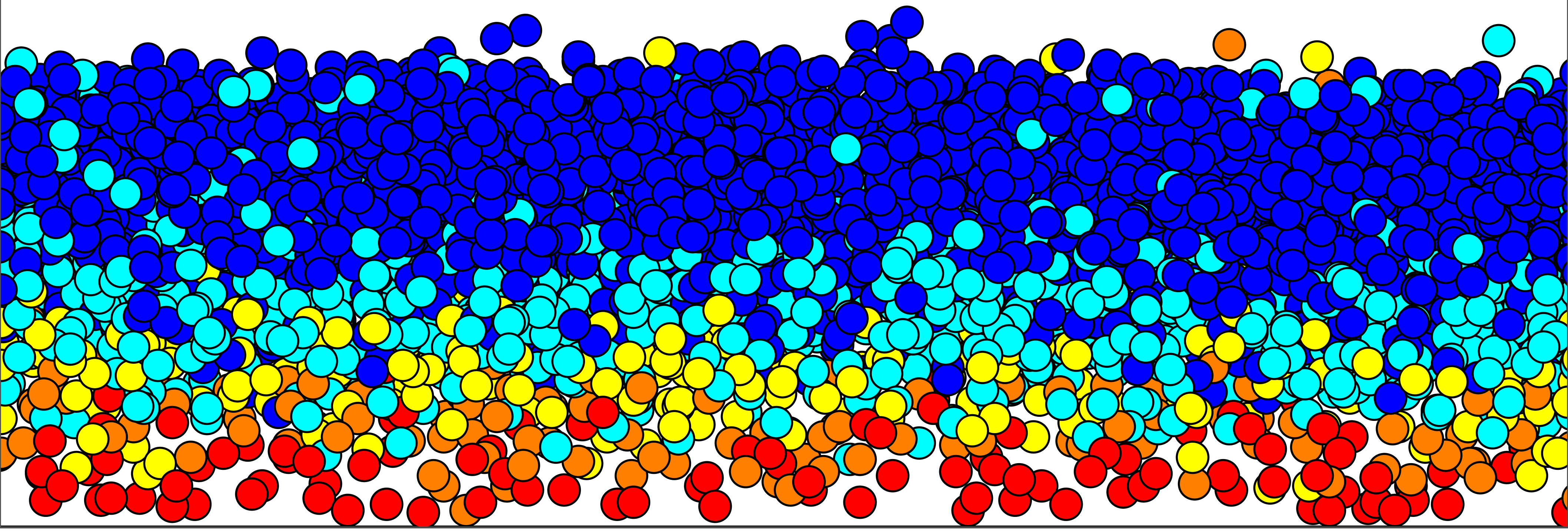
UNIVERSITY OF TWENTE.
Multi Scale Mechanics

Nicolás Rivas (n.a.rivas@utwente.nl)
German-Dutch Granular Workshop
Cologne, 2 April, 2013

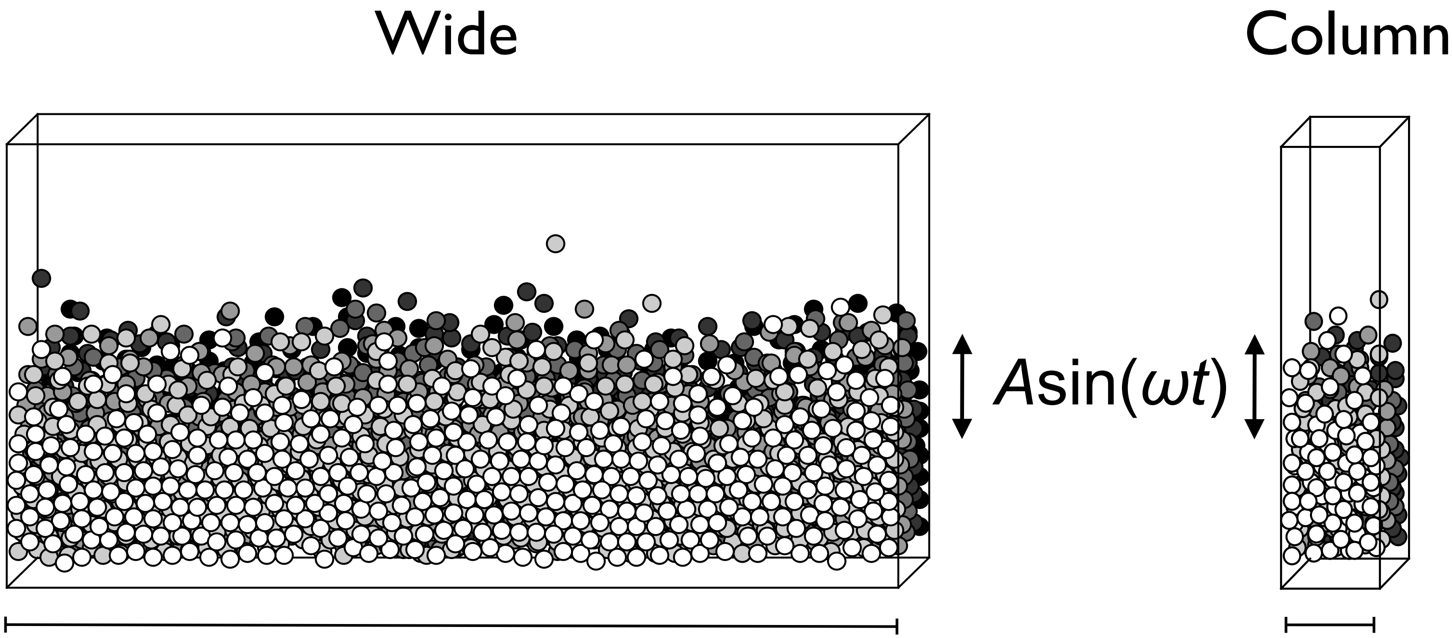
Vertically vibrated grains and continuum models



MOTIVATION

- How do granular materials flow?
- Discrete to continuum transition
- Collective dynamics of many-particle systems

SYSTEM GEOMETRY



$L_X = 50d$

$N = 3000$

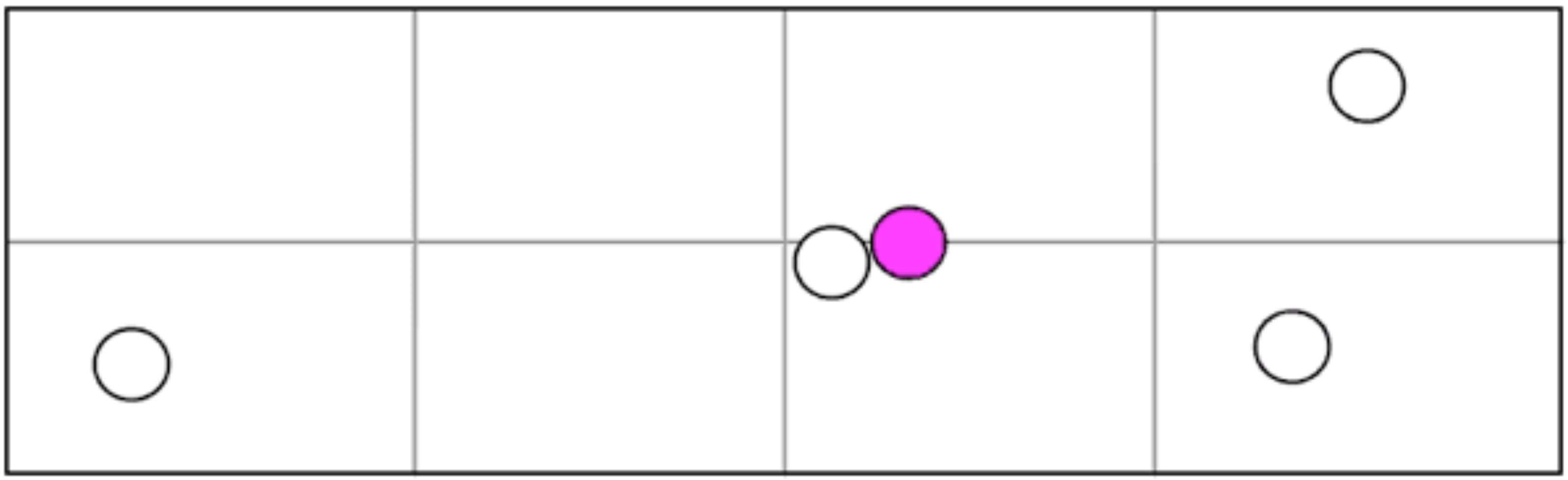
$L_X = 5d$

$N = 300$

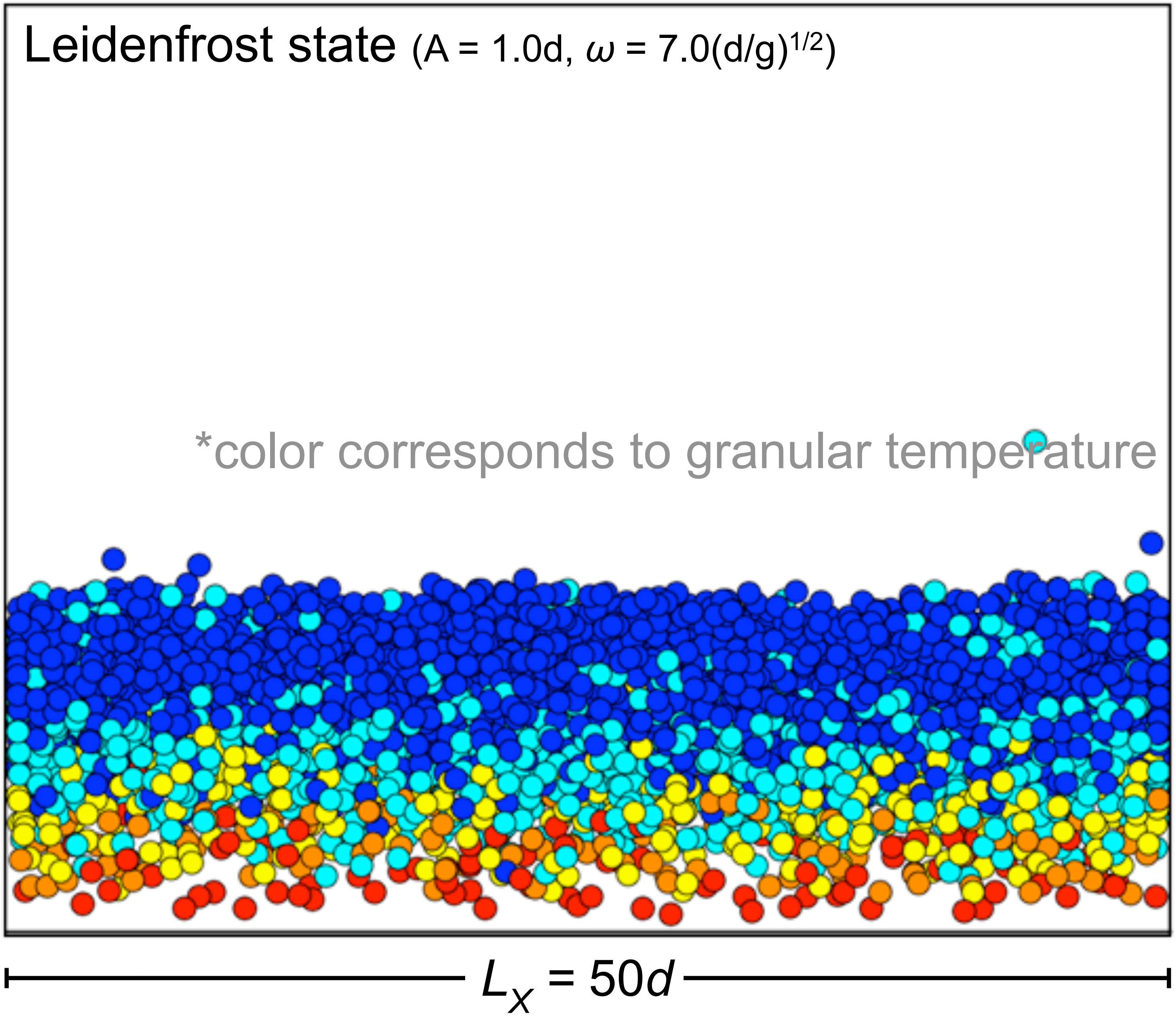
control parameter $S \equiv A^2\omega^2/gd \in (20,400)$

SIMULATIONS

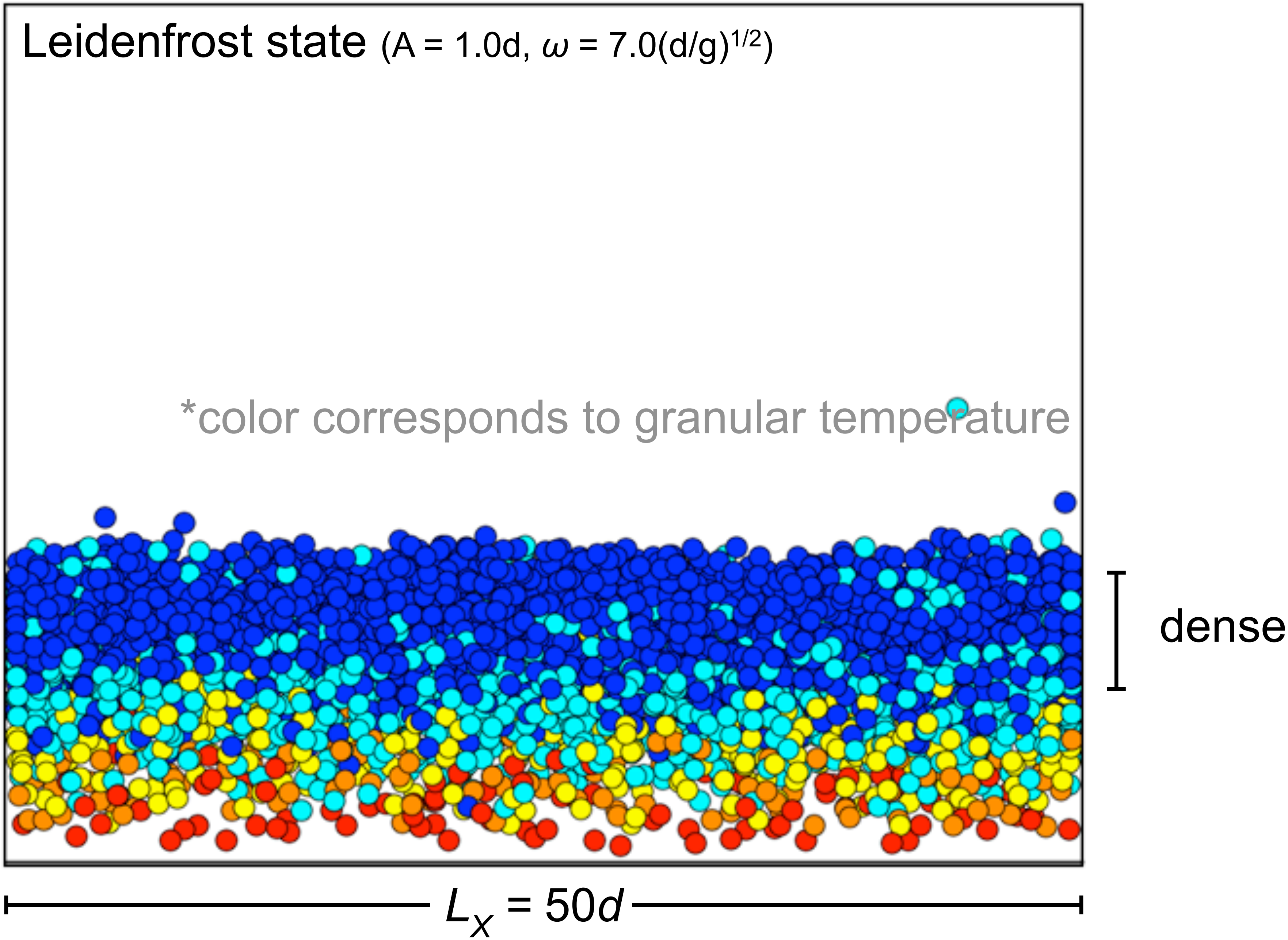
- Event-driven algorithm
- Perfect hard spheres
- Collisions modeled by ε_N, ε_T and μ_S, μ_D
- Solid walls boundary conditions (no top)
- Bi-parabolic sine interpolation



PHASES

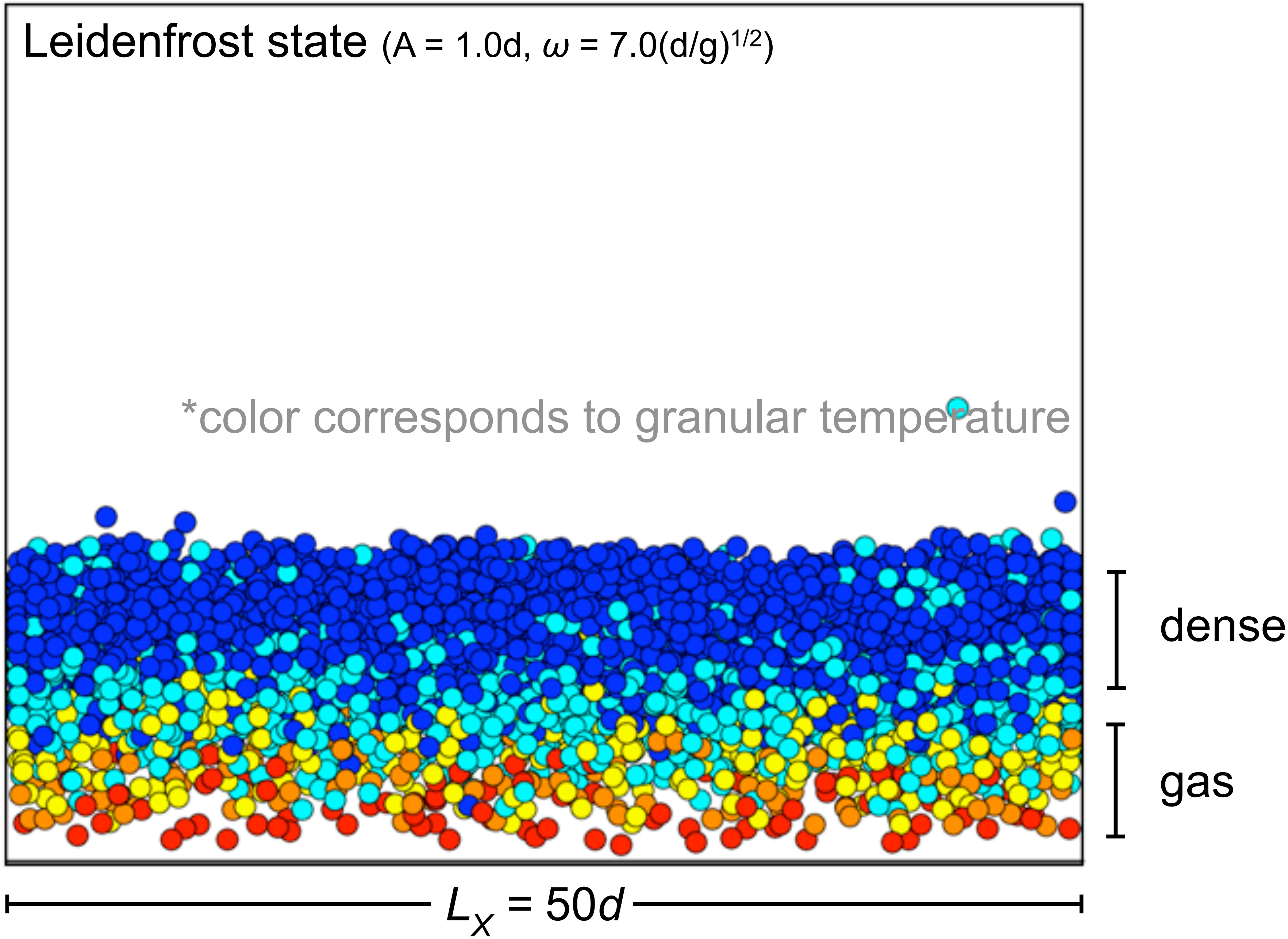

Leidenfrost state (A = 1.0d, ω = 7.0(d/g)^1/2)
*color corresponds to granular temperature
L_X = 50d

PHASES



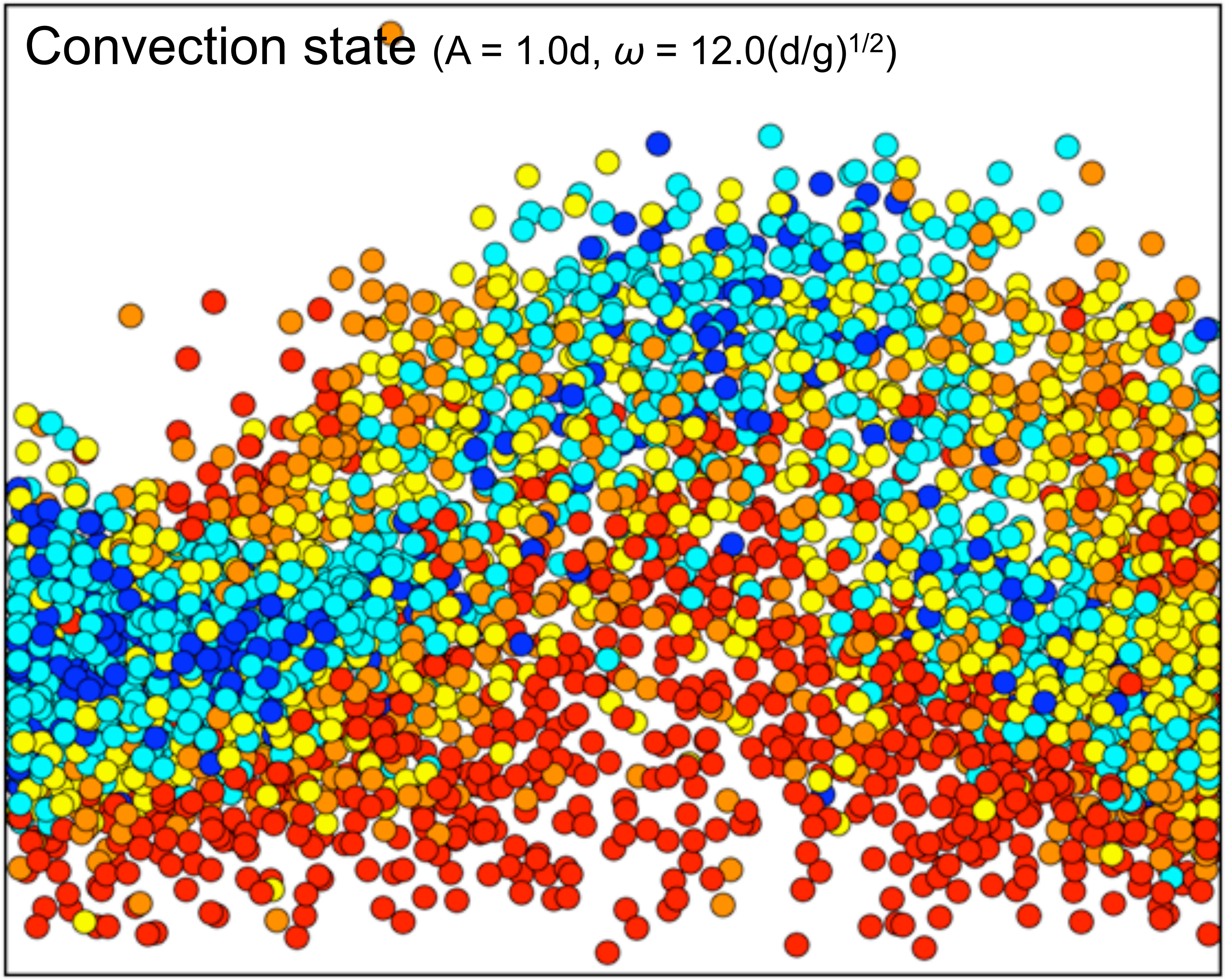
Leidenfrost state (A = 1.0d, ω = 7.0(d/g)^1/2)
*color corresponds to granular temperature
dense
$L_x = 50d$

PHASES



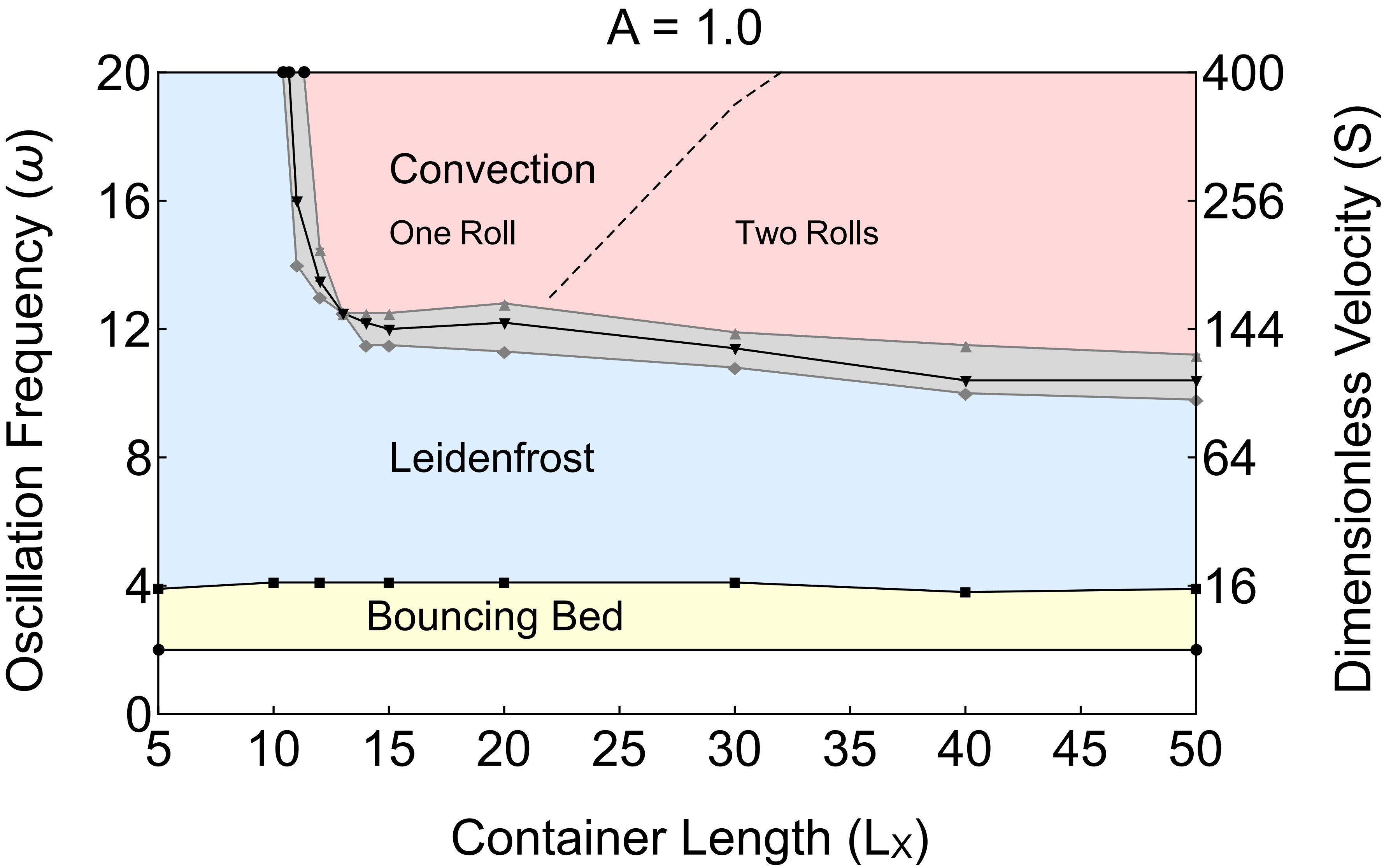
Leidenfrost state (A = 1.0d, ω = 7.0(d/g)^1/2)
*color corresponds to granular temperature
dense
gas
$L_x = 50d$

PHASES

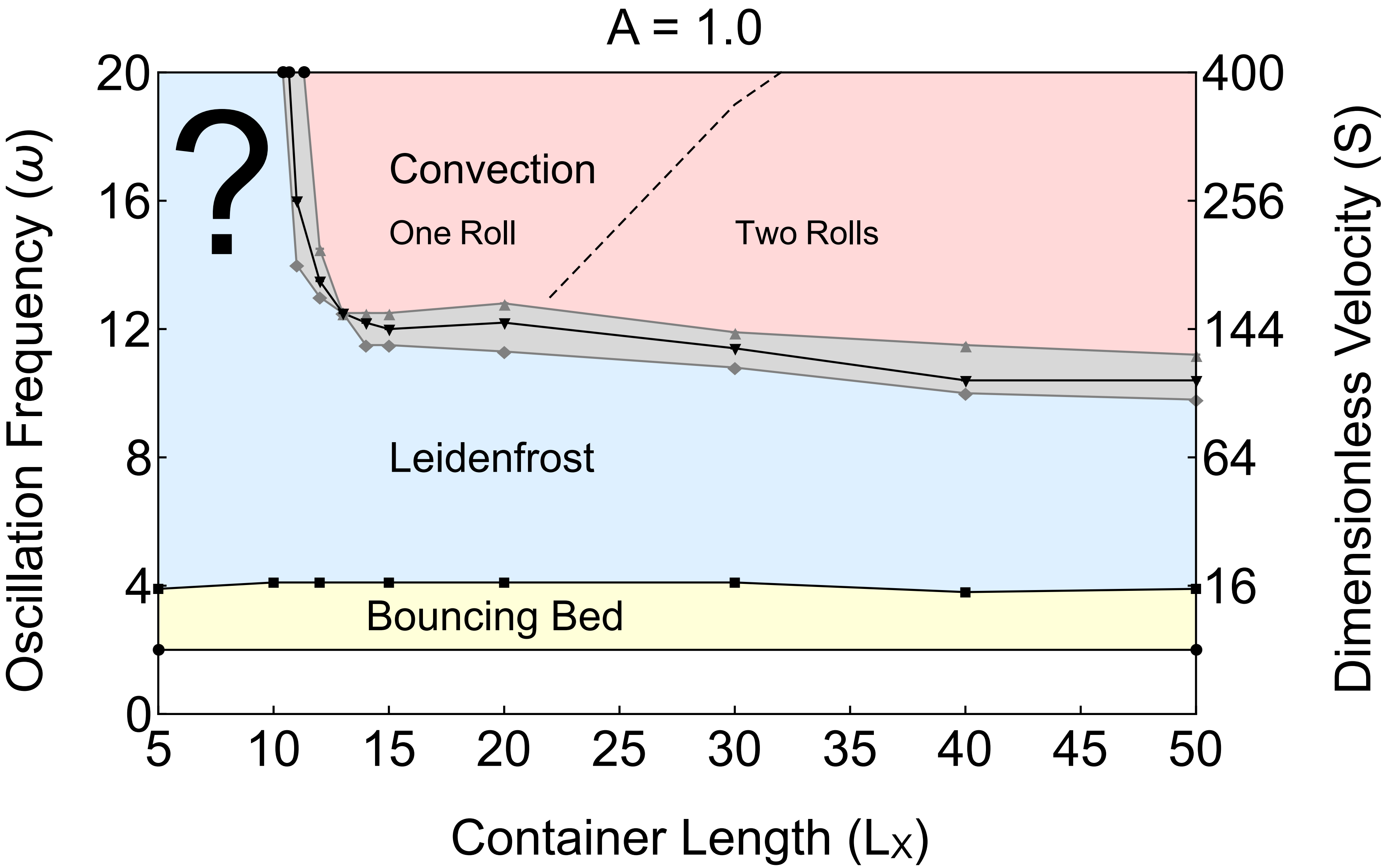


$L_x = 50d$

PHASES


A = 1.0
Convection
One Roll
Two Rolls
Leidenfrost
Bouncing Bed
Oscillation Frequency (ω)
Dimensionless Velocity (S)
Container Length (Lx)
20
16
12
8
4
0
400
256
144
64
16
5
10
15
20
25
30
35
40
45
50

PHASES



A = 1.0
?
Convection
One Roll
Two Rolls
Leidenfrost
Bouncing Bed
Oscillation Frequency (ω)
Dimensionless Velocity (S)
Container Length (Lx)
20
16
12
8
4
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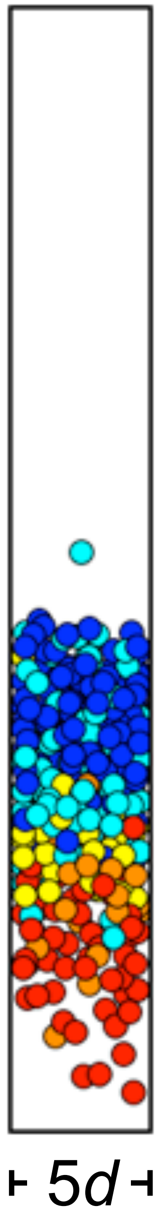
256
144
64
16
5
10
15
20
25
30
35
40
45
50

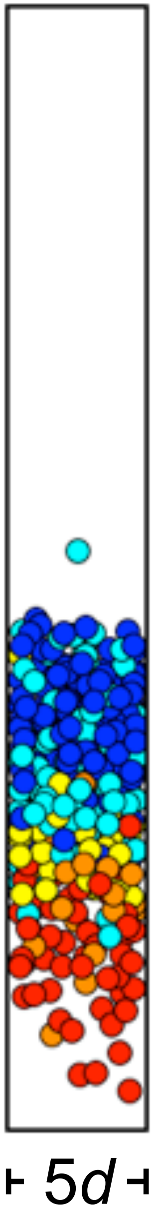
LFO's

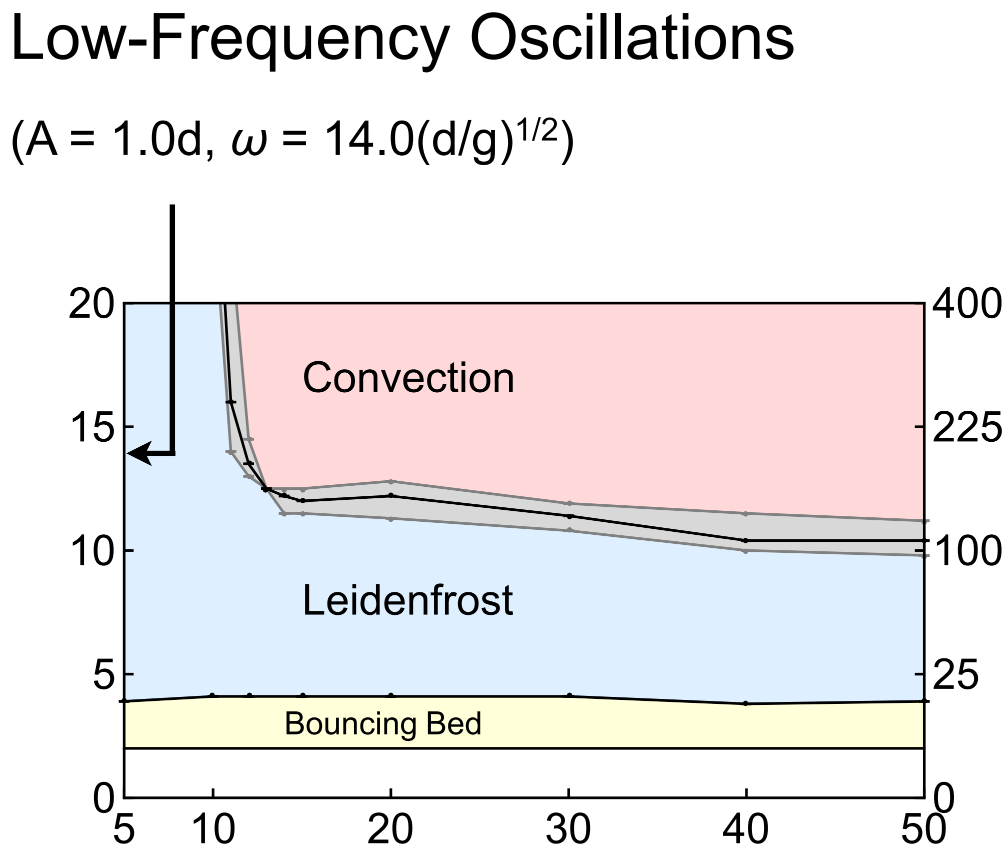


Low-Frequency Oscillations

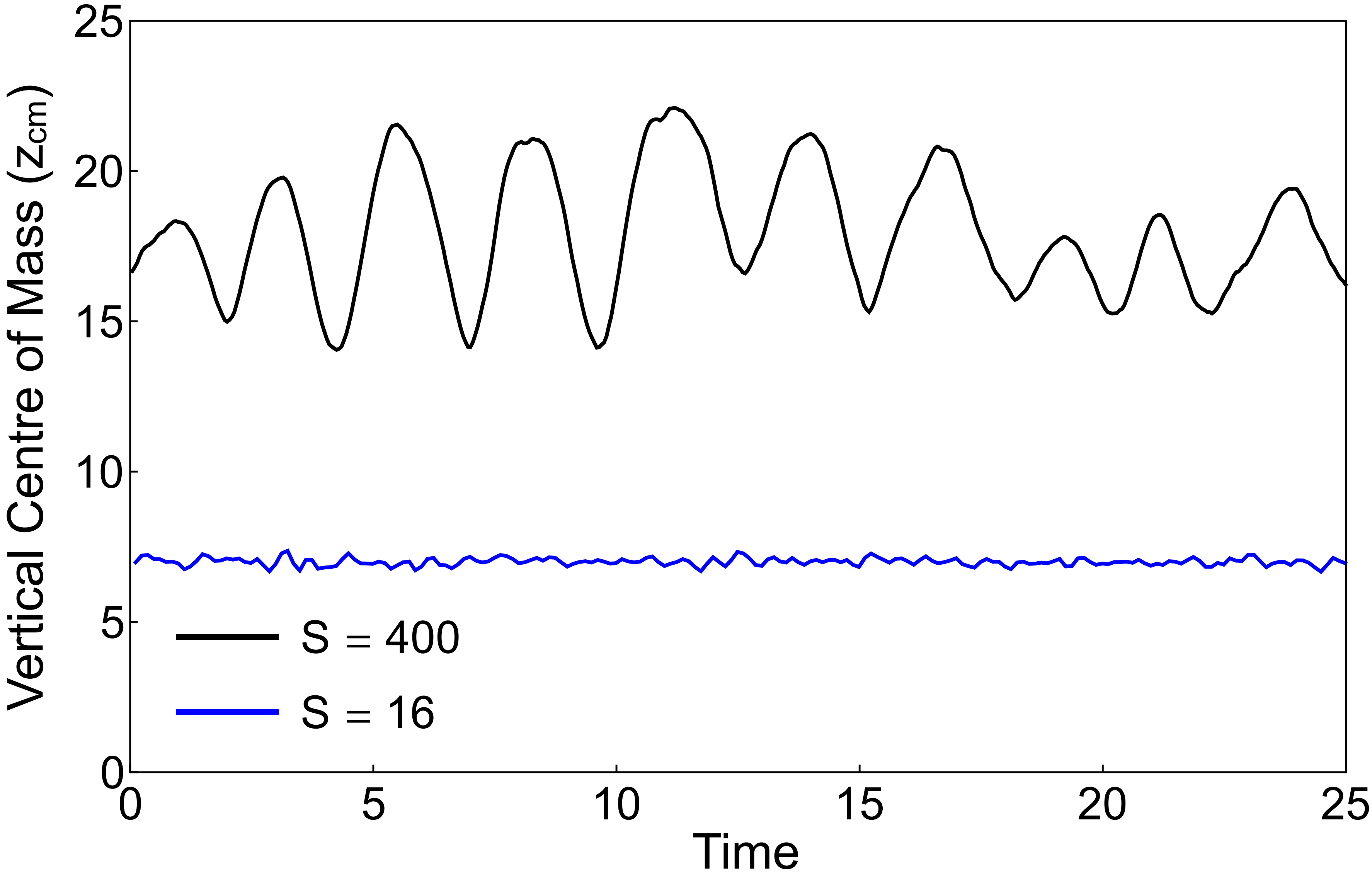
($A = 1.0d$, $\omega = 14.0(d/g)^{1/2}$)

LFO's


5d

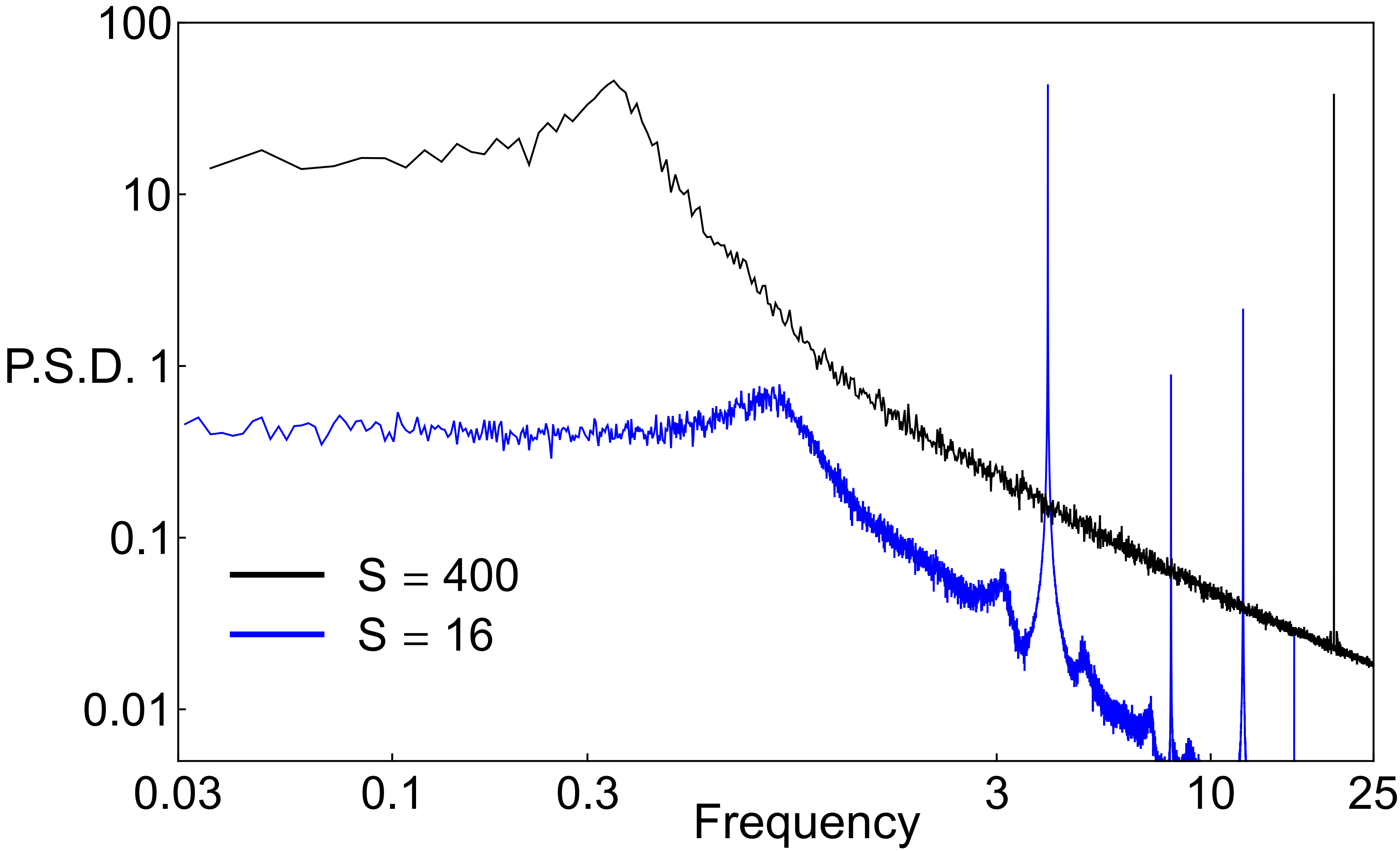

Low-Frequency Oscillations
(A = 1.0d, ω = 14.0(d/g)^1/2)
Convection
Leidenfrost
Bouncing Bed
20
15
10
5
0
400
225
100
25
0
5
10
20
30
40
50

LFO's

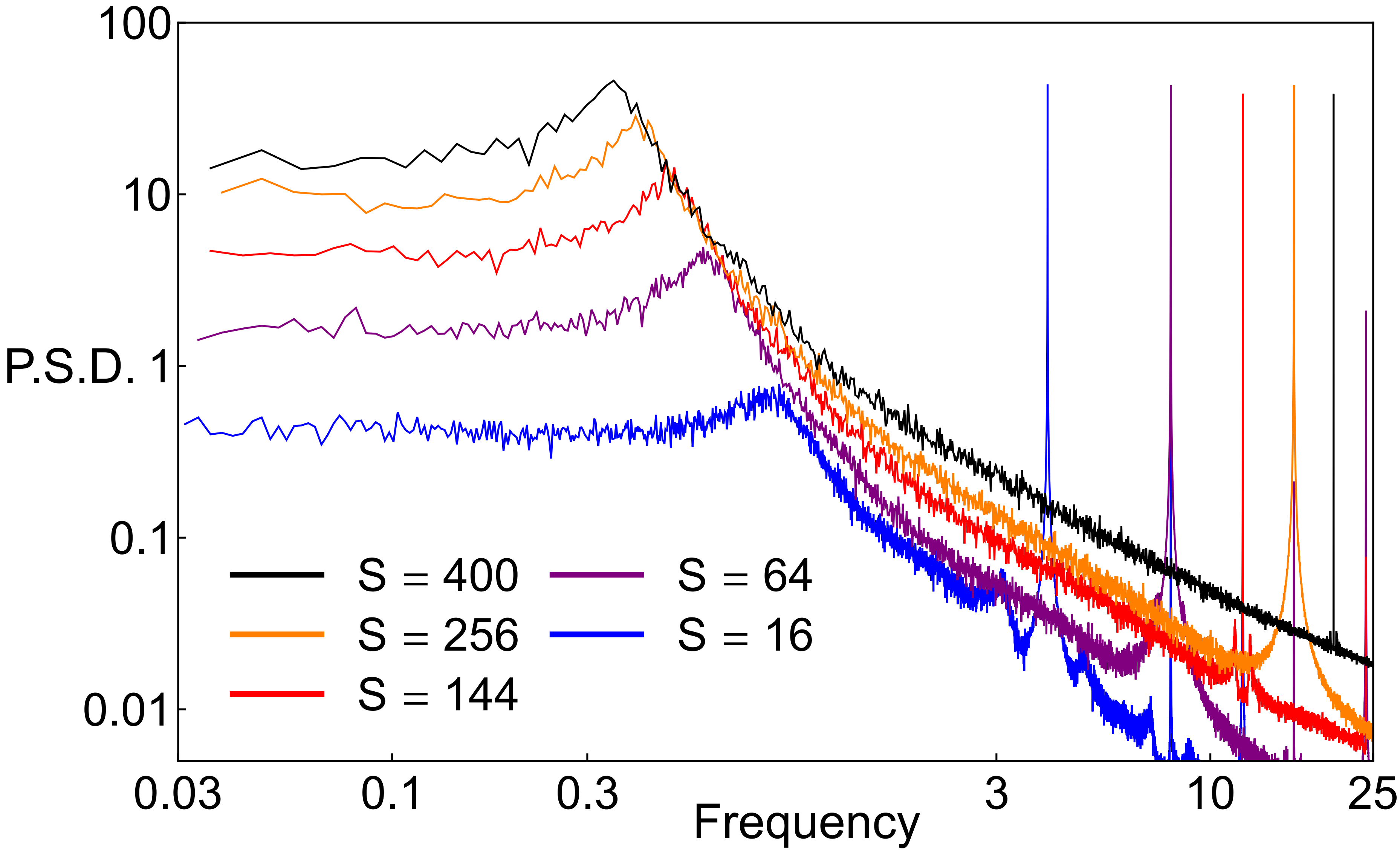


Vertical Centre of Mass (z_{cm})
Time
S = 400
S = 16
0
5
10
15
20
25
0
5
10
15
20
25

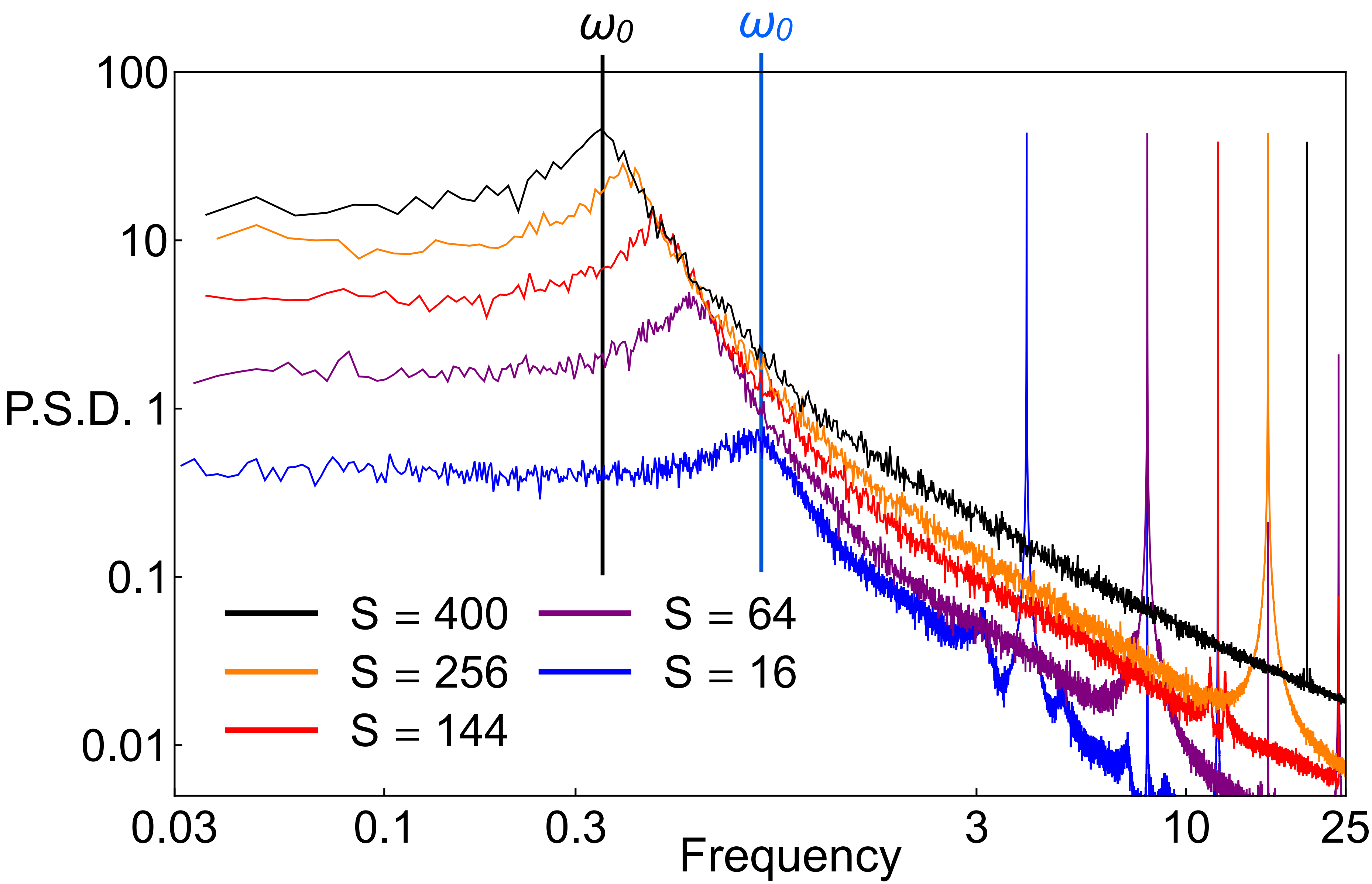
LFO's



LFO's

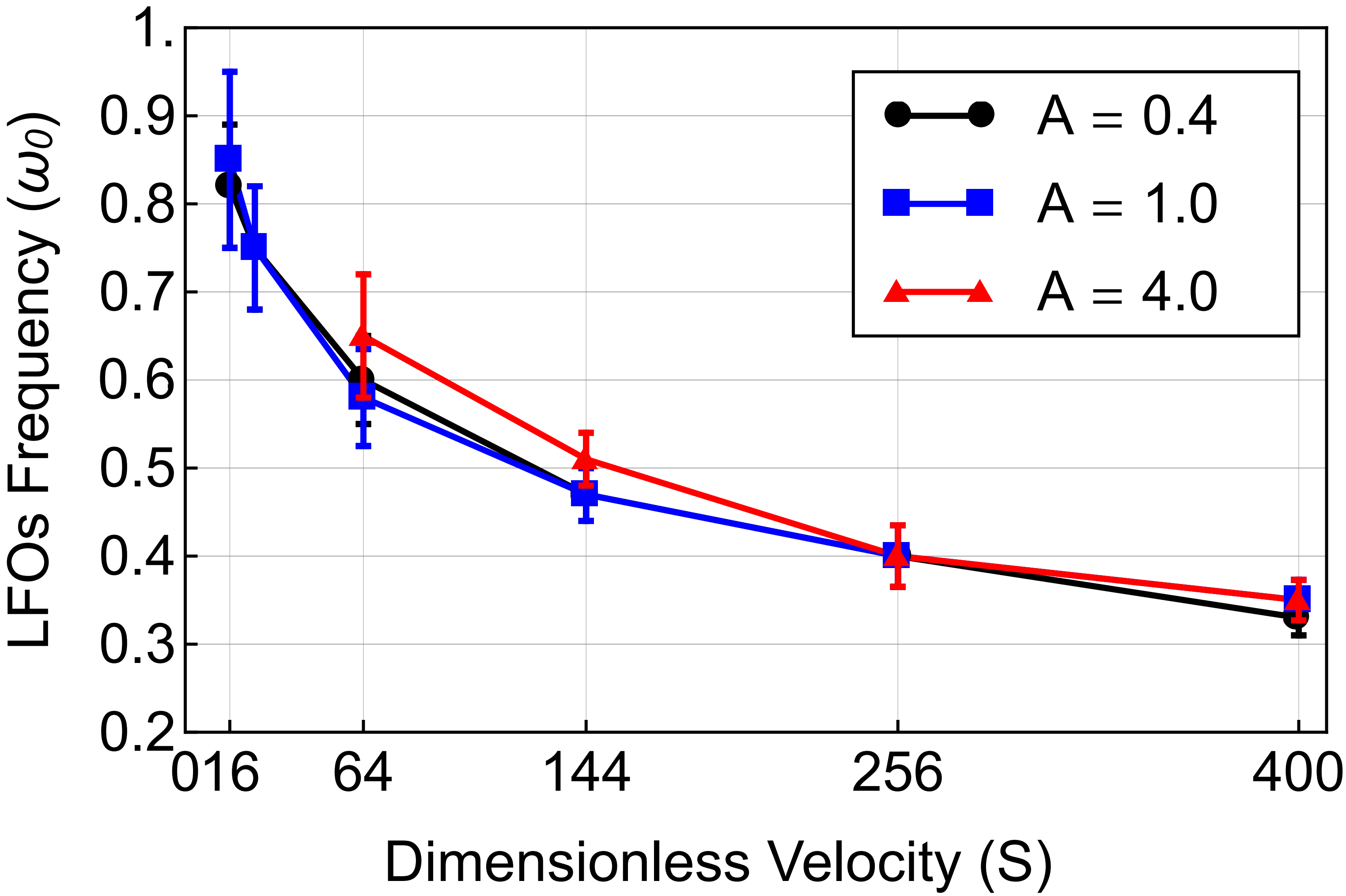

100
10
P.S.D. 1
0.1
0.01
S = 400
S = 256
S = 144
S = 64
S = 16
0.03
0.1
0.3
3
10
25
Frequency

LFO's



ω0
ω0
100
10
P.S.D. 1
0.1
0.01
S = 400
S = 256
S = 144
S = 64
S = 16
0.03
0.1
0.3
3
10
25
Frequency

LFO's



LFOs Frequency (ω_0)
1.
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
016
64
144
256
400
Dimensionless Velocity (S)
A = 0.4
A = 1.0
A = 4.0

LFO's MODEL

Cauchy's equations

$$D_t\tilde{\rho} + \rho(\nabla \cdot \tilde{\vec{u}}) = 0,$$
$$D_t(\rho\tilde{\vec{u}}) = \nabla \cdot \tilde{\sigma} + \rho\vec{B},$$

ASK AUTHOR →

Forced harmonic oscillator

$$\ddot{\xi} + \omega_{0_m}^2 \xi = \frac{1}{m_s} A_{f_m} \cos(\omega_{f_m} t) + C,$$

$$\omega_{0_m}^2 = 4g\rho_g/\rho_s h_s$$

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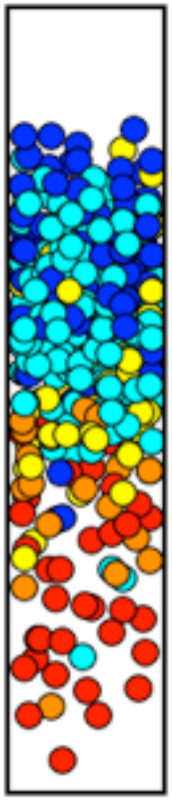

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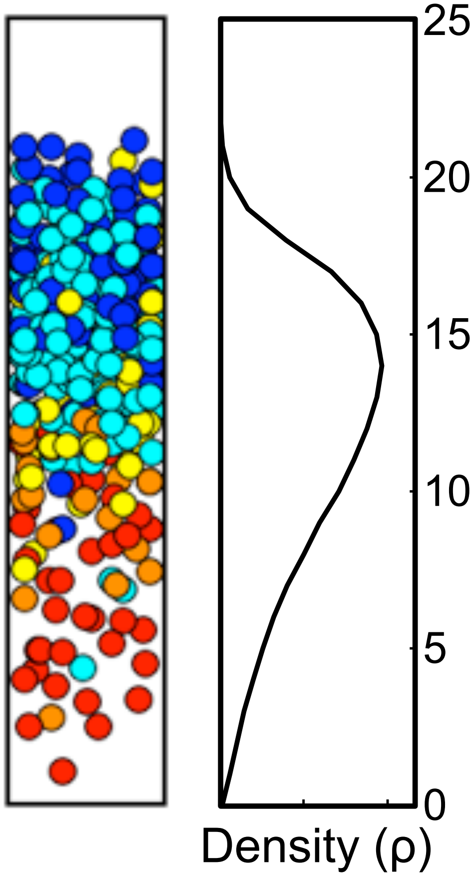

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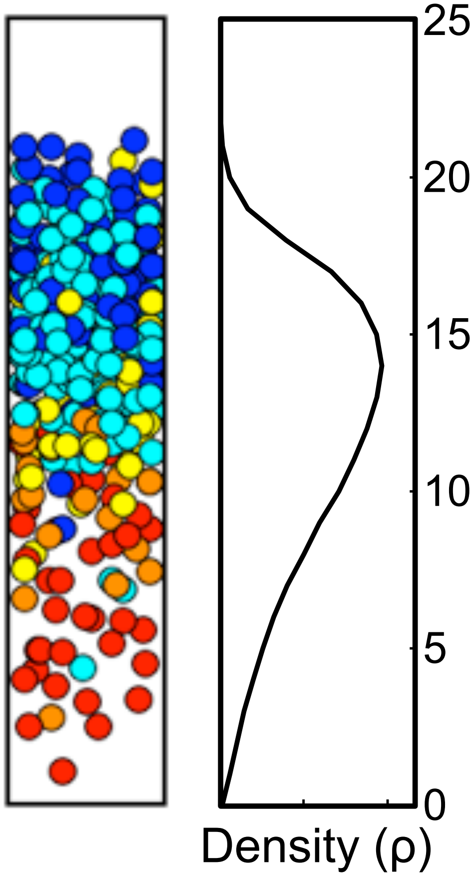

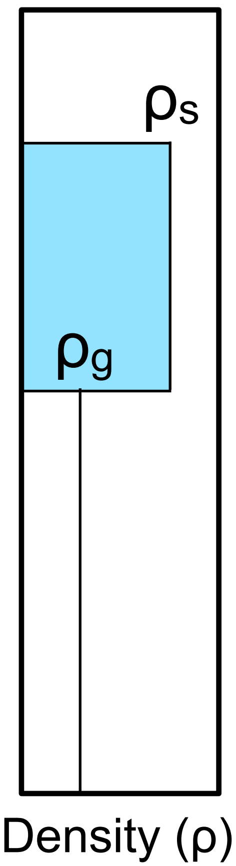

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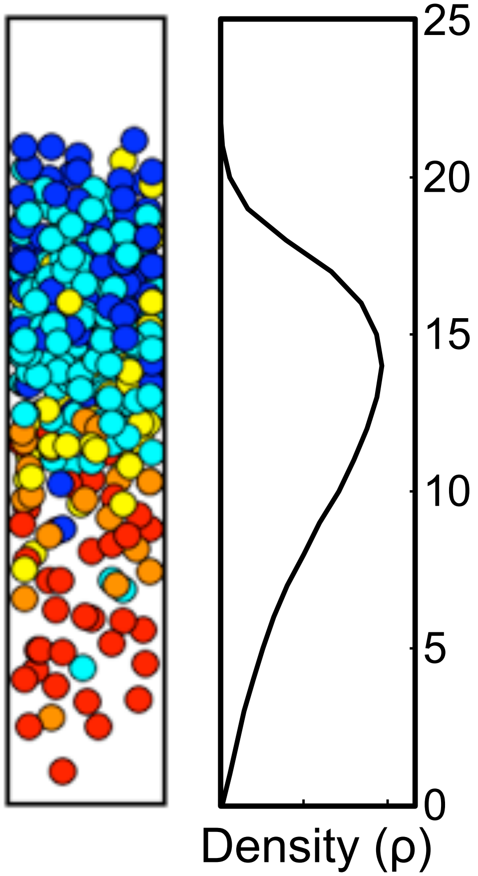

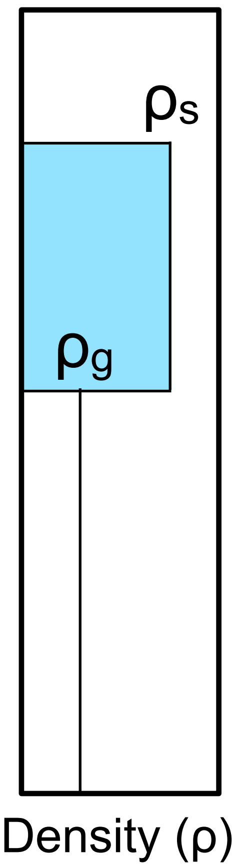

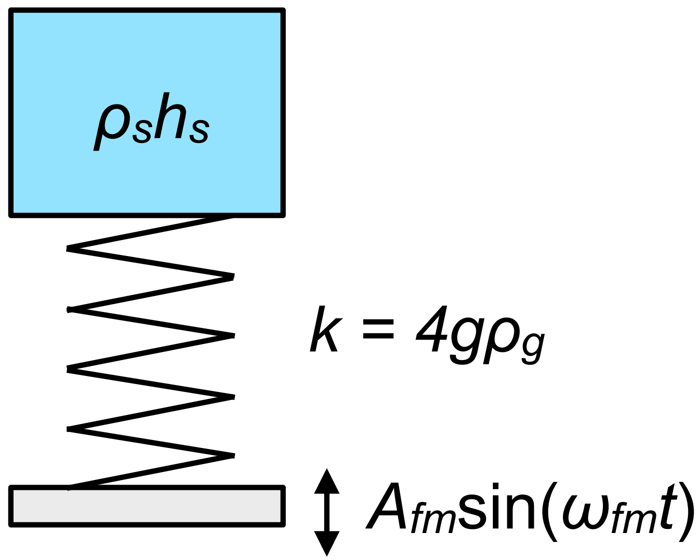

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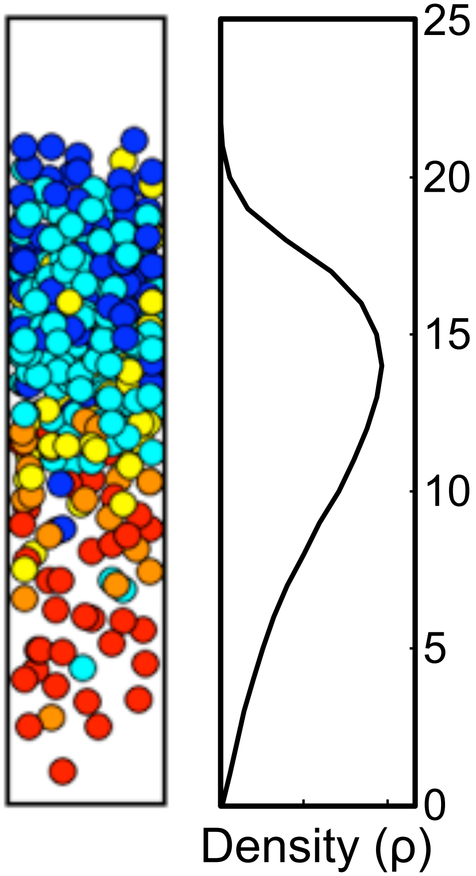

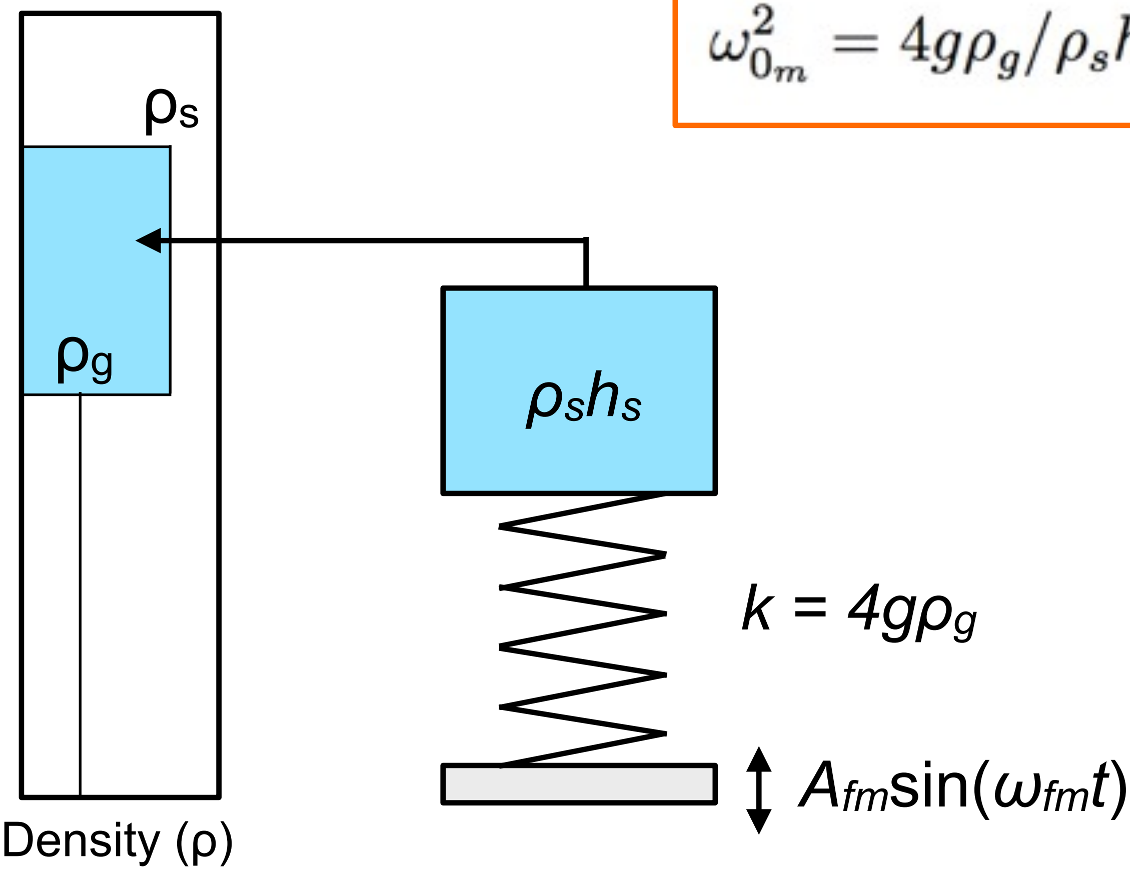

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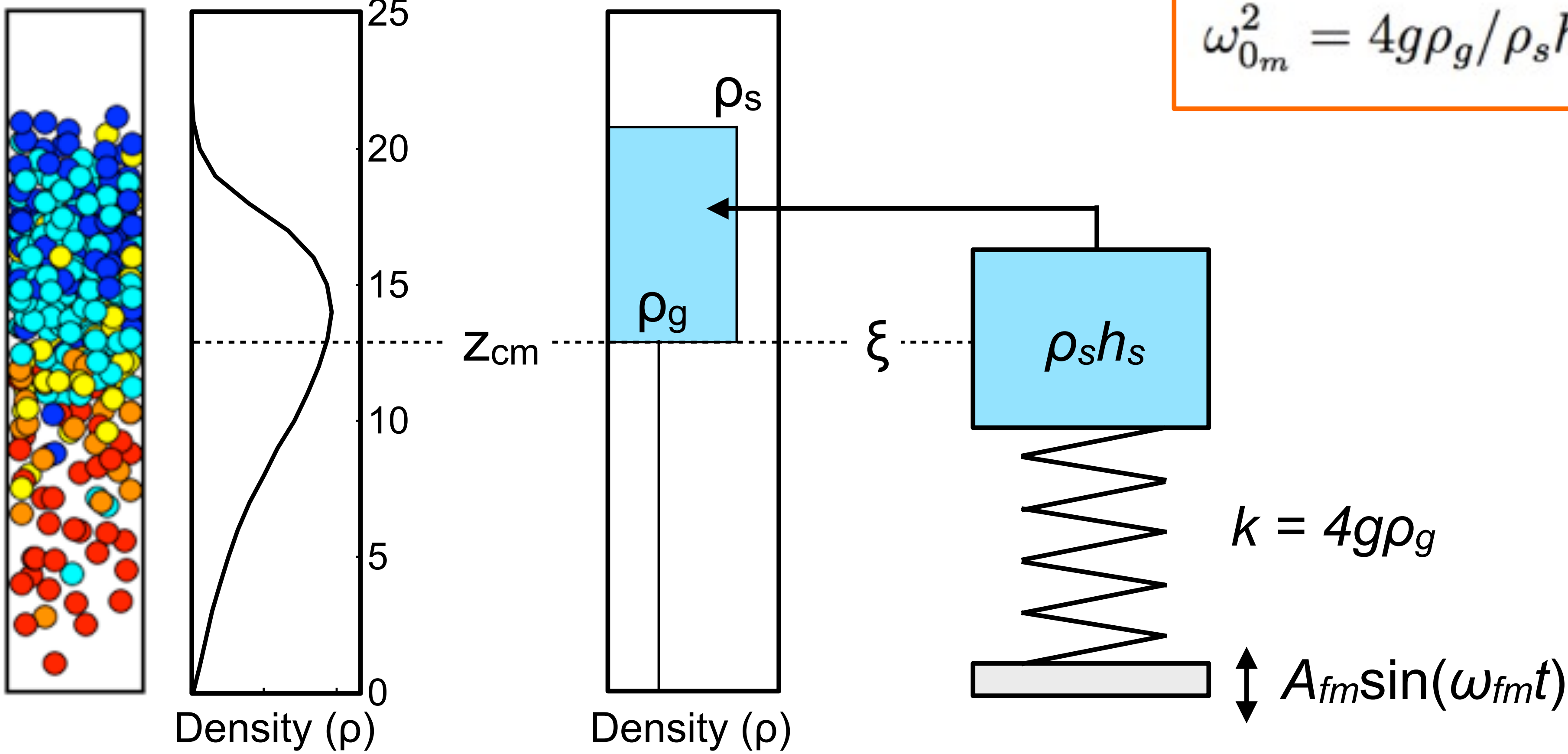

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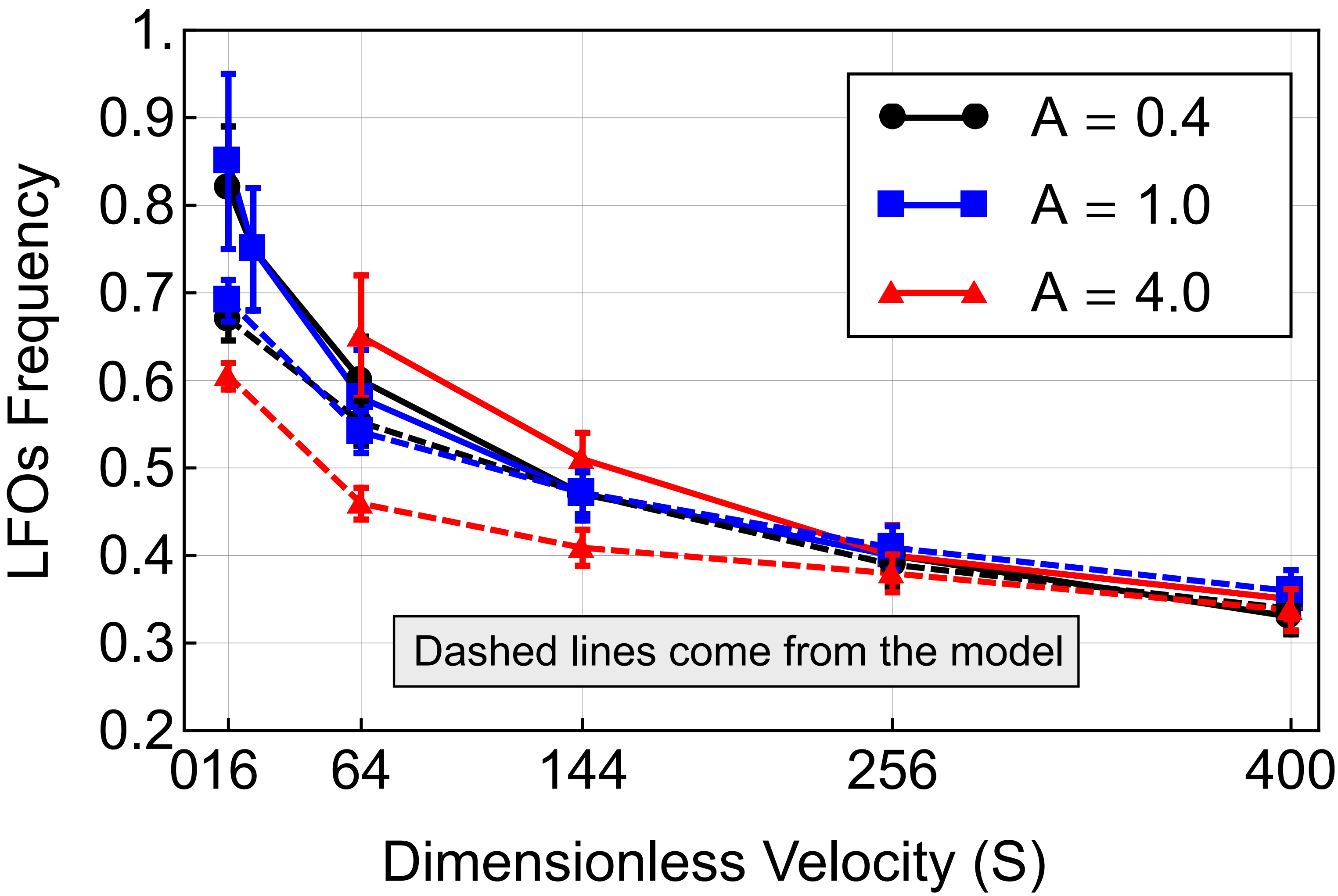

CONCLUSIONS

- Vertically driven granular matter in density inverted states present low-frequency oscillations (LFOs).

- A forced oscillator model, obtained from considering a two phases continuum medium, agrees remarkably well with simulation measurements.

PROSPECTIVE WORK

- Expand the model
- Study relevance of LFOs in wider systems
- Make experiments
- Hydrodynamic simulations