Twente Granular Matter Days
at University of Twente on Feb.01-02, 2012.

Slow dynamics in jamming transition

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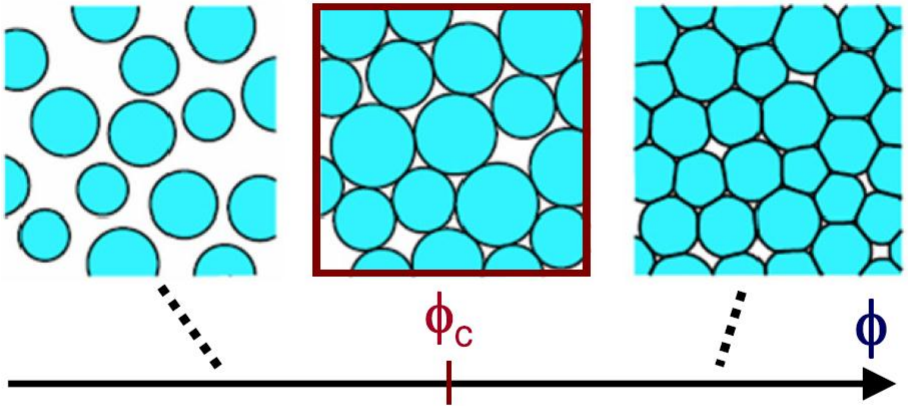
Introduction



Jamming transition - thermal & athermal systems

- Constituents are arrested in the disordered state
- Glass, emulsions, colloids, and grains
- Temperature, density, and external loads

Critical scaling of athermal jamming

- Density ϕ $\longrightarrow$ measure $\Delta\phi = \phi - \phi_c$
- Excess coordination number, pressure, bulk & shear moduli

$$\Delta z \sim \Delta\phi^{1/2} \quad p \sim \Delta\phi^{\psi} \quad G \sim \Delta\phi^{\lambda} \quad K \sim \Delta\phi^{\gamma}$$

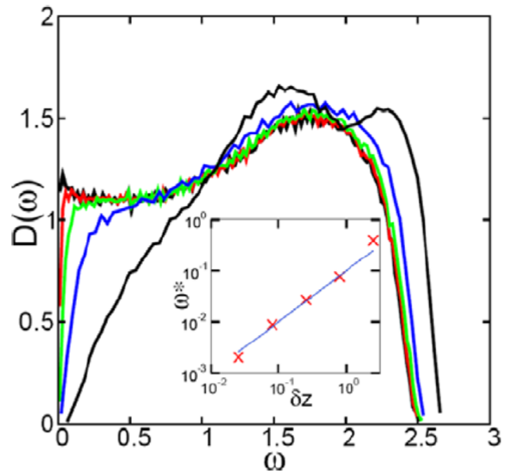
Structural signature (monodisperse case)

- The first peak of the radial distribution function

$$g_1 \sim \Delta\phi^{-1} \qquad g(r) \propto 1/\sqrt{r-1}$$

Density of vibrational modes

- Untrivial plateau in low frequencies
- Diverging length scale



[M. van Heck, J. Phys.: Condens. Matter **22** (2010) 033101]

Method

Our purpose

- To study static and dynamic properties of jamming of bidisperse *dissipative* particles.

Our approach – numerical model of dissipative particles

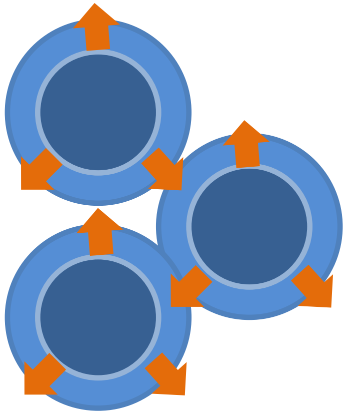
- DEM simulation
- Periodic boundary condition
- 2-dimensional bidisperse disks
- Linear spring & dashpot model

$$m_i(t)\ddot{\mathbf{x}}_i = \sum_j \{k_n \delta_{ij} - \eta_n (\mathbf{v}_{ij} \cdot \mathbf{n}_{ij})\} \mathbf{n}_{ij}$$

- Radius growth; Lubachevsky-Stillinger (LS) algorithm

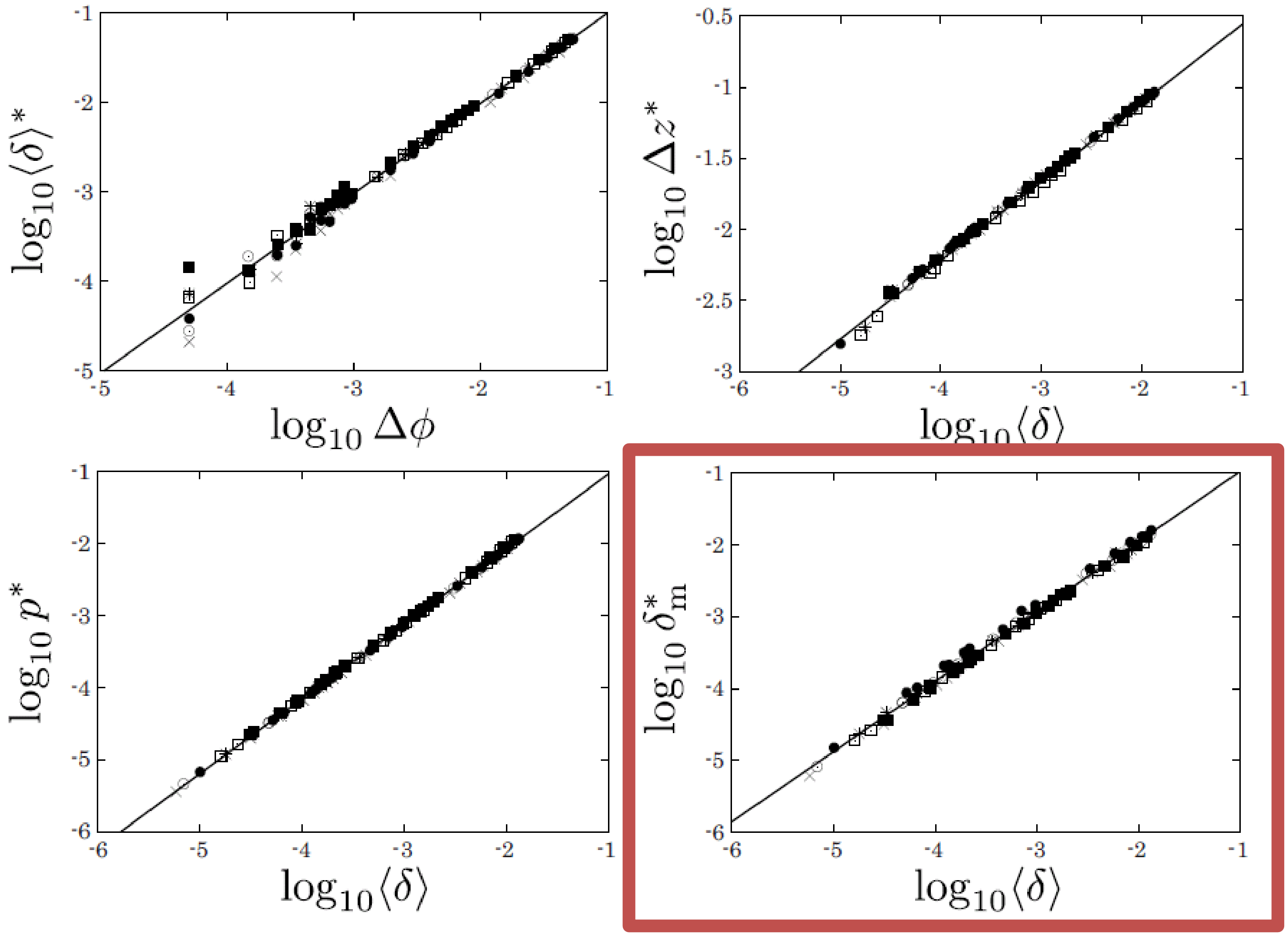
$$\dot{\sigma}_i(t) = g_r \sigma_i(0)$$

- The number of disks N = 32768
- Restitution coefficient e = 0.99
- Parameters; $\phi, \rho \equiv \sigma_L / \sigma_S$ $\quad 0.8 \leq \phi \leq 0.9 \quad 1.2 \leq \rho \leq 2.2$



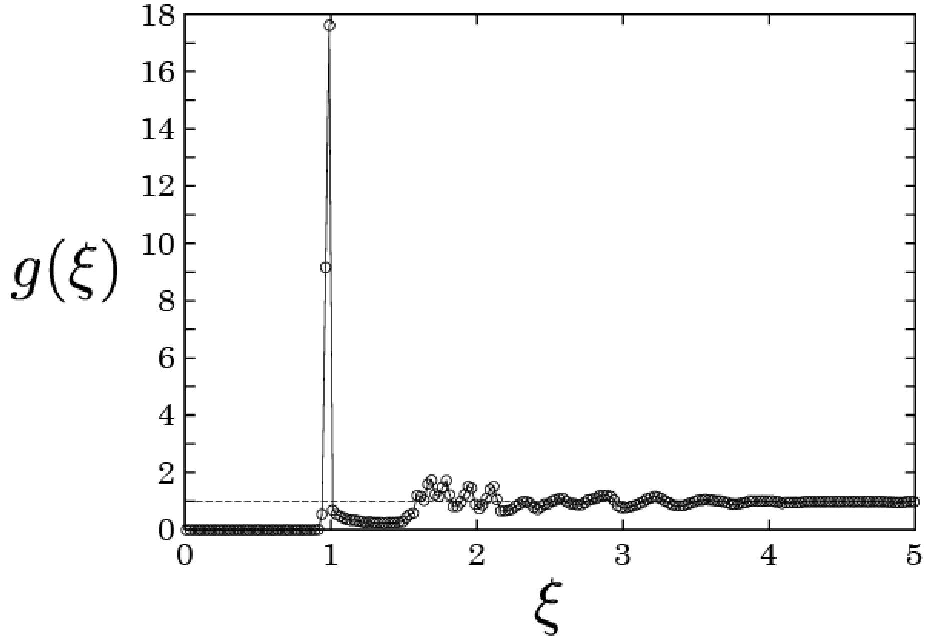
isotropic compression, or
the tunable-diameter colloids.

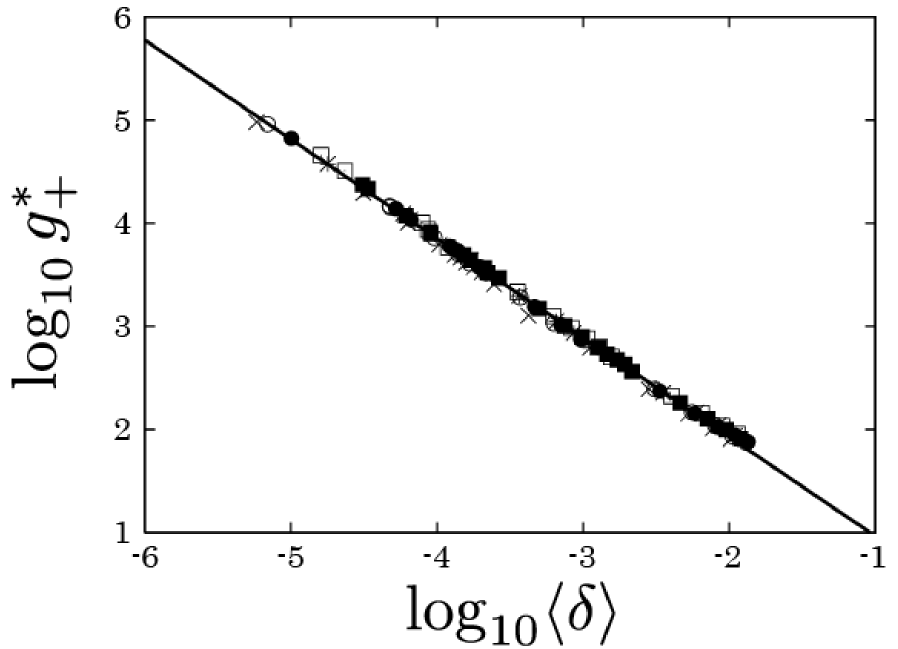
Critical scaling above jamming



$$\Delta z \sim \Delta\phi^{1/2} \qquad \langle\delta\rangle, p \sim \Delta\phi, \qquad \delta_{\mathrm{m}} \sim \Delta\phi$$

Structural signature above jamming





- Radial distribution function of scaled distance

$$\xi = \frac{r}{r_i + r_j}$$

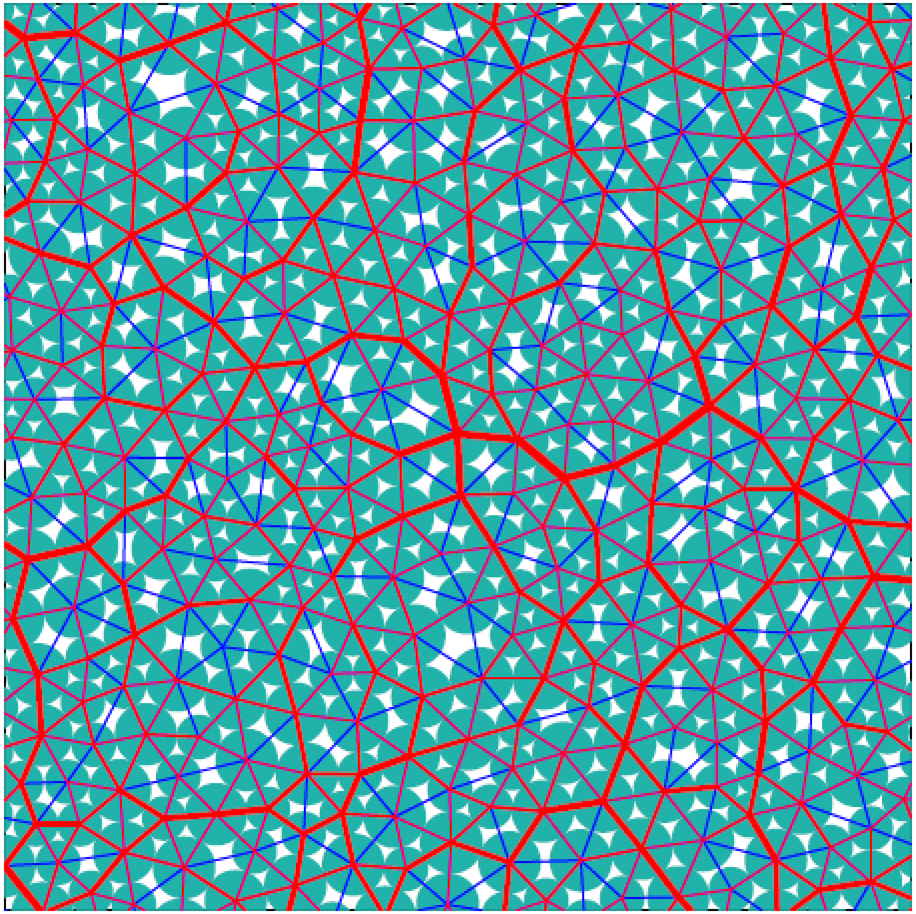
- The first peak

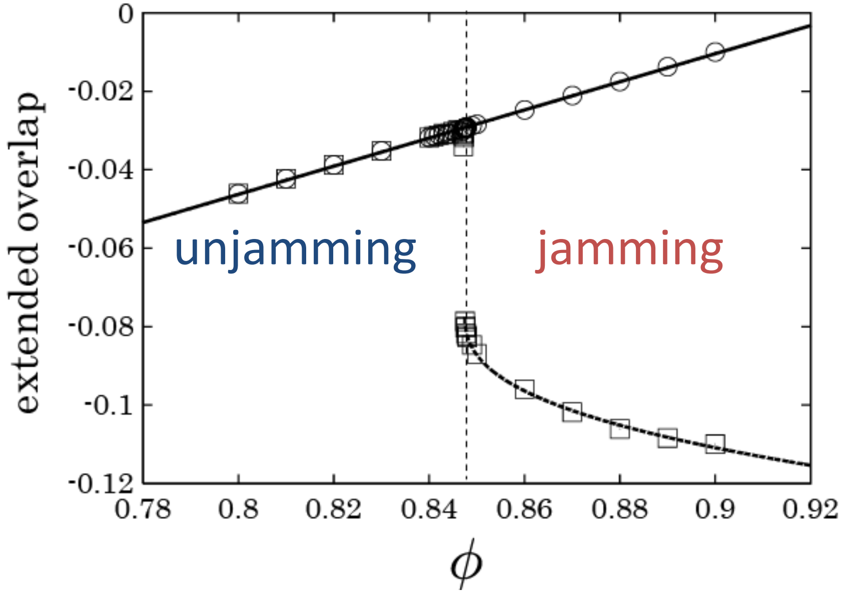
$$g_+ \sim \Delta\phi^{-1} \longrightarrow g_+\langle\delta\rangle \approx \mathrm{const}$$

Extended overlaps

real = Force chain network

real + *virtual* = the Delaunay graph

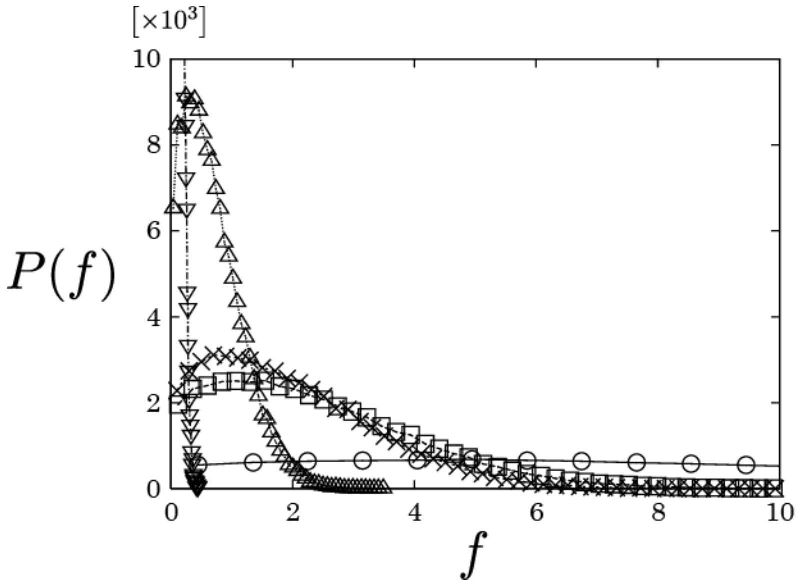


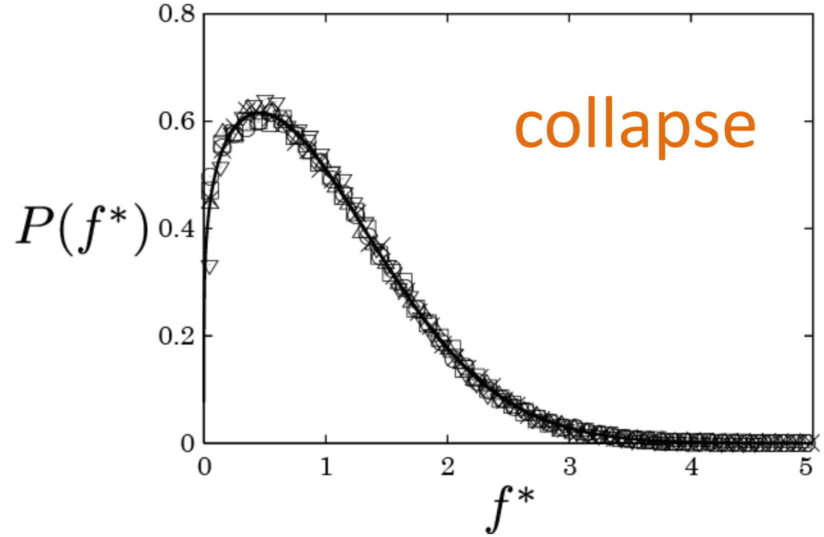


- *Overlap* $\delta = r_i + r_j - |\boldsymbol{x}_i - \boldsymbol{x}_j|$
- Mean *overlap* extended to negative $< \delta >_G$
- Mean negative *overlap* $< \delta >_-$

→ Defined both *below* and *above* jamming

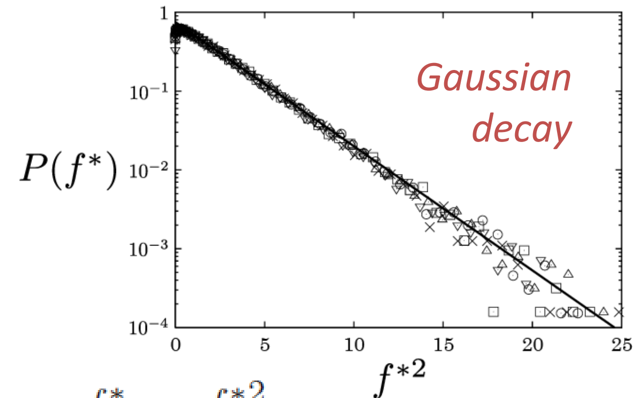
PDF of forces





Probability distribution function (PDF) of forces

- Signature of jamming transition
- Collapsed to one master curve near jamming



$$\Delta\phi < 10^{-2}$$

$$P(f^*) = (A_0 + A_1 f^{*a_0}) e^{-a_1 f^* - a_2 f^{*2}}$$

$$\Delta\phi = 1.3 \times 10^{-2}, 5.5 \times 10^{-4}, 4.5 \times 10^{-4}, 2.5 \times 10^{-4}, 5.0 \times 10^{-5}$$

Slow dynamics above jamming – early stage

Critical slowing down?

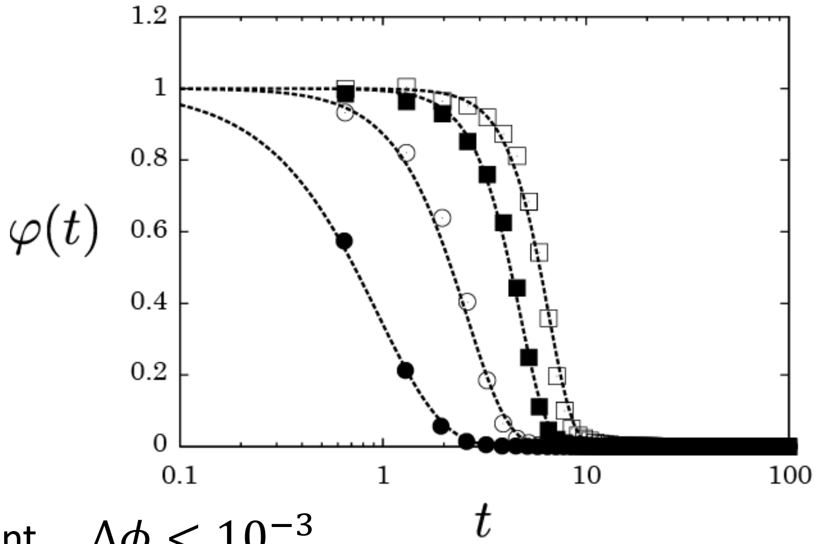
$\Delta\phi = 1.2 \times 10^{-2}, 2.5 \times 10^{-3}, 4.4 \times 10^{-4}, 1.4 \times 10^{-4}$

- Ratio of the kinetic energy to the elastic potential

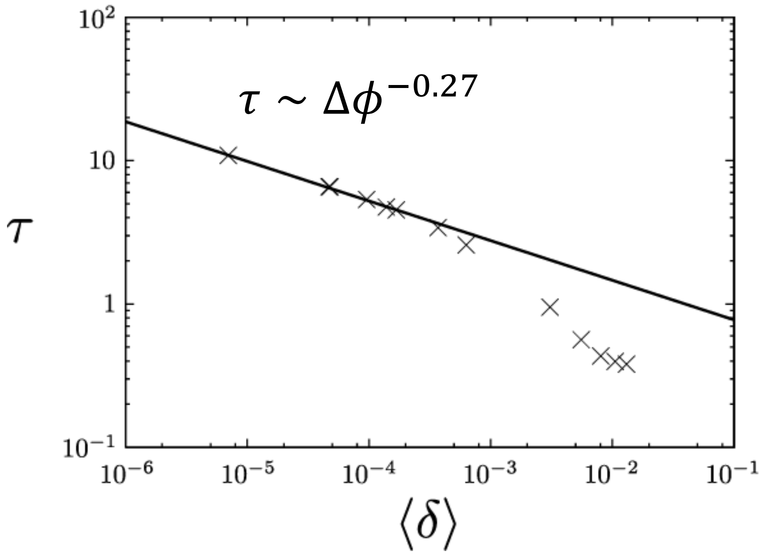
$$\varphi(t) \equiv \frac{K(t)/U(t)}{K(0)/U(0)}$$

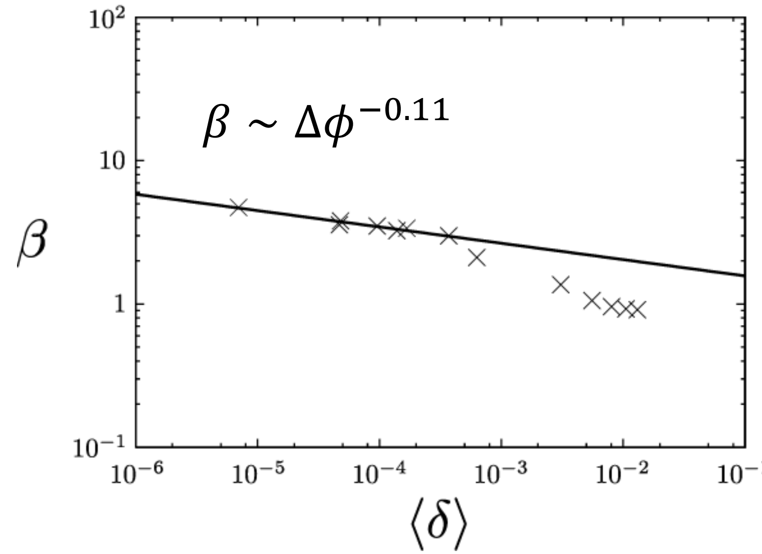
- Stretched dynamics

$$\varphi(t) = e^{-(t/\tau)^{\beta}}$$



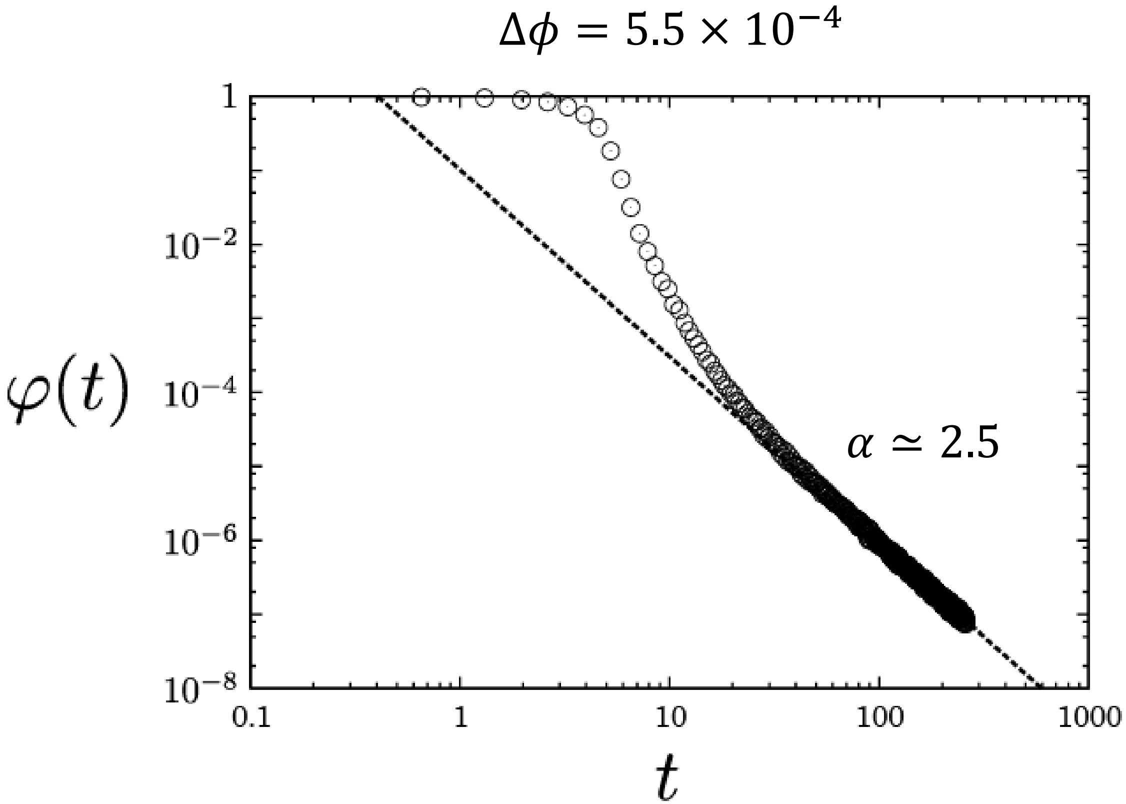
- Scalings of the relaxation time and stretched exponent $\Delta\phi < 10^{-3}$





Slow dynamics above jamming – late stage

Long time tail



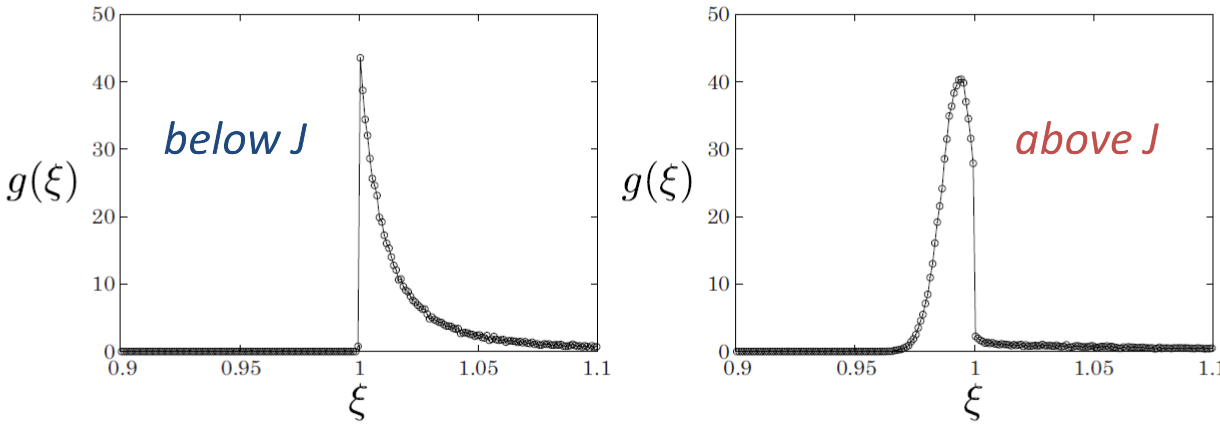
- Power law decay

$$\varphi(t) = C_L t^{-\alpha t}$$

- Faster than the tail observed in the free cooling of granular gases

$$\alpha > 2.0$$

Discussion & future works



Discussion

- How to understand the slow dynamics near jamming?
  - *Approach; response theory? dynamic critical phenomena?*
- System size
- Initial condition
- Growth rate
- Restitution coefficient

Future works

- How to understand the discontinuities of g(\xi) around 1?
- 3-dimension
- Monodisperse case
- Polydisperse case

Conclusion

- We numerically investigvated the jamming of 2-dimensional bidisperse dissipative particles.
- We systematically studied the effect of the size ratio. $1.2 \leq \rho \leq 2.2$
- We found the new scaling of the maximum overlap. $\delta_m \sim \Delta\phi$
- We showed the divergence of the first peak of g(\xi). $g_+ \sim \Delta\phi^{-1}$
- We found the slow dynamics of K(t)/U(t) slightly above the jamming point.

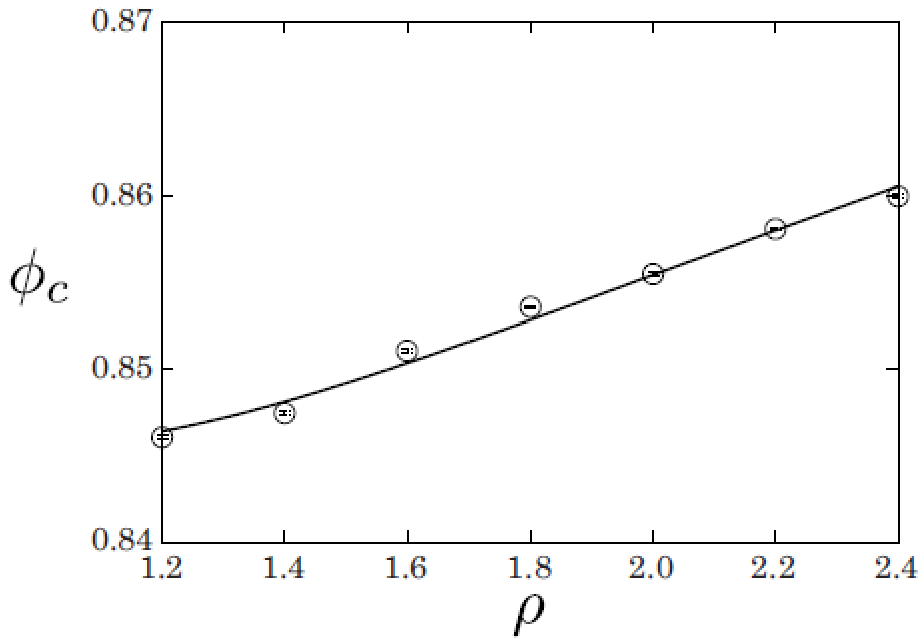
$$\tau \sim \Delta\phi^{-0.27} \qquad \beta \sim \Delta\phi^{-0.11}$$

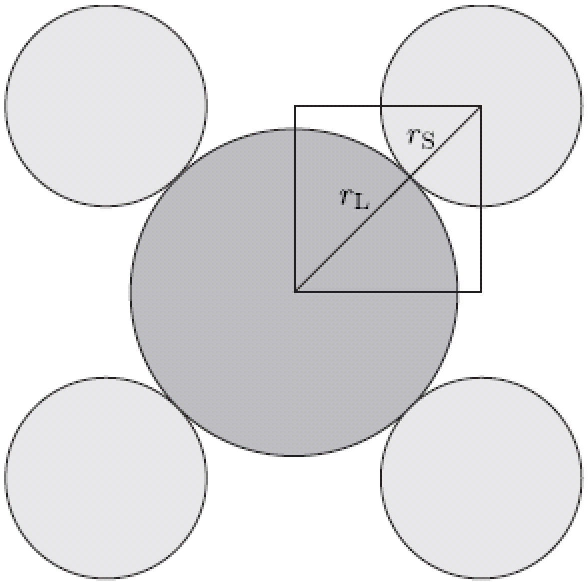
- We also found the long time tail of K(t)/U(t).

$$\alpha \simeq 2.5$$

Thank you for your attention!

Critical area fraction





$$\phi_1 = \frac{\pi}{2}\frac{\rho^2+1}{(\rho+1)^2}$$

$$\phi_2 = \pi/4.$$

$$\phi_c = w_1\phi_1 + w_2\phi_2$$

$$w_1 = 0.1213 \text{ and } w_2 = 0.9550 \ (w_1 + w_2 \simeq 1)$$