Anisotropy in 2D granular media under cyclic shear

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April 1, 2013

Introduction

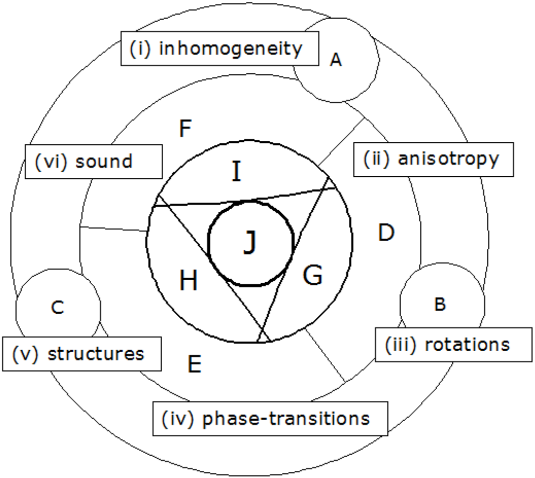
Goal:

Develop a simple local constitutive model based on observations from DPM simulations

Current subgoal:

Study anisotropy in 2D granular media and compare with existing constitutive models

Scope within the VICI project



Outline

DPM

$$\vec{a}_i = \frac{1}{m_i}\sum_{i\neq j}\vec{F}_{ij}$$

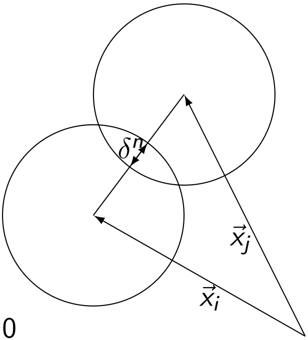
Where:

$$\delta_{ij} = r_i + r_j - |\vec{x}_i - \vec{x}_j|$$

$$\vec{n}_{ij} = \frac{\vec{x}_i - \vec{x}_j}{|\vec{x}_i - \vec{x}_j|}$$

$$dv_{ij}^n = \left(\frac{\partial \vec{x}_i}{\partial t} - \frac{\partial \vec{x}_j}{\partial t}\right)\cdot \vec{n}_{i,j}$$

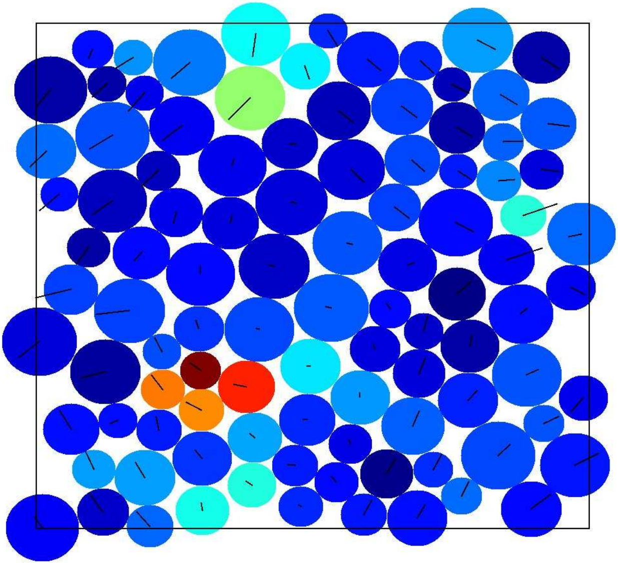
$$\vec{F}_{ij} = \begin{cases} \left(k^n\delta_{ij} + \gamma^n dv_{ij}^n\right)\vec{n}_{ij} & \text{if } \delta_{i,j} > 0 \\ \vec{0} & \text{if } \delta_{i,j} < 0 \end{cases}$$



Simulations details

- 2D soft cylinders (10^4 Particles)
- Polydisperse ($r_{large} = 2r_{small}$)
- Quasi steady ($8 \cdot 10^4 \, \mathrm{t_c/cycle}$)
- Bi-axial box
- Periodic walls
- Linear normal forces and dissipation (data based on small particles)
  - Collision time ($t_c = 6.5 \cdot 10^{-4} \, \mathrm{s}$)
  - Coefficient of restitution ($r = 0.8$)
- No tangential forces
- Small background friction ($\gamma_{bg} = 0.1\gamma_{pp}$)

Video



Forces

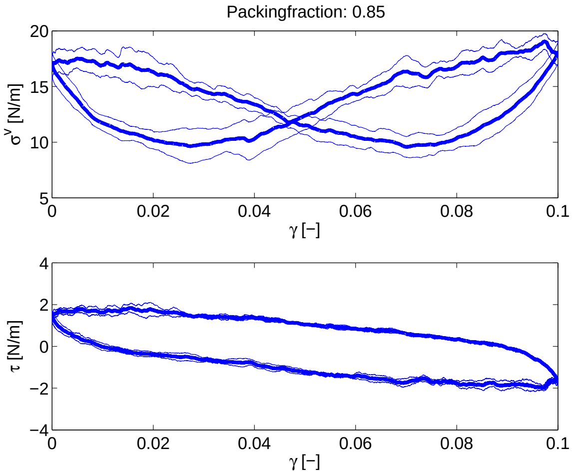
$$\bar{\bar{\sigma}} = \frac{1}{A} \sum_{C} \left(R_i + R_j - \delta_{ij} \right) \vec{F}_{ij} \otimes \vec{n}_{ij}$$

Pressure and shear stress:

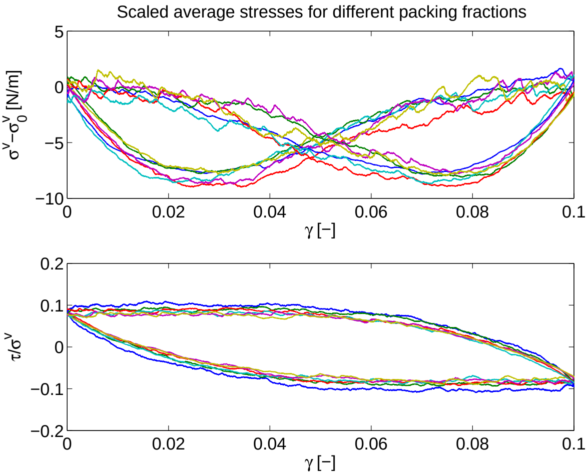
$$\begin{aligned} \sigma^{v} &= \frac{\sigma_1 + \sigma_2}{2} \\ \tau &= \frac{\sigma_1 - \sigma_2}{2} \end{aligned}$$

With σ_1 and σ_2 the eigenvalues of $\bar{\bar{\sigma}}$.

Typical result (average)

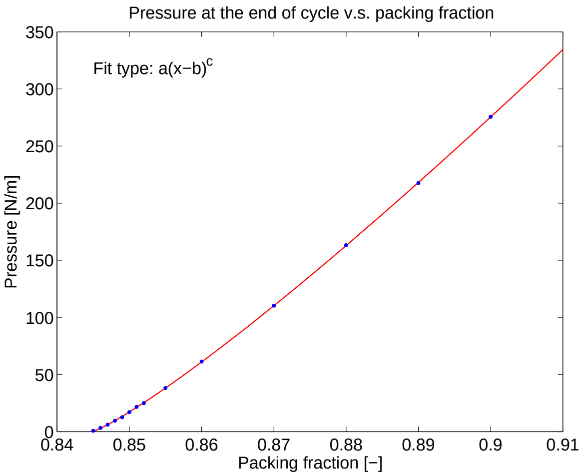


Results different packing fractions



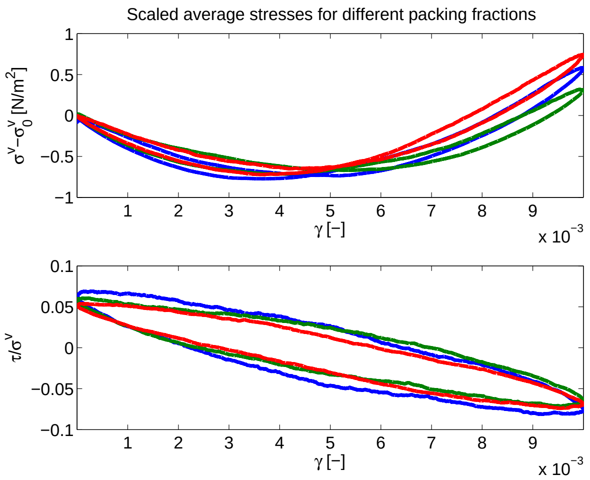
Packing fractions vary from 0.85 (blue) till 0.90 (yellow).

Pressure power law



Critical packing fraction: 0.845, with exponent: 1.165

Small amplitude



Packing fractions vary from 0.848 (blue) till 0.850 (red).

Conclusion DPM simulations

- Scaling works well for packing fractions of 0.848 and higher.
- For lower packing fractions unjammed states occur
- Information about the initial state vanishes after a few cycles (not shown here)
- Pressures at the end of cycles nicely shows power law behaviour

Model

Basic equation:

$$\begin{bmatrix} \delta\sigma^v \\ \delta\tau \end{bmatrix} = \begin{bmatrix} 2B & A \\ A & 2G \end{bmatrix} \begin{bmatrix} \delta\varepsilon^v \\ S\delta\gamma \end{bmatrix} \tag{1}$$

Anisotropy:

$$\frac{dA}{d\gamma} = \beta_A \left(A_{max} - \mathrm{sign}\left(\delta\gamma\right) A \right) \tag{2}$$

Stress saturation:

$$S = 1 - \frac{\tau}{\sigma^v} \frac{\mathrm{sign}\left(\delta\gamma\right)}{s^d_{max}} \tag{3}$$

Model modifications

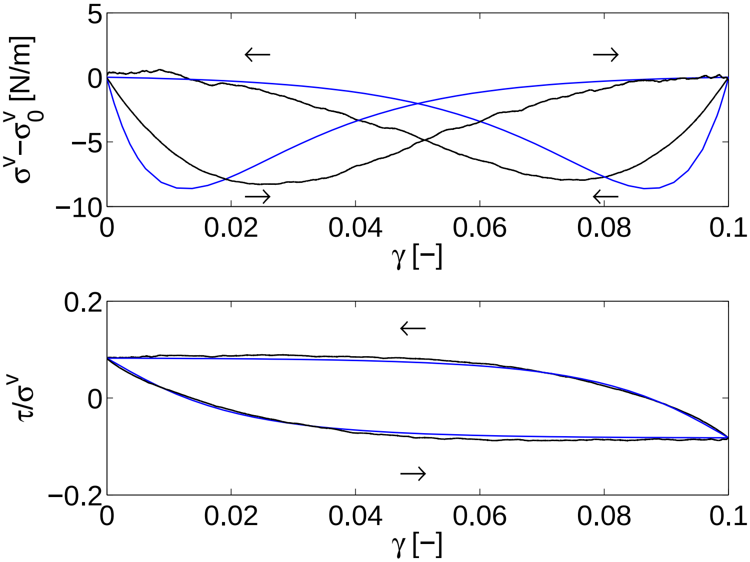
To also show the transient pressure stabilization term is required:

$$\beta_p \left(\sigma^h_{steady} (\phi) - \sigma^h \right) |\delta\gamma| \tag{4}$$

Limit of small oscillations around mean pressure (σ^h_{steady}):

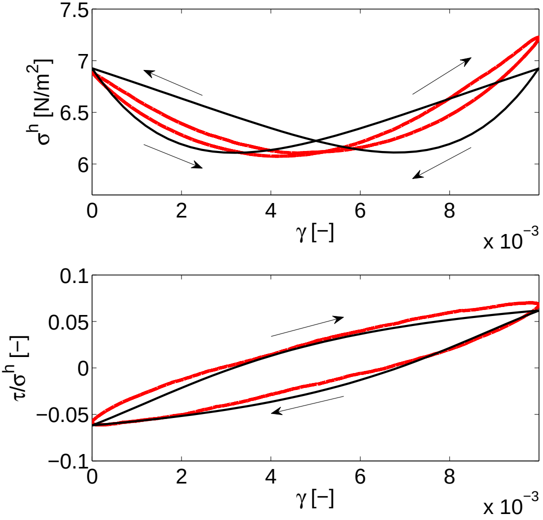
$$2G = \beta_A \sigma^h_{steady} s^d_{max} \tag{5}$$

Model fit



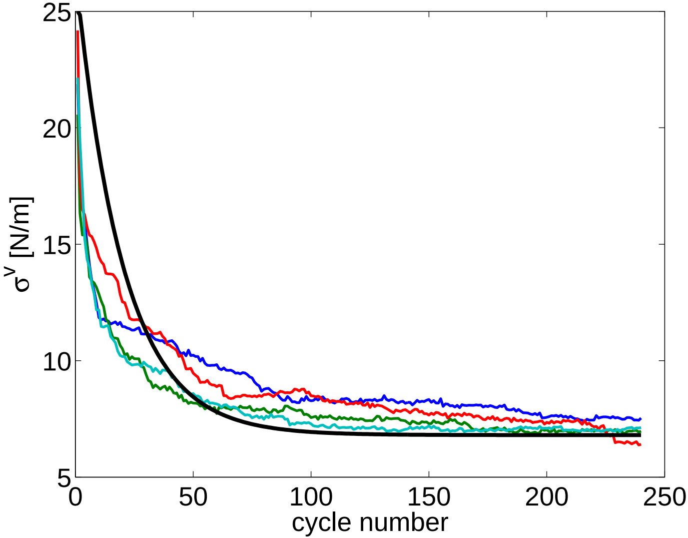
Blue line: model, black line: average results

Model fit



Blue line: model, black line: average results

Model fit



Black line: model, other linse: simulation results

Conclusion

- Model is able to predicts stresses qualitatively during the transient as well as during the limit cycles
- (For large strains) Location of minimum in pressure in not correctly predicted
- (For small strains) Anisotropy in pressure seems to be to high

Model with non-constant parameters may produce better results.

Outlook

- Apply model with non-constant parameters
- Try different models (like the granular hydrodynamics model)
- Study influence of tangential forces
- Study influence of 3th dimension