Faculty Engineering Technology
Program Mechanical Engineering

UNIVERSITY OF TWENTE.

COLLOQUIUM

Group: **Engineering Fluid Dynamics**

As part of his MSc thesis assignment

G.H.P. Campmans

will give a presentation, entitled:

An Exploratory Study of Multigrid Techniques for Hyperbolic Systems of Equations

Date: Thursday December 19, 2013

Time: 09:00

Room: **Horstgebouw C.101**

Summary:
In Computational Fluid Dynamics (CFD) flows are simulated for a wide range of applications. The numerical models necessary to give sufficiently accurate solutions often lead to large systems of equations. These systems are non-linear and behave hyperbolically for large Reynolds numbers. Solving such a system requires very robust techniques which at present are often not very efficient: the computational work increases quadratically or faster with the dimension of the system.

Multigrid has shown to be very efficient for many types of problems, including (systems of) elliptic partial differential equations. Convergence rates are independent of grid size, therefore the computational work scales nearly linearly with the problem size. However the efficiency of standard multigrid rapidly deteriorates when applied to hyperbolic problems.

The goal of this study is to explore the problems and potential remedies for the use of multigrid for hyperbolic systems of equations. Several test cases have been investigated, starting with the relatively easy elliptic Poisson problem, followed by the advection-diffusion, Cauchy-Riemann and finally the Euler equations. The origin and behaviour of typical problems like odd-even decoupling and non-diagonal dominant system matrices are investigated.

Quite some modifications to multigrid for standard elliptic problems have to be made for getting good results. The advection-diffusion problem rapidly diverges for low diffusivity, but with multigrid as preconditioner used in a GMRES algorithm, convergence is maintained down to much smaller diffusivity. The Cauchy-Riemann equations suffer from odd-even decoupling, and non-diagonal dominance. The phenomenon that causes odd-even decoupling is investigated, and some solutions are given. Resulting oscillatory components may be removed with an alternative relaxation method. Another possibility is to tackle the cause of the odd-even decoupling by discretising on a staggered grid, or by adding artificial dissipation. For the latter good multigrid convergence is shown. For any discretisation Kaczmarz relaxation will be stable, also for non-diagonal dominant system matrices. Multigrid has not yet been successfully applied to the Euler equations. Single grid convergence is achieved with Kaczmarz relaxation, but multigrid performs poorly and hardly accelerates convergence. Future research will have to address the phenomena behind the slow convergence for this system. Especially constructing the coarse grid operator is expected to improve convergence.

Assessment committee:
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