Signal Propagation in Granular Matter – Order and Disorder

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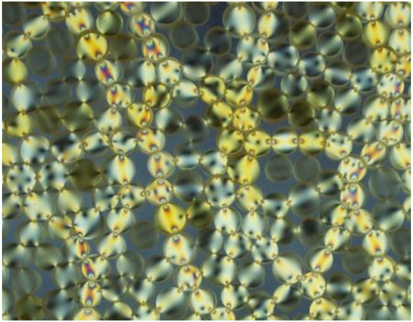
VICI Progress Report 2012

Introduction



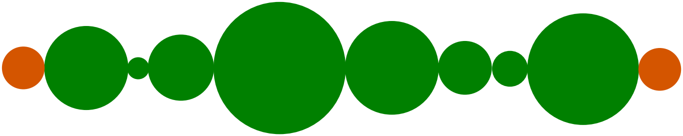
How might small disturbances propagate through such a medium?

- Challenges:
  - Dense
  - Discrete
  - Inhomogeneous
  - Anisotropic
  - Frictional
  - Dissipative

Overview

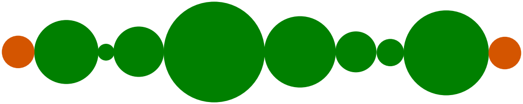
- One–dimensional chains
  - Model system and equations
  - Introduction of mass–disorder
    - Disorder magnitude
    - System excitation
  - Linear vs. nonlinear
  - Role of contact disorder
- Three–dimensional crystalline packings
  - Anisotropy
  - Disorder effects

One–dimensional chains



- Convenient model
  - Analytically accessible
  - Isolation of mass–disorder
- Significant attention in literature
  - Nonlinear oscillators
  - Soliton–like waves

One–dimensional chains



- Force–displacement model:

$$\tilde{F}_{(i,j)} = \tilde{\kappa}_{(i,j)} \tilde{\delta}^{1+\beta}$$

$$\tilde{m}^{(i)} \frac{d^2 \tilde{x}^{(i)}}{d\tilde{t}^2} = \tilde{F}_{(i,i-1)} + \tilde{F}_{(i,i+1)}$$

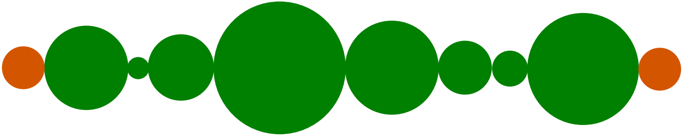
- Scaling:

$$\text{Mass: } \tilde{m}_o \qquad (1)$$

$$\text{Length: } \tilde{\ell}$$

$$\text{Time: } \tilde{t}_c = \frac{1}{\tilde{\ell}^{\beta/2}} \sqrt{\frac{\tilde{m}_o}{\tilde{\kappa}_o}}$$

One–dimensional chains



- General equation of motion:

$$b^{(i)}\frac{d^2u^{(i)}}{d\tau^2} = \kappa_{(i-1,i)}\left[\Delta_{(i-1,i)} - u^{(i)} + u^{(i-1)}\right]^{1+\beta}$$
$$-\kappa_{(i+1,i)}\left[\Delta_{(i+1,i)} + u^{(i)} - u^{(i+1)}\right]^{1+\beta}$$

$$b \equiv \tilde{m}^{(i)}/\tilde{m}_o$$

$$\tau \equiv \tilde{t}/\tilde{t}_c$$

$$\kappa_{(i,j)} \equiv \tilde{\kappa}_{(i,j)}/\tilde{\kappa}_o$$

One–dimensional chains

- General (nonlinear) equation of motion:

$$b^{(i)} \frac{d^2 u^{(i)}}{d\tau^2} = \kappa_{(i-1,i)} \left[\Delta_{(i-1,i)} - u^{(i)} + u^{(i-1)} \right]^{1+\beta} - \kappa_{(i+1,i)} \left[\Delta_{(i+1,i)} + u^{(i)} - u^{(i+1)} \right]^{1+\beta}$$

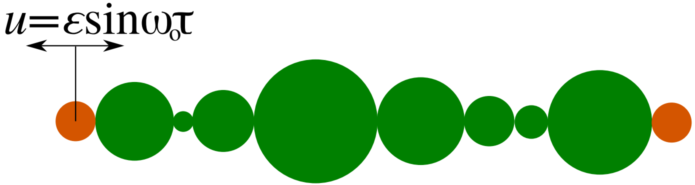
- Linearized model:

$$\mathbf{M} \frac{\mathrm{d}^2 \mathbf{u}}{\mathrm{d}\tau^2} = \mathbf{K}\mathbf{u}$$

$$u^{(p)}(\tau) = \sum_{j=1}^{N} \frac{S_{pj} S_{1j}}{\left(\omega_j^2 - \omega_o^2\right)} \left(\sin \omega_o \tau - \frac{\omega_o}{\omega_j} \sin \omega_j \tau \right)$$

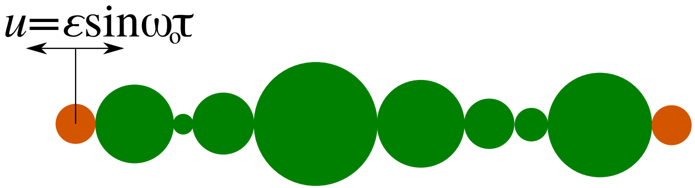
One–dimensional chains

- Harmonically driven:



- Mass–disorder: Normal (Gaussian) distribution
  - Mean mass $\rightarrow b = 1$
  - Standard deviation $\rightarrow \sigma = \xi$
- Pre–stress $\rightarrow$ equilibrium overlap $\rightarrow$ NOT sonic vacuum

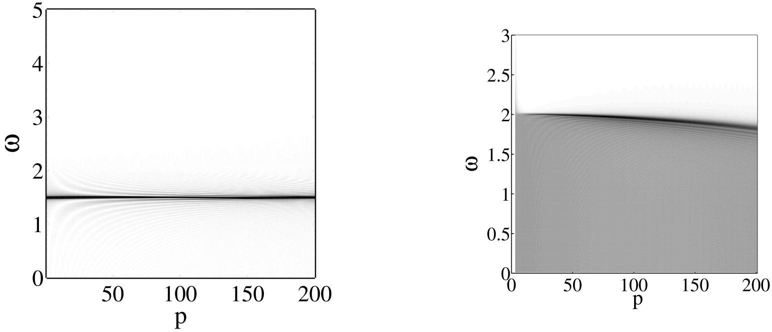
Disordered chains as a frequency filter?



How do signals propagate in such systems?

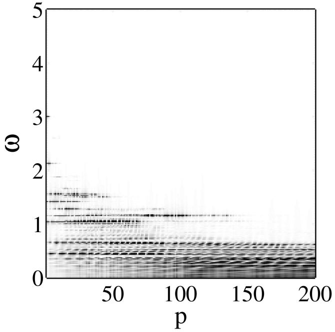
- Disorder ξ
- Input frequency ω_o
- Mass distribution
- Contact order/disorder
- Linear vs. nonlinear

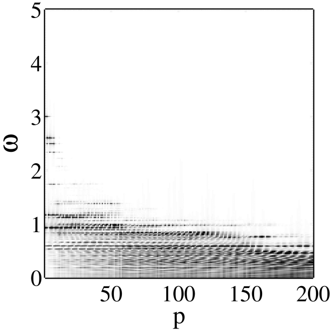
Base case– perfect chain



- Linear
- Uniform stiffness $\kappa_{(i,j)} = k_n$
- $\xi = 0.0$

Disordered chains as a frequency filter?

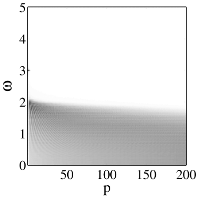




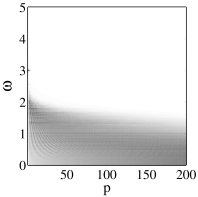
- Linear
- Uniform stiffness $\kappa_{(i,j)} = k_n$
- $\omega_o = 3.0$
- $\xi = 0.5$

Disordered chains as a frequency filter?

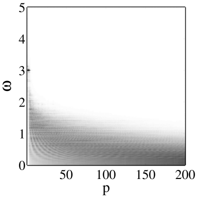
- Consider an ensemble of chains, fix $\omega_o = 3.0$, and vary ξ:



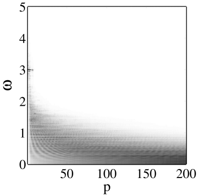
(a) $\xi = 0.1$



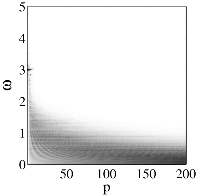
(b) $\xi = 0.2$



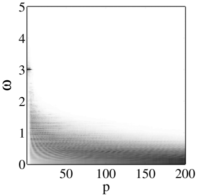
(c) $\xi = 0.35$



(d) $\xi = 0.45$

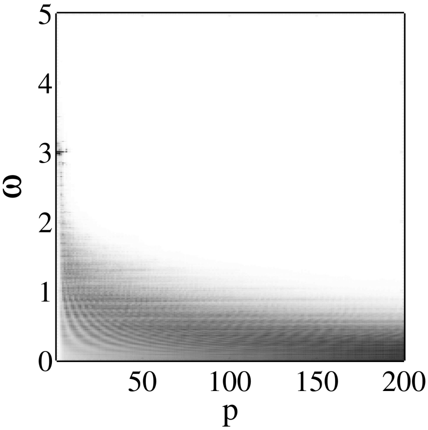


(e) $\xi = 0.5$



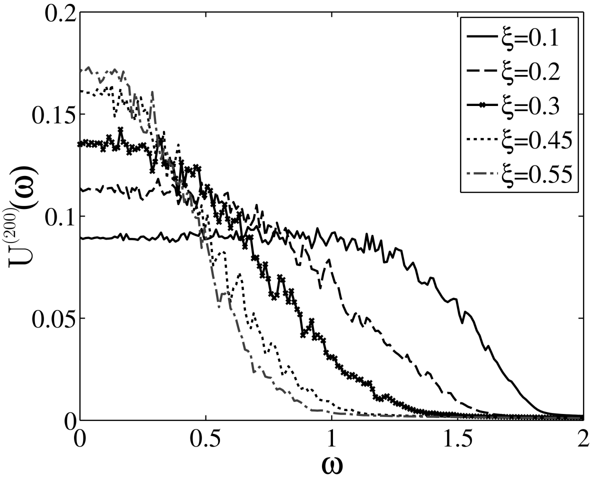
(f) $\xi = 0.55$

Disordered chains as a frequency filter?



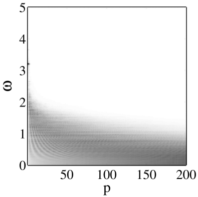
$\xi = 0.5$

Disordered chains as a frequency filter?

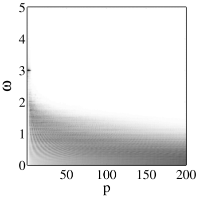


Disordered chains as a frequency filter?

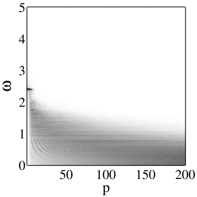
- Fix ξ, vary ω_o:



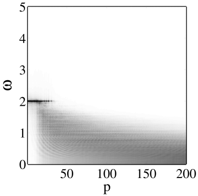
(a) $\omega_o = 3.2$



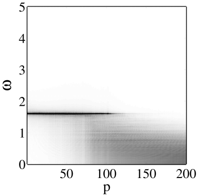
(b) $\omega_o = 3.0$



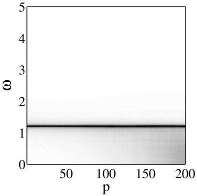
(c) $\omega_o = 2.4$



(d) $\omega_o = 2.0$



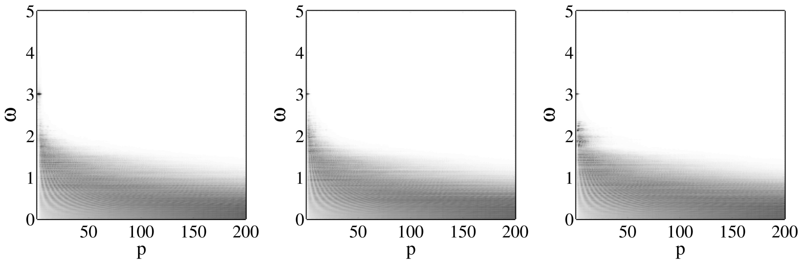
(e) $\omega_o = 1.6$



(f) $\omega_o = 1.2$

Disordered chains as a frequency filter?

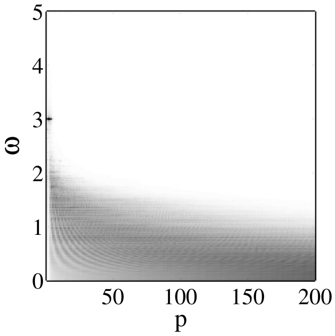
- Compare mass distributions:
  - Match moments $M_n^{(q)} = \int_{-\infty}^{\infty} b^n f^{(q)}(b)\ \mathrm{d}b$



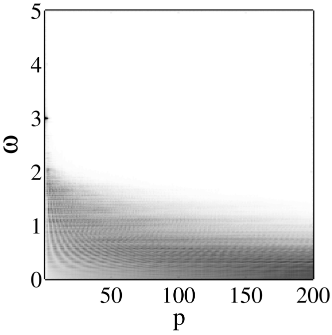
(a) Normal (b) Uniform (c) Binary

- Linear
- Uniform stiffness $\kappa_{(i,j)} = k_n$
- $\omega_o = 3.0$
- $\xi = 0.3$

Contact disorder

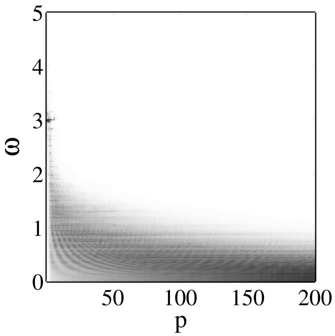


No contact disorder

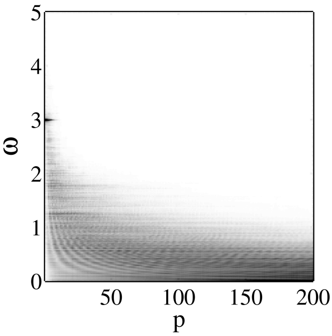


WITH contact disorder

Linear vs. Nonlinear (Hertzian)

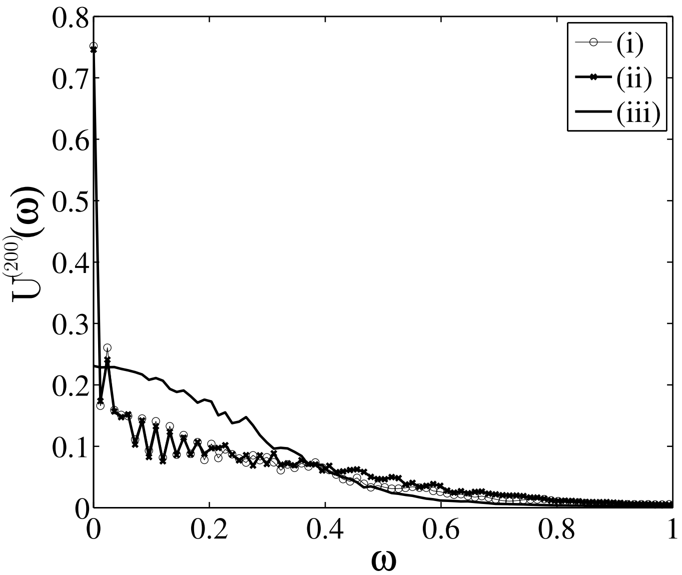


Linear, contact disorder

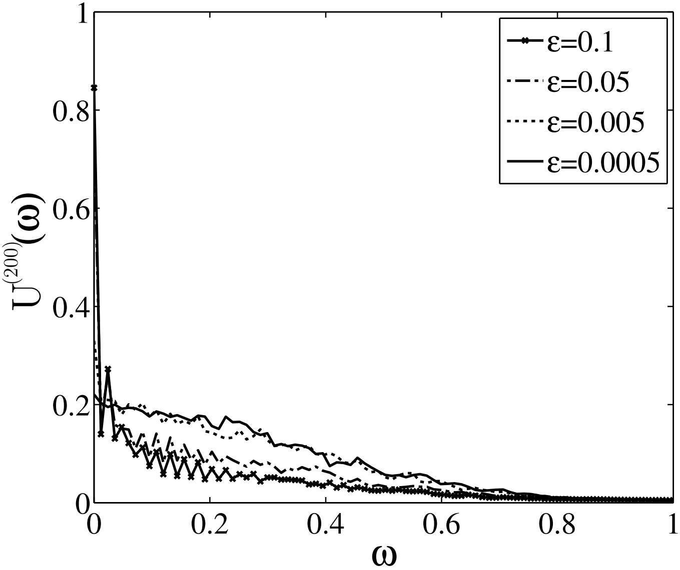


Nonlinear, contact disorder

Linear vs. Nonlinear (Hertzian)



Linear vs. Nonlinear (Hertzian)

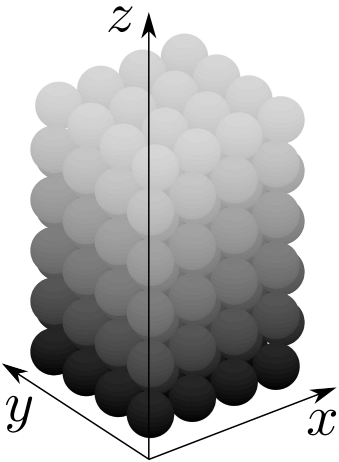


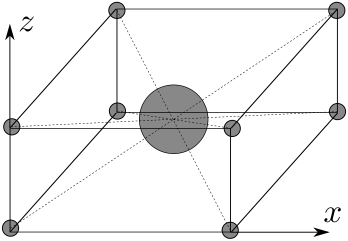
- Conclusions:
  - $\uparrow$ disorder ξ, $\downarrow$ transmission bandwidth.
    - Threshold value of ξ
  - Lower input ω_o, improved transmission
    - Low frequencies less sensitive to mass arrangements
  - Mass–distribution: only moments
  - Nonlinear coupling $\rightarrow$ more power in lowest frequencies
    - Small ϵ $\rightarrow$ recovery of linear behavior

Overview

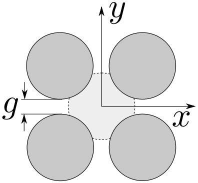
- One–dimensional chains
  - Model system and equations
  - Introduction of mass–disorder
    - Disorder magnitude
    - System excitation
  - Linear vs. nonlinear
  - Role of contact disorder
- **Three–dimensional crystalline packings**
  - Anisotropy
  - Disorder effects

Long-short-short (LSS) geometry





Unit cell



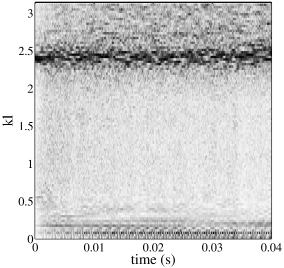
Top view

Ordered LSS system

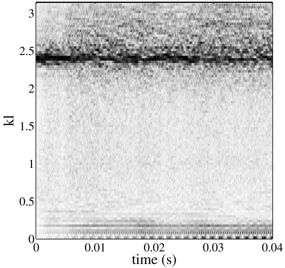
(loading...)

Disordered LSS system

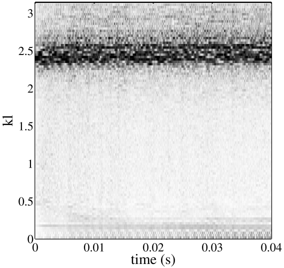
(loading)



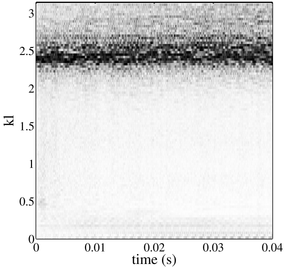
(a) $\gamma = 31°$



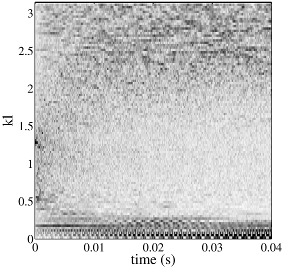
(b) $\gamma = 35°$



(c) $\gamma = 39°$



(d) $\gamma = 43°$



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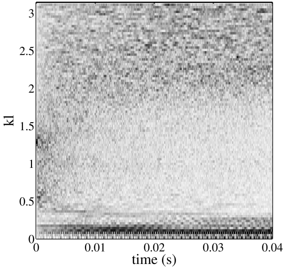
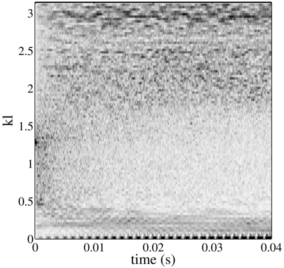


(b) $\gamma = 35°$



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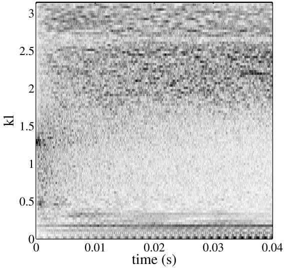

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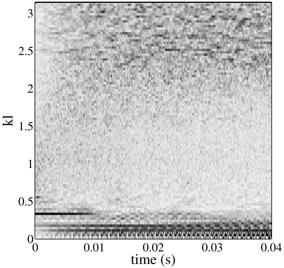

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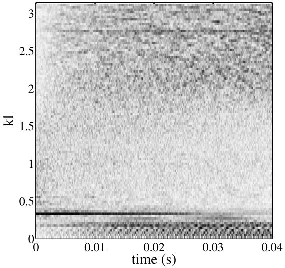

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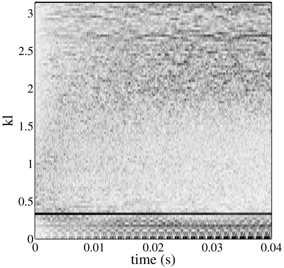

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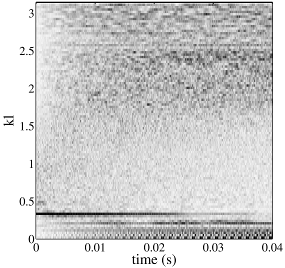

(d) $\gamma = 43°$

Future work

- Relate anisotropy to particle motion
  - Energy transfer
  - Mode conversion, coupling, micro parameters
- Statistical descriptions of disorder
  - Incorporating anisotropy into these