Progress Report on VICI
Kuniyasu Saitoh

A. Description of my project

Jamming, granular flow, instability of granular flow and dynamic heterogeneities in driven granular systems are my project.

B. Recent talk
(Please see the attached PDF file)

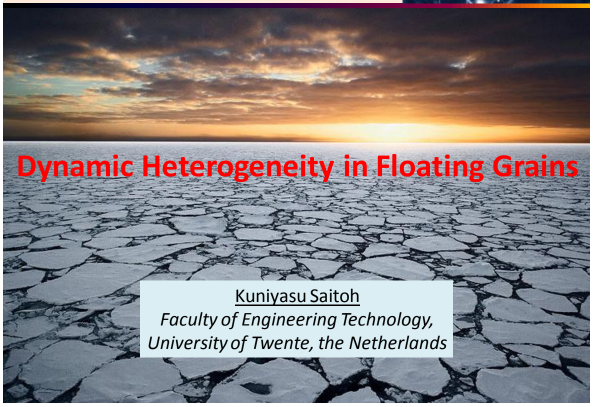
C. Recent papers

Kuniyasu Saitoh and Hisao Hayakawa, "**Quantitative test of the time dependent Ginzburg-Landau equation for sheared granular flow in two dimension**", Phys. Fluids (2012), in press.

*We have derived the TDGL equation from the hydrodynamic equations of granular gases and adopted to the shear flow. We compare the numerical solutions of the TDGL equation and the hydrodynamic fields, where both results quantitatively same.*

Kuniyasu Saitoh, Vanessa Magnanimo, and Stefan Luding, "**Slow dynamics near jamming**", AIP Conf. Proc. **1501** (2012) 1038, *28th International Symposium on Rarefied Gas Dynamics.*

*We have studied the jamming transition and critical scaling, especially, the dependence on the size ratio.*



# Dynamic Heterogeneity in Floating Grains

Kuniyasu Saitoh
*Faculty of Engineering Technology,*
*University of Twente, the Netherlands*

## Contents






Ceyda Sanli
OIST Graduate University


Kuniyasu Saitoh
University of Twente


Devaraj van der Meer
University of Twente

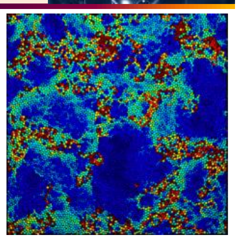

Stefan Luding
University of Twente

## Introduction – *Dynamic heterogeneity*



*Supercooled Liquids*

Control parameter
- *Temperature*


Garrahan (2011) *PNAS*

Time scale
- Self-intermediate scattering function
- Structural relaxation (rearrangement)
- Divergence near the glass transition

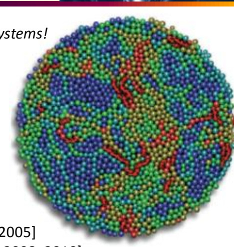
Length scale
- Microstructure of glass is almost same with that of liquid (The *static* correlation function does not work)
- The 4-point *dynamic* correlation function
- Heterogeneous structure of displacements
- Divergence near the glass transition temperature

## Introduction – *Dynamic heterogeneity*



*Dynamic heterogeneity is universal in disordered systems!*

*Driven Granular Systems*


Durian *et al.* (2007) *Nat. Phys.*

Control parameters
- Driving force
- Density

Examples
- Air-fluidized grains [Keys, et.al. 2007]
- Sheared granular systems [Dauchot, et.al. 2005]
- Horizontally vibrated grains [Lechenault, F. 2008, 2010]

*Our Aim*

***Understanding of dynamic heterogeneity in more complex systems.***

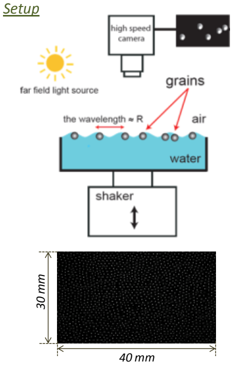
***Floating grains***

***Driven by standing wave, cohesive forces, flow with convection***

## Experiment

UNIVERSITY OF TWENTE

*Setup*




*Grains*
Polystyrene beads distributed on water (mean diameter σ=0.62mm, polydisperse)

*Driving force*
Standing wave generated by a shaker

*Control parameter*
Area fraction of grains ϕ

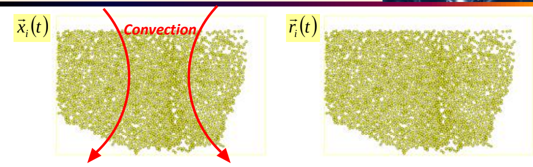
*Interaction between grains*
Capillary force -> ***"cohesive"***

*Data*
Trajectories projected on 2-dimensions

*Units*
time = sec. length = σ

## Coarse Graining Method

UNIVERSITY OF TWENTE




$\vec{x}_i(t)$ $\vec{r}_i(t)$

*Trajectories of grains*

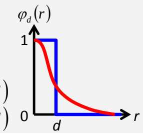
$$\vec{x}_i(t)=\vec{r}_i(t)+\int_0^t \vec{u}(\vec{x}_i(s),s)ds \quad (i=1,\cdots,N)$$

Fluctuation — Transport by convection

*Coarse Graining (CG)*

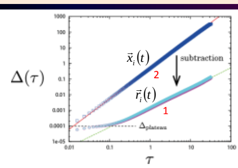
*Velocity field* $\vec{u}(\vec{x},t)\equiv\frac{\sum \vec{v}_i(t)\varphi_d(\vec{x}-\vec{x}_i(t))}{\sum \varphi_d(\vec{x}-\vec{x}_i(t))}$

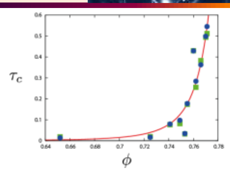
*CG function* $\varphi_d(\vec{x})=e^{-(|\vec{x}|/d)^2}$ *or* $\begin{cases}1 & (|\vec{x}|\le d)\\ 0 & (|\vec{x}|>d)\end{cases}$




## Diffusion

UNIVERSITY OF TWENTE




*Mean Square Displacement*

$\vec{x}_i(t)$ : Convection is dominant & almost *ballistic*.
$\vec{r}_i(t)$ : Crossover from *sub-diffusion* ( $C_2\tau^{\beta}$ ) to *diffusion* ( $C_1\tau^{\alpha}$ ).

*Crossover time* $\tau_c$ defined as

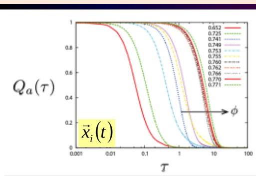
$$C_1\tau_c^{\alpha}=C_2\tau_c^{\beta}\equiv\Delta_{plateau}$$

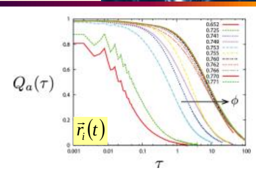
diverges near the estimated *jamming point* $\phi_c=0.82$ as

$$\tau_c\sim\Delta\phi^{-3.9} \quad (\Delta\phi\equiv\phi_c-\phi)$$

## Order Parameter

UNIVERSITY OF TWENTE




*Displacement*

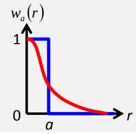
$$r_i(t,\tau)\equiv|\vec{X}_i(t+\tau)-\vec{X}_i(t)| \quad \textit{where} \quad \vec{X}_i(t)=\vec{x}_i(t) \ \text{or} \ \vec{r}_i(t)$$

*Order Parameter*

*Overlap function* $w_a(r)=e^{-(r/a)^2}$ *or* $\begin{cases}1 & (r\le a)\\ 0 & (r>a)\end{cases}$

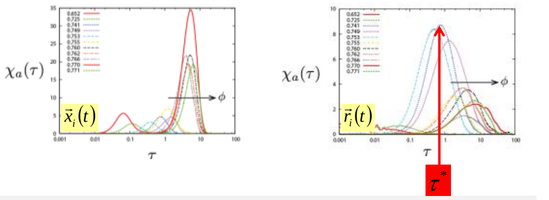
*Mobility* $q_a(t,\tau)\equiv\frac{1}{N}\sum_{i=1}^{N}w_a(r_i(t,\tau))$

*Order parameter* $Q_a(\tau)=\langle q_a(t,\tau)\rangle_t$



## Dynamic susceptibility

UNIVERSITY OF TWENTE




*Dynamic Susceptibility*

$$\chi_a(\tau) \equiv N\left\{\left\langle q_a(t,\tau)^2\right\rangle_t - \left\langle q_a(t,\tau)\right\rangle_t^2\right\}$$

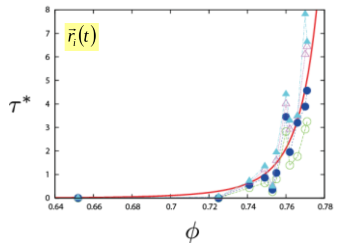
is defined as a *deviation* of mobility & has a *single peak* at $\tau^*$.

$\vec{x}_i(t)$ : $\tau^*$ is not monotonous.

$\vec{r}_i(t)$ : $\tau^*$ monotonously depends on ϕ.

## Time Scale

UNIVERSITY OF TWENTE



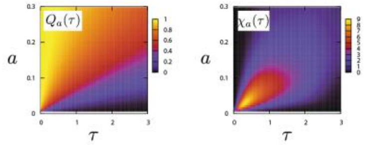

*Time Scale*

*Characteristic time* $\tau^*$ given by $\vec{r}_i(t)$ diverges near the jamming point as

$$\tau^* \sim \Delta\phi^{-4.2}$$

## Width of Overlap Function

UNIVERSITY OF TWENTE




*Criteria for width "a"*

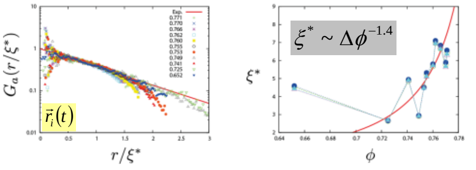
1. $a > \sqrt{\Delta_{plateau}}$
2. The maximum amplitude of $\chi_a(\tau^*)$.

| CG func. | Overlap func. | a (width of OF) |
|---|---|---|
| None | Step | $\sigma$ |
| | Gauss | $\sigma$ |
| Step (d = $1.6\sigma$) | Step | $0.046\sigma$ |
| | Gauss | $0.042\sigma$ |
| Gauss (d = $\sigma$) | Step | $0.042\sigma$ |
| | Gauss | $0.038\sigma$ |

**Cf.) Horizontally vibrated grains (Lechenault & Dauchot et.al.)** $a \sim 10^{-2}\sigma$

## 4-point correlation function

UNIVERSITY OF TWENTE




*Definition*

*Volume integral* $\quad \chi_a(\tau^*) \equiv 2\pi \int_0^\infty r g_a(r,\tau^*) dr \quad$ *Number density*

$$g_a(r,\tau) = \frac{1}{2\pi r N}\left\langle \sum_{i,j} \delta\left(r - \left|\vec{r}_i(t) - \vec{r}_j(t)\right|\right) w_a(r_i(t,\tau)) w_a(r_j(t,\tau)) \right\rangle_t - \rho\left\langle q_a(t,\tau)\right\rangle_t^2$$

*Scaling*

$$g_a(r,\tau^*) = c r^{-\beta} e^{-r/\xi^*} \Rightarrow G_a(r/\xi^*) = c^{-1} r^{\beta} g_a(r,\tau^*) \quad (\beta \approx 0.01)$$

**Cf.) Horizontally vibrated grains (Lechenault & Dauchot et.al.)** $\beta << 0.15$

## Dynamic criticality

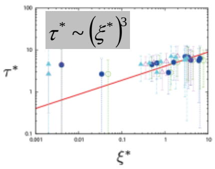
UNIVERSITY OF TWENTE

*Dynamic Criticality of Floating Grains*

$$\tau^* \sim \Delta\phi^{-\lambda}$$

$$\xi^* \sim \Delta\phi^{-\mu}$$

$$\tau^* \sim \left(\xi^*\right)^{\lambda/\mu}$$




*Exponents for Driven Granular Systems*

| | $\lambda$ | $\mu$ | $\lambda/\mu$ | Dimension |
|---|---|---|---|---|
| **Horizontally Vibrated Grains** | **0.33** | **0.50** | **0.65** | **Quasi-2D** |
| **Air-fluidized Grains** | **1.0** | **1.7** | **0.6** | **Quasi-2D** |
| **Sheared Colloids** | **4** | **4/3** | **3** | **Quasi-2D** |
| **Floating Grains** | **4.2** | **1.4** | **3** | **Quasi-2D** |

## Summary

UNIVERSITY OF TWENTE

*Dynamic Heterogeneity in Floating Grains*

- ***Floating grains*** are driven by *standing wave* & flow with *convection*
- ***CG method*** successfully subtracts the additional displacements by *convection*
- ***Crossover time*** diverges near *the jamming point*
- ***Time scale*** given by susceptibility also diverges near *the jamming point*
- ***Dynamic correlation length*** diverges near *the jamming point*
- ***Exponents*** obtained from our analysis are consistent with previous works
- ***Our results do not*** depend on the choose of CG & overlap functions

*Outlook*

- The maximum susceptibility $\chi_a\left(\tau^*\right)$
- Self-intermediate scattering function
- Self-van Hove function
- Effect of convection