

# Quantification of uncertain bed roughness under design conditions and propagation to the design water levels, a case study for the river Rhine

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**ABSTRACT:** Hydrodynamic river models are applied to design and evaluate measures for purposes such as safety against flooding. The modeling of river processes involves numerous uncertainties, resulting in uncertain model results. Knowledge of the type and magnitude of these uncertainties is crucial for a meaningful interpretation of the model results. The aim of this study is to quantify the uncertainty due to the hydraulic roughness predictor for the river bed in the main channel and assess its effect on the uncertainty in the modelled water levels under design discharge conditions. This uncertainty in the main channel roughness predictor consists of the uncertainty between different roughness predictors and the uncertainty within each roughness predictor. To assess the uncertainty between the selected roughness predictors, we compared five roughness predictors that computed the hydraulic resistance of the river bed in the Dutch river Rhine, based on measurements of bed forms characteristics and water levels for different discharges. The propagation of this uncertainty to the water levels is carried out using a two dimensional hydrodynamic river model with dimensions similar to the river Rhine in The Netherlands. It is shown that different roughness predictors result in significantly different roughness values for the same measurements of bed form and flow characteristics.

To determine the uncertainty between the different roughness predictors under design conditions, the Generalized Extreme Value distribution is used to extrapolate the predicted roughness values for each roughness predictor to design conditions. The uncertainty between and within the roughness predictors are combined to show that with 95% confidence the Nikuradse roughness for the main channel of the river Rhine under design conditions lies between 0.32 m to 1.03 m. The uncertain roughness results in a large uncertainty range in the design water levels of 71 cm.

*Keywords: Uncertainty; Monte Carlo Simulation; Hydraulic roughness; Hydrodynamic modeling; Generalized Extreme Value distribution; River Rhine*

## 1 INTRODUCTION

Hydrodynamic river models are applied to design and evaluate measures for purposes such as safety against flooding. These numerical models are all based on a deterministic approach. However, the modeling of river processes involves numerous uncertainties, resulting in uncertain model outcomes. Knowledge of the type and magnitude of uncertainties is crucial for a meaningful interpretation of the model results and the usefulness of results in decision making processes.

The Dutch flood defenses along the river Rhine are designed to withstand a flood with a probability of exceedance of 1/1250 per year. This number is laid down in the 1995 flood protection act (Ministry of Transport, Public Works and Water Management, 1995). The water levels that occur during such a flood are computed with the deterministic hydrodynamic model, WAQUA, us-

ing a design discharge as input. However, uncertainty resides in all parts of this model. To quantify the uncertainty in the design water levels we want to estimate the uncertainties in these parts of the model that have the largest contribution to the uncertainty in the design water levels.

Elicitation of expert opinions showed that the uncertainty due to the imposed design discharge and the hydraulic roughness formulation of the main channel contributes most to the uncertainty in the model outcomes (Warmink et al., 2009). Also, the studies by Chang et al. (1993), Bates et al. (1996), Hall et al. (2005) and Pappenberger et al. (2008) show that the hydraulic roughness of the river bed is one of the main uncertainties in river models.

The hydraulic roughness in the main channel of the Dutch river Rhine is dominated by the resistance due to bed forms that develop on the river bed and increase in height with increasing dis-

charge. Different predictors exist that relate flow and dune characteristics to hydraulic roughness. In previous research the differences between results of different roughness predictors were found to be large (Van Rijn 1993, Julien et al. 2002). Noordam et al. (2005) showed that different roughness predictors resulted in substantial different roughness values for a series of measured bed forms and flow characteristics in flumes. Also, Van der Mark (2009) showed that different roughness predictors resulted in different roughness predictions for uniform and alluvial flume data. The results from the study by Julien et al. (2002) show that also for field measurements in the Dutch river Rhine, different roughness predictors result in different roughness values. Furthermore, they showed that there is a strong hysteresis effect between bed form roughness and discharge.

The studies by Noordam et al. (2005) and Julien et al. (2002), however, only give an indication of the uncertainty in the bed roughness for measured discharges. We want to study bed roughness for the design discharge. Therefore, the aim of this study is 1) to quantify the uncertainty in the bed roughness of the main channel under design conditions, by extrapolation of field measurements and 2) to quantify the contribution of this uncertain roughness to the uncertainty in the water levels at design discharge.

## 2 UNCERTAINTY

Uncertainty is defined by Walker et al. (2003) as being any departure from the unachievable ideal of complete determinism. They presented a classification based on the classifications by Janssen et al. (1990) and Van Asselt and Rotmans (2002). The classification method by Walker et al. (2003) distinguishes between the *nature* of uncertainty, the *location* of the uncertainty in the model and the *degree* of uncertainty.

Two natures of uncertainty can be distinguished: variability (inherent uncertainty) and limited knowledge (epistemic uncertainty). Variability represents the randomness of variations in nature and limited knowledge is a property of the state of knowledge in general or of the modeller. The second dimension is the location where the uncertainties manifest themselves within the model, its context or the input and parameters of the model, which are actually parts of the model itself. The third feature is the degree of uncertainty, which deals with the different levels of knowledge, ranging from complete deterministic understanding up to total indeterminacy (in case we do not know what we do not know). Table 1 shows the classifi-

cation of three sources of uncertainty following the Walker classification matrix.

Table 1: Uncertainty matrix after Walker et al. (2003). The matrix is filled with examples of uncertainties in roughnesses, which occur in the WAQUA model. Grain refers to resistance due to grain on the river bed. Form refers to the equation used for the prediction of bed roughness and Calib. refers to the data used for calibration of the parameters in the roughness predictor in the WAQUA model.

Uncertainty	1. Nature		2. Location				3. Degree			
	Epistemic	Variability	Context	Model		Input	Parameters	Quantifiable	Scenario	Recognised ignore.
				Structure	Technical					
Grain.		X				X		X		
Form.	X			X						X
Calib.		X					X		X	

## 3 DATA

We used the data from the studies by Julien et al. (2002) and Wilbers (2004) to quantify the uncertainty in the hydraulic roughness due to bed forms. The data from Julien et al. (2002) are measured during the 1998 peak discharge in the river Rhine. Longitudinal and cross-sectional profiles of the bed elevation are recorded by single and multi-beam echosounding. Bedform data are recorded about twice a day for ten days during the flood wave and about every three days for the next twelve days. The data are processed and classified into primary and secondary dunes using the procedure described by Ten Brinke et al. (1999). Next to the bedform measurements also the water level, bed material and flow velocities are recorded.

The data from Wilbers (2004) are measured during the flood waves of 1995, 1997 and 1998 using a single-beam echosounder for the 1995 discharge, a single and multi-beam echosounder for the 1997 discharge and a multi-beam echosounder for the 1998 discharge. Both measurements are conducted in the river Rhine near Lobith. Wilbers used a program called DT2D (Wilbers, 2004) to calculate the dune characteristics from the measured bed elevation profiles. For this study only the data from the rising limb of the flood waves are taken into account.

Julien et al. (2002) and Wilbers (2004) used different methods to calculate dune characteristics from measured bed elevations. This means that the absolute values from both methods are not comparable. Therefore, the data from Julien et al. (2002)

are used to compute the performance of the roughness predictors and the extrapolation of the roughness predictors is carried out using the data of Wilbers (2004).

## 4 METHOD

To determine the uncertainty in the bed roughness under design conditions, we consider two different sources of uncertainty: firstly, the uncertainty due to different roughness predictor, that is the between predictor uncertainty and secondly the uncertainty within the roughness predictor itself, that is due to the variability of the measured data around roughness predictor.

Firstly, we will determine the uncertainty between the different roughness predictors. Many different roughness predictors exist. Therefore, we first selected five roughness predictors that were compared. Subsequently, a Generalized Extreme Value (GEV) (Coles 2001) distribution is fitted through the data and extrapolated to design conditions. Secondly, we determine the uncertainty within each roughness predictor and finally, both sources of uncertainty are combined and propagated through the WAQUA model to yield the uncertainty in the water levels.

### 4.1 Selection of roughness predictors

Many different roughness predictors exist to compute bed roughness based on characteristics of the bed, bedforms and flow characteristics. These can be classified in analytical, semi-analytical and empirical roughness predictors. The analytical predictors (Yalin, 1964; Engelund, 1966) are based directly on the mass and momentum conservation laws. The semi-analytical roughness predictors (Engelund, 1977; Karim, 1999; Van der Mark 2009) are based on the conservation laws, but are calibrated to fit measured (flume) data. The empirical roughness predictors (Vanoni-Hwang, 1967; Van Rijn, 1984, 1993; Engelund-Hansen, 1967, Haque-Mahmood, 1983; Wright-Parker, 2004) are empirical relations between bedform and flow characteristics and measured bed roughness. The (semi-)analytical predictors are developed using flume data, while the empirical predictors are best fitted on both alluvial flume data and field data. The Engelund-Hansen and Wright-Parker predictors are sensitive to the flow velocity and grain characteristics, while the other predictors are most sensitive to measured bedform characteristics.

The data from Julien et al. (2002) are used to compare these roughness predictors for the 1998 peak flood event in the river Rhine. The measured

roughness is computed using the Darcy-Weisbach friction factor:

$$f = \frac{8ghS_b}{V^2} \quad (1)$$

where  $g$  = gravity acceleration,  $h$  = water depth at river axis,  $S_b$  = bed slope, and  $V$  = flow velocity.

### 4.2 Fitting the extreme value distribution

To extrapolate the predicted roughness values to design conditions, we fit an extreme value distribution through the predicted roughness values, because we do not know the characteristics of the bed forms under design conditions. Furthermore, it is uncertain which processes in the bed form development are dominant and, therefore, how the bed forms will develop.

Most statistical distributions assume that the data are independent. However, the data from Wilbers (2004) are measured during three peak discharge waves and therefore contain only three independent discharge waves. Therefore, we generate independent roughness data based on parameterization of the predicted roughness values and an independent discharge series.

The historically measured discharges of the last 100 years for the river Rhine are used. These discharges are the annual maximum measured values at Lobith over the last 100 years. The first step is to generate a series of annual maximum roughness data by parameterization of the predicted roughness values for each selected roughness predictor. The parameterization is carried out by fitting a linear relation to the predicted roughness values. Next, for all historical discharges, a roughness value is computed based on the parameterized relation. Figure 1 shows an example of the parameterization.

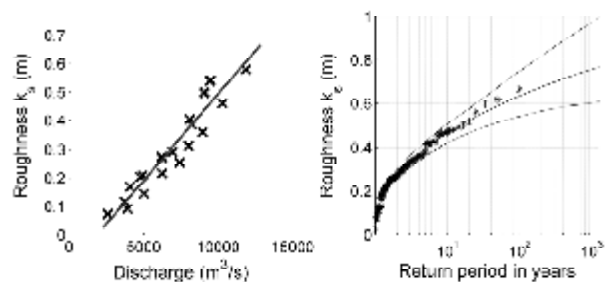


Figure 1. Illustration of the method used to determine the roughness values and fit a distribution for a single roughness predictor. The left panel shows the parameterization of the predicted roughness values. The right panel shows the fitting of an extreme value distribution and associated confidence intervals.

For each roughness predictor, a Generalized Extreme Value (GEV) distribution is fitted (Coles 2001). The fitting of the distribution is carried out using the “Orca routines”, a MatLab package for extreme value statistics (Deltares, 2010) using

probability weighted moments. The fitted GEV distributions are extrapolated to the return period of 1250 years. Figure 1 shows an example of a fitted distribution on the parameterized data. The confidence intervals are computed by bootstrapping based on the variances of the fitted parameters of the GEV distribution.

Thirdly, we included the uncertainty due to the variability in the roughness predictors. This uncertainty is caused by the simplification of the roughness predictor and the uncertainties in the measurements of the bed form and flow characteristics and account for the performance of the roughness predictors for the river Rhine data. We assign a weight to the different roughness predictors based on their performance for the Julien et al. (2002) data. The performances of the roughness predictors,  $i$  are expressed by the Root Mean Squared Errors (RMSE). The weights are then computed by:

$$w_i = \frac{\left(\frac{1}{RMSE_i}\right)^a}{\sum_{i=1}^5 \left(\frac{1}{RMSE_i}\right)^a} \quad (2)$$

In this equation the roughness predictors with low RMSE values are assigned a large weight. Subsequently, the normalized weights are multiplied by the total number of samples that are computed. We assume  $a$  to be one so the weights decrease linearly with the value of the RMSE.

#### 4.3 Propagation of uncertainties to the design water levels

The final step is to compute the contribution of the uncertain roughness under design conditions to the uncertainty in the water levels. Therefore, a Monte Carlo Simulation has been carried out using the numerical, two dimensional, depth averaged river model, WAQUA (Rijkswaterstaat 2009).

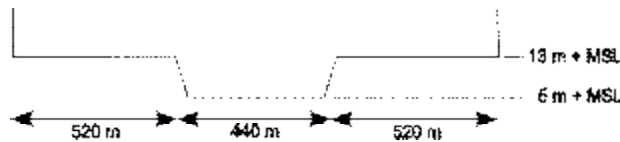


Figure 2. Cross-section of the model WAQUA. The dimensions are similar to the river Rhine.

A model is created with dimensions similar to those of the river Rhine upstream of the first bifurcation in The Netherlands (figure 2). We use grid cells of 40 m and the model represents a straight river reach of 60 km long. The downstream

boundary condition is set at a fixed water depth of 12 m in the main channel.

Thousand samples are drawn from the extrapolated log-normally distributed roughness values at the design return period. The number of samples ( $m$ ) for every selected roughness predictor,  $i$ , are computed by:

$$m_i = Nw_i \quad (3)$$

where  $N=1000$  is the total number of samples drawn from the distributions. The number of samples for each roughness predictor are rounded towards the nearest integer, so that the sum of the samples equals  $N$ .

The roughness for the floodplains is set constant at 0.6 m, which is approximately the average of the vegetated floodplains in the WAQUA model. The model is run with a constant design discharge of 16000 m<sup>3</sup>/s (Parment et al., 2001) as input.

## 5 RESULTS

### 5.1 Selection of roughness predictors

In this study we selected 5 roughness predictors that are considered applicable to predict the roughness for the main channel of the river Rhine under design conditions. Figure 3 shows the performance of ten roughness predictors for the Julien et al. (2002) data. It is shown that the analytical and semi-analytical roughness predictors perform worse than the empirical predictors. This was expected, because the (semi-)analytical predictors do not account for variability in the bedforms characteristics.

Furthermore, Figure 3 shows that the Vanoni-Hwang, Van Rijn, Engelund66, Engelund77, Wright-Parker, Haque-Mahmood and Yalin roughness predictors yield the best results. These predictors are calibrated partly on field measurements of lowland rivers, therefore they are assumed to be able to predict the roughness in other lowland rivers, such as the river Rhine.

We selected the Van Rijn, Vanoni-Hwang, Engelund77, Haque-Mahmood and the Wright-Parker roughness predictors. The Yalin and Engelund66 roughness predictors are omitted because the assumptions of abrupt flow expansion are not valid under field conditions. Furthermore, these predictors are adapted by Engelund77 by calibration of the analytical roughness predictors on irregular bed forms. The Wright-Parker roughness predictor is based on the Engelund-Hansen and adapted using field measurements. Therefore, the Wright-Parker predictor is more appropriate to

predict bed roughness for the river Rhine. Furthermore, the Karim and Van der Mark roughness predictors are not taken into account, because they are only calibrated on flume data and perform poorly for the data in this case study.

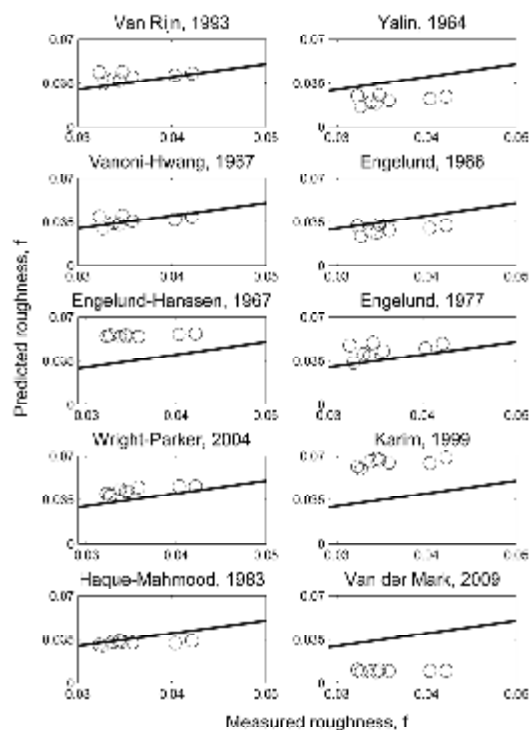


Figure 3. Performance of the different roughness predictors for the rising limb data measured during the 1998 peak discharge in the river Rhine, from Julien et al. (2002). The straight line represents the line of perfect agreement.

Figure 4 shows the predicted roughness values for the data from Wilbers (2004) using the five selected roughness predictors and the relation between predicted roughness and discharge that is used as parameterization. This figure shows that for all roughness predictors the Nikuradse value increases with increasing discharge. This was also expected, because the bedforms increase in height and length with increasing discharge. Furthermore, there is a large scatter in roughness values, especially for larger discharges. At a discharge near  $12000 \text{ m}^3/\text{s}$ , which is 75% of the design discharge, the predicted roughnesses range from 0.22 for the Haque-Mahmood roughness predictor to 0.65 for the Engelund77 roughness predictor.

The Wright-Parker roughness predictor shows a convex shaped trend of roughness with discharge, while the other three roughness predictors indicate a more linear increase. This is because the Wright-Parker roughness predictor is based on flow characteristics, while the other roughness predictors take bed form characteristics as input. Note that in this figure, only the rising limb data of the flood wave is taken into account.

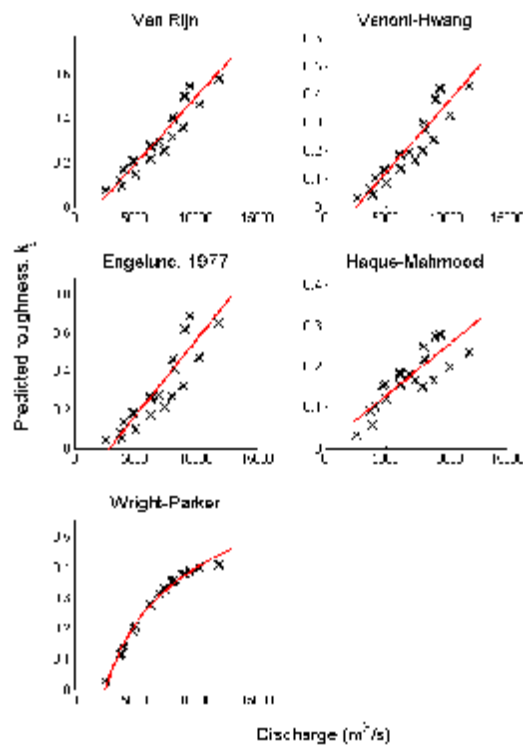


Figure 4. Parameterization of the five selected roughness predictors

## 5.2 Extreme value distribution fitting and confidence interval

Figure 5 shows the fitted GEV distributions for the five selected roughness predictors. The fitted GEV distributions are extrapolated to the return period of 1250 years. The roughness values at the design discharge range from 0.36 m for the Haque-Mahmood roughness predictor to 0.92 m for the Engelund77 roughness predictor. The figure shows that all hydraulic roughness predictors show a clear increase in the hydraulic roughness with increasing return periods. However, the GEV distributions slightly underestimate the predicted roughness values at higher return periods.

The tail behaviour of the GEV is determined by the data and not fixed a priori. The trend in the GEV distribution shows a convex shape, which is expected as dunes flatten with increasing discharge and the roughness is reduced. This is in accordance with the expectation that the hydraulic roughness does not infinitely grow with increasing discharge (Van Rijn, 1984; Julien and Klaassen 1995). Furthermore, figure 5 shows the 95% confidence intervals for the fitted distributions.

Figure 6 shows the samples drawn from the lognormal distributions around the extrapolated roughness at the design discharge for the five roughness predictors. This figure shows that the

maximum range of Nikuradse roughness varies between 0.2 and 1.4 m.

Figure 6 also shows that with 95% confidence the roughness under design conditions lies approximately between 0.32 m and 1.03 m with a mean of 0.57 m, which is a range of 71 cm. Furthermore, the smaller 95% confidence intervals for the Haque-Mahmood and Wright-Parker roughness predictors in figure 5 result in high and narrow peaks in figure 6, while the Van Rijn, Vanoni-Hwang and Engelund77 roughness predictors show less high and wider peaks in figure 6.

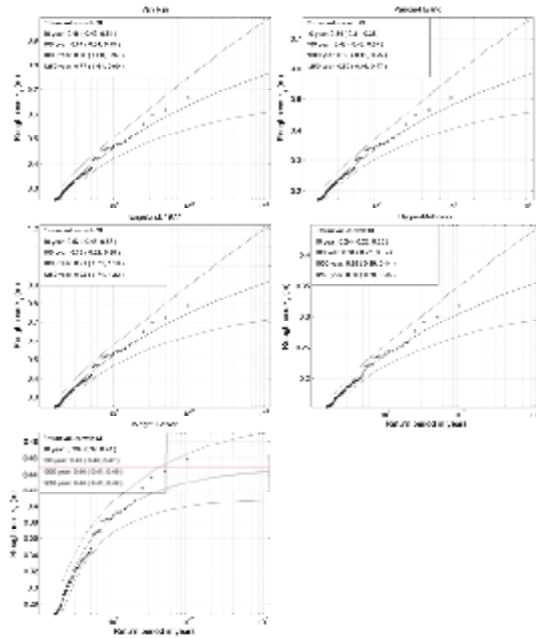


Figure 5. Fitted GEV distributions on the five roughness predictors. The dotted lines show the 95% confidence interval.

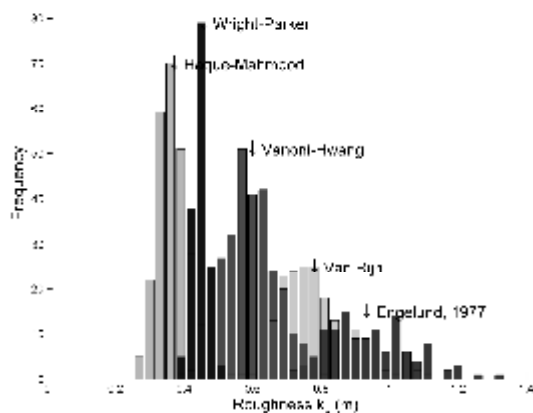


Figure 6. Uncertainty in the hydraulic roughness predictors under design conditions. The samples are drawn from the lognormal distribution at the design return period.

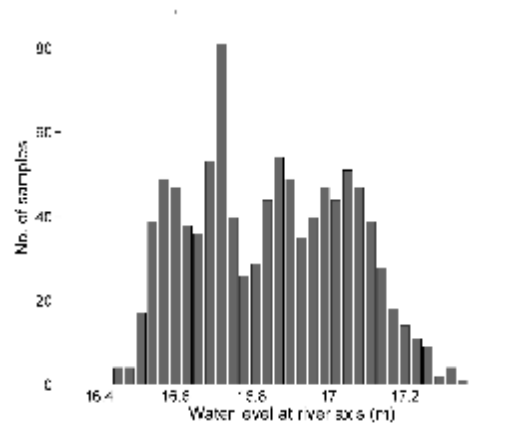


Figure 7. Uncertainty in water levels in the river Rhine under a constant design discharge.

### 5.3 Monte Carlo Simulation

A simulation with the WAQUA model is carried out for each of the samples in figure 6 to determine the contribution of the uncertain roughness to the uncertainty in the design water levels. Figure 7 shows the uncertainty in the water levels at Lobith (the location of the cross-section). This figure is the sum of five normally distributed peaks for each roughness predictor. The left peak results mainly from the Haque-Mahmood and Wright-Parker roughness predictors. The wide peak on the right is the combination of the other roughness predictors that overlap.

The figure shows that the uncertainty in the water level has a maximum range of 88.3 cm. If we only take a single roughness predictor into account (not shown in the figure), this range is approximately 20 to 40 cm depending on the selected roughness predictor. The uncertainty between the different roughness predictors, therefore, has a large influence on the uncertainty in the modelled water levels. Furthermore, figure 7 shows that with 95% confidence the design water level at the observed location lies approximately between 16.52 m and 17.22 m above MSL with a mean of 16.85 m, which is a range of 70 cm.

## 6 DISCUSSION

The study by Julien et al. (2002) shows that hysteresis has a large effect on the relation between roughness and discharge. We did not correct for hysteresis, which is therefore still present in the discharge-roughness relation. However, because only rising limb data of the flood waves is used, there is a constant lag between discharge and bed-form roughness. This may result in a shift of the average roughness values and may affect the abso-

lute values of the extrapolated roughness. However, the hysteresis effect results in a constant lag and, therefore, does not affect the uncertainty around the mean values. In further research, this hysteresis effect must be accounted for.

The uncertainty in the water levels is sensitive to the selected roughness predictors that are included in the uncertainty analysis and the selected distribution used to extrapolate the hydraulic roughness to the design discharge. The selection of the hydraulic roughness predictors has been carried out with care by using only the predictors that are valid for the river Rhine. However, several other roughness predictors exist and the uncertainty due to the roughness predictor should therefore be considered a lower bound for the uncertainty.

In future research, also the contribution of the uncertain roughness to the water levels will be computed for a design discharge wave instead of a constant design discharge. Also, the uncertainty due to the shape of the discharge wave will be studied and we will account for hysteresis.

## 7 CONCLUSIONS

The aim of this study is to quantify the uncertainty due to the hydraulic roughness predictor for the river bed in the main channel and assess its effect on the uncertainty in the modelled water levels for design discharge conditions. This uncertainty consists of uncertainty between different roughness predictors, the uncertainty due to extrapolation and the uncertainty within each roughness predictor. To quantify the uncertainty between the selected roughness predictors, we selected the Van Rijn, Vanoni-Hwang, Engelund, 1977, Haque-Mahmood and Wright-Parker roughness predictors that are appropriate to predict bed roughness for the river Rhine, based on a comparison of the performance of these roughness predictors for measurements of bed forms characteristics and water levels under varying discharge. It is shown that different roughness predictors result in a wide spreading of roughness values for the same measurements of bedform and flow characteristics.

The Generalized Extreme Value distribution is used to extrapolate the predicted roughness values for each roughness predictor to design conditions. The Generalized Extreme Values distribution accounts for the fact that we deal with extreme values. The three different sources of uncertainty are quantified and combined to show that the 95% confidence interval of the Nikuradse roughness for the main channel of the river Rhine under design conditions ranges from 0.32 m to 1.03 m, which is a range of 56 cm. A Monte Carlo Simulation shows that this result in a 95% confidence interval

for the design water levels with a range of 70 cm. The uncertain hydraulic roughness of the main channel has therefore a significant influence on the modelled water for the Dutch river Rhine. Improving the roughness predictions has the potentials to significantly reduce the uncertainties in modelled (design) water levels.

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