UNIVERSITY OF TWENTE.
Multiscale Mechanics (MSM) group

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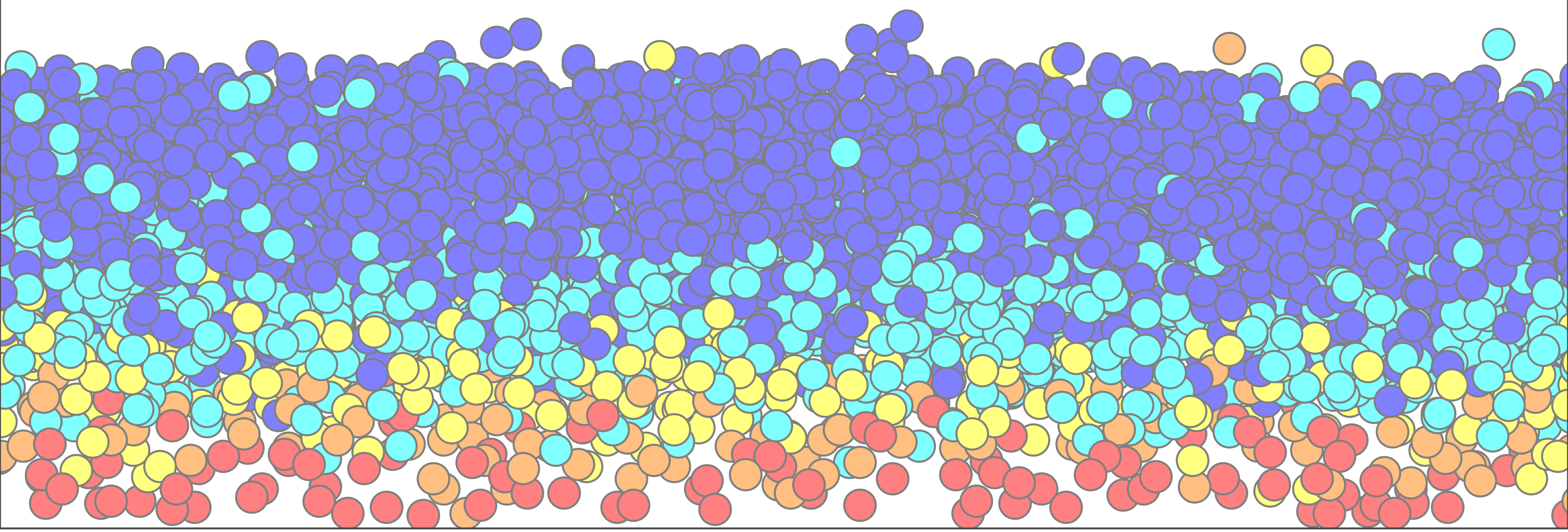
VICI Progress meeting
Enschede, Netherlands, January 2014

Particle size scalings in vertically vibrated granular media

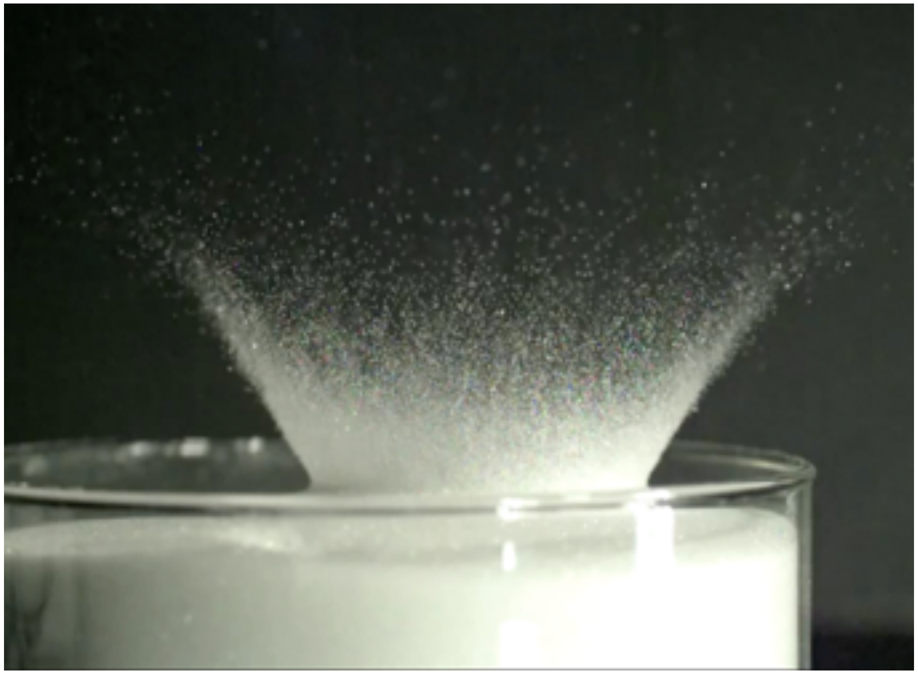



SIXTYSYMBOLS

Granular splash and jet

D. Lohse et al., Nature (London), 2004

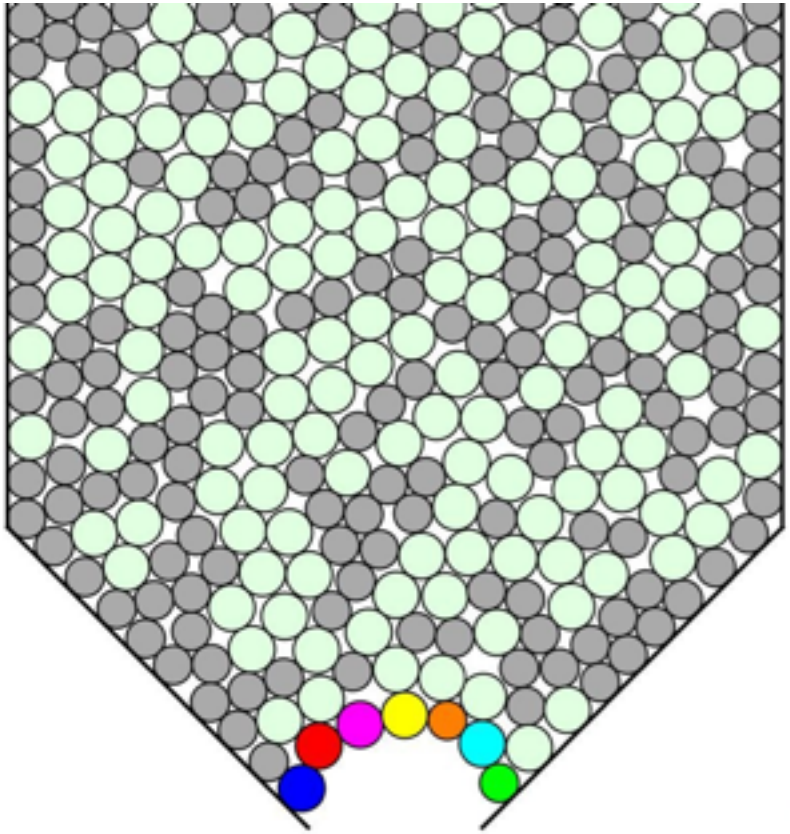
J. R. Royer et al., Nature Phys., 2005



by SixtySymbols

Arching

A. Drescher et al., Powder Technol, 1995



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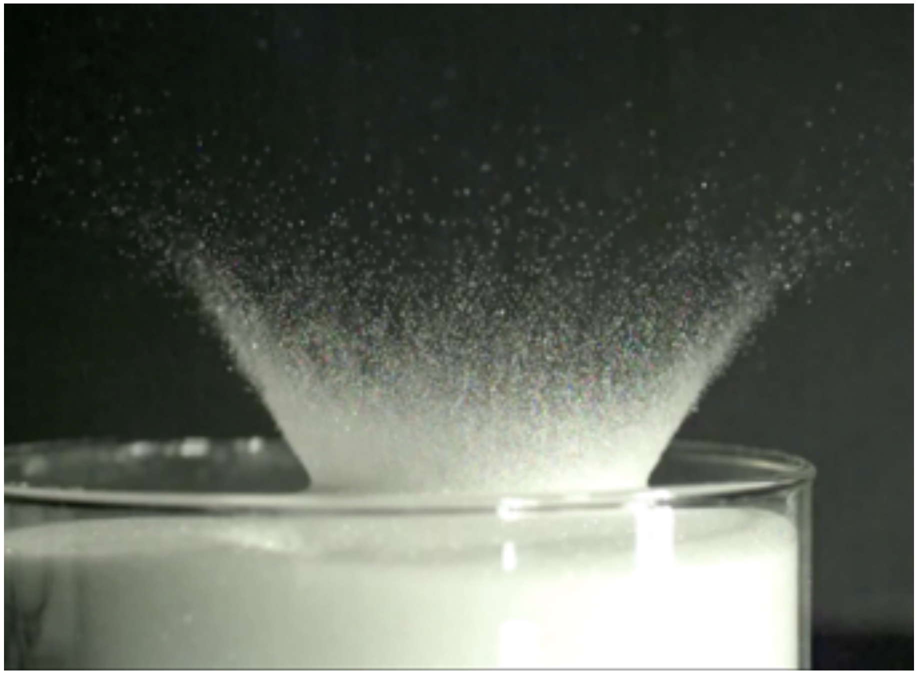

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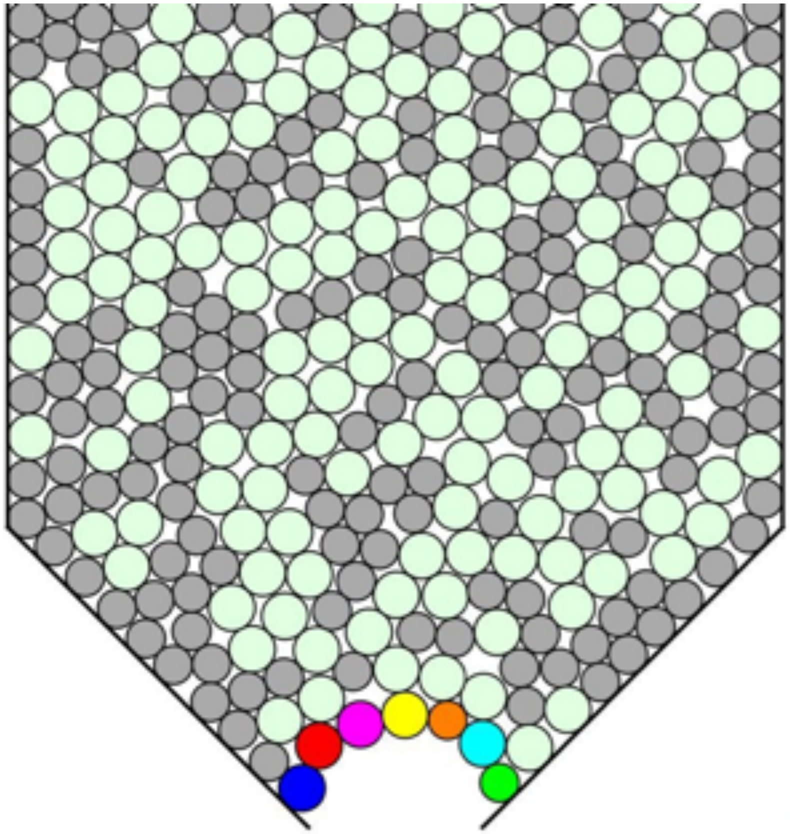

When/how do granular materials flow?

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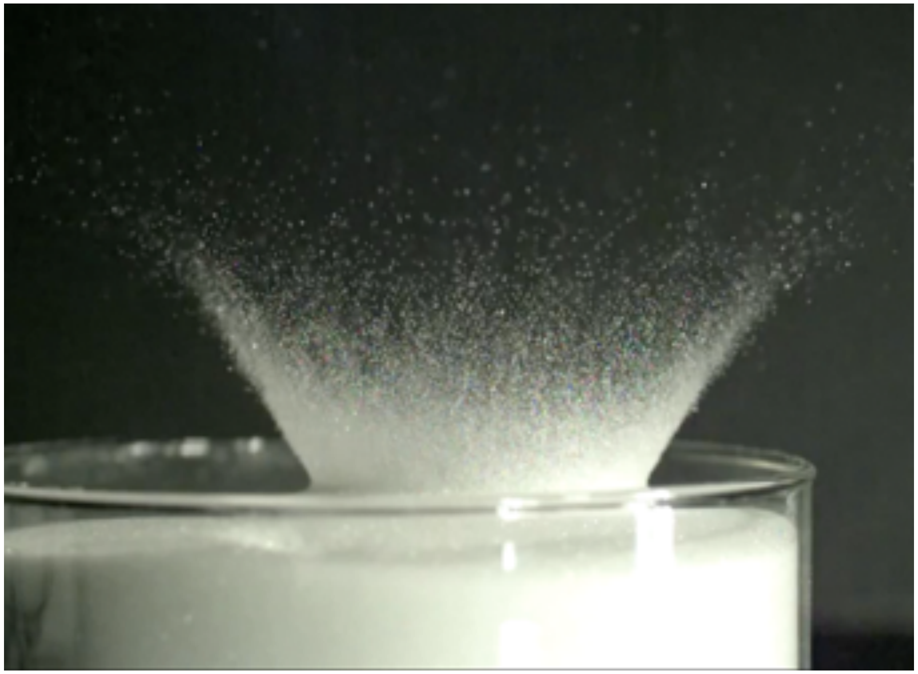

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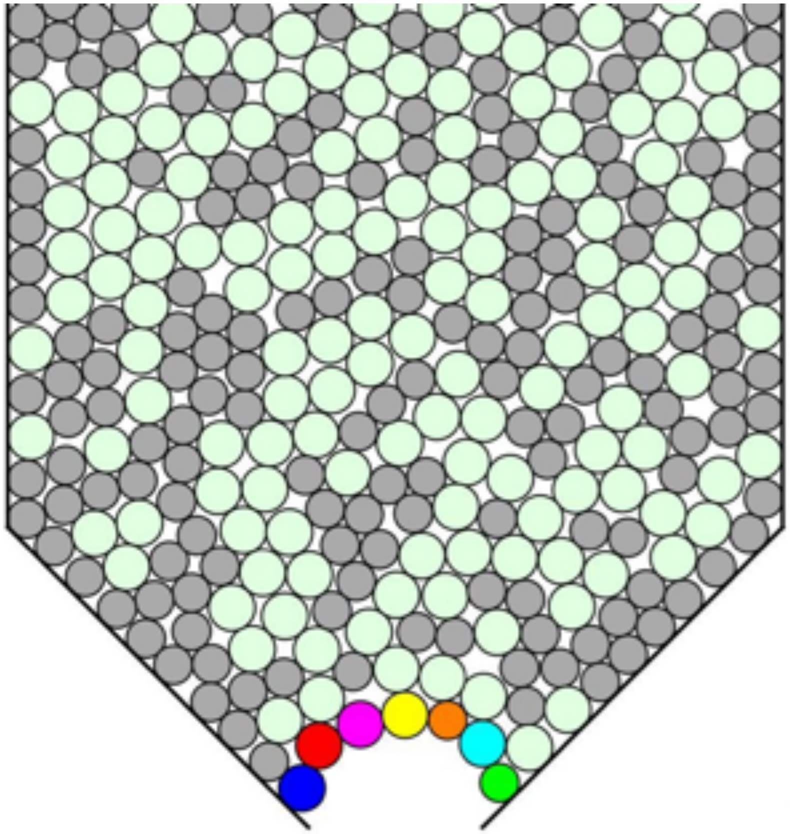
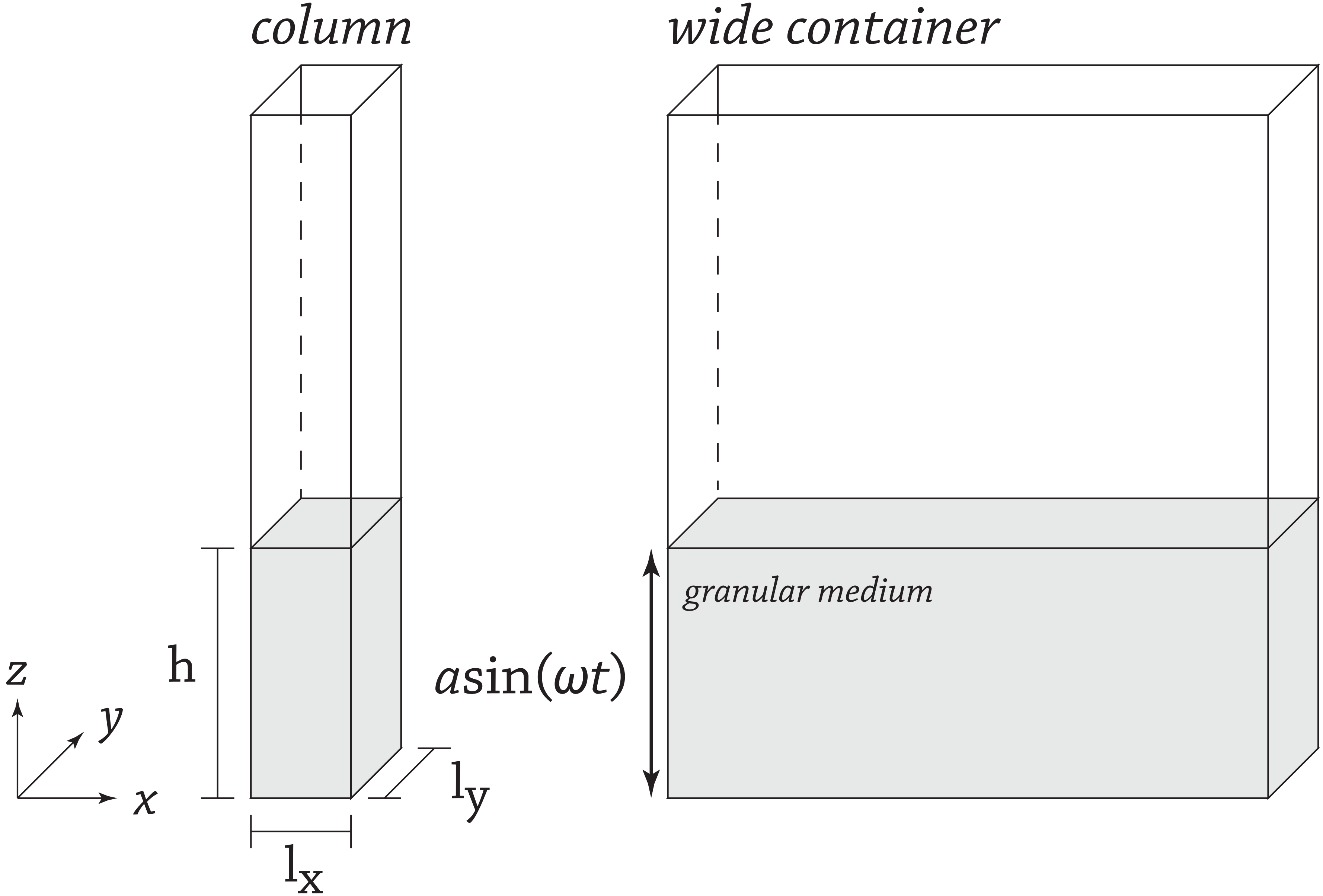


When/how do granular materials flow?

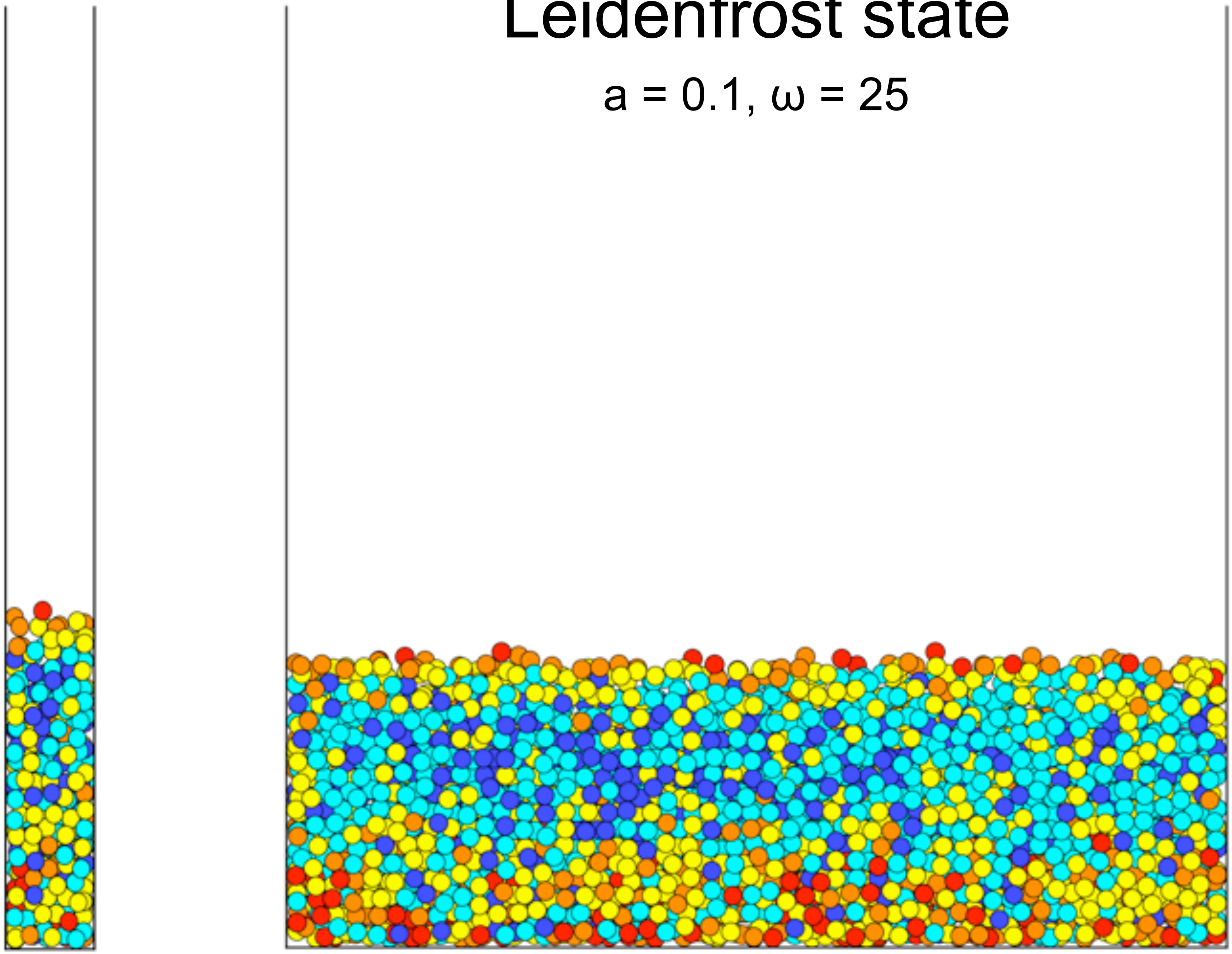
How do granular dynamics depend on particle size?



$$d = 1;\ l_x = (5, 50),\ l_y = 5,\ N = (300, 3000)$$

Leidenfrost state

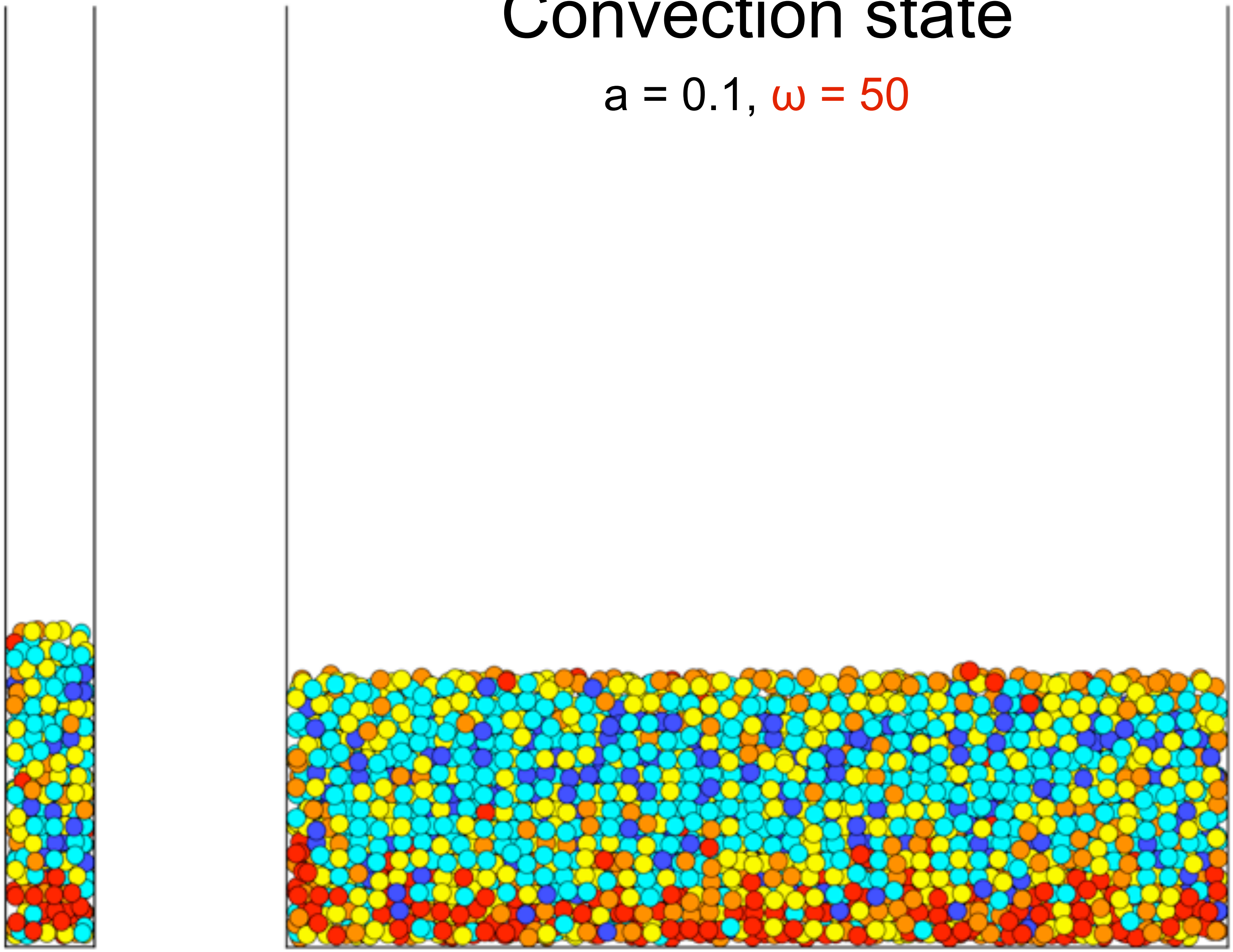
$a = 0.1, \omega = 25$



Color is kinetic energy

Convection state

a = 0.1, ω = 50



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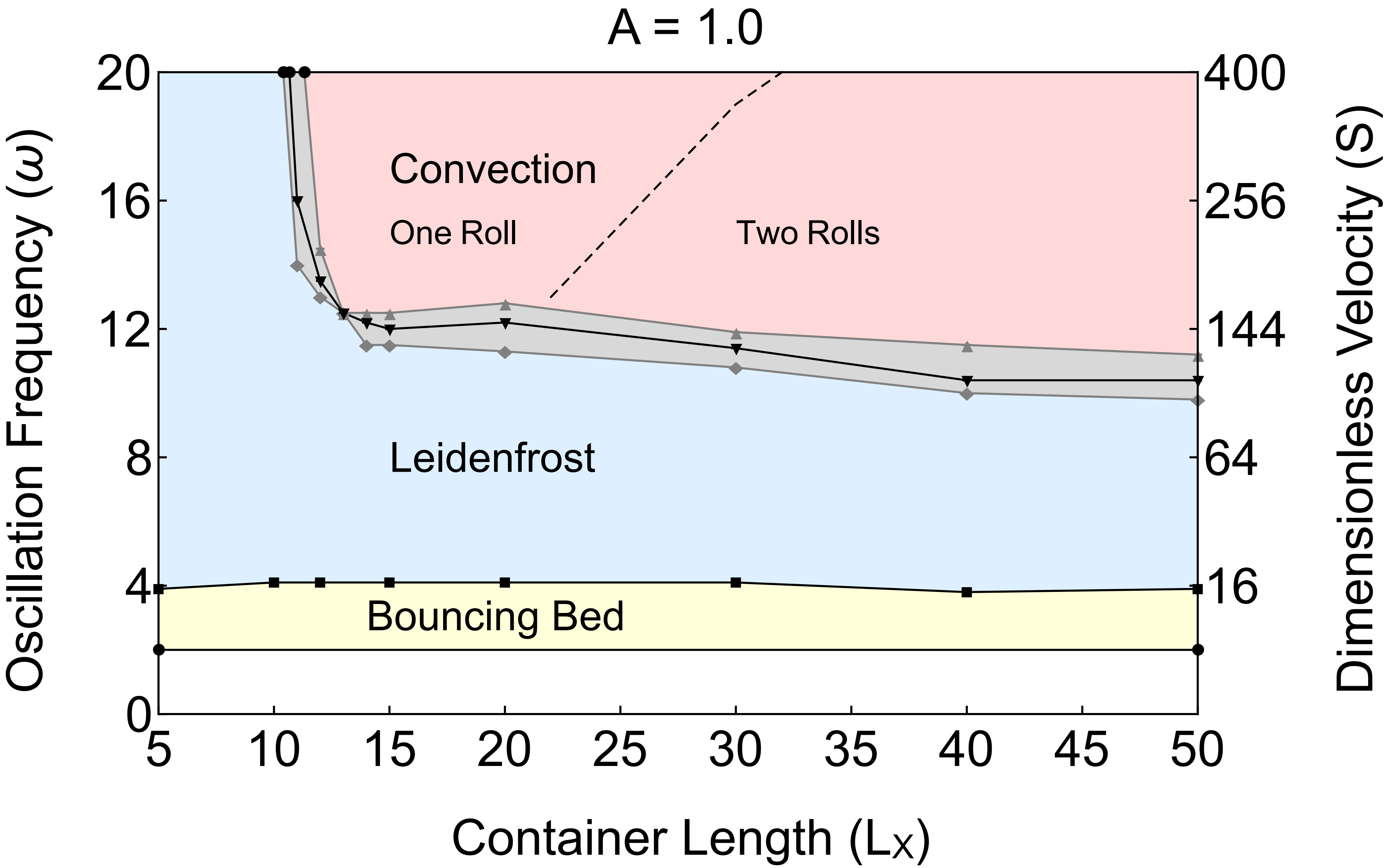

Further description of states: *P. Eshuis et al.* Granular Matter, vol. 15, pp. 893-911.

How do granular dynamics depend on particle size?

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Modify ‘d’ while retaining the macroscopic phenomenology

How do granular dynamics depend on particle size?

Modify ‘d’ while retaining the
macroscopic phenomenology

=

granular hydrodynamics equations

particle conservation

$$\frac{\partial n}{\partial t} + \vec{u} \cdot \nabla n + n \nabla \cdot \vec{u} = 0$$

momentum conservation

$$m\,n \left(\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} \right) = -\nabla p + \nabla \cdot \left(\mu \left[\nabla \vec{u} + (\nabla \vec{u})^T \right] \right) + \nabla (\lambda \nabla \cdot \vec{u}) + mn\vec{g}$$

energy conservation

$$n \frac{\partial T}{\partial t} + n\vec{u} \cdot \nabla T = \nabla \cdot \vec{J} - p(\nabla \cdot \vec{u}) + \tau \cdot \nabla \vec{u} - I$$

particle conservation

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energy conservation

$$n \frac{\partial T}{\partial t} + n\vec{u} \cdot \nabla T = \nabla \cdot \vec{J} - p(\nabla \cdot \vec{u}) + \tau \cdot \nabla \vec{u} - I$$

energy conservation in steady state

$$0 = \nabla \cdot \vec{J} - I$$

Fourier's law

$$\vec{J} = -\kappa_0(T)\,\nabla T,$$

$$\kappa_0(T) = \kappa\, T^{1/2}.$$

Dissipation by particle-particle collisions

$$I = \gamma\, n^2\, T^{3/2}$$

energy conservation in steady state

$$\nabla \cdot \left(T^{1/2} \nabla T\right) = \frac{\gamma}{\kappa} n^2 T^{3/2}$$

energy conservation in steady state

$$\nabla \cdot (T^{1/2} \nabla T) = \frac{\gamma}{\kappa} n^2 T^{3/2}$$

energy conservation in steady state

$$\nabla \cdot (T^{1/2} \nabla T) = \frac{\gamma}{\kappa} n^2 T^{3/2}$$

transport coefficients

$$\gamma_{2D} = \frac{\varepsilon}{\gamma_c n l \sqrt{m}},$$

$$\kappa_{2D} = \frac{n(0.6l + d)^2}{l\sqrt{m}}.$$

$$l_{2D} = \frac{n_c - n}{\sqrt{8} n d (n_c - 0.6n)}.$$

scalings with d

$$n \propto d^{-2}$$

$$m \propto d^2$$

energy conservation in steady state

$$\nabla \cdot \left(T^{1/2} \nabla T\right) = \frac{\gamma}{\kappa} n^2 T^{3/2}$$

restitution coefficient scaling with d

$$\frac{\gamma_{2D}}{\kappa_{2D}} n^2 \propto \varepsilon\, d^{-2}$$

$$r_{2D}(d) = \sqrt{1 - \left(\frac{d}{d_1}\right)^2 (1 - r_1^2)}$$

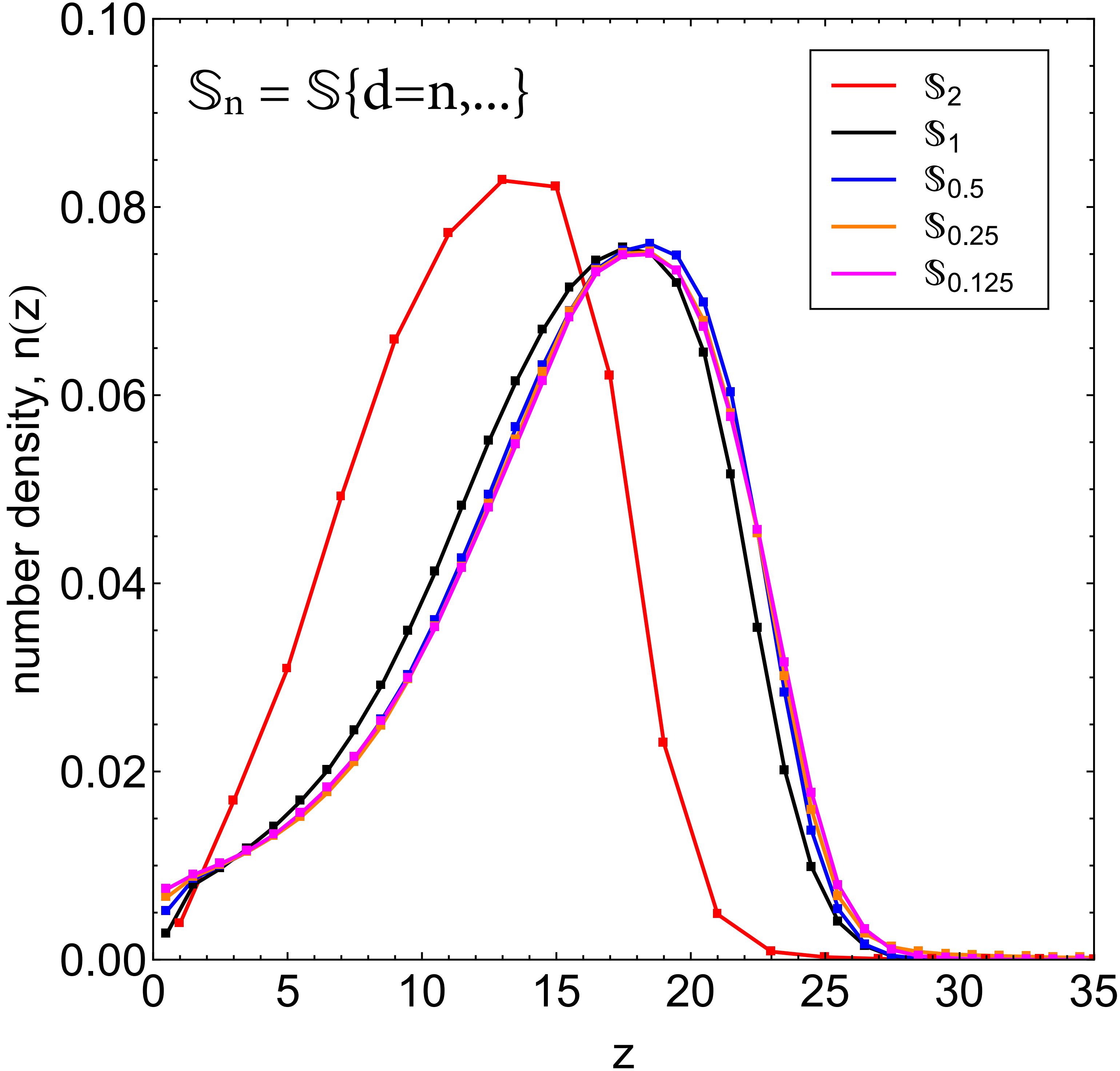
equivalent systems $\mathbb{S}(\mathrm{d};\ \mathrm{l_x},\ \mathrm{l_y},\ \mathrm{N},\ \mathrm{r},\ \mathrm{a}, \omega)$

$$a = 0.1d \qquad r_{2D}(d) = \left[1 - \left(\frac{d}{d_1}\right)^2 (1 - r_1^2)\right]^{1/2}$$

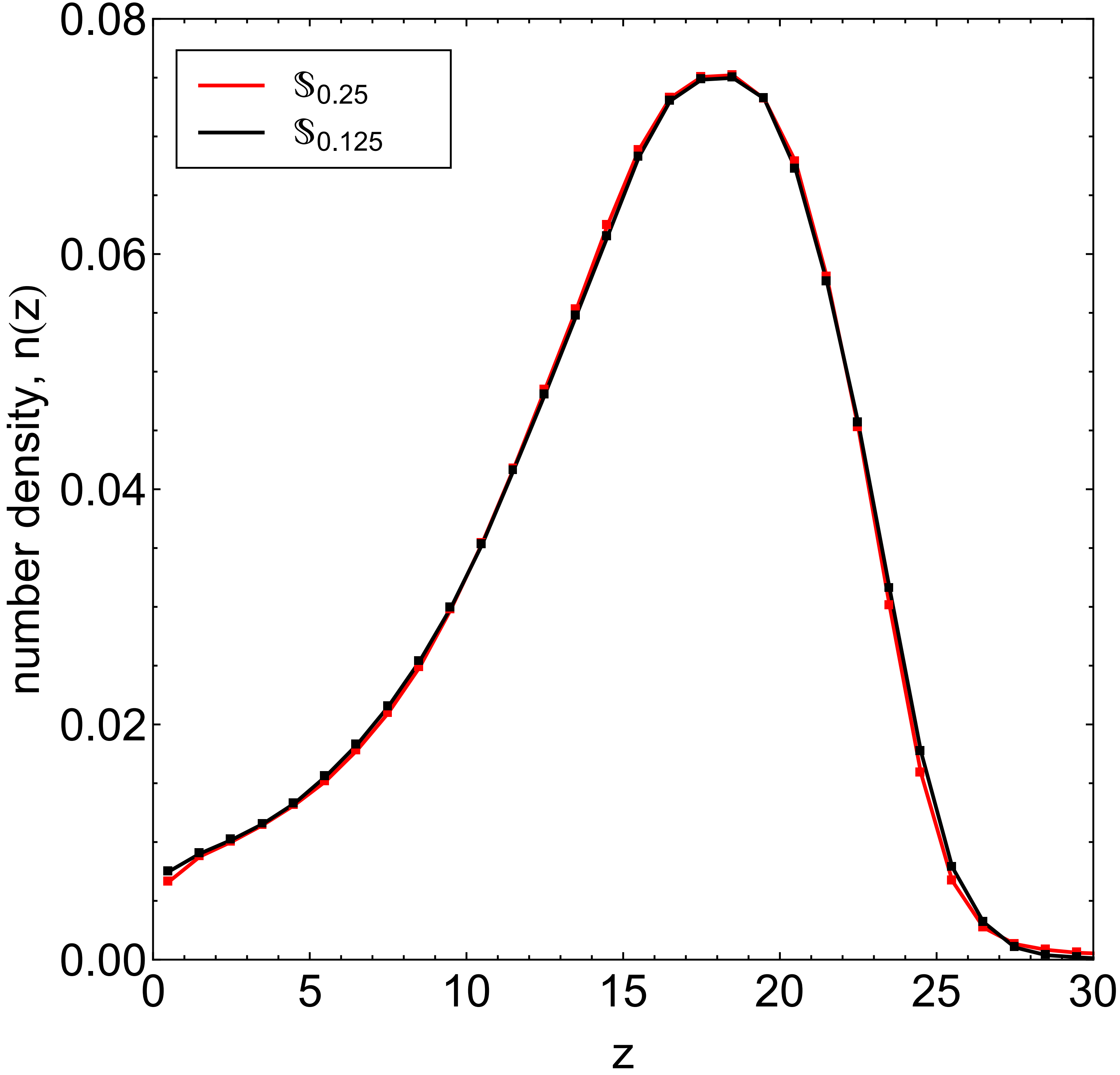
$$l_y = 5d \qquad \omega(d) = \omega_1 \left(\frac{d_1}{d}\right)^{1/2}$$

$$l_x = 5d \qquad N(d) = N_1 \frac{d_1}{d}$$

$$N(d) = N_1 \left(\frac{d_1}{d}\right)^2$$

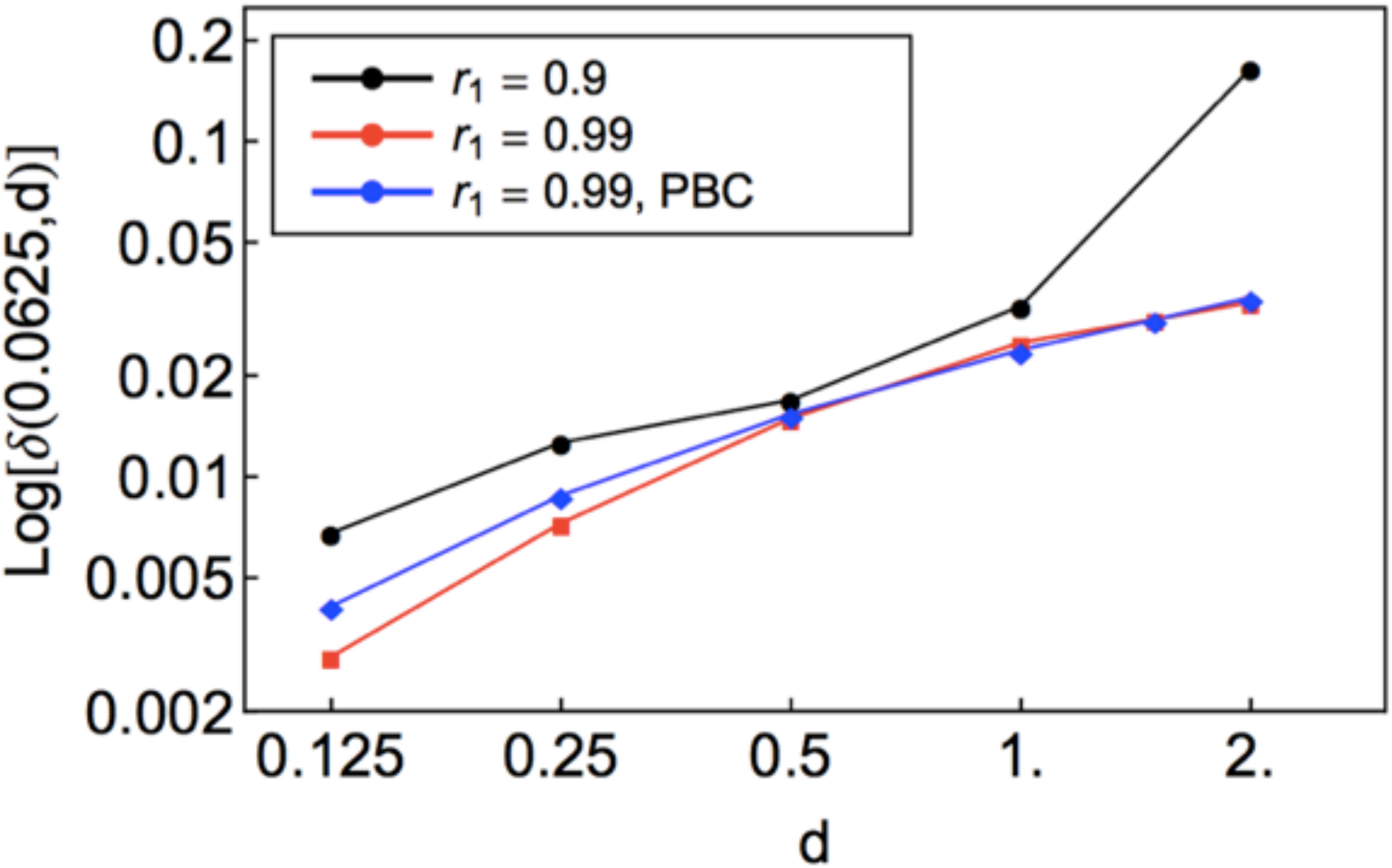


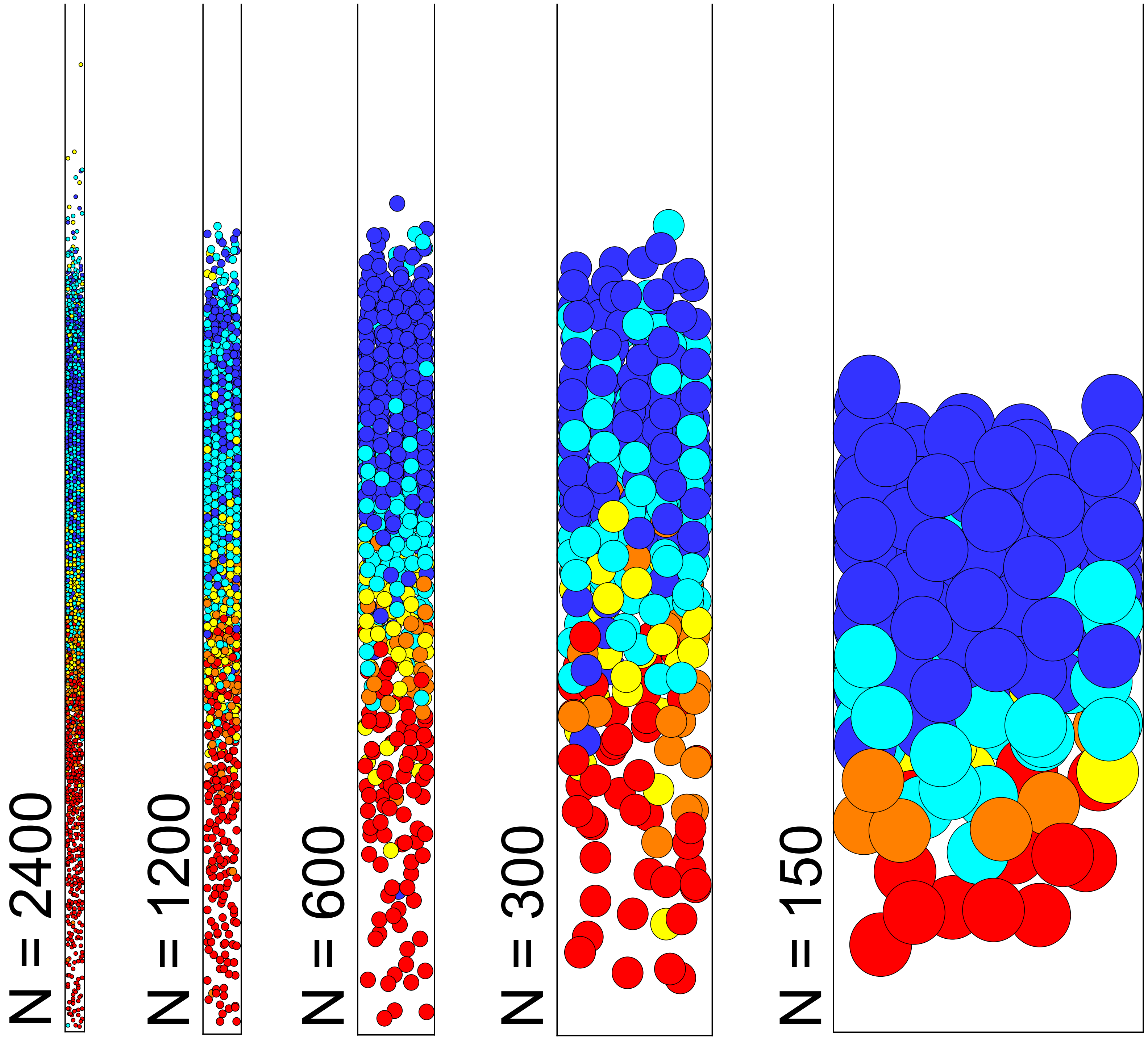
$\mathbb{S}_n = \mathbb{S}\{d=n,...\}$
$\mathbb{S}_2$
$\mathbb{S}_1$
$\mathbb{S}_{0.5}$
$\mathbb{S}_{0.25}$
$\mathbb{S}_{0.125}$
number density, n(z)
z
0.00
0.02
0.04
0.06
0.08
0.10
0
5
10
15
20
25
30
35



0.08
0.06
0.04
0.02
0.00
0
5
10
15
20
25
30
z
number density, n(z)
$\mathbb{S}_{0.25}$
$\mathbb{S}_{0.125}$

$$\delta(d_1, d_2) = \frac{1}{2}\int_0^\infty |\bar{n}_{d_1}(z) - \bar{n}_{d_2}(z)| dz$$

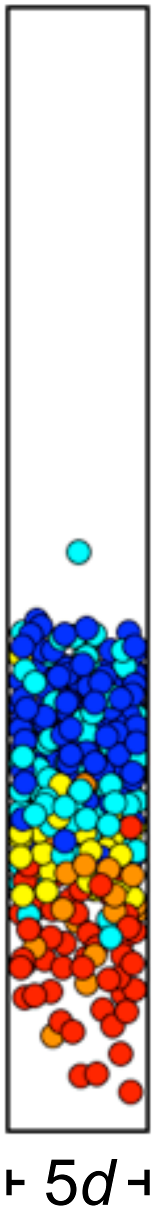




N = 2400
N = 1200
N = 600
N = 300
N = 150

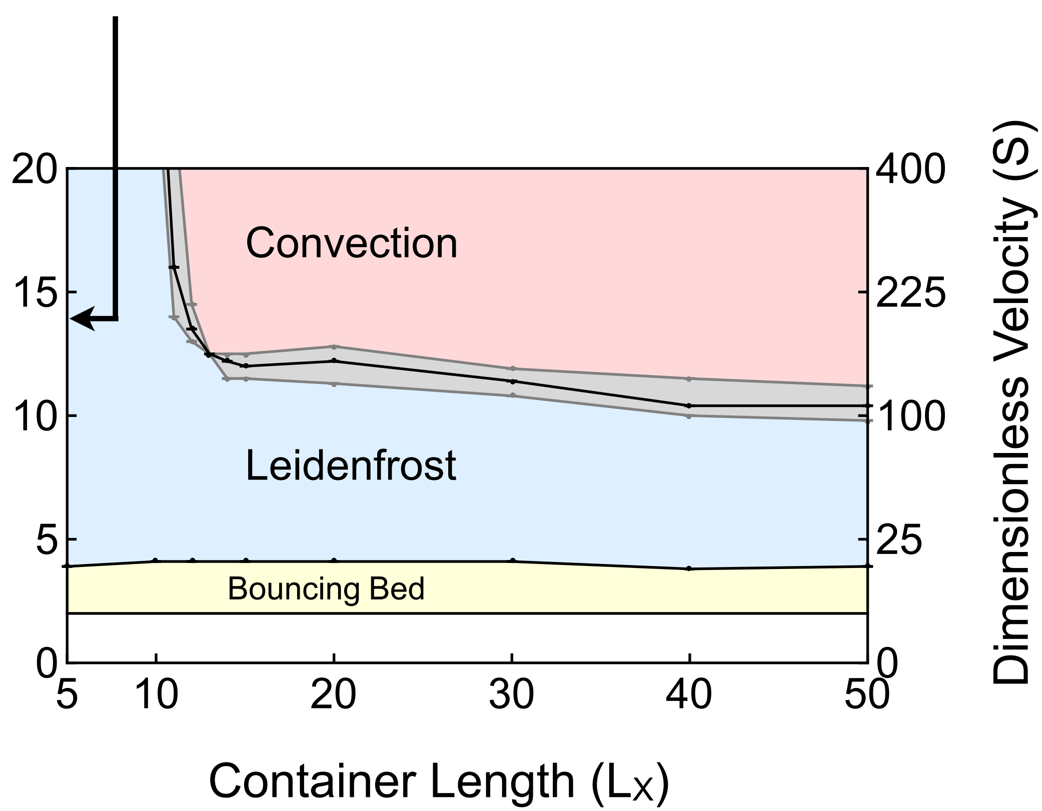
What can equivalent macroscopic systems with different number of particles show us?

low-frequency oscillations

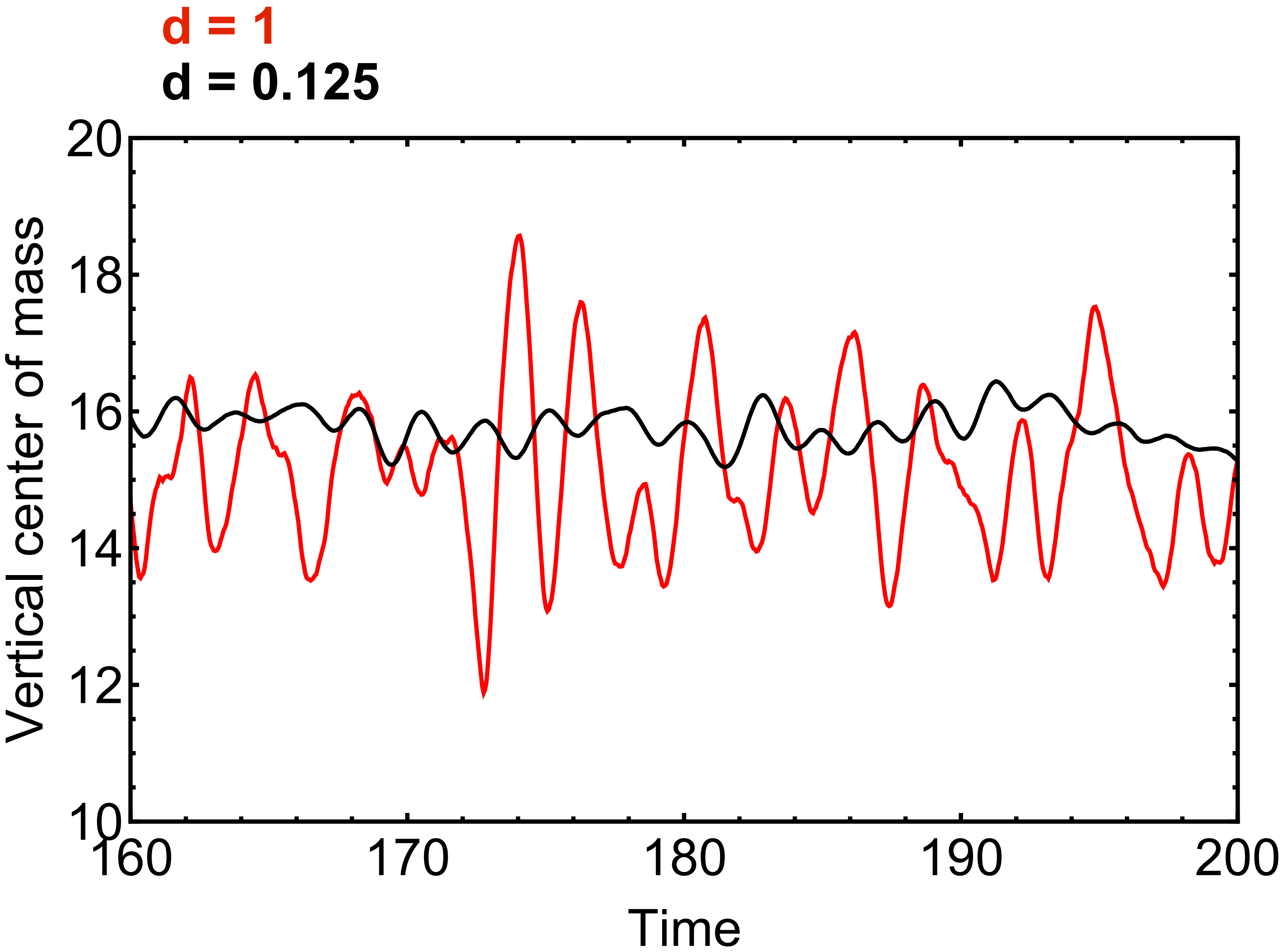


$A = 1.0d$, $\omega = 14.0(d/g)^{1/2}$

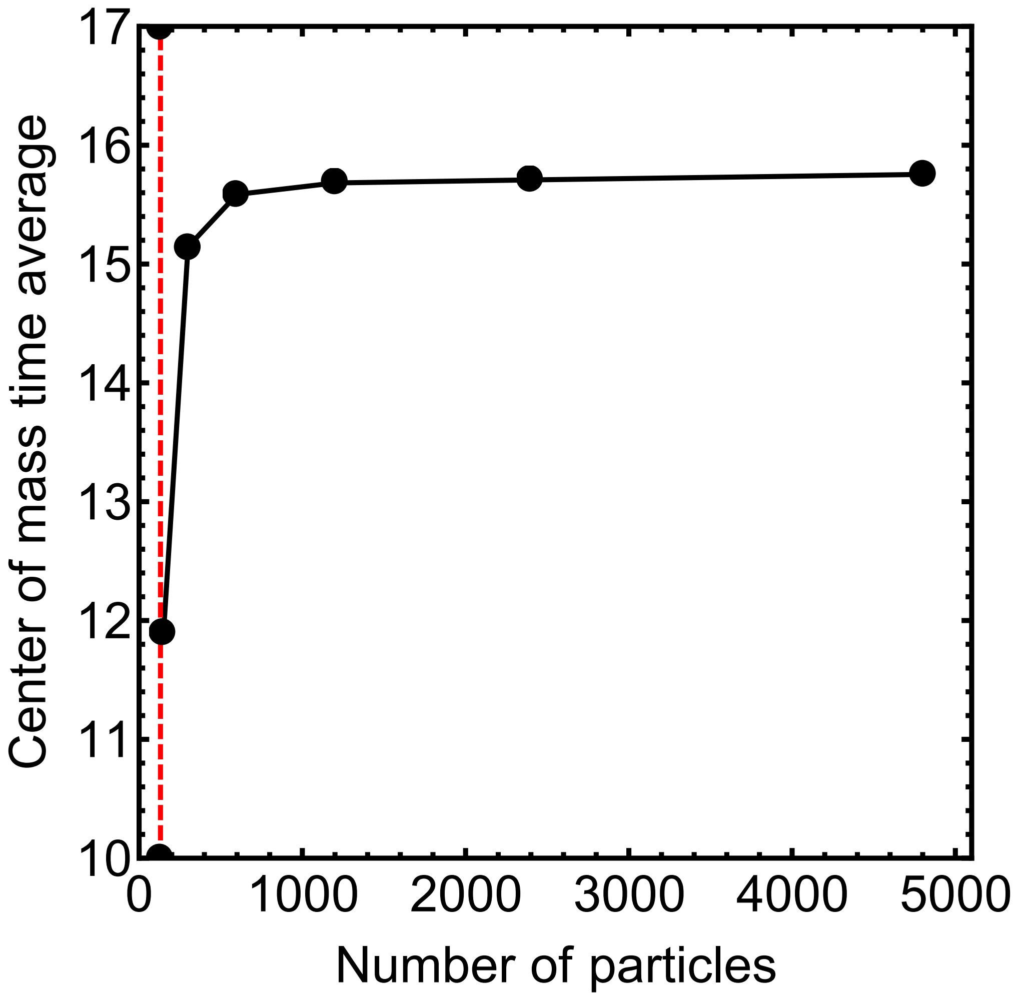
(40 Hz for 5mm particles)

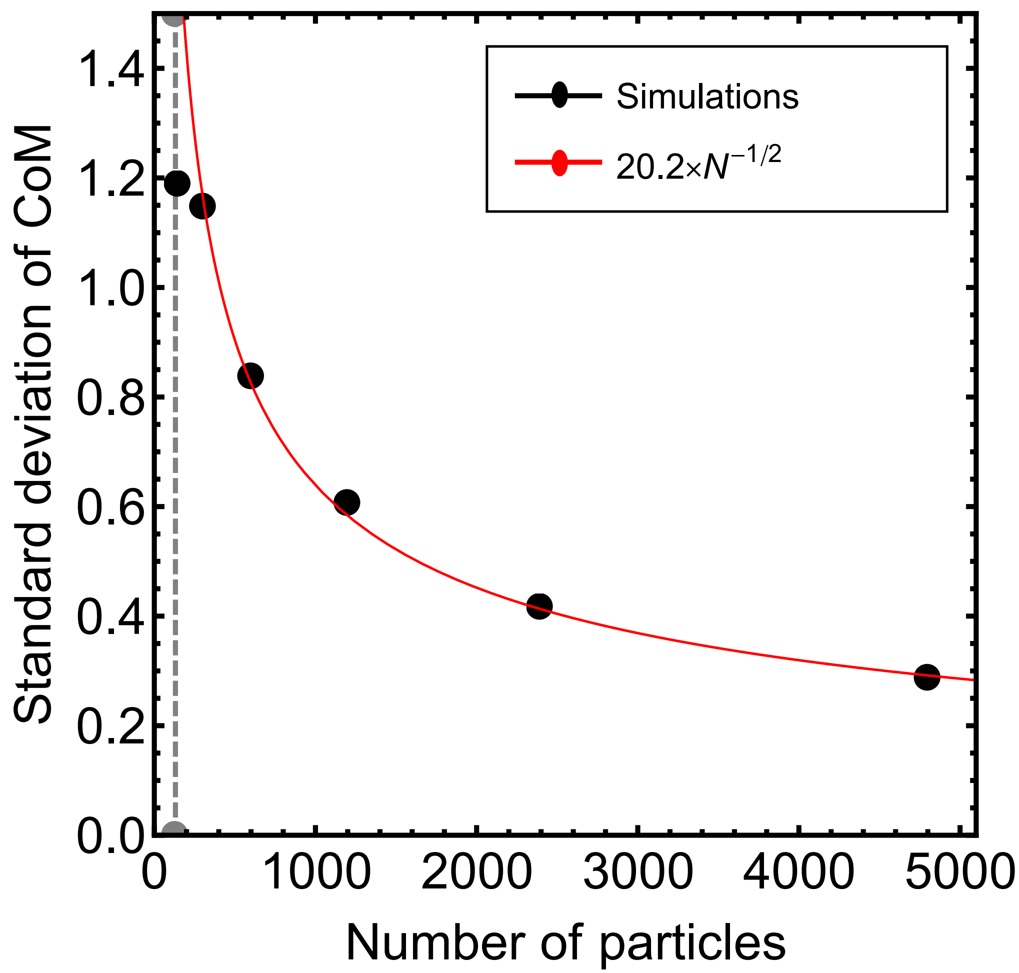


low-frequency oscillations

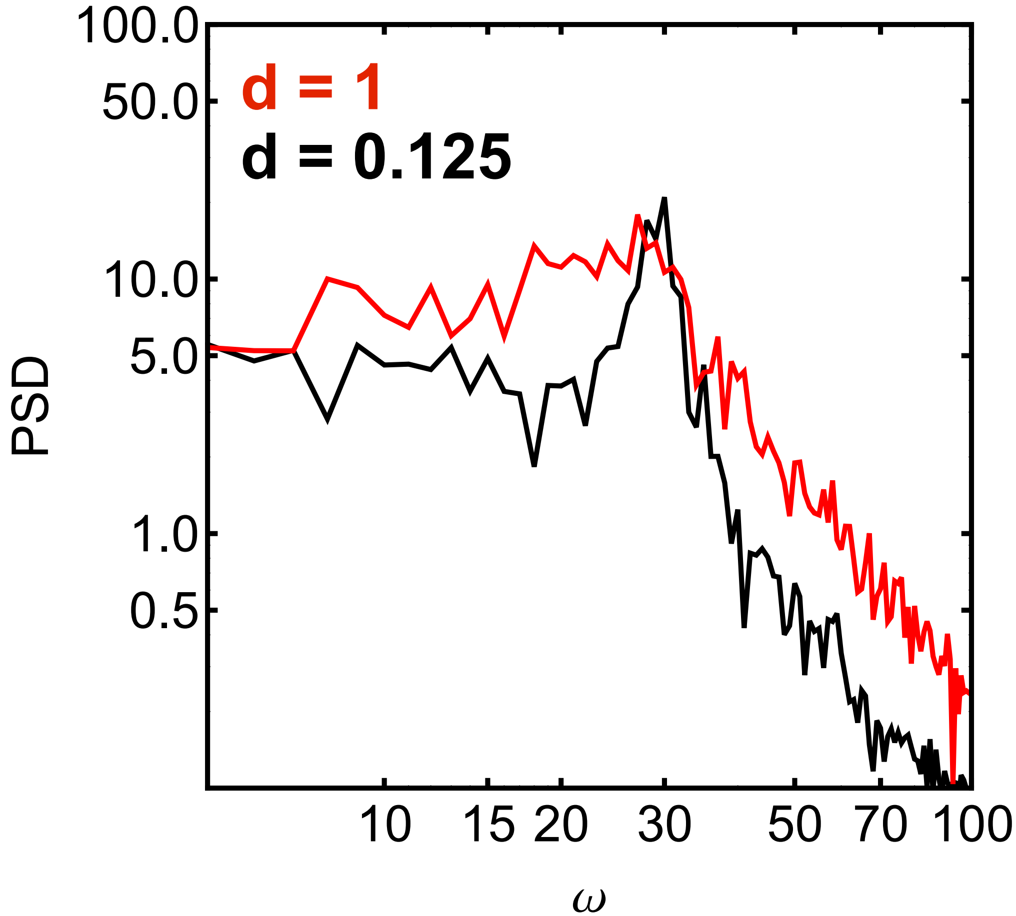


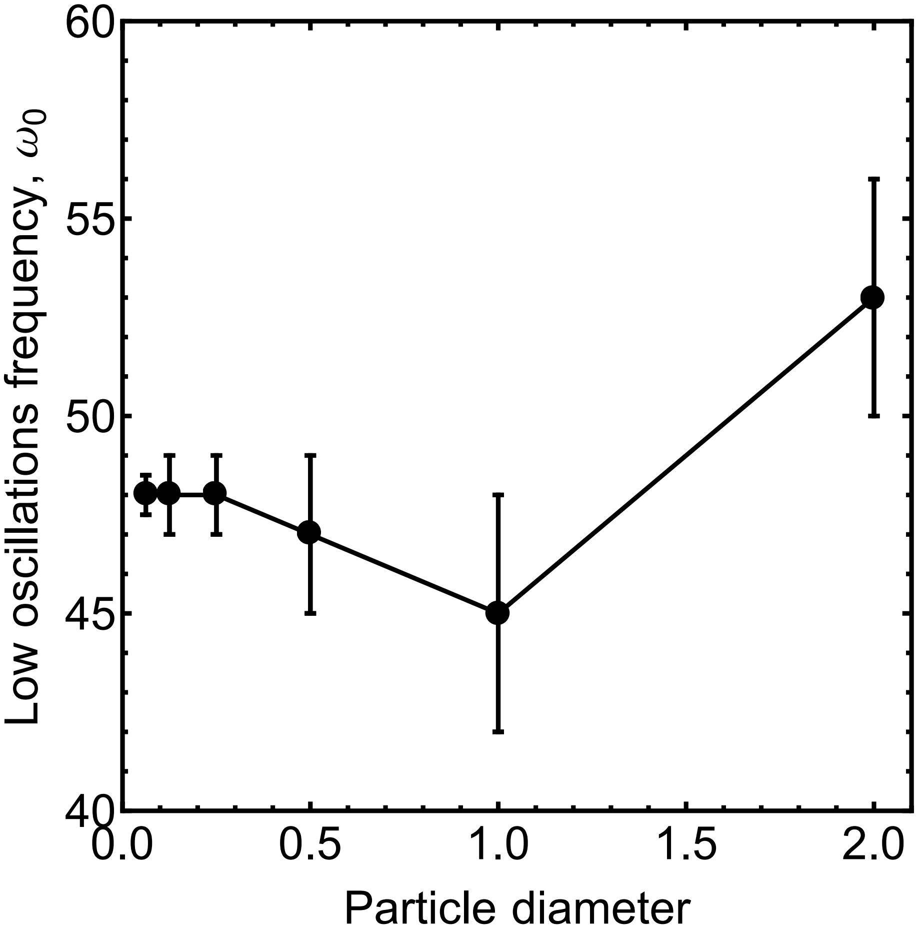
Amplitude of the oscillations decreases...





...but frequency converges





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The mode of the LFOs is invariant,

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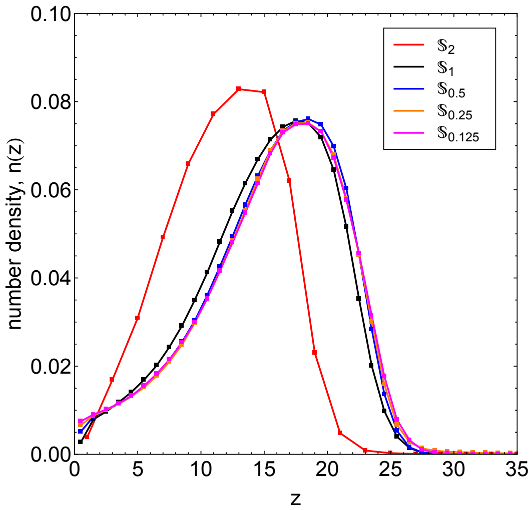
while the amplitude seems to depend on fluctuations.

$$\omega_{0_m}^2 = \frac{g\rho_g}{m_s}$$

N. Rivas et al., New Journal of Physics, 2013



Particle size scalings in vertically vibrated granular media

- The obtained granular hydrodynamic scalings with particle size lead to convergence of macroscopic fields as ‘d’ diminishes.

- Low-frequency oscillations amplitude is driven by fluctuations.

Particle size scalings in vertically vibrated granular media

Current work:

- Comparison with solution of granular hydrodynamics.
- Leidenfrost/convection transition for different ‘d’: