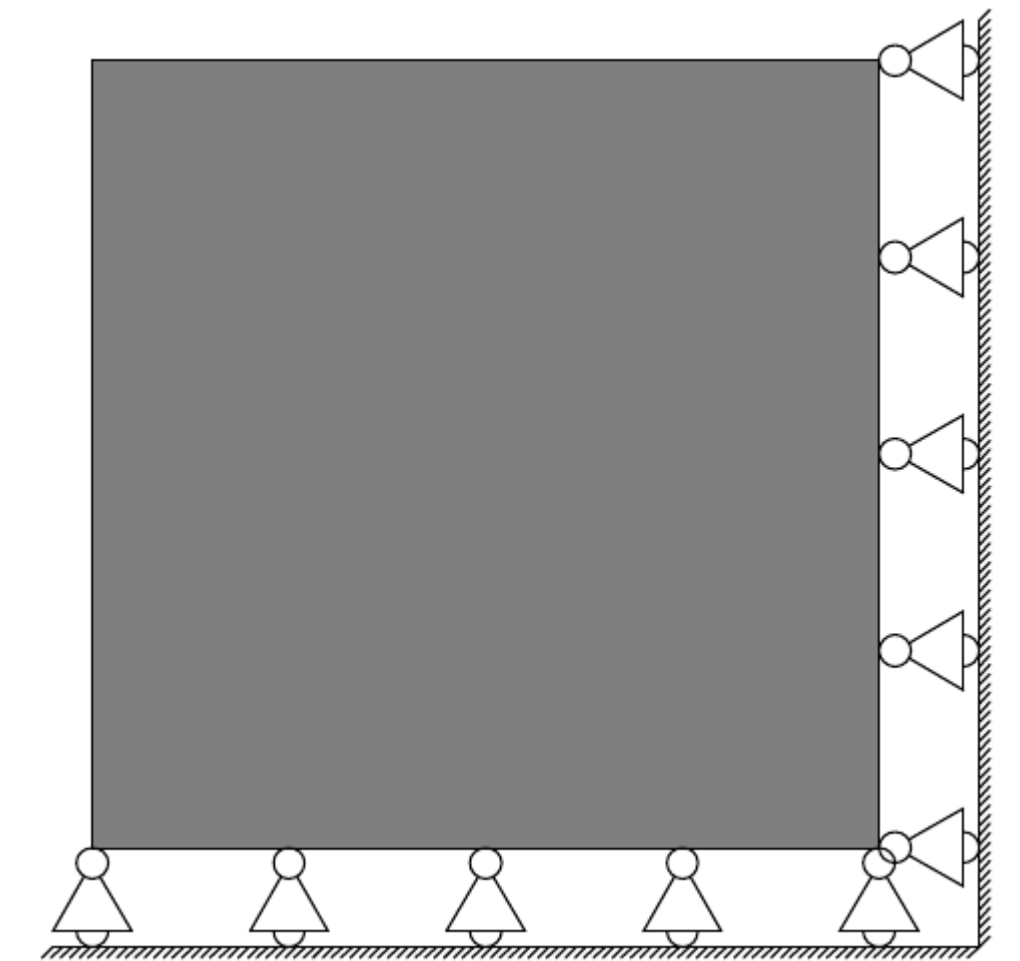


# Linear Elastic Mechanics

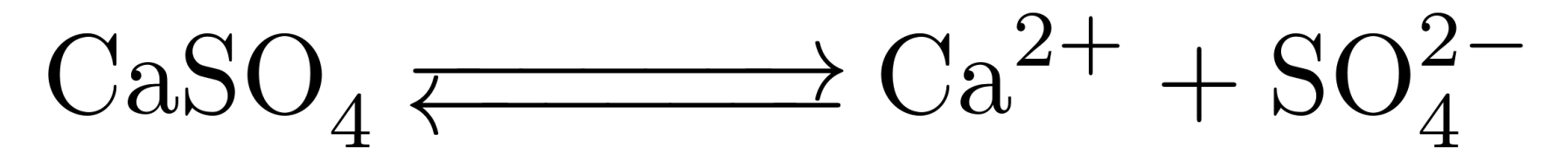
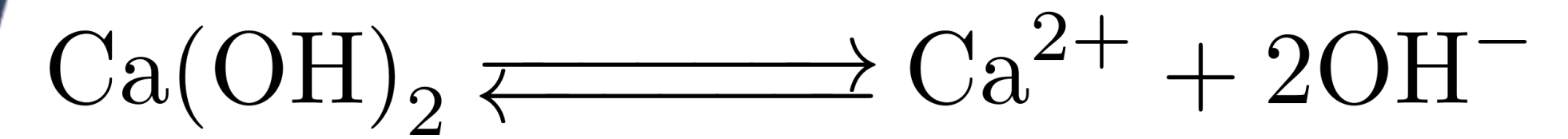


$$\begin{aligned} \frac{\partial[CH]}{\partial t} &= + k_{CH}Q_{CH} \\ \frac{\partial[CaSO_4]}{\partial t} &= + k_{CaSO_4}Q_{CaSO_4} \\ \frac{\partial[Ca^{2+}]}{\partial t} &= D_0 \nabla^2 [Ca^{2+}] - k_{CH}Q_{CH} - k_{CaSO_4}Q_{CaSO_4} \\ \frac{\partial[OH^-]}{\partial t} &= D_0 \nabla^2 [OH^-] - 2k_{CH}Q_{CH} \\ \frac{\partial[SO_4^{2-}]}{\partial t} &= D_0 \nabla^2 [SO_4^{2-}] - k_{CaSO_4}Q_{CaSO_4} \end{aligned}$$

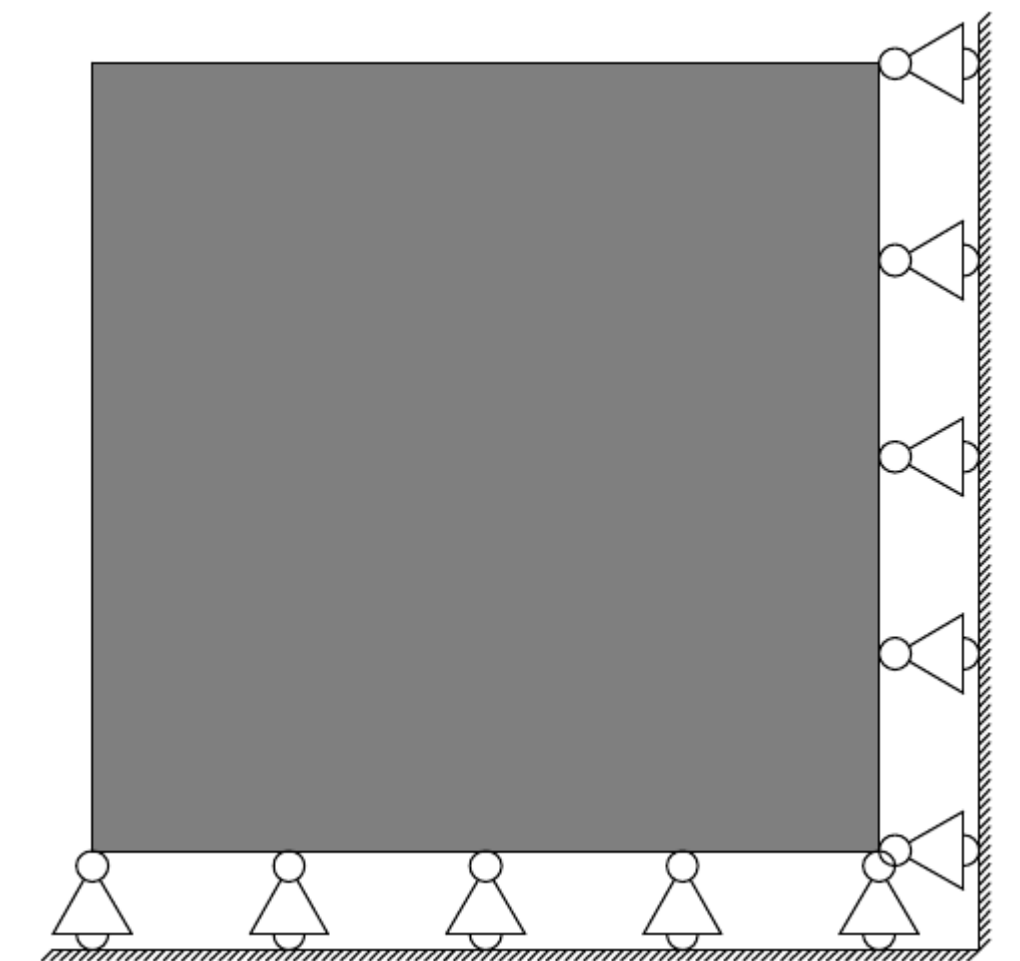
$$\begin{aligned} \boldsymbol{\sigma} &= {}^4\mathbb{C}_0 \boldsymbol{\varepsilon}^e \\ \text{where: } \boldsymbol{\varepsilon}^e &= \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^g \\ &= \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^g [CaSO_4] \mathbf{I} \end{aligned}$$



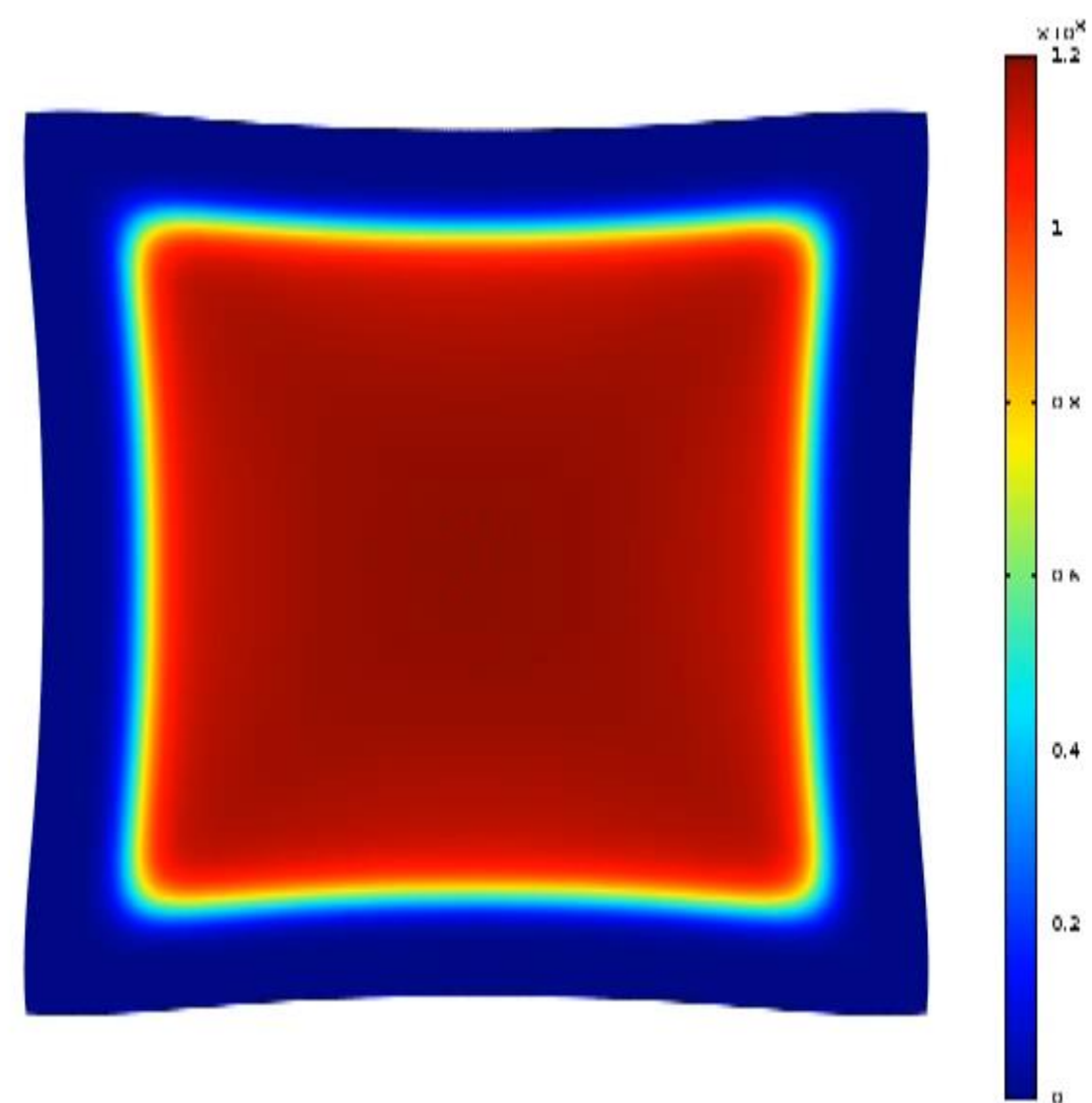
# Linear Elastic Mechanics



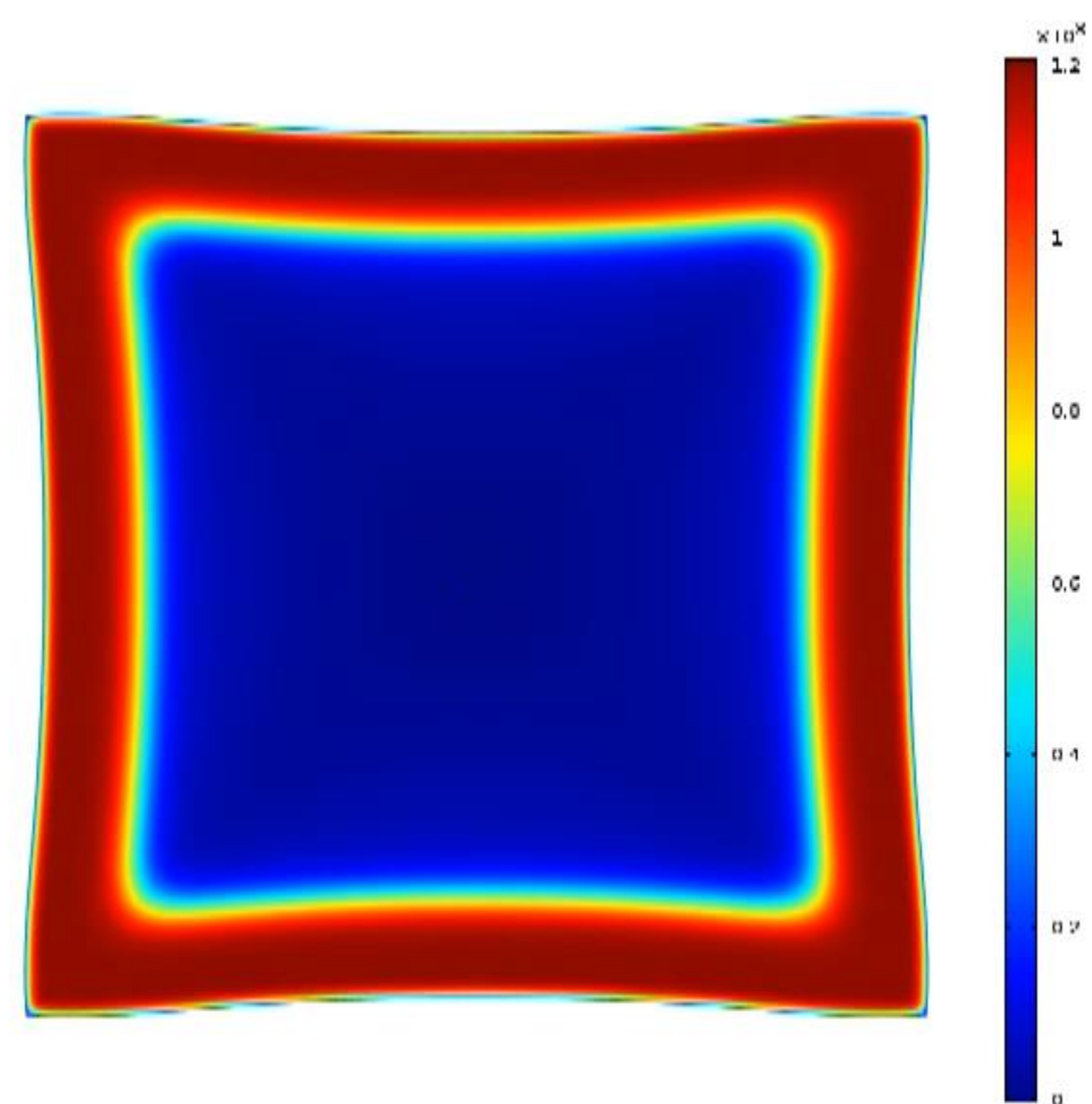
$$\begin{aligned} \frac{\partial[CH]}{\partial t} &= + k_{CH}Q_{CH} \\ \frac{\partial[CaSO_4]}{\partial t} &= + k_{CaSO_4}Q_{CaSO_4} \\ \frac{\partial[Ca^{2+}]}{\partial t} &= D_0 \nabla^2 [Ca^{2+}] - k_{CH}Q_{CH} - k_{CaSO_4}Q_{CaSO_4} \\ \frac{\partial[OH^-]}{\partial t} &= D_0 \nabla^2 [OH^-] - 2k_{CH}Q_{CH} \\ \frac{\partial[SO_4^{2-}]}{\partial t} &= D_0 \nabla^2 [SO_4^{2-}] - k_{CaSO_4}Q_{CaSO_4} \\ \boldsymbol{\sigma} &= {}^4\mathbb{C}_0 (\boldsymbol{\varepsilon} - \varepsilon^g [CaSO_4] \mathbf{I}) \end{aligned}$$





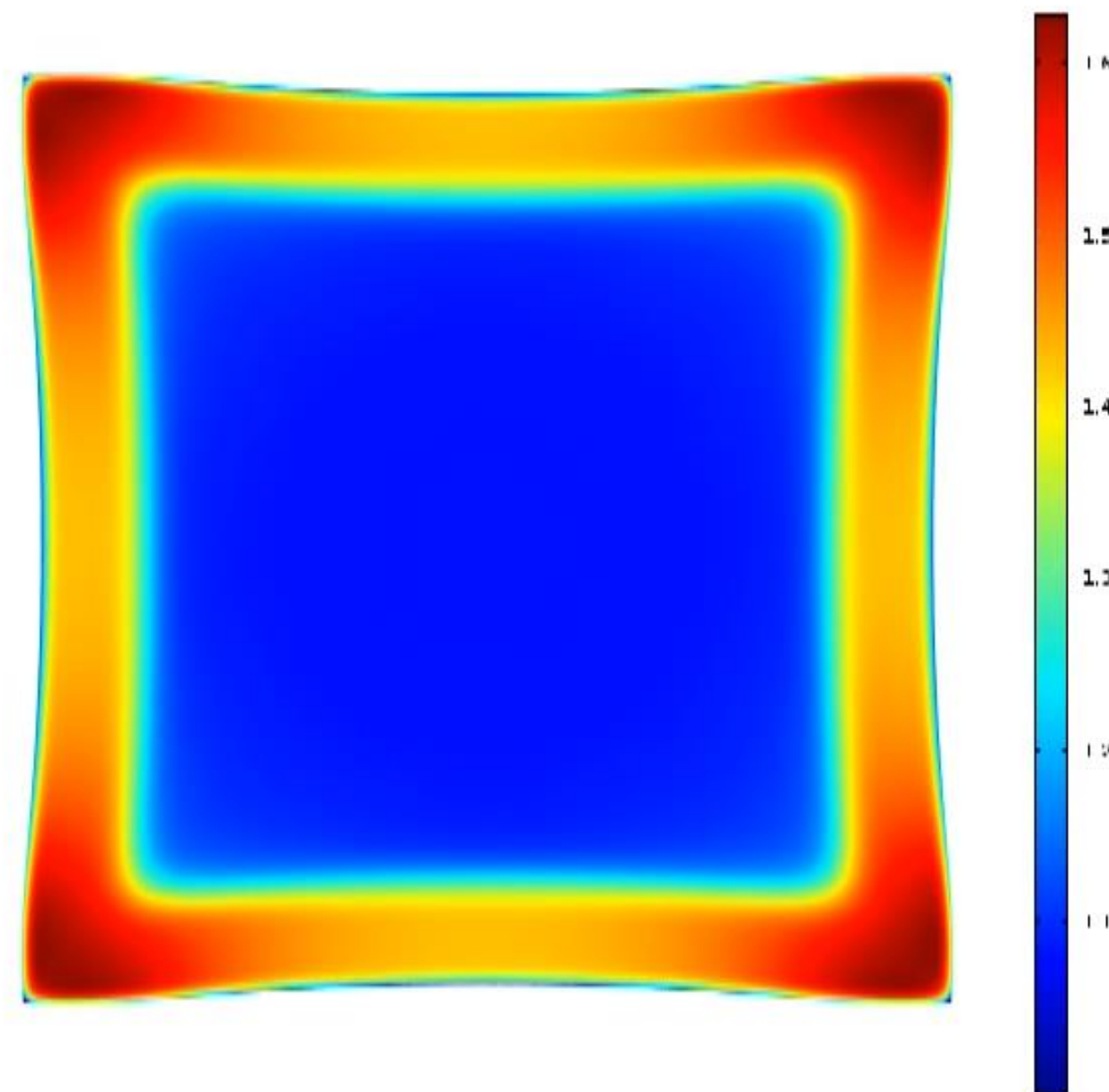


CH

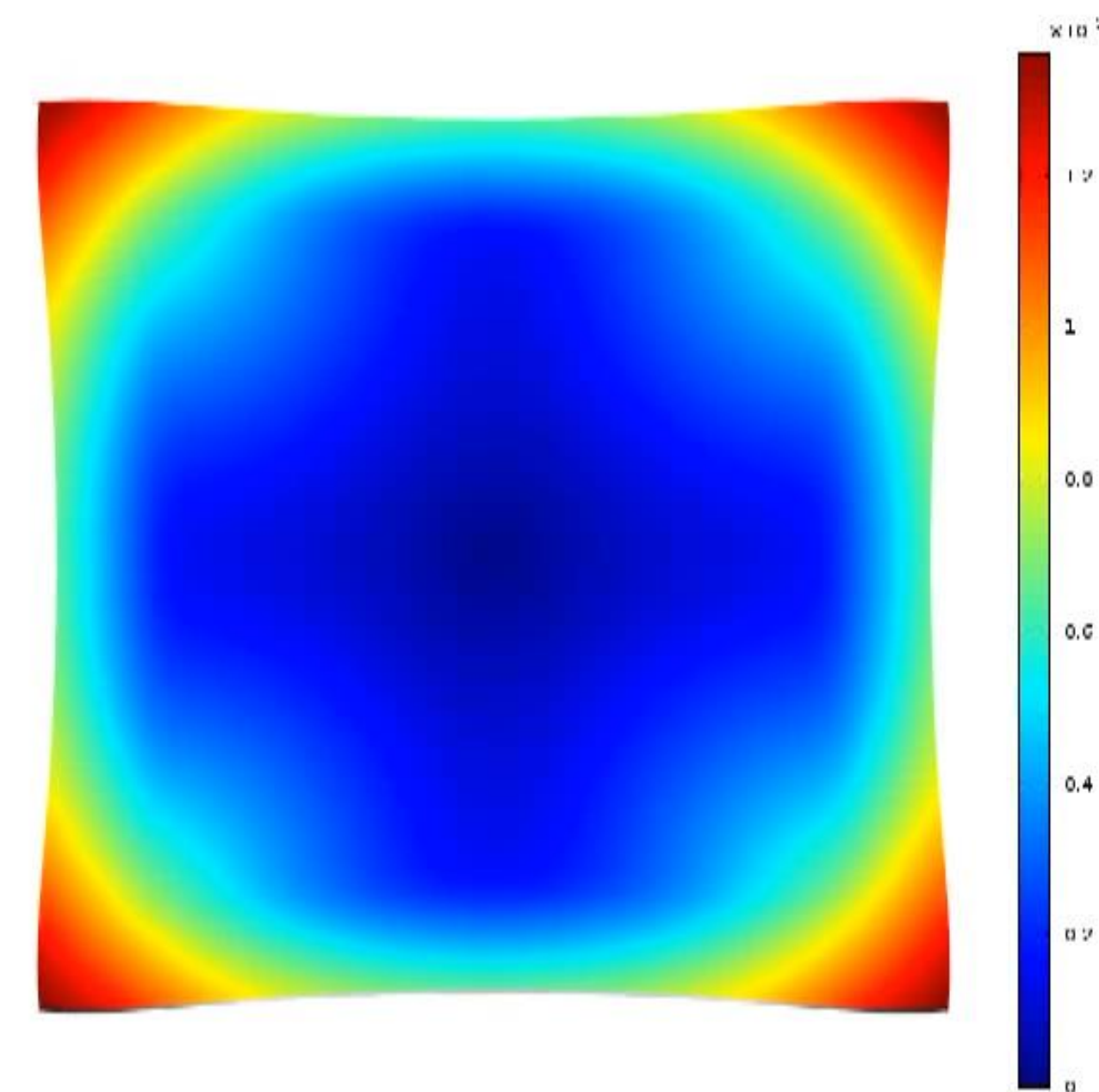


CaSO4

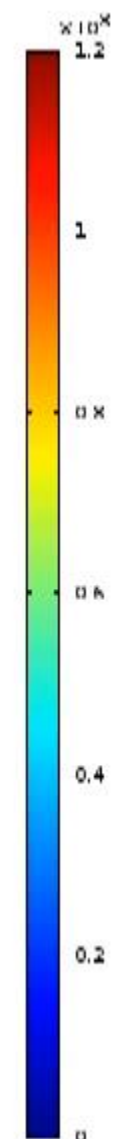
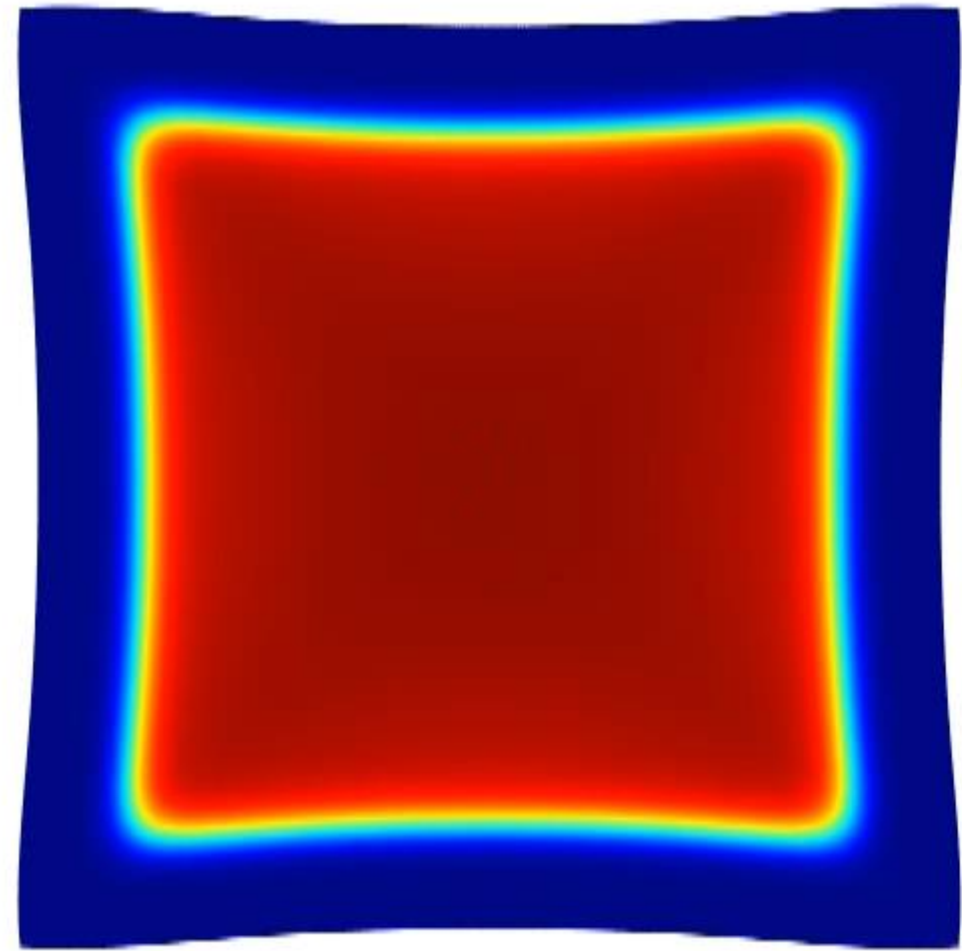
Vol. Strain



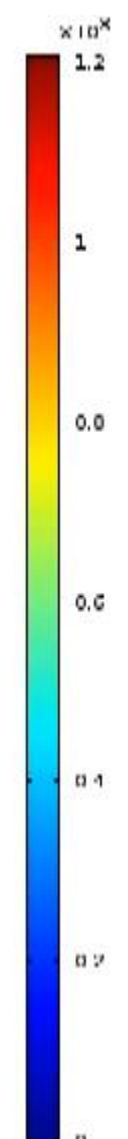
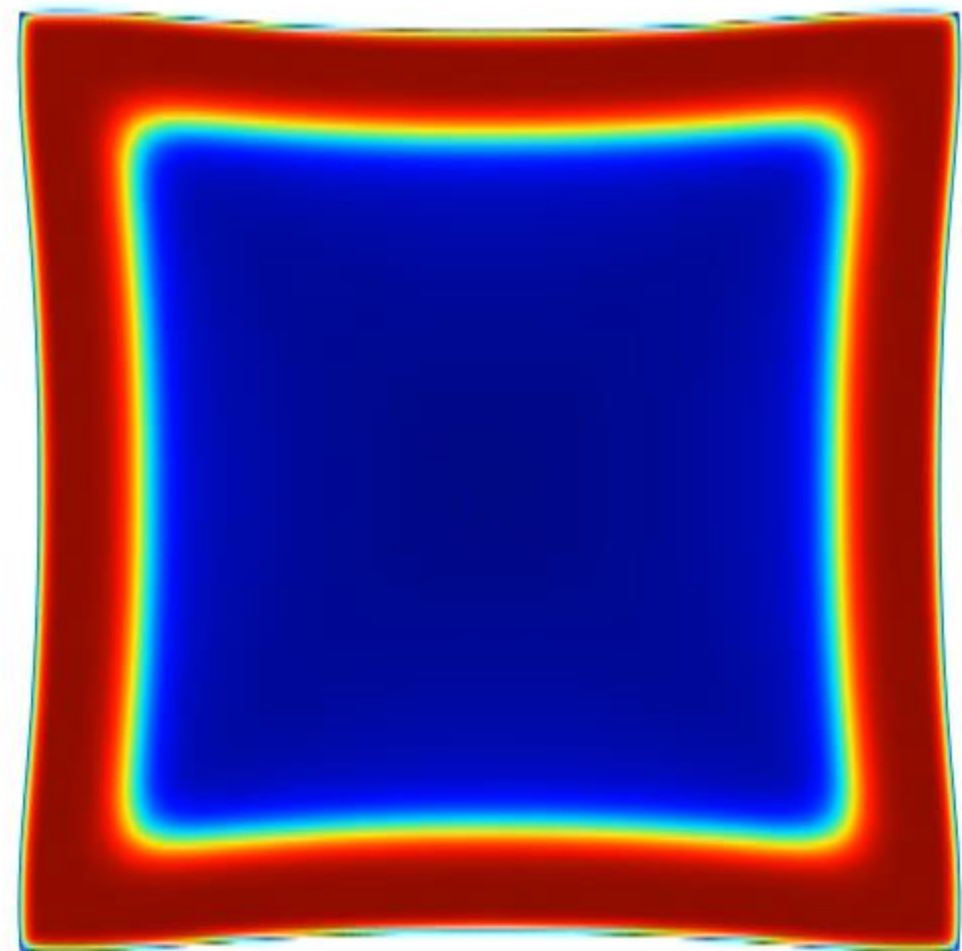
Displacements





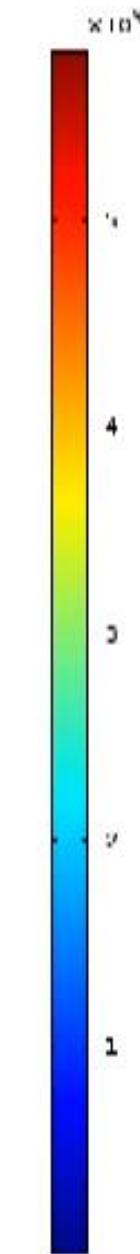
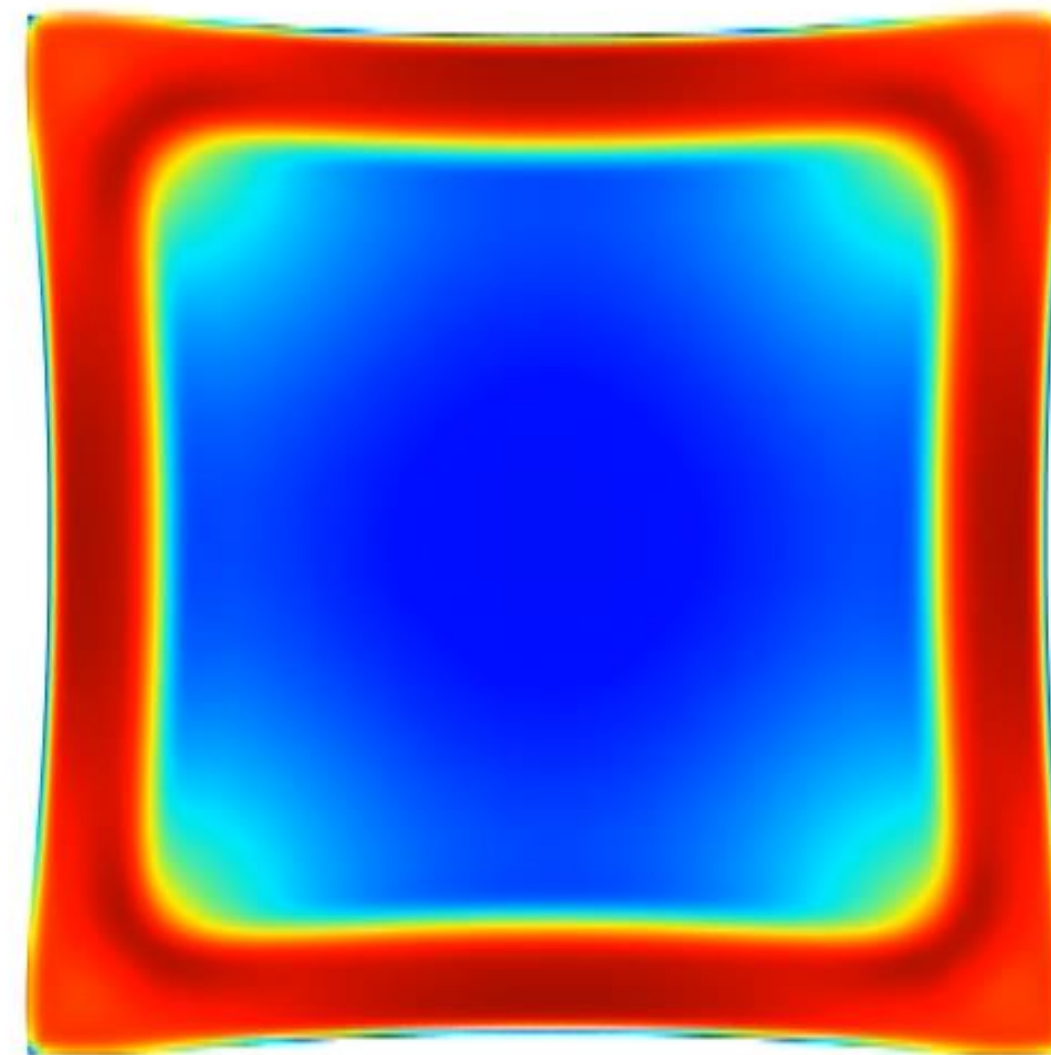


CH

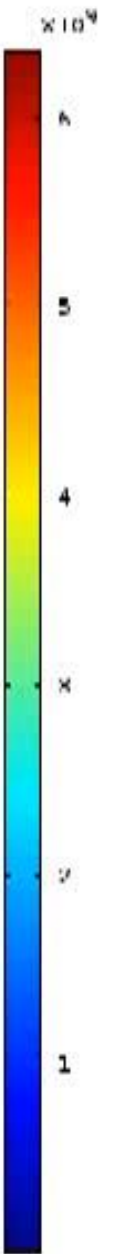
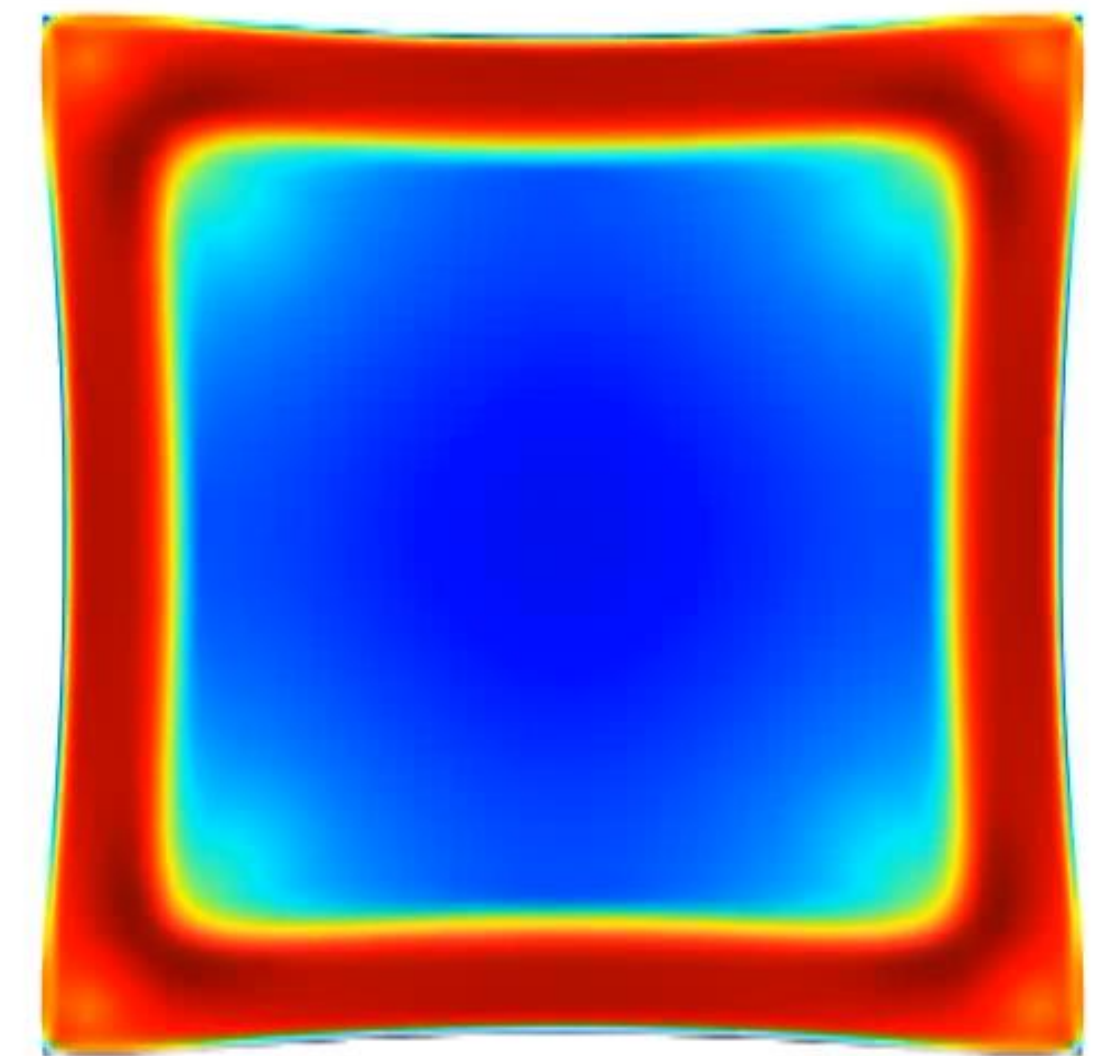


CaSO4

Stress (von Mises)

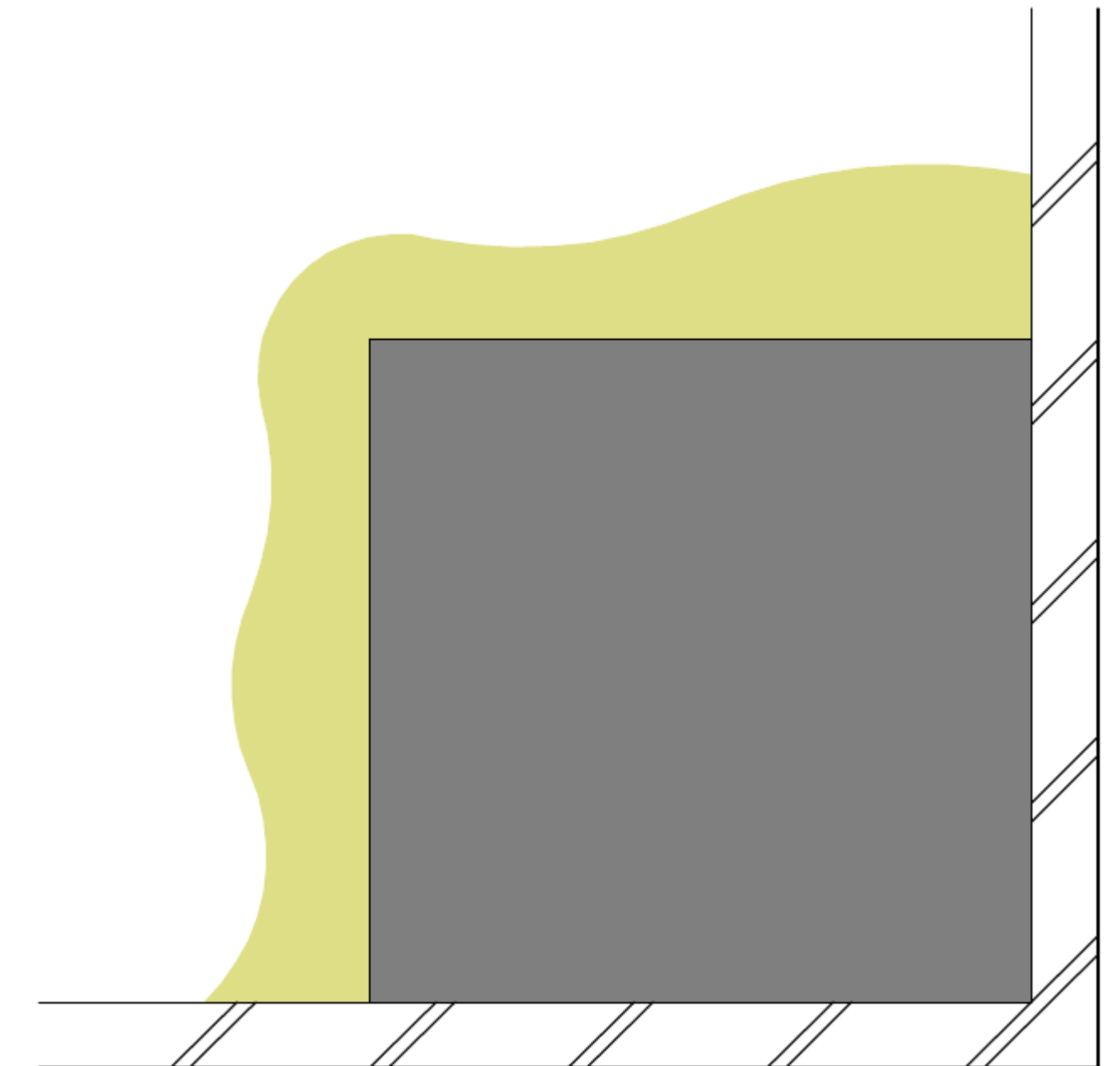


Stress (Tresca)



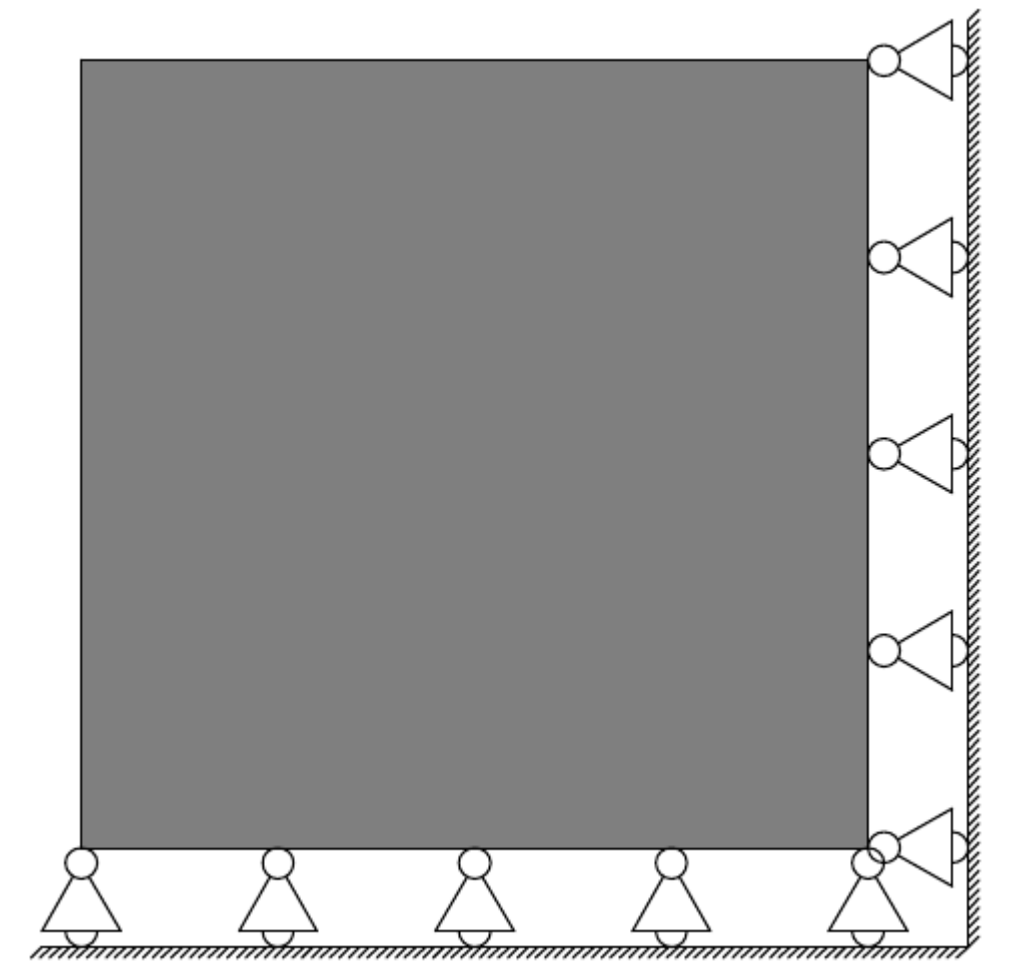
# Continuum Damage Mechanics

$$\begin{aligned}
 \frac{\partial[CH]}{\partial t} &= + k_{CH} Q_{CH} \\
 \frac{\partial[CaSO_4]}{\partial t} &= + k_{CaSO_4} Q_{CaSO_4} \\
 \frac{\partial[Ca^{2+}]}{\partial t} &= \nabla \cdot (D(\omega) \nabla [Ca^{2+}]) - k_{CH} Q_{CH} - k_{CaSO_4} Q_{CaSO_4} \\
 \frac{\partial[OH^-]}{\partial t} &= \nabla \cdot (D(\omega) \nabla [OH^-]) - 2k_{CH} Q_{CH} \\
 \frac{\partial[SO_4^{2-}]}{\partial t} &= \nabla \cdot (D(\omega) \nabla [SO_4^{2-}]) - k_{CaSO_4} Q_{CaSO_4} \\
 \boldsymbol{\sigma} &= (1 - \omega) {}^4C_0 (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^g [CaSO_4] \mathbf{I})
 \end{aligned}$$

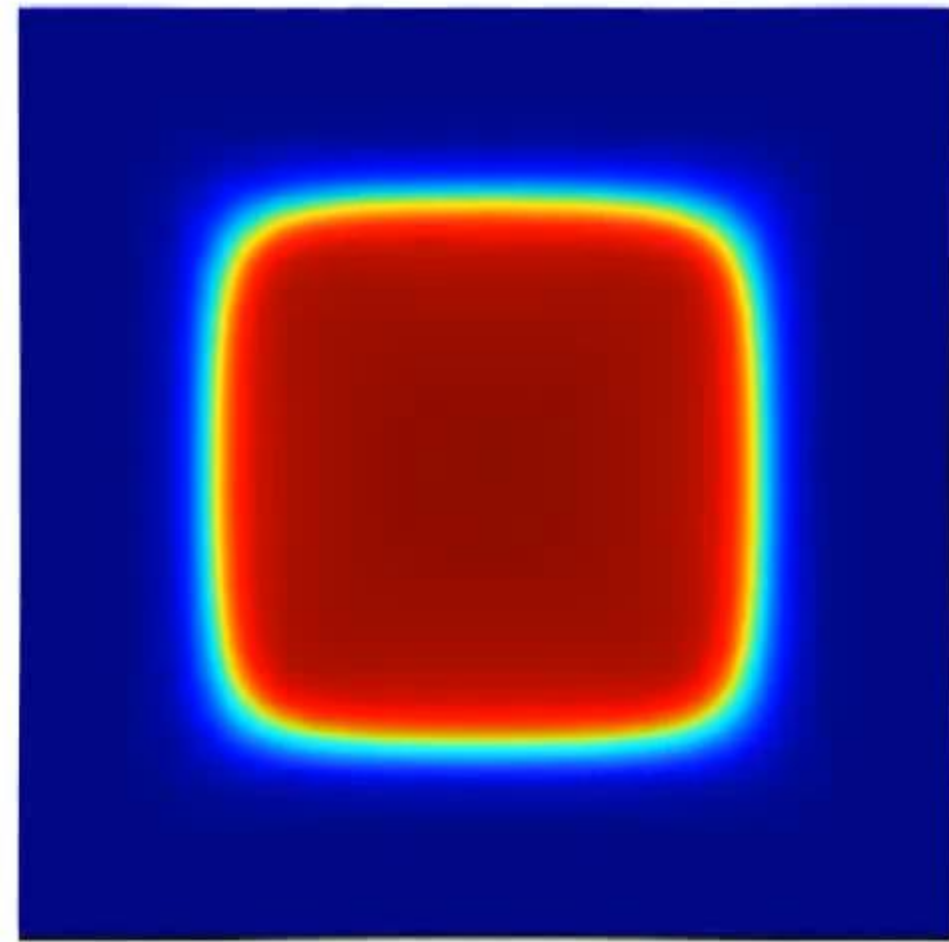


# Continuum Damage Mechanics

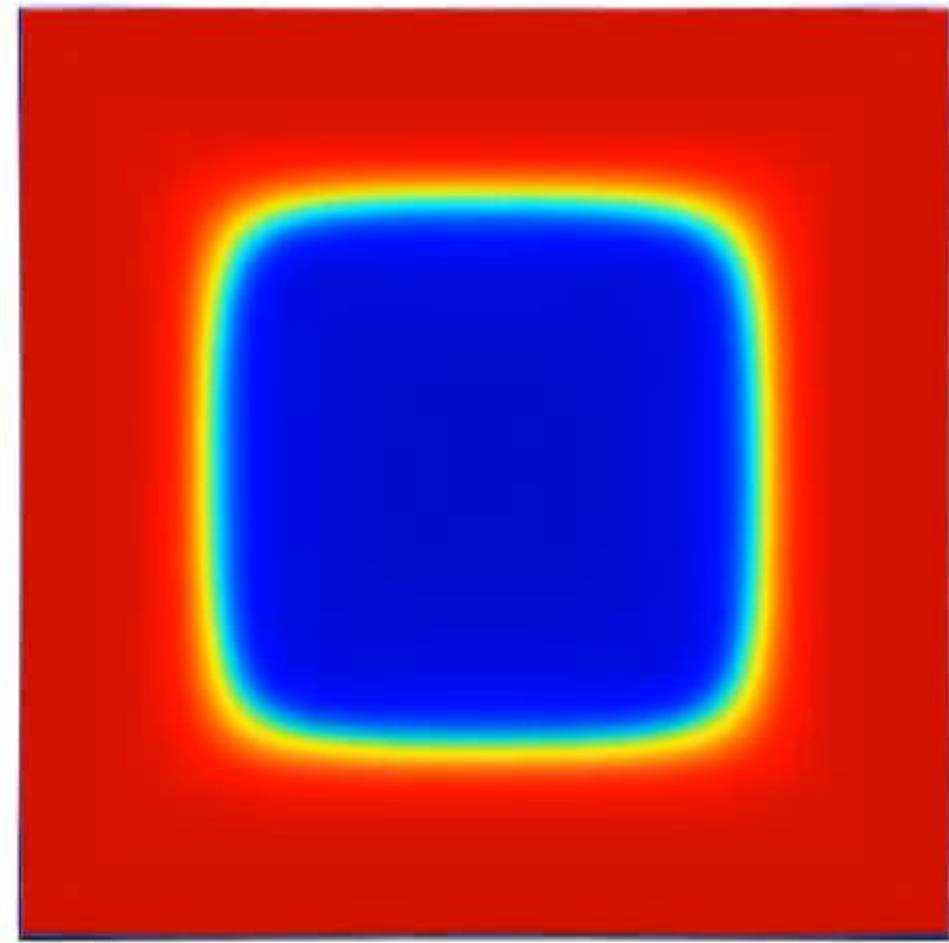
$$\begin{aligned}
 \frac{\partial[CH]}{\partial t} &= + k_{CH}Q_{CH} \\
 \frac{\partial[CaSO_4]}{\partial t} &= + k_{CaSO_4}Q_{CaSO_4} \\
 \frac{\partial[Ca^{2+}]}{\partial t} &= \nabla \cdot (D(\omega)\nabla[Ca^{2+}]) - k_{CH}Q_{CH} - k_{CaSO_4}Q_{CaSO_4} \\
 \frac{\partial[OH^-]}{\partial t} &= \nabla \cdot (D(\omega)\nabla[OH^-]) - 2k_{CH}Q_{CH} \\
 \frac{\partial[SO_4^{2-}]}{\partial t} &= \nabla \cdot (D(\omega)\nabla[SO_4^{2-}]) - k_{CaSO_4}Q_{CaSO_4} \\
 \boldsymbol{\sigma} &= (1 - \omega) {}^4C_0 (\boldsymbol{\varepsilon} - \varepsilon^g[CaSO_4]\mathbf{I})
 \end{aligned}$$





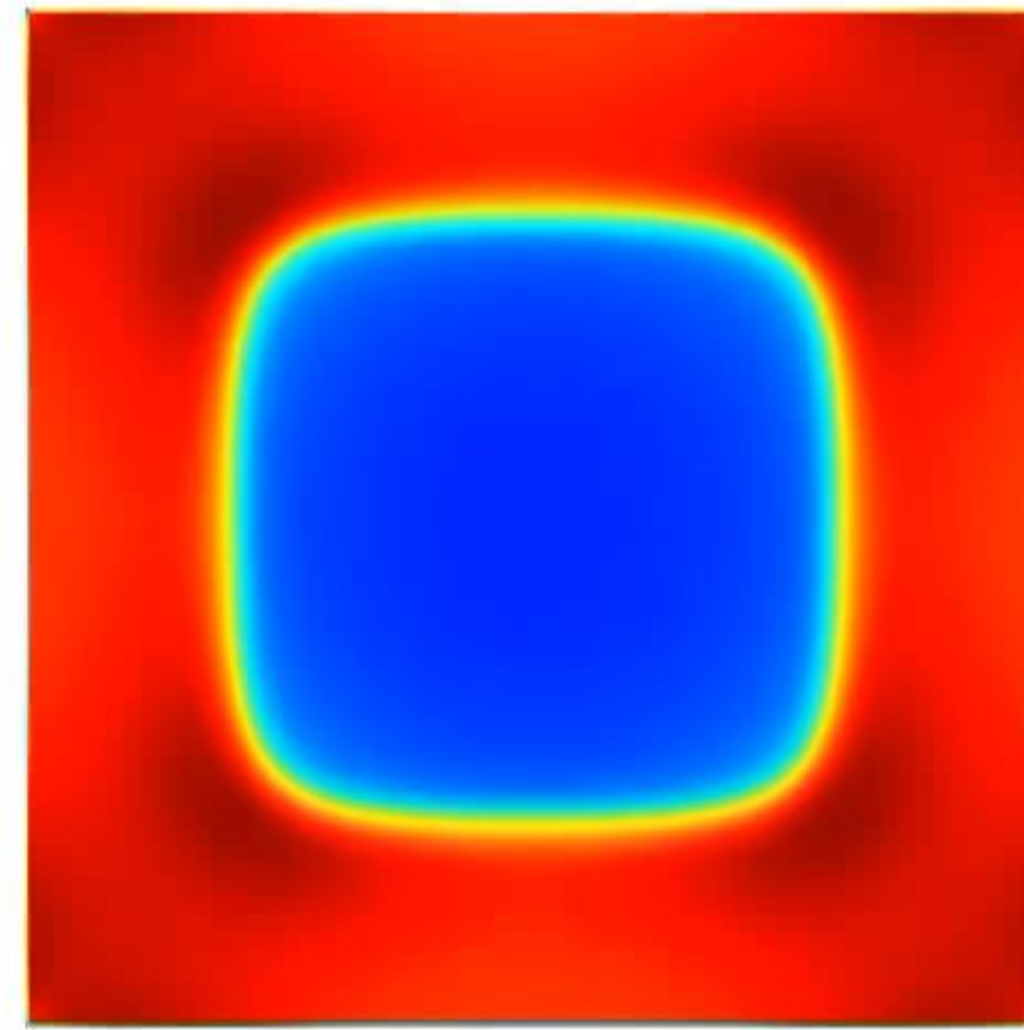


CH

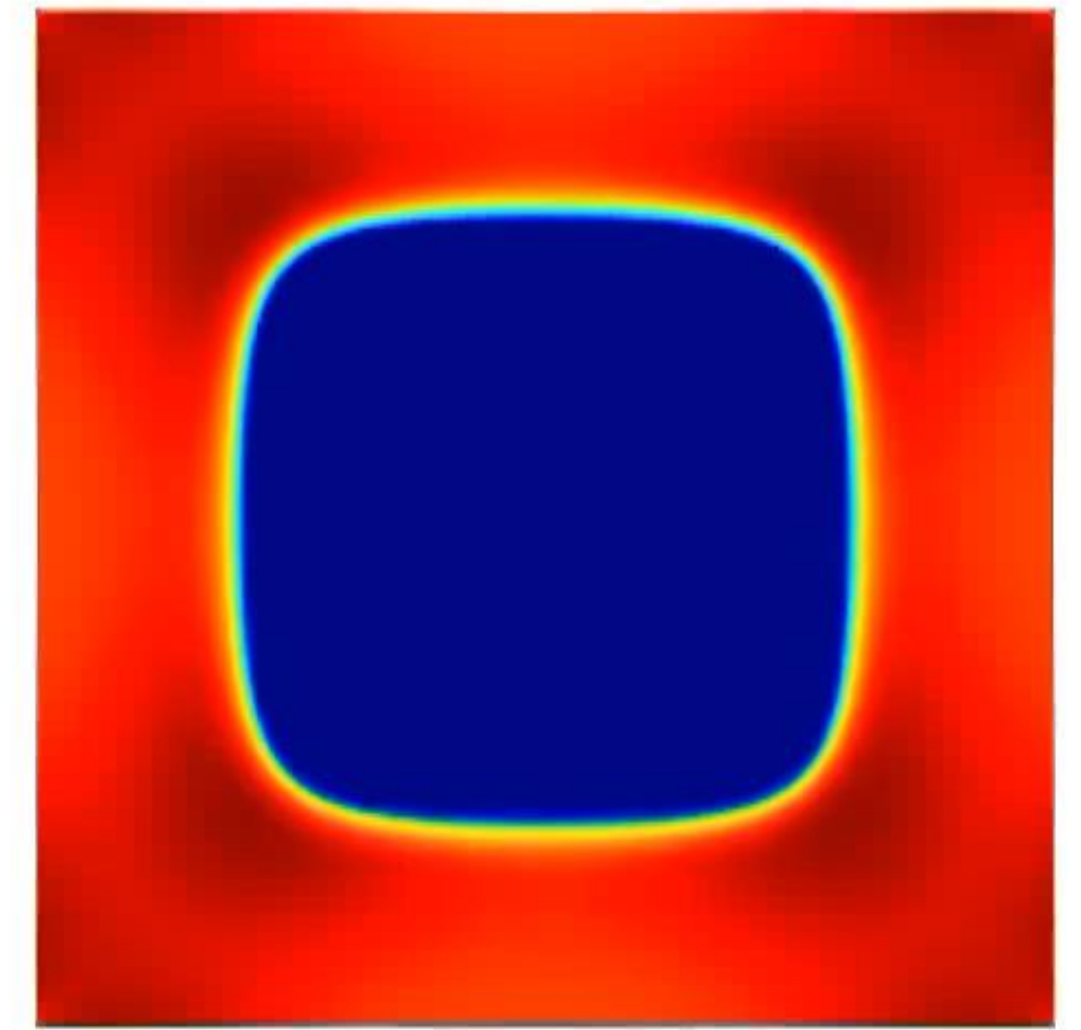


CaSO4

Equivalent strain



Damage Parameter



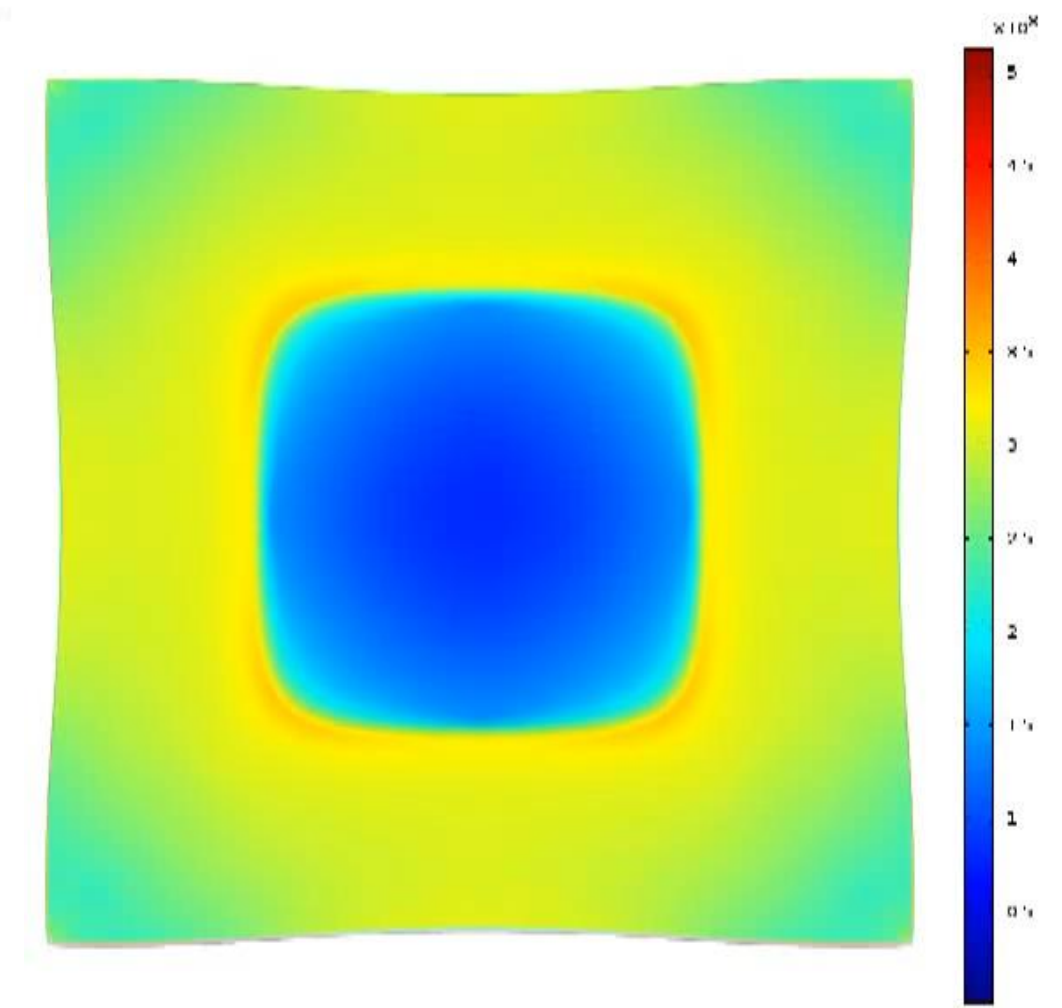
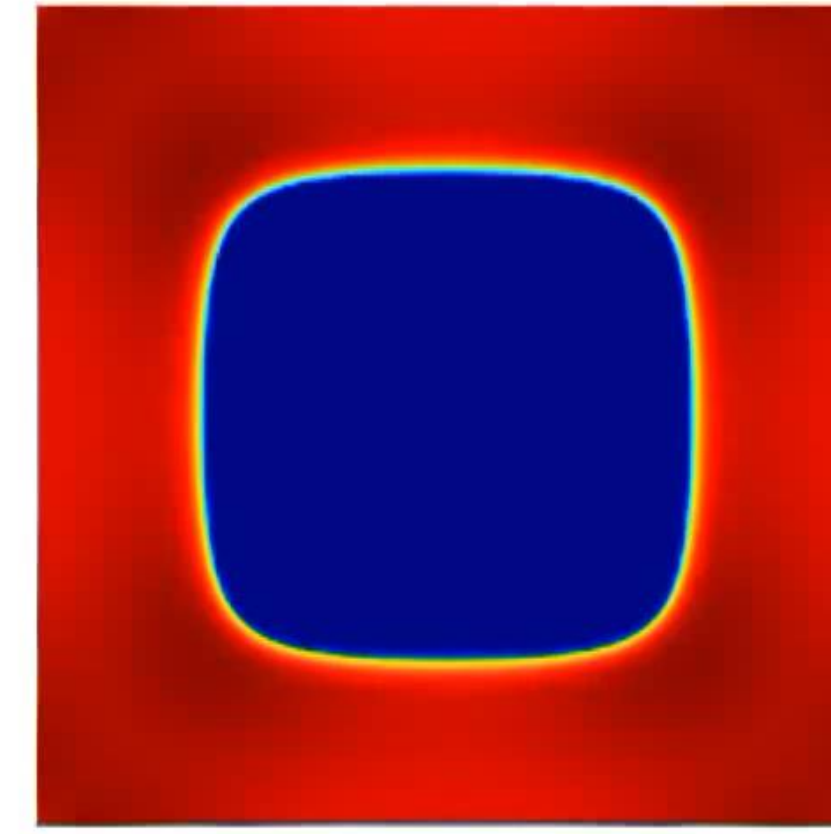
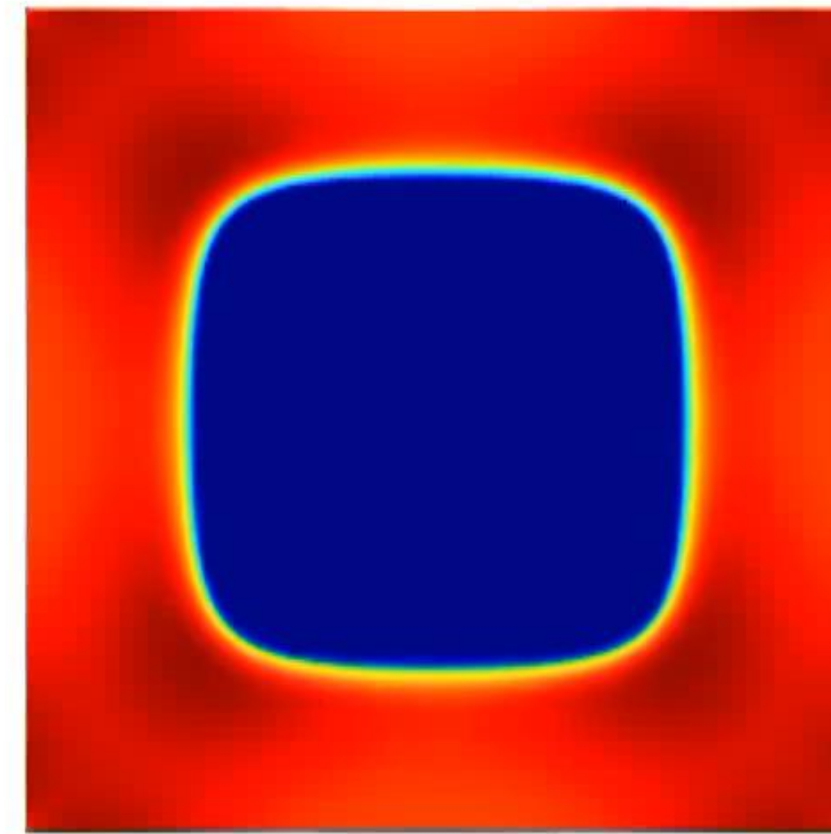
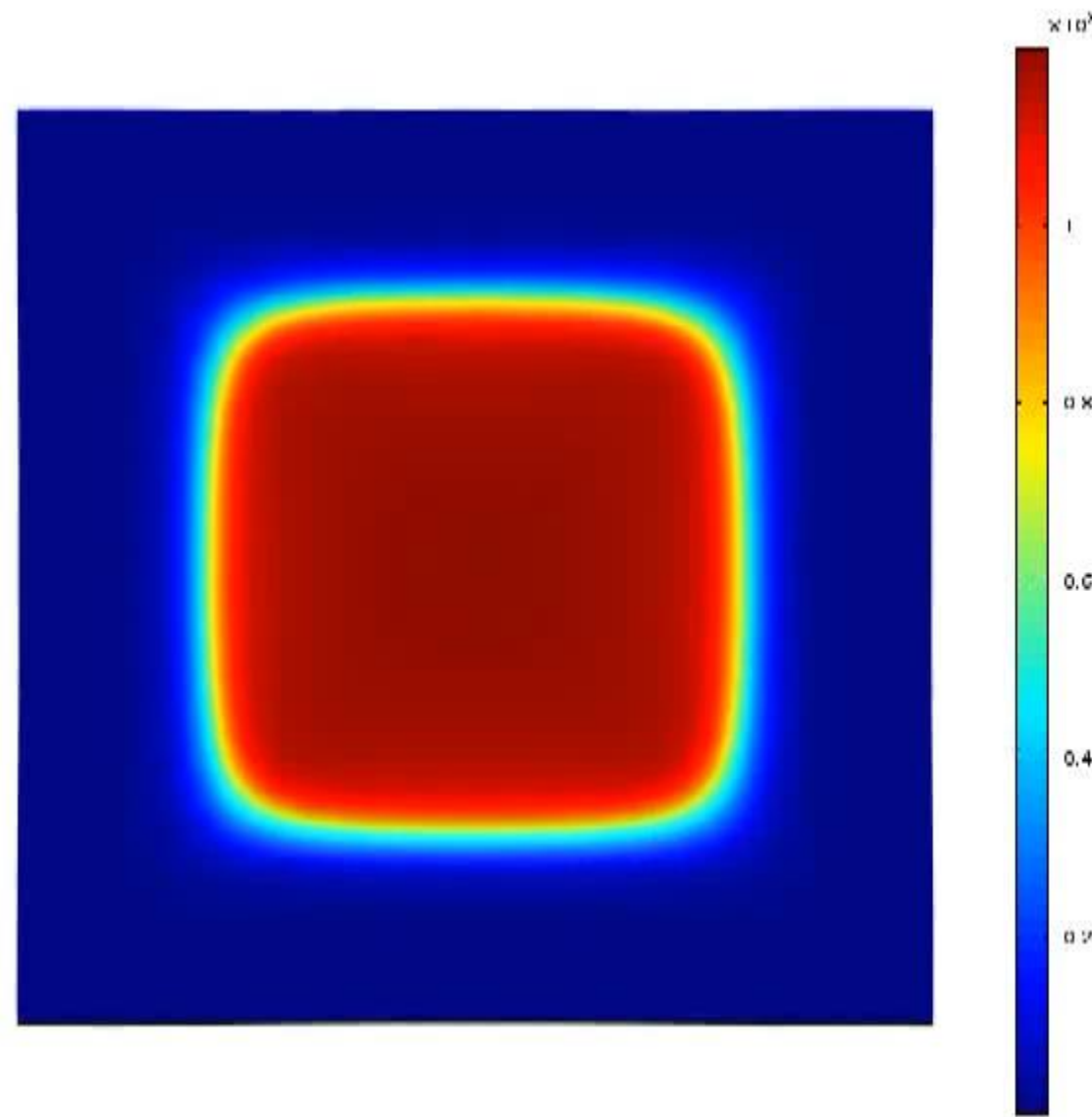


CH

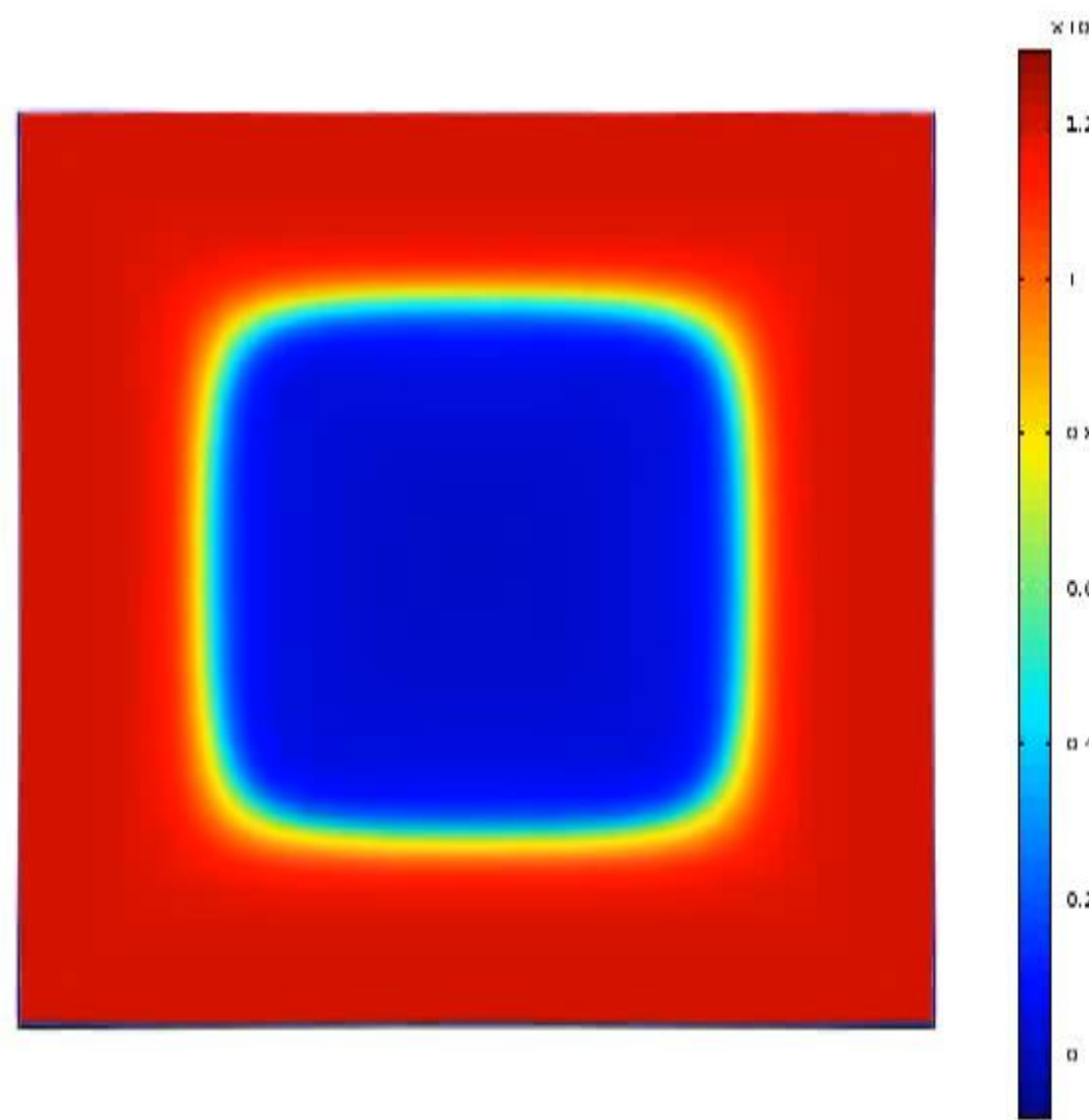
Damage Parameter

Diffusion Coefficient

Stress (Tresca)



CaSO4



# Parameters

$$\begin{aligned}
 \frac{\partial[CH]}{\partial t} &= + k_{CH}Q_{CH} \\
 \frac{\partial[CSH]}{\partial t} &= + k_{CSH}Q_{CSH} \\
 \frac{\partial[CaSO_4]}{\partial t} &= + k_{CaSO_4}Q_{CaSO_4} \\
 \frac{\partial[Ca^{2+}]}{\partial t} &= D_0 \nabla^2[Ca^{2+}] - k_{CH}Q_{CH} - k_{CaSO_4}Q_{CaSO_4} \\
 \frac{\partial[OH^-]}{\partial t} &= D_0 \nabla^2[OH^-] - 2k_{CH}Q_{CH} - 2k_{CSH}Q_{CSH} \\
 \frac{\partial[SO_4^{2-}]}{\partial t} &= D_0 \nabla^2[SO_4^{2-}] - k_{CaSO_4}Q_{CaSO_4} \\
 \sigma &= (1 - \varphi)(1 - \omega) {}^4C_0 (\epsilon - \epsilon^g[CaSO_4])
 \end{aligned}$$

# Future Developments

## Experiments

- Concrete casting starts end of June

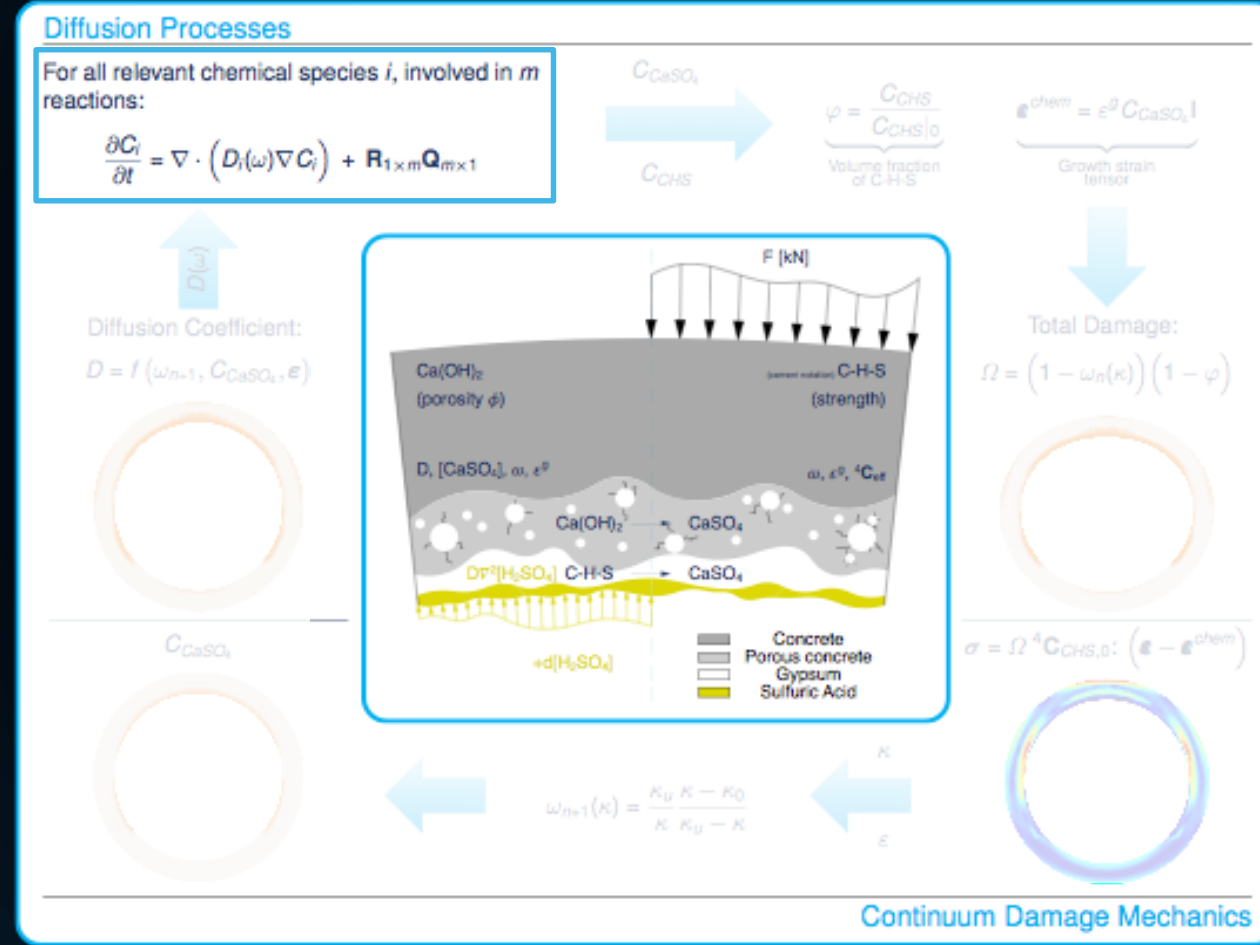
## Numerical Simulations

- Finishing and combining last parameters (chemical damage, CSH)
- Calibrating and validating small numerical models
- Benchmarking the model on literature
- Finishing first paper

# Questions?



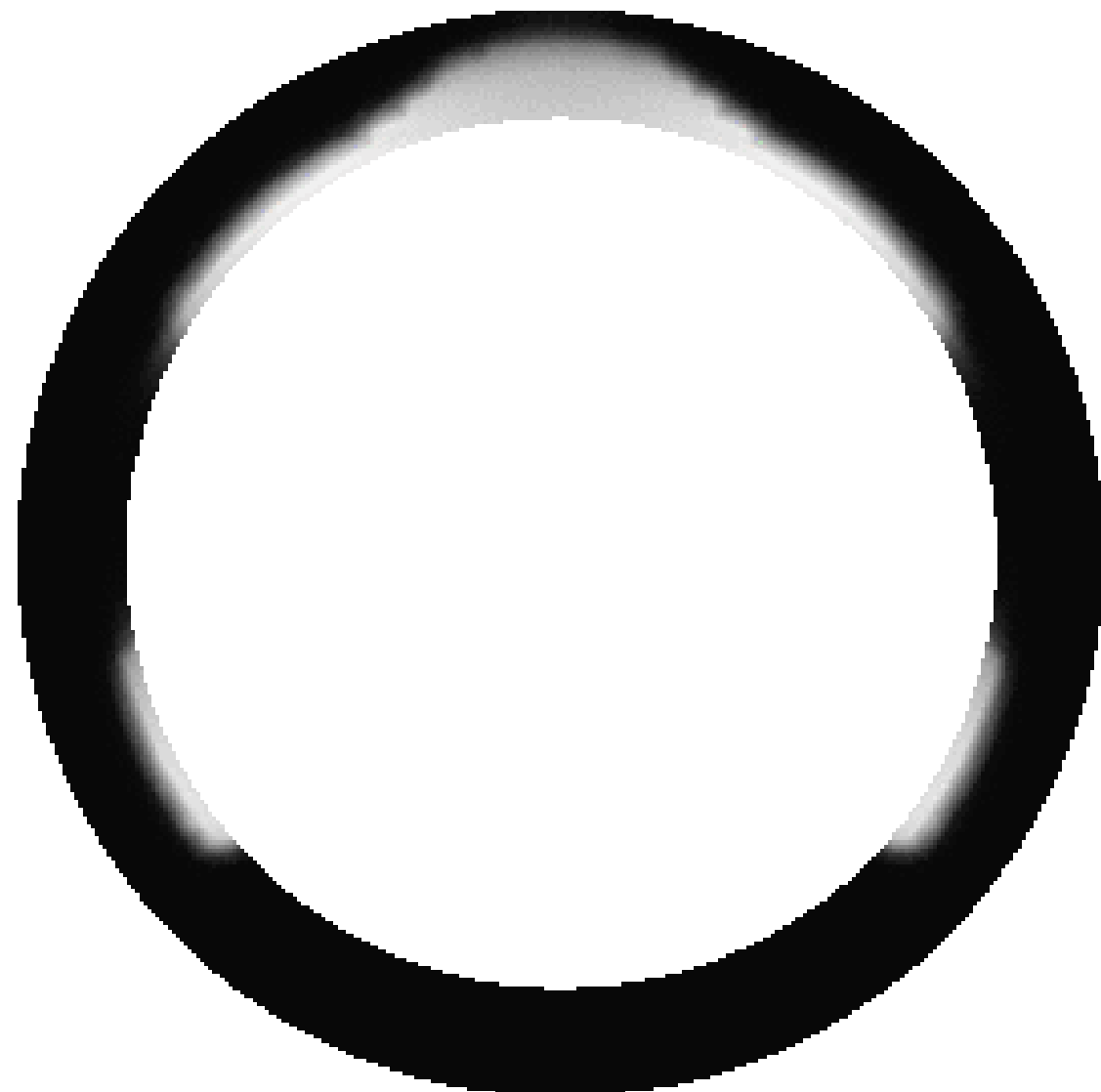
# Additional / Explanatory Slides



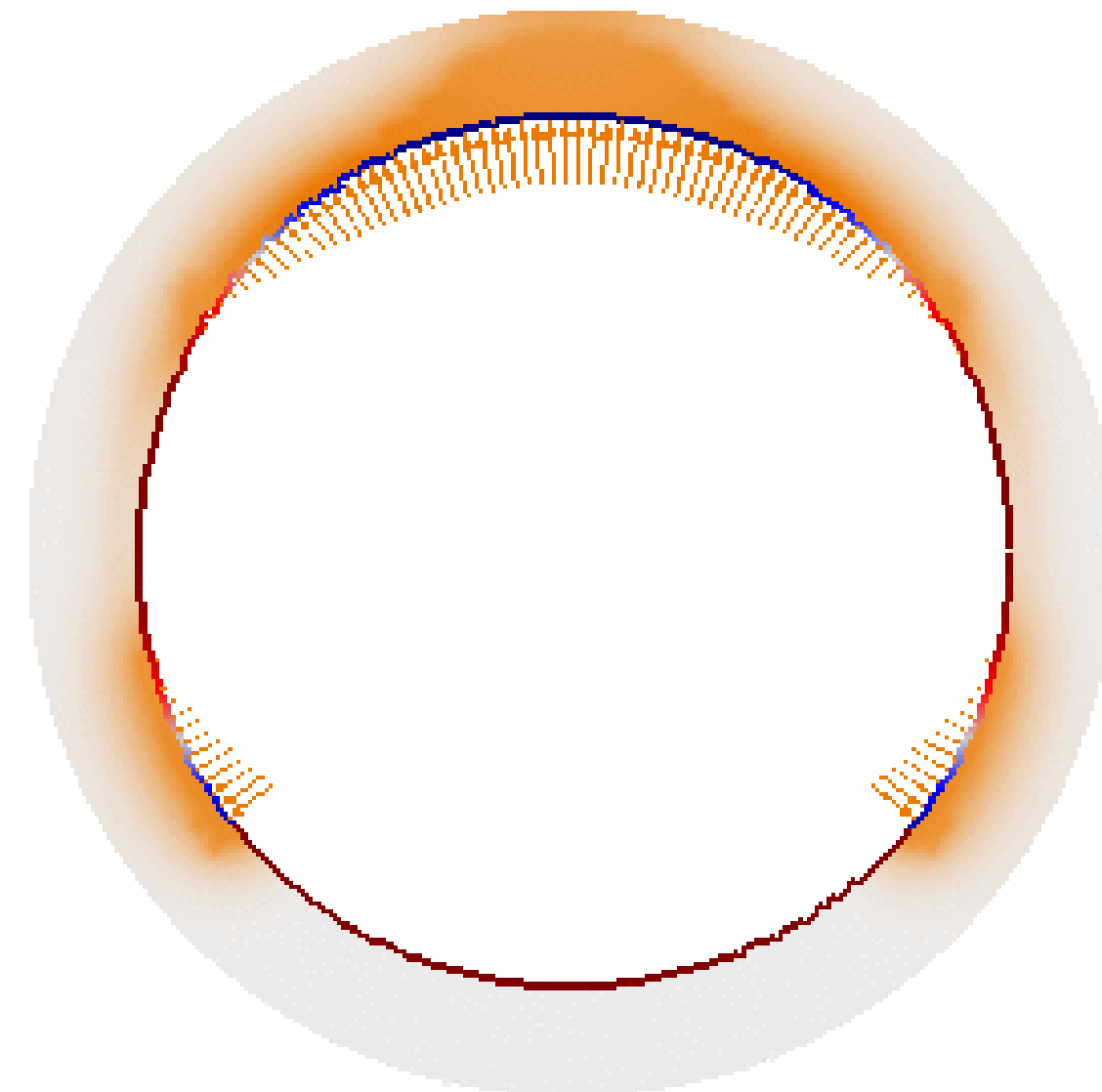
## Diffusion Models

$$\frac{\partial}{\partial t} \begin{bmatrix} [Ca(OH)_2] \\ [Ca^{2+}] \\ [OH^-] \\ [SO_4^{2-}] \\ [CaSO_4] \end{bmatrix} = \nabla \cdot \left( D \nabla \begin{bmatrix} 0 \\ [Ca^{2+}] \\ [OH^-] \\ [SO_4^{2-}] \\ 0 \end{bmatrix} \right) + \mathbf{R}_{5 \times 4} \mathbf{Q}_{4 \times 1}$$

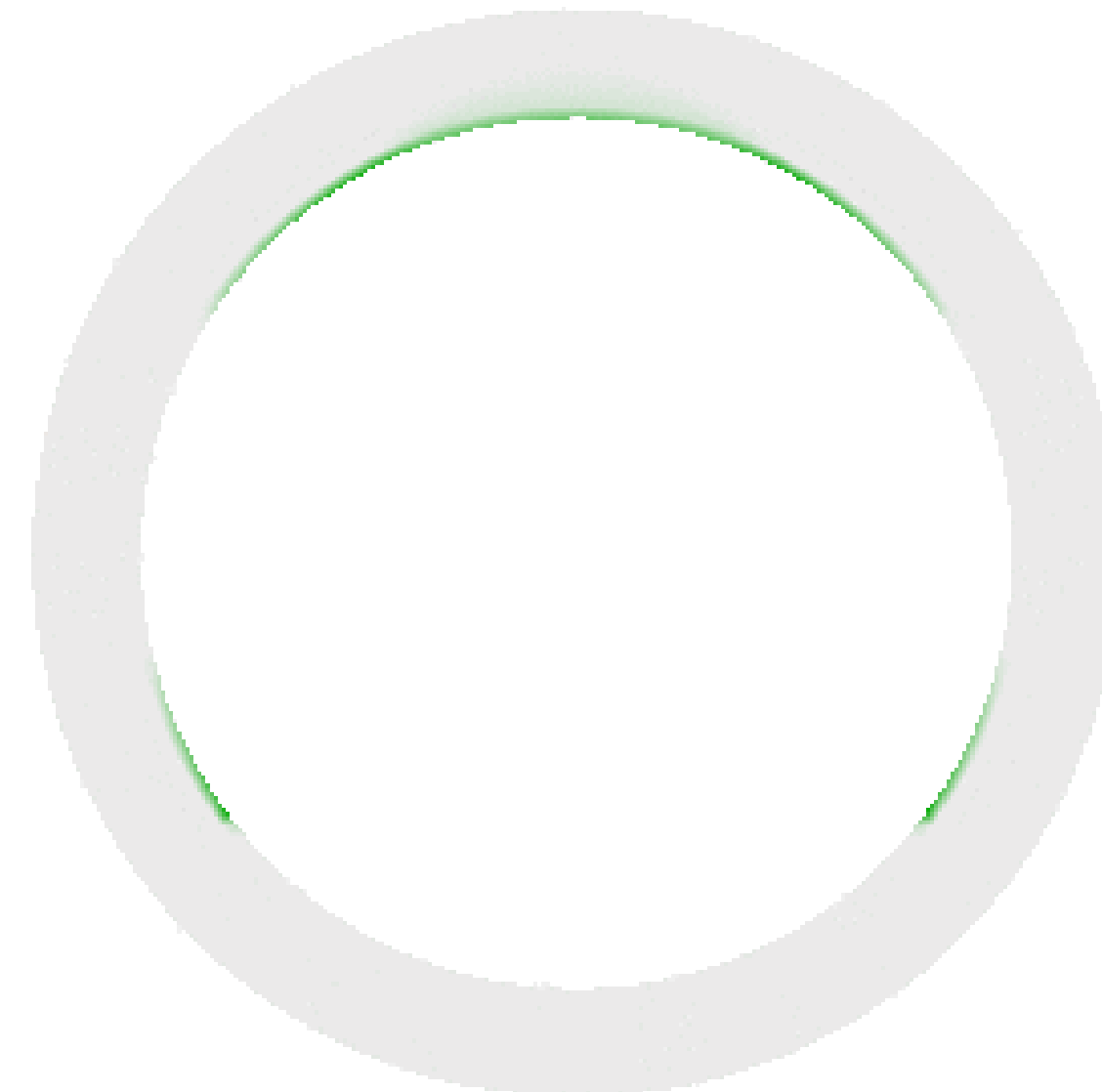
Calcium Hydroxide  
[Ca(OH)<sub>2</sub>]

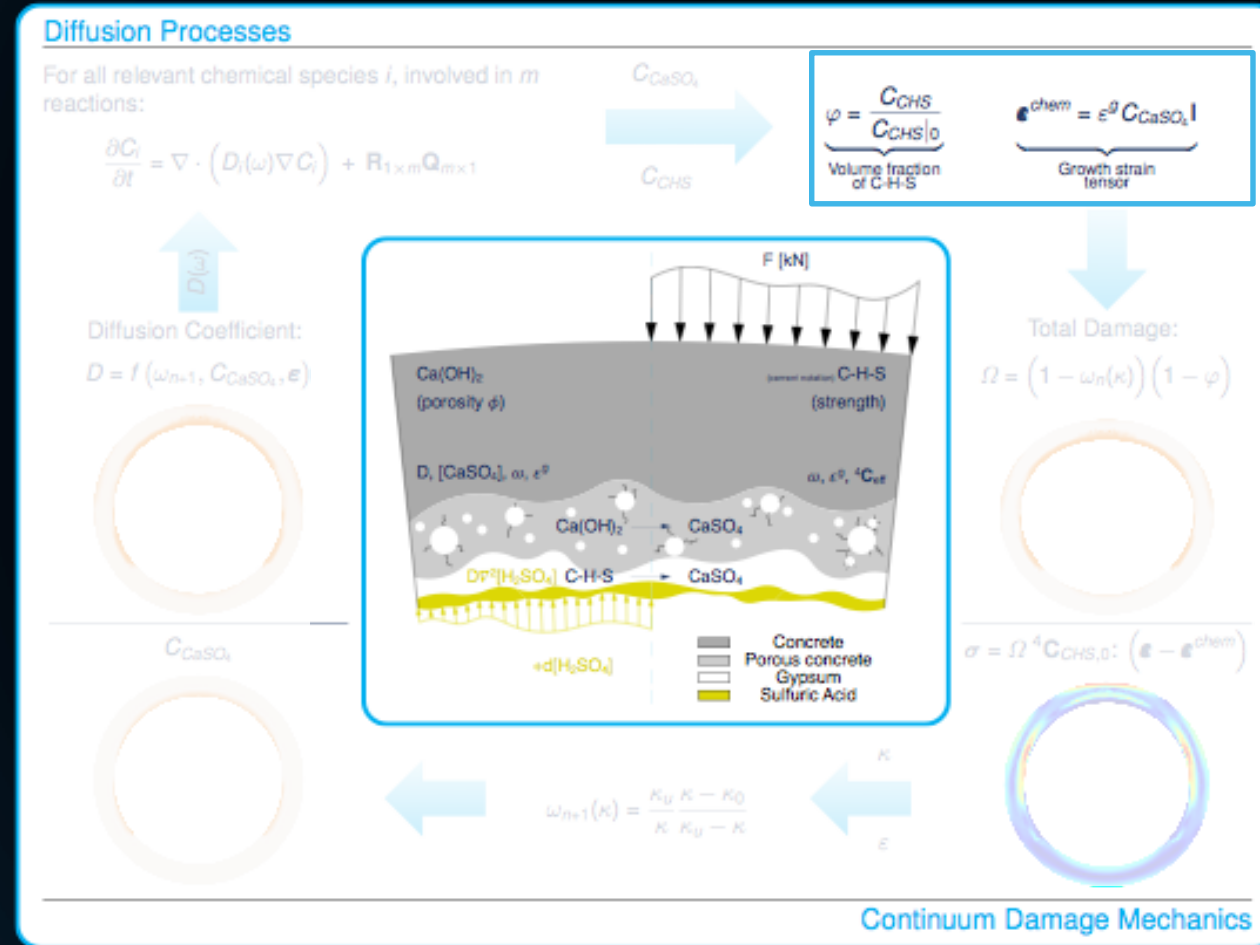


Sulfate ions  
[SO<sub>4</sub><sup>2-</sup>]



Gypsum  
[CaSO<sub>4</sub>]





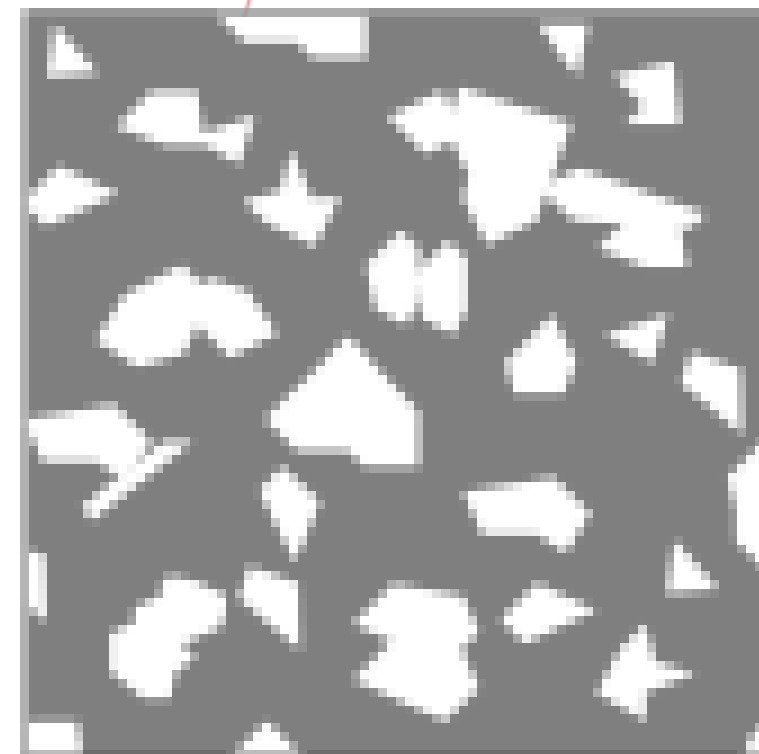
## Chemical Damage

$$\varphi = \frac{[CSH]}{[CSH]_0}$$

$$\varphi = 1$$

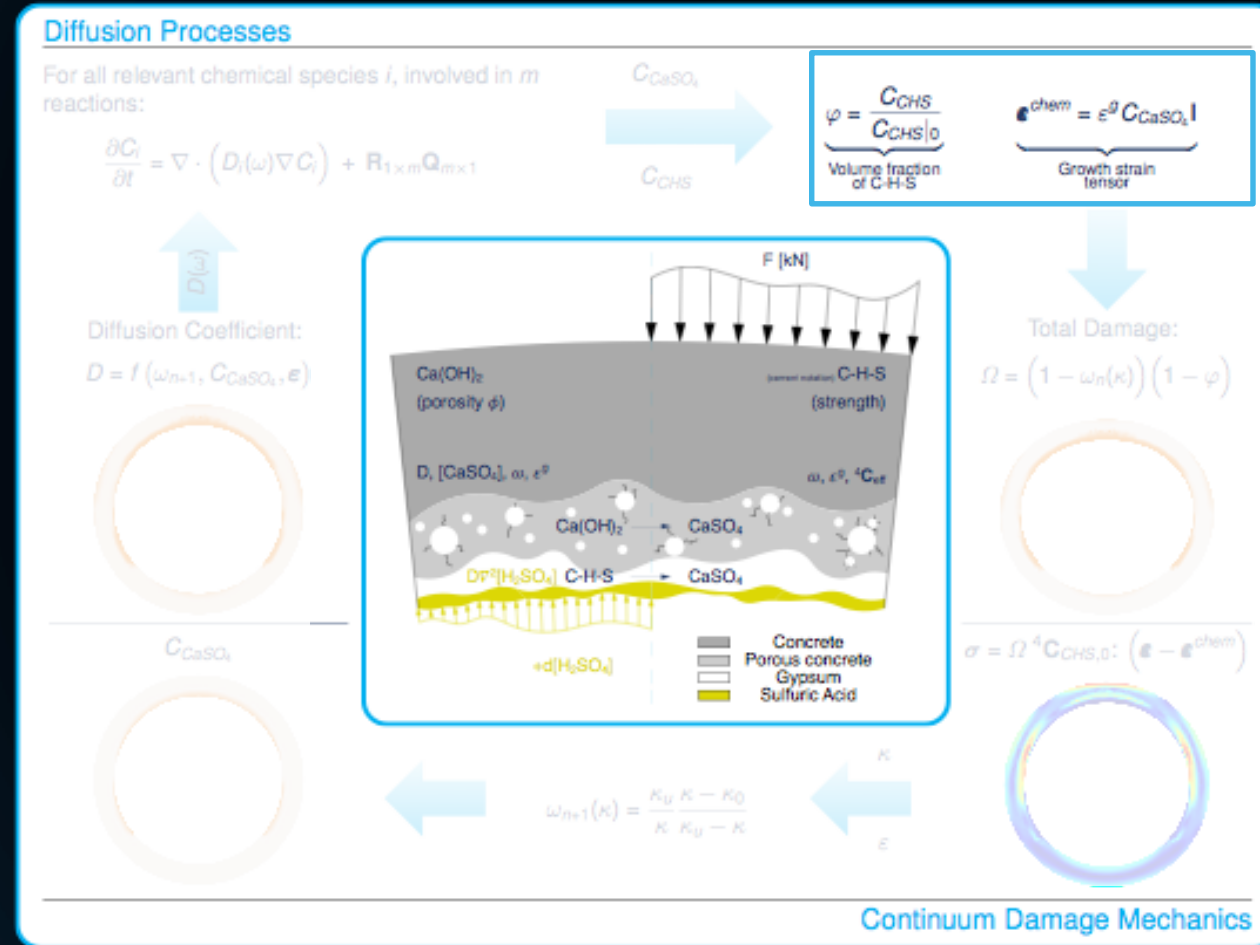


$$\varphi = 0.5$$



$$\varphi = 0$$





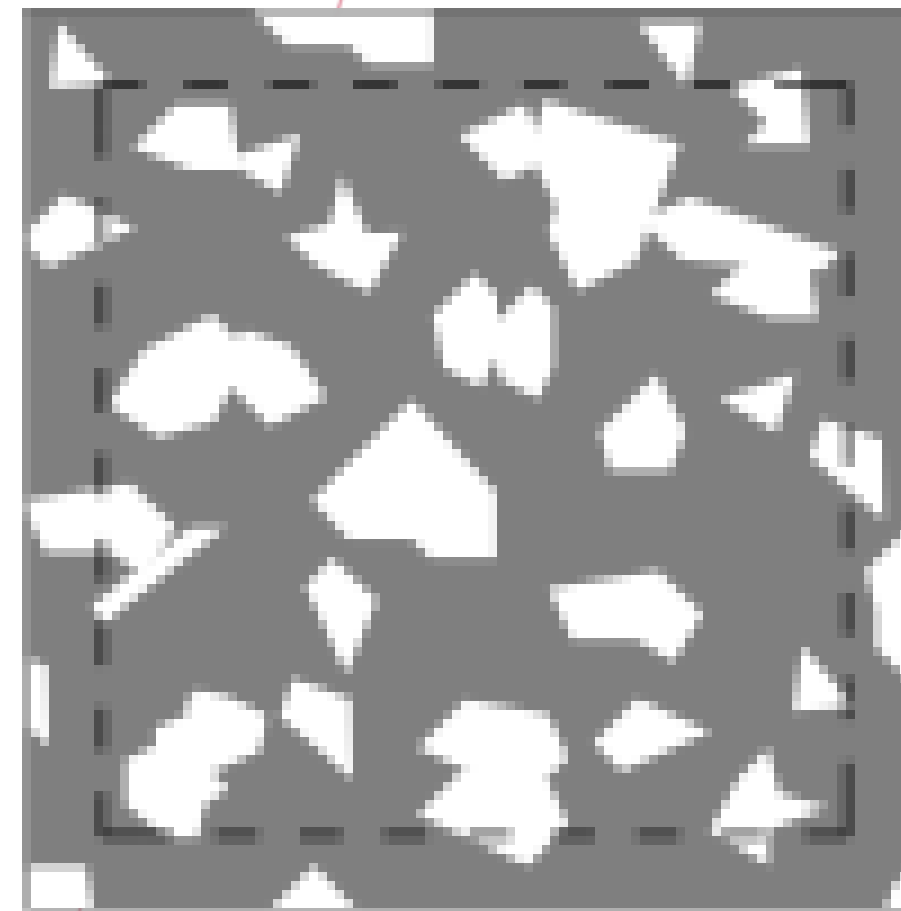
## Gypsum Expansion

$$\epsilon^{chem} = \epsilon^g [CaSO_4] \mathbf{I}$$

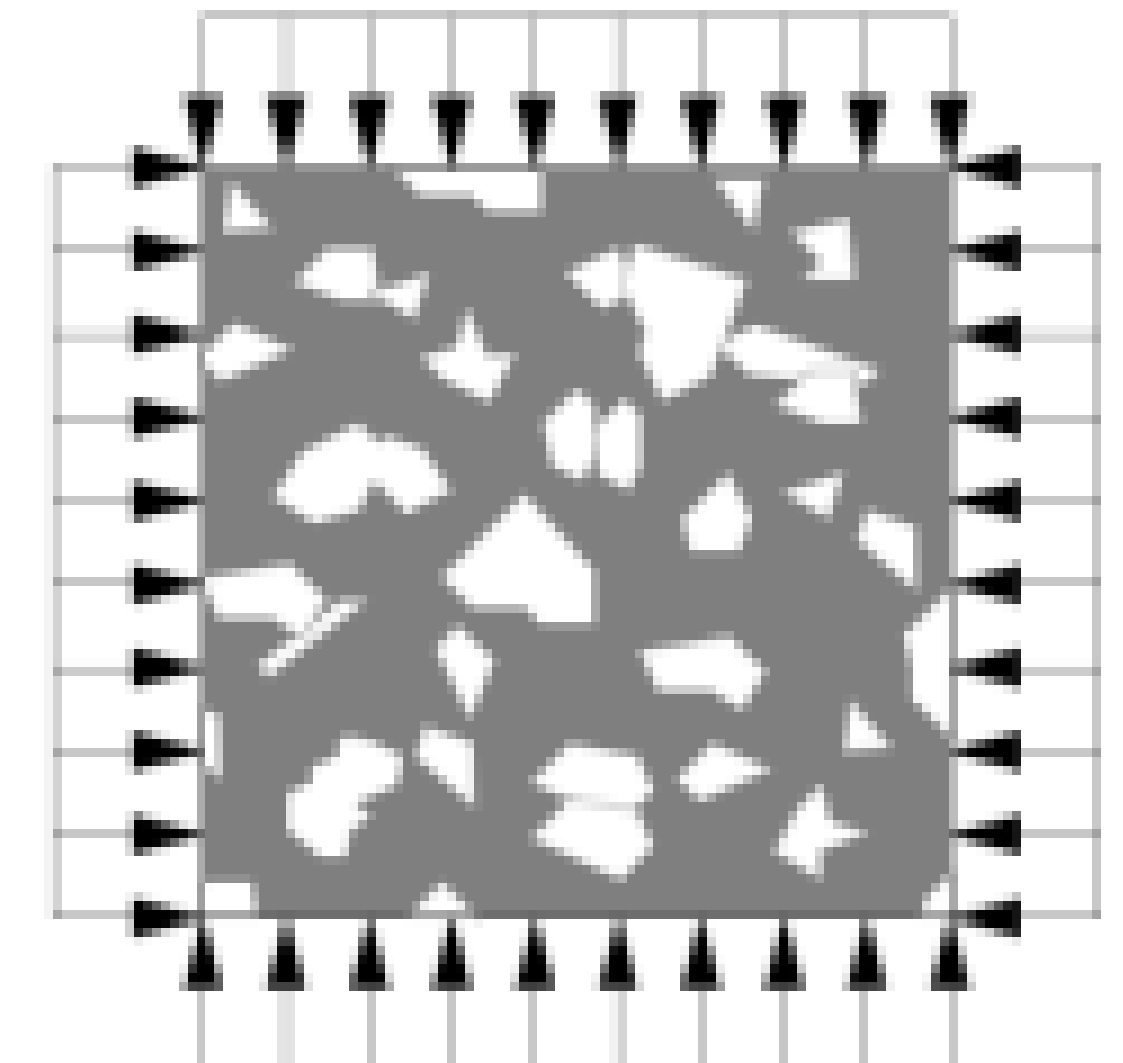
$$\epsilon^{chem} = 0$$



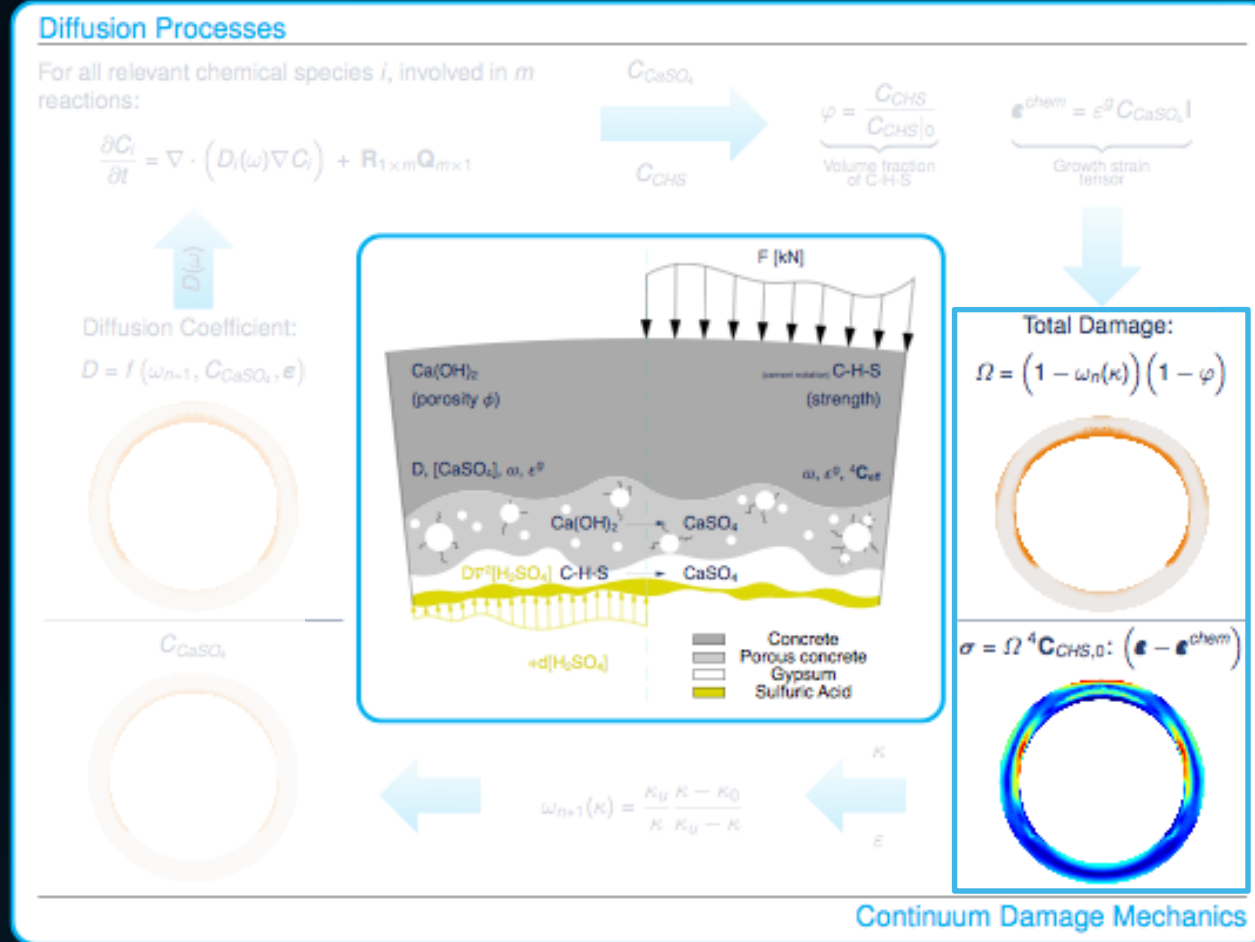
$$\epsilon^{chem} = \epsilon^g [CaSO_4] \mathbf{I}$$



$$\epsilon^{tot} = \epsilon^{chem} + \epsilon^e$$

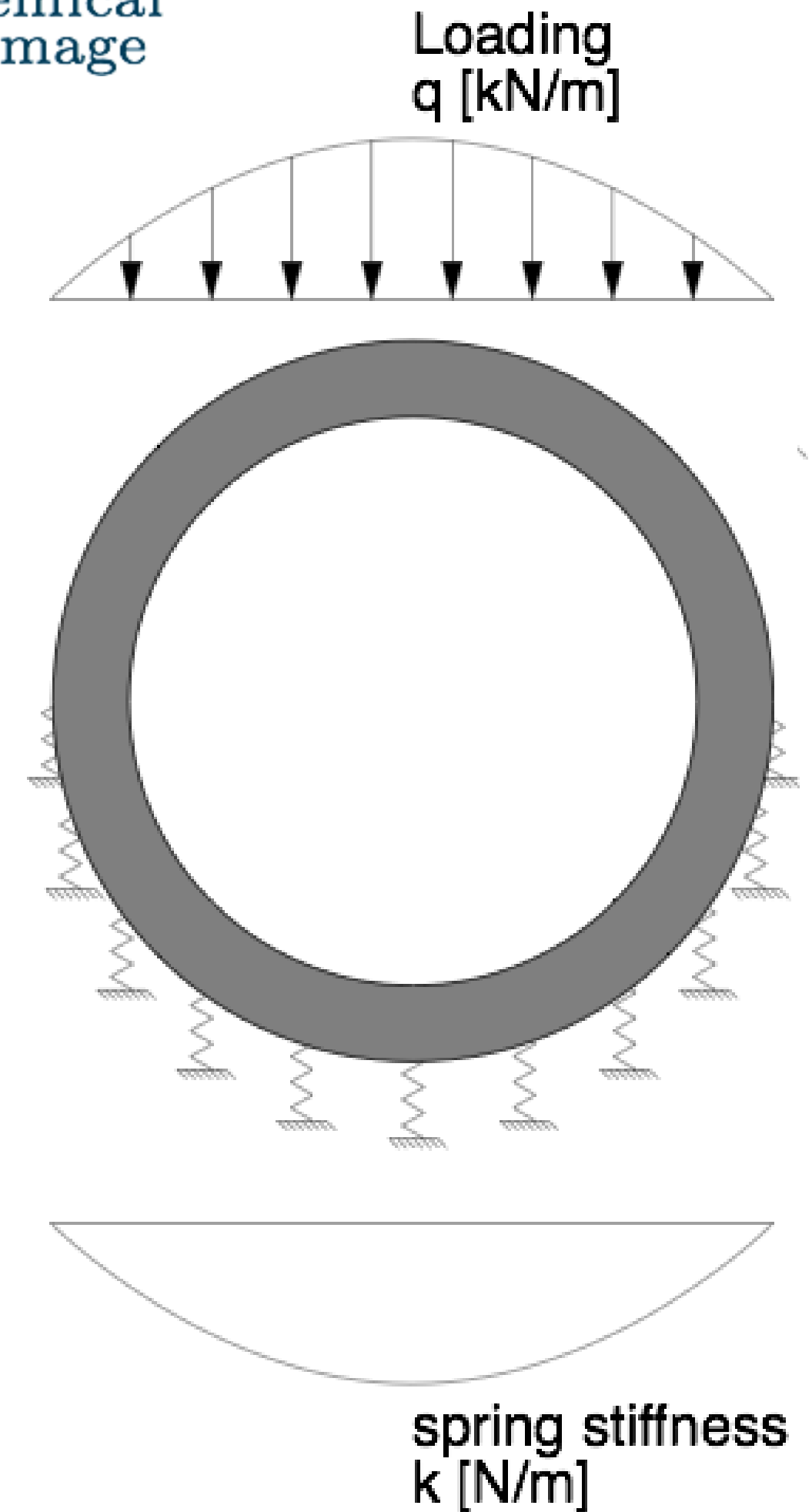


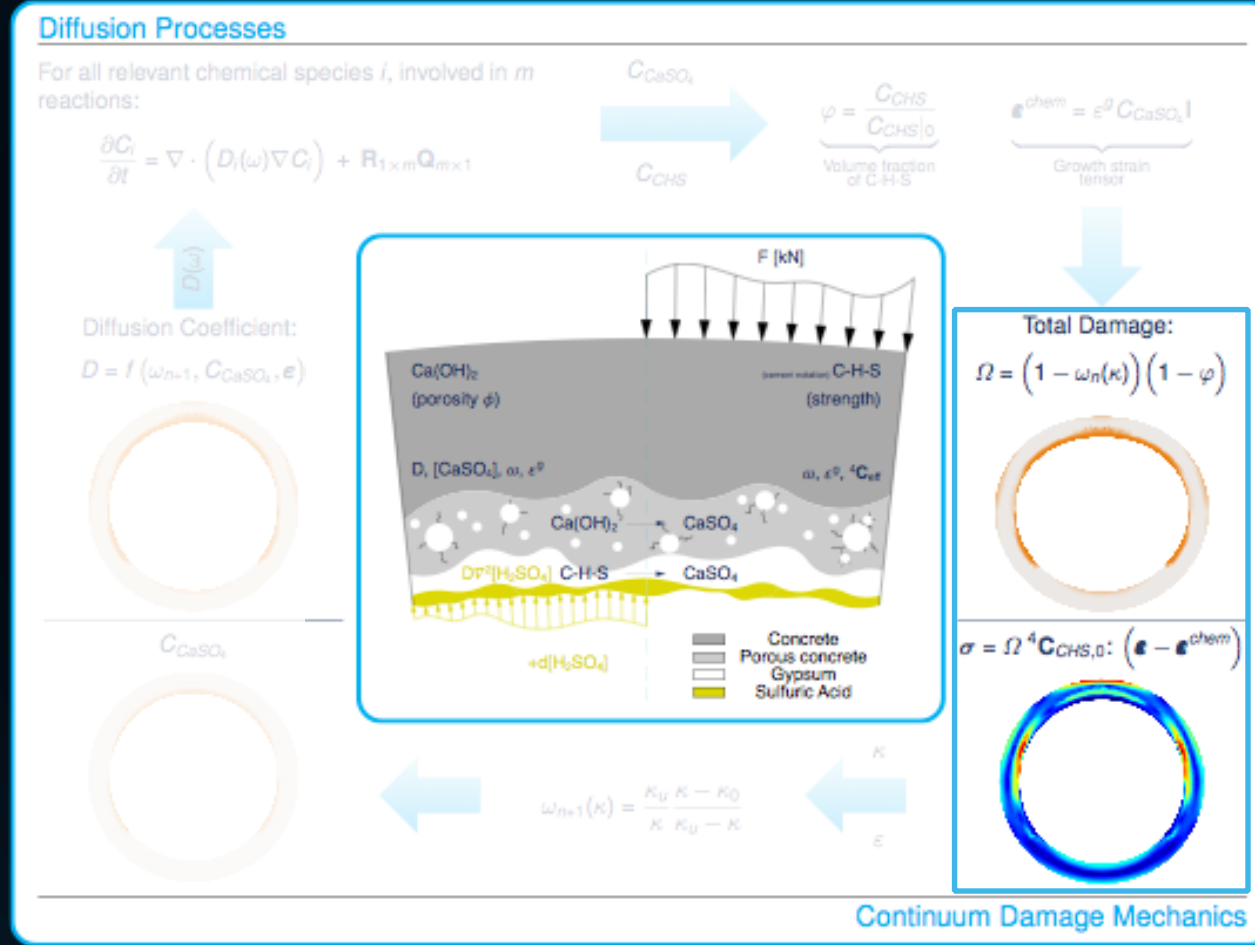




## (Continuum Damage) Mechanical Model

$$\sigma = \underbrace{\left(1 - \omega_n(\kappa)\right)}_{\text{Mechanical Damage}} \underbrace{\left(\varphi\right)}_{\text{Chemical Damage}} {}^4C_0: (\epsilon - \epsilon^{chem})$$



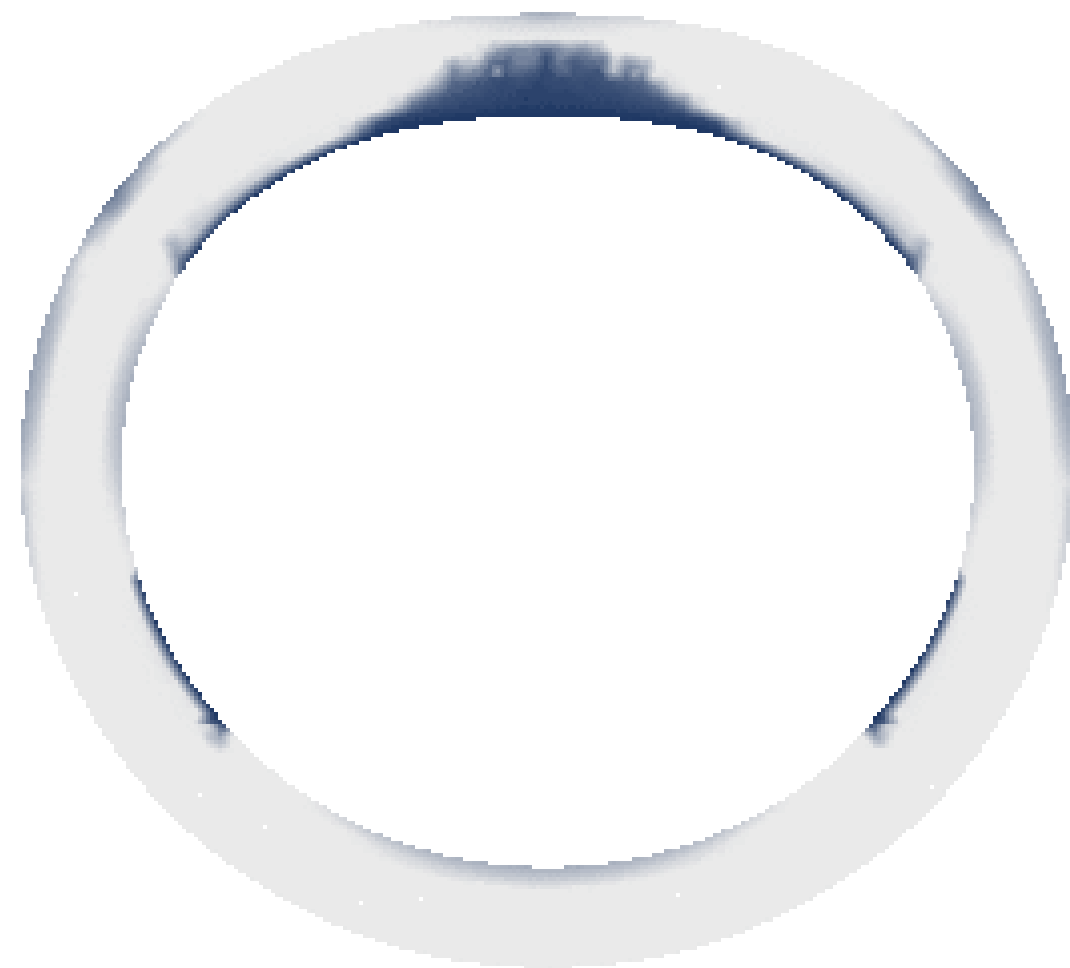


## (Continuum Damage) Mechanical Model

$$\sigma = \underbrace{\left(1 - \omega_n(\kappa)\right)}_{\text{Mechanical Damage}} \underbrace{\left(\varphi\right)}_{\text{Chemical Damage}} \mathbb{C}_0 : (\epsilon - \epsilon^{chem})$$

Total damage

Total Damage ( $\Omega$ )



=

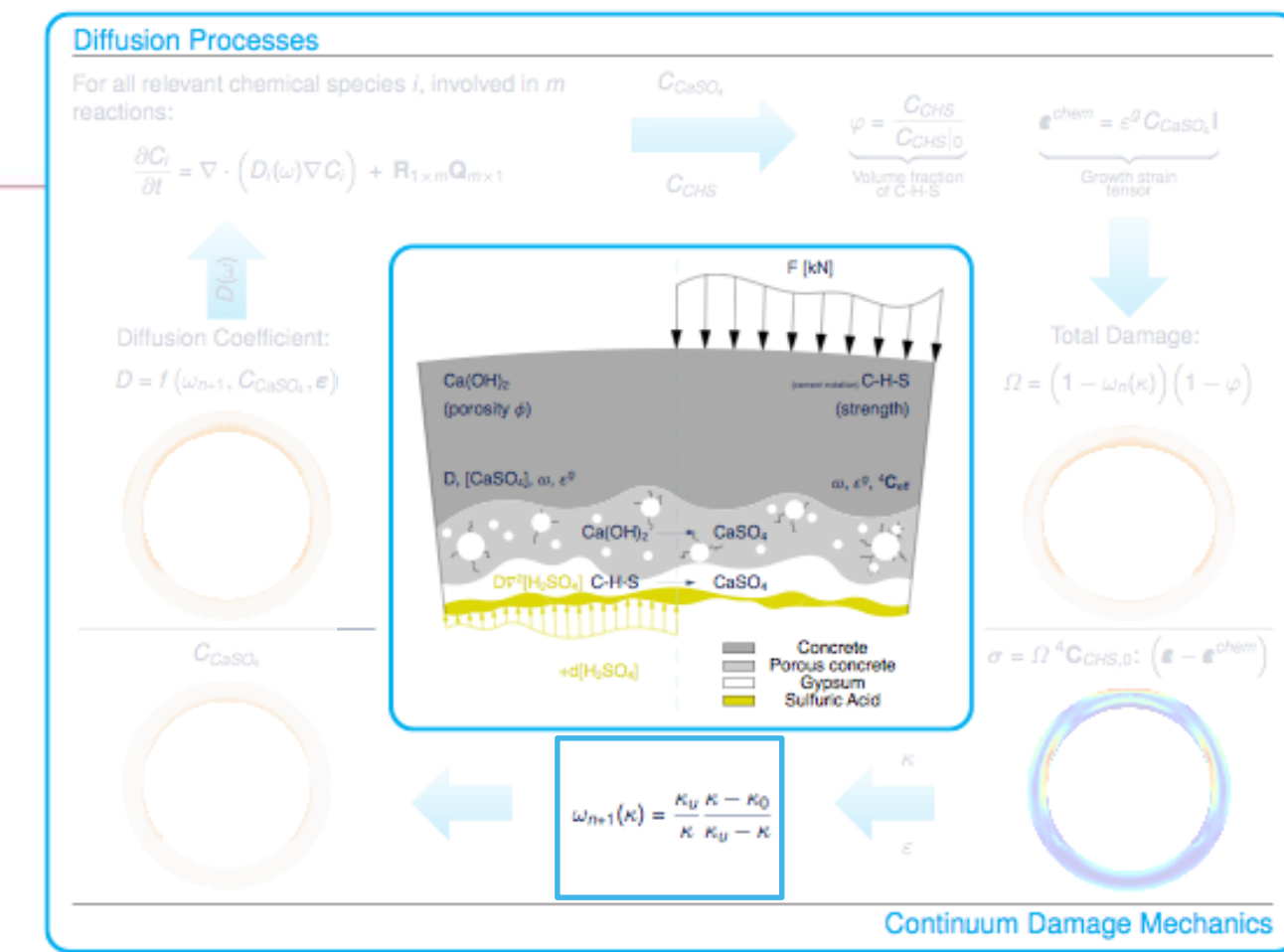
Mechanical damage ( $\omega$ )



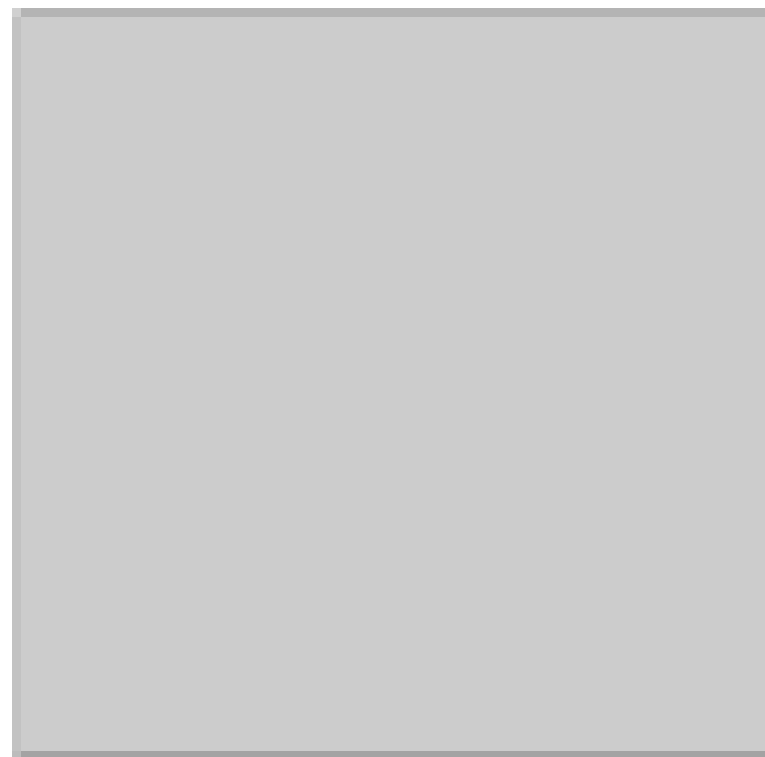
Chemical damage ( $\varphi$ )



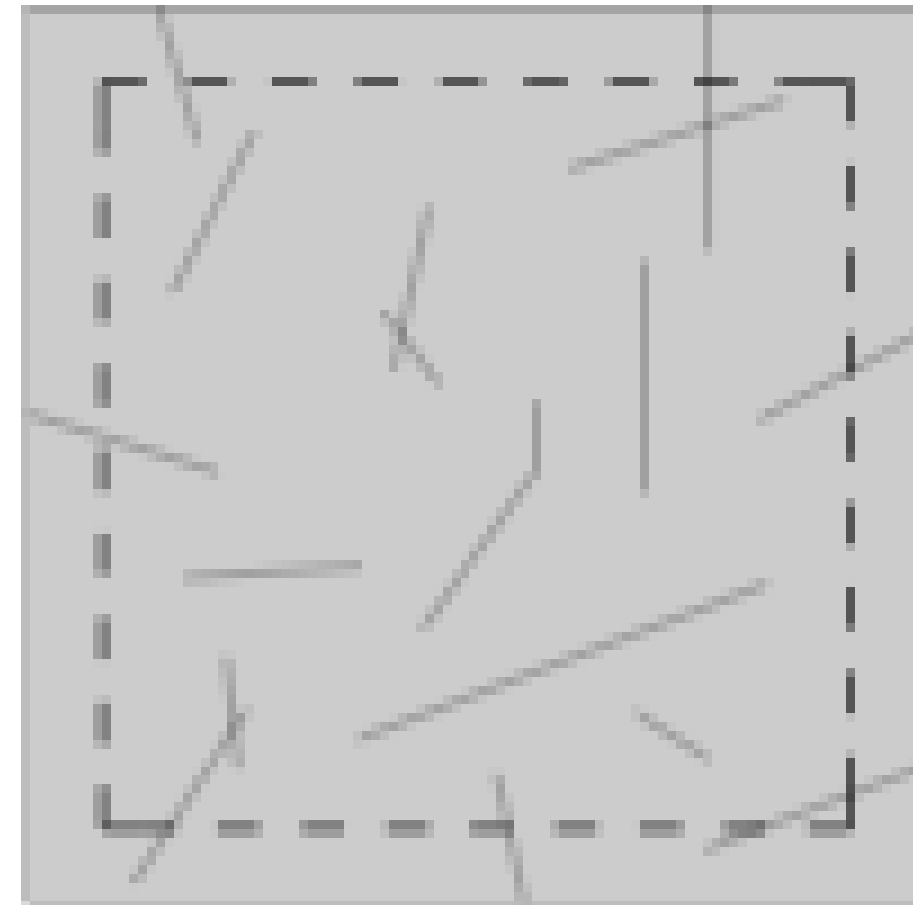
# Mechanical Damage



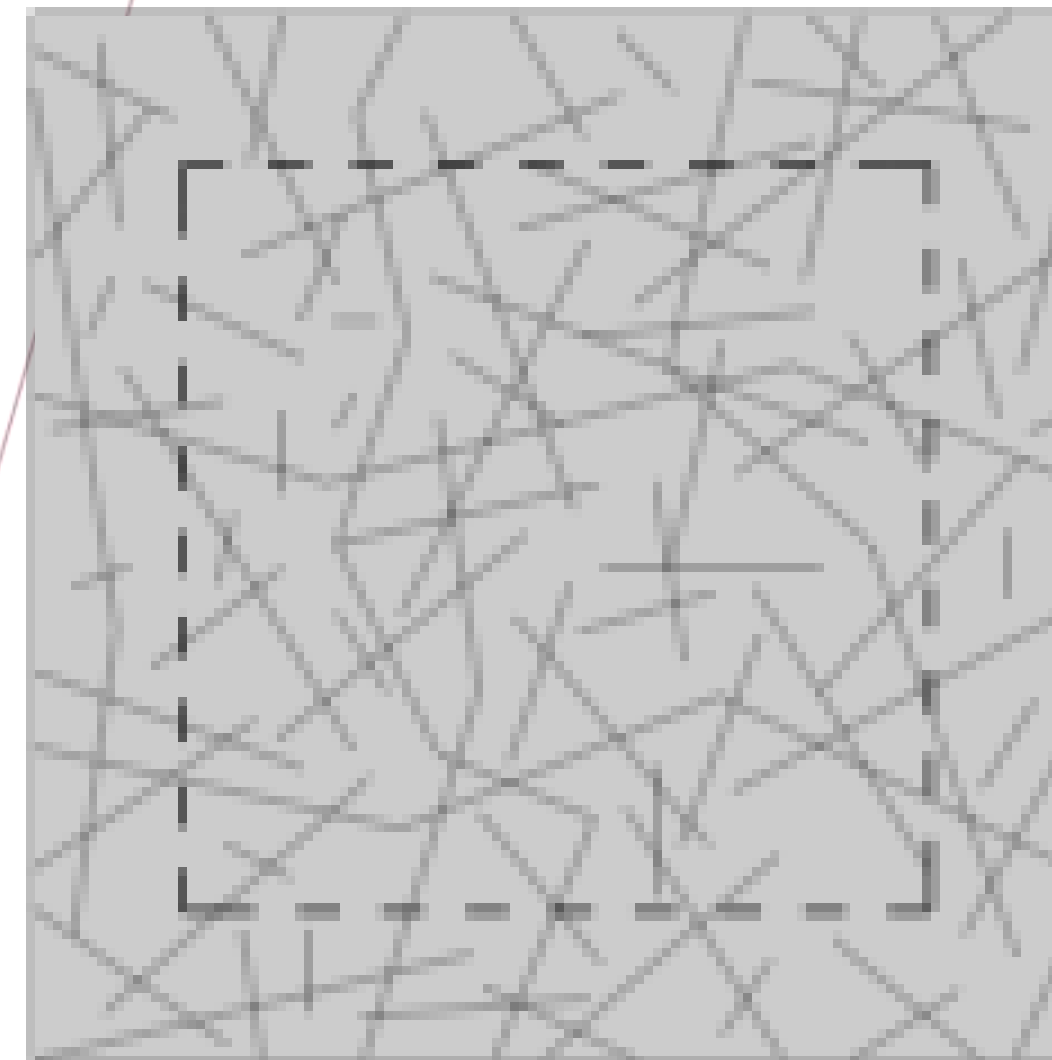
$$\begin{aligned} \tilde{\epsilon} &= 0 \\ \kappa &= \kappa_0 \\ \omega(\kappa) &= 0 \end{aligned}$$



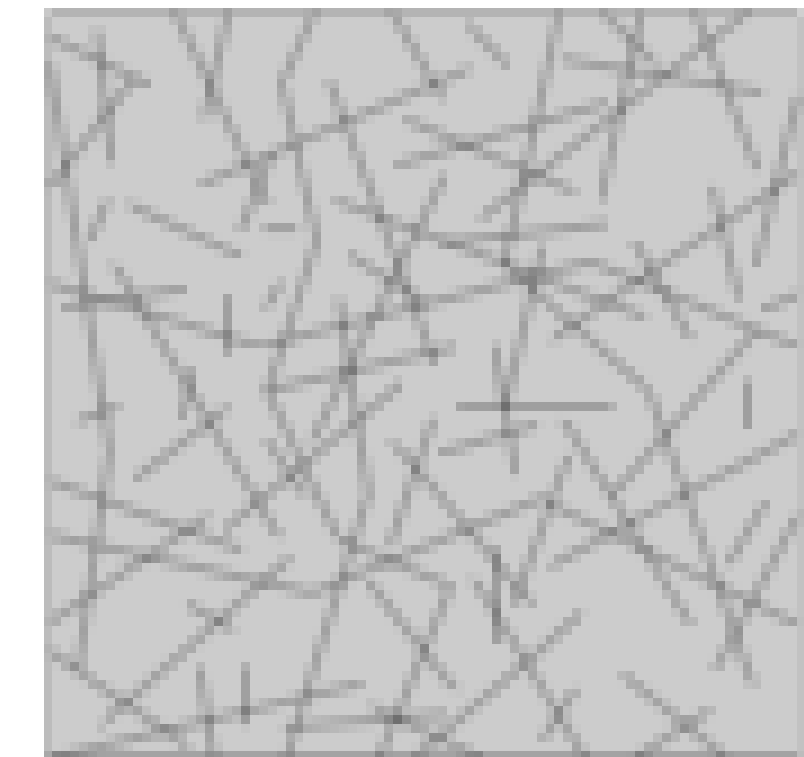
$$\begin{aligned} \tilde{\epsilon} &= \tilde{\epsilon}_t \\ \kappa_0 &< \kappa < \kappa_u \\ 0 &< \omega(\kappa) < 1 \end{aligned}$$



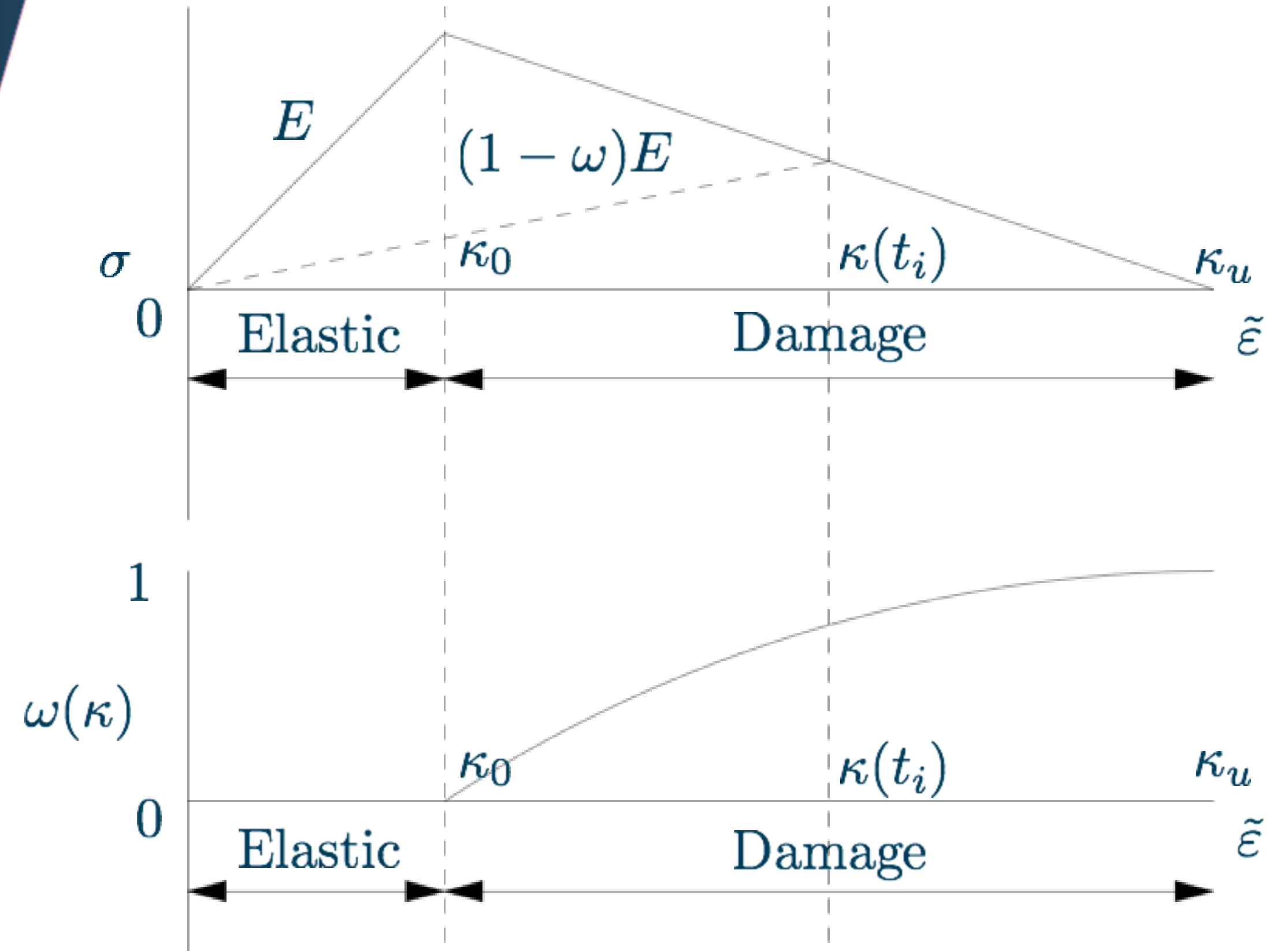
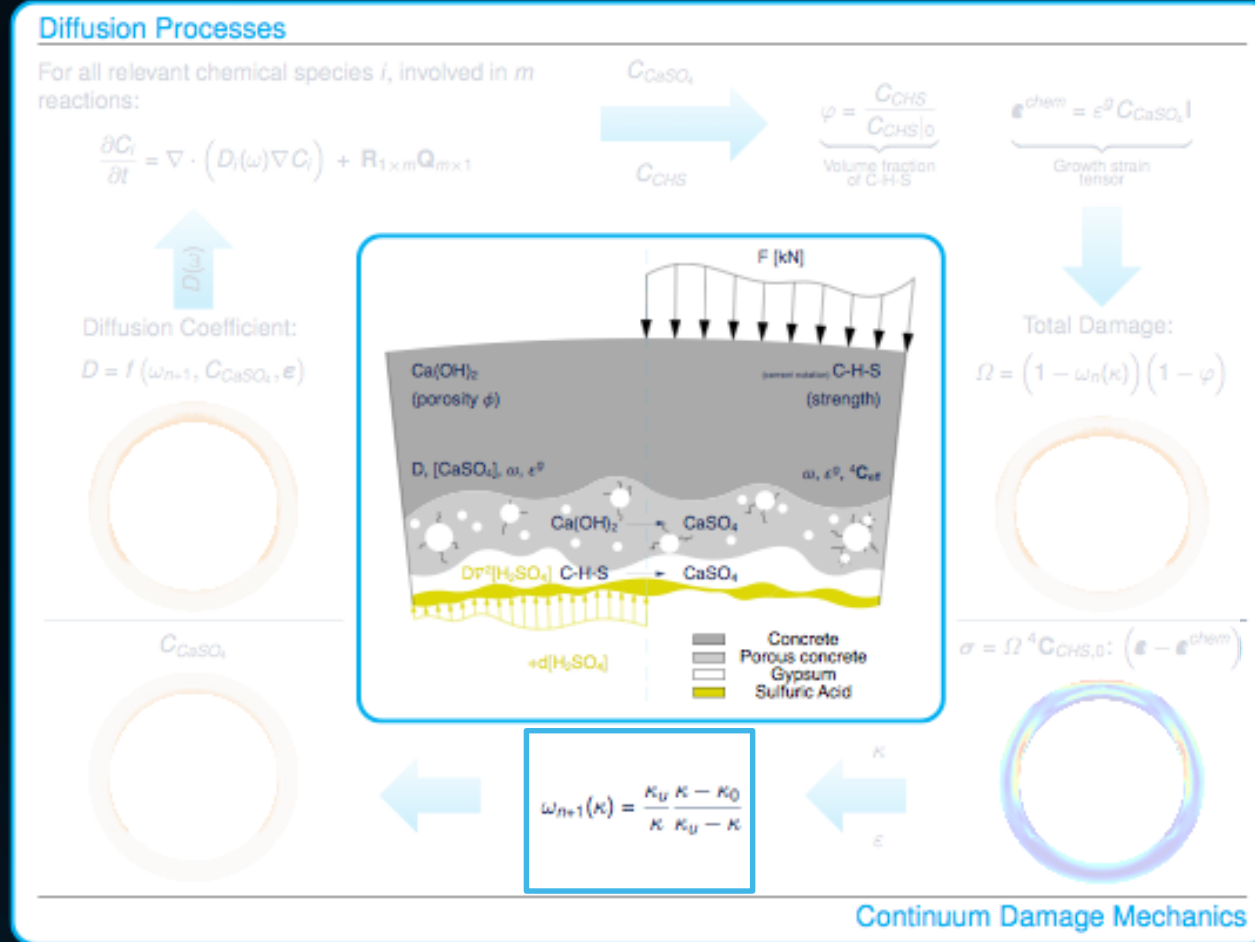
$$\begin{aligned} \tilde{\epsilon} &= \tilde{\epsilon}_{t+1} \\ \kappa &= \kappa_u \\ \omega(\kappa) &= 1 \end{aligned}$$



$$\begin{aligned} \tilde{\epsilon} &= 0 \\ \kappa &= \kappa_u \\ \omega(\kappa) &= 1 \end{aligned}$$



# Mechanical Damage



$$\omega(\kappa) = \frac{\kappa_u(\kappa - \kappa_0)}{\kappa(\kappa_u - \kappa_0)}$$



# Diffusion Properties

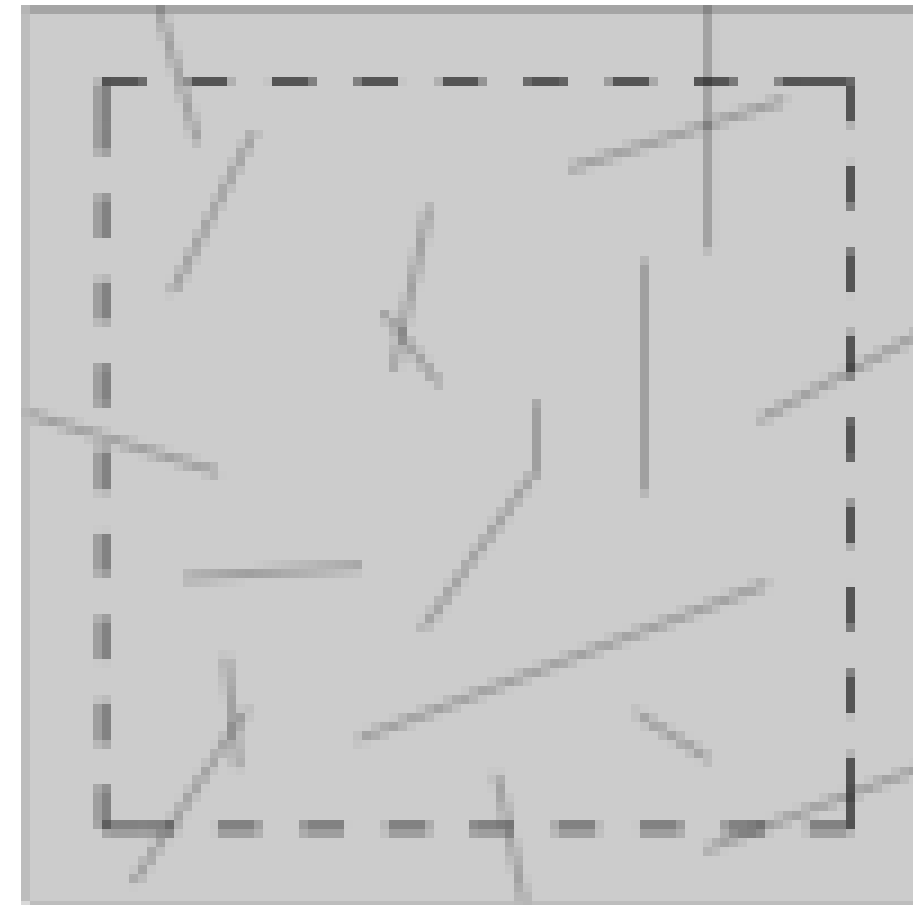
$$\omega = 0$$

$$D(\omega) = D|_0$$



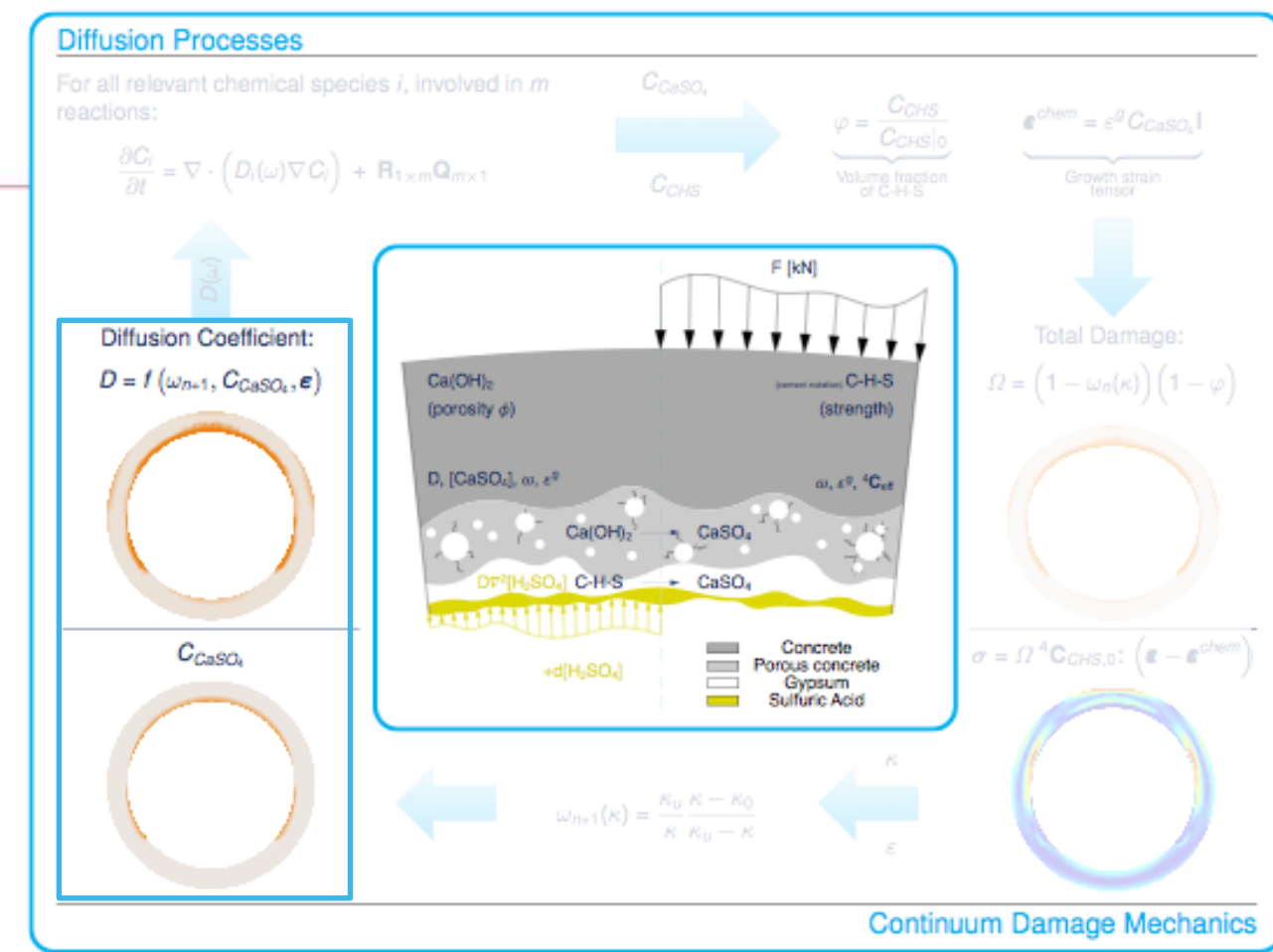
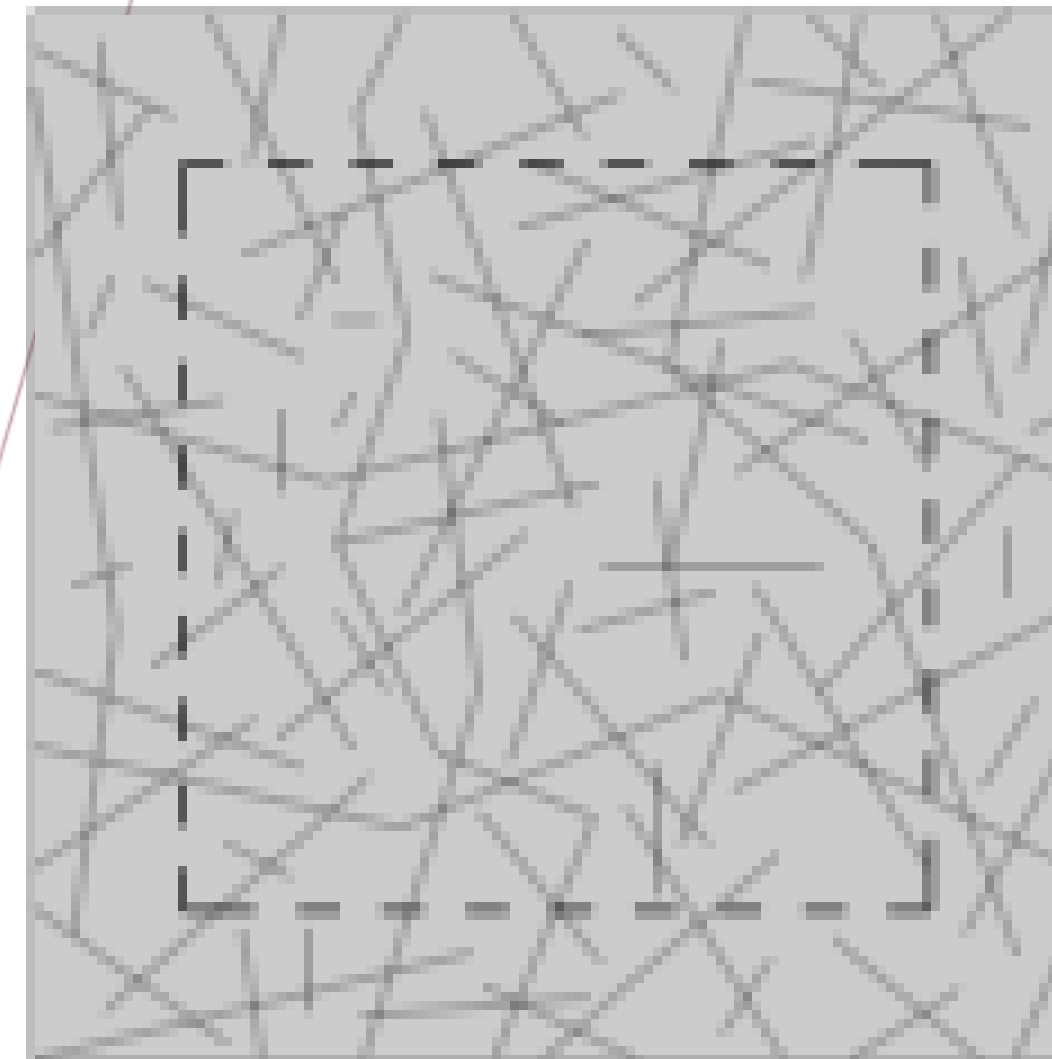
$$0 < \omega < 1$$

$$D(\omega) = (1 + \omega)D|_0$$



$$\omega = 1$$

$$D(\omega) = 2D|_0$$



$$\omega = 1$$

$$D(\omega) = 2D|_0$$

