Anisotropy in 2D granular media under cyclic shear

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Introduction

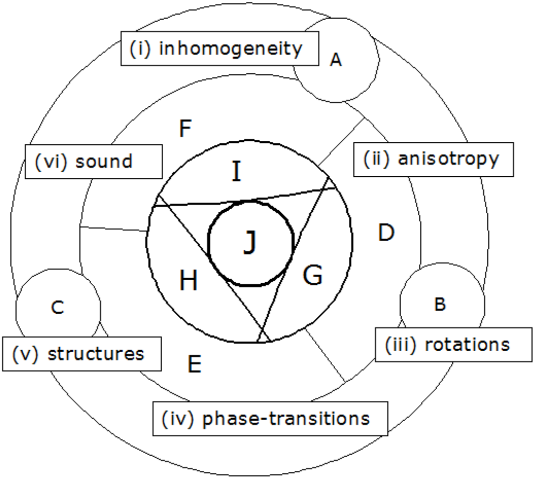
Goal:

Develop a simple local constitutive model based on observations from DEM simulations

Current subgoal:

Study anisotropy in 2D granular media and compare with existing constitutive models

Scope within the VICI project



Outline

DEM

$$\vec{a}_i = \frac{1}{m_i} \sum_{i \neq j} \vec{F}_{ij}$$

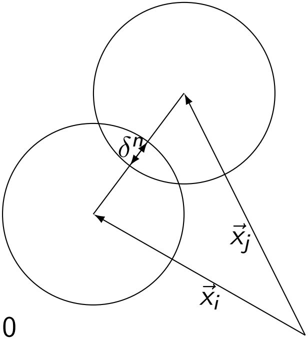
Where:

$$\delta_{ij} = r_i + r_j - |\vec{x}_i - \vec{x}_j|$$

$$\vec{n}_{ij} = \frac{\vec{x}_i - \vec{x}_j}{|\vec{x}_i - \vec{x}_j|}$$

$$dv_{ij}^n = \left(\frac{\partial \vec{x}_i}{\partial t} - \frac{\partial \vec{x}_j}{\partial t} \right) \cdot \vec{n}_{i,j}$$

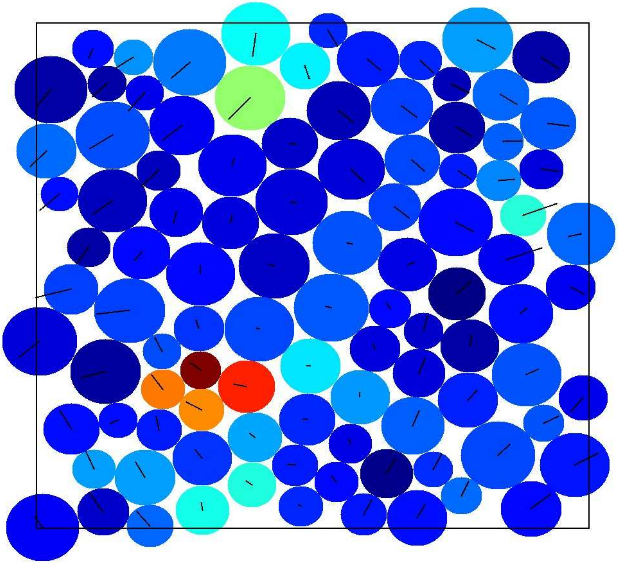
$$\vec{F}_{ij} = \begin{cases} \left(k^n \delta_{ij} + \gamma^n dv_{ij}^n \right) \vec{n}_{ij} & \text{if } \delta_{i,j} > 0 \\ \vec{0} & \text{if } \delta_{i,j} < 0 \end{cases}$$



Simulations details

- 2D soft cylinders (10^4 Particles)
- Polydisperse ($r_{large} = 2r_{small}$)
- Quasi steady ($8 \cdot 10^4\ t_c/\text{cycle}$)
- Bi-axial box
- Periodic walls
- Linear normal forces and dissipation (data based on small particles)
  - Collision time ($t_c = 6.5 \cdot 10^{-4}$ s)
  - Coefficient of restitution ($r = 0.8$)
- No tangential forces
- Small background friction ($\gamma_{bg} = 0.1\gamma_{pp}$)

Video



Forces

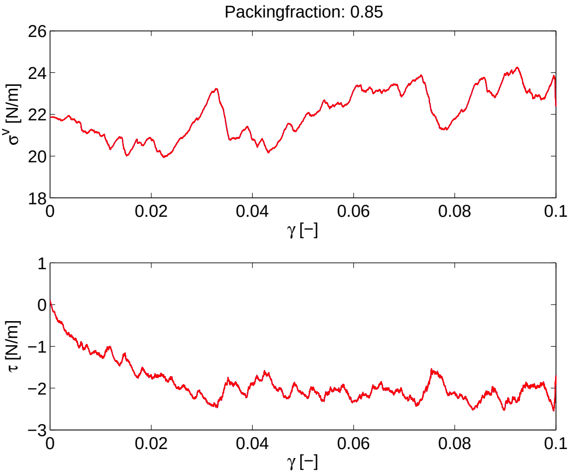
$$\bar{\bar{\sigma}} = \frac{1}{A} \sum_{C} (R_i + R_j - \delta_{ij}) \vec{F}_{ij} \otimes \vec{n}_{ij}$$

Pressure and shear stress:

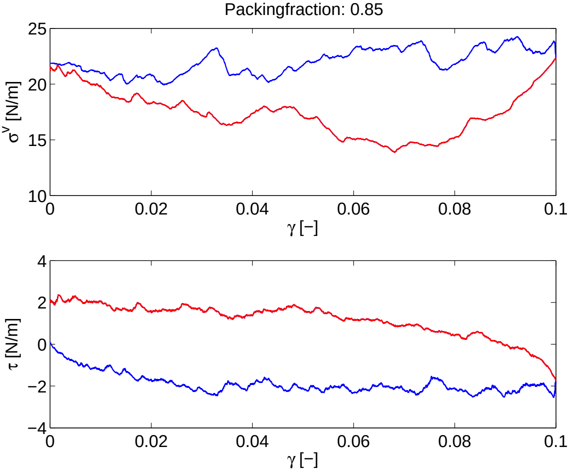
$$\begin{aligned} \sigma^v &= \frac{\sigma_1 + \sigma_2}{2} \\ \tau &= \frac{\sigma_1 - \sigma_2}{2} \end{aligned}$$

With σ_1 and σ_2 the eigenvalues of $\bar{\bar{\sigma}}$.

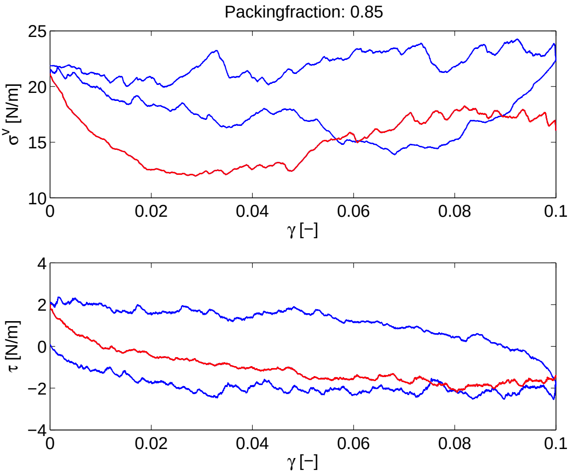
Typical result (1)



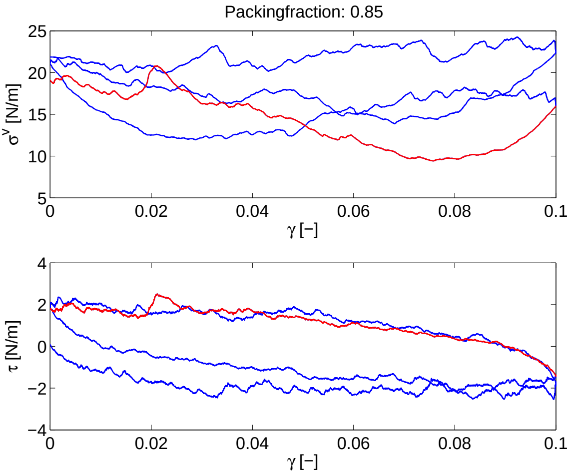
Typical result (2)



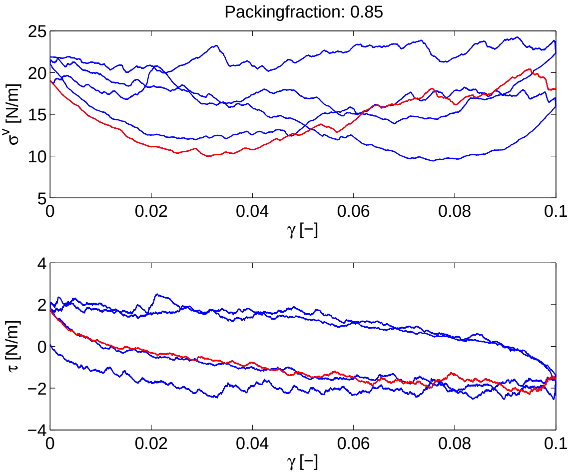
Typical result (3)



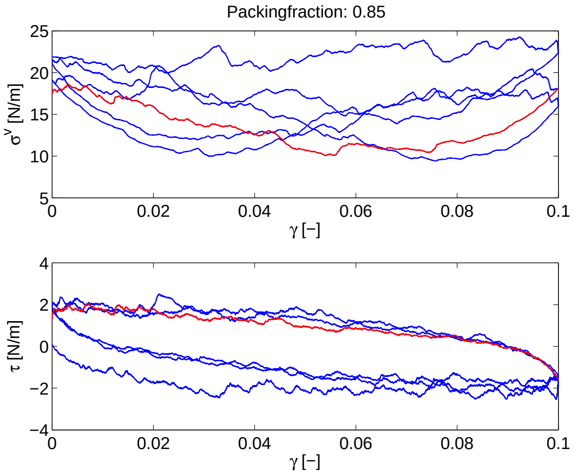
Typical result (4)



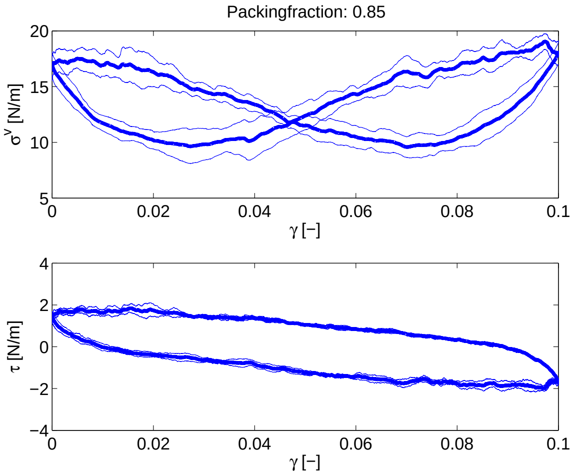
Typical result (5)



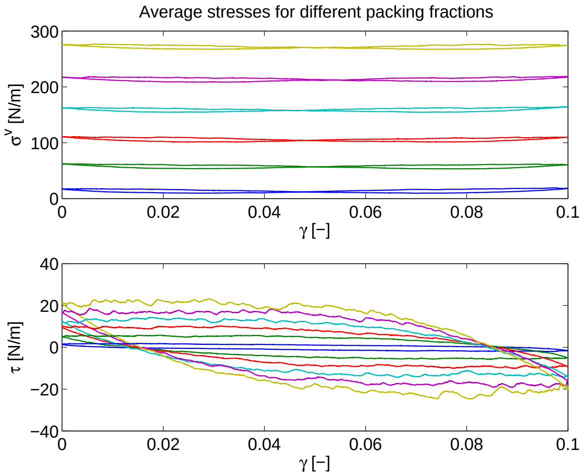
Typical result (6)



Typical result (average)

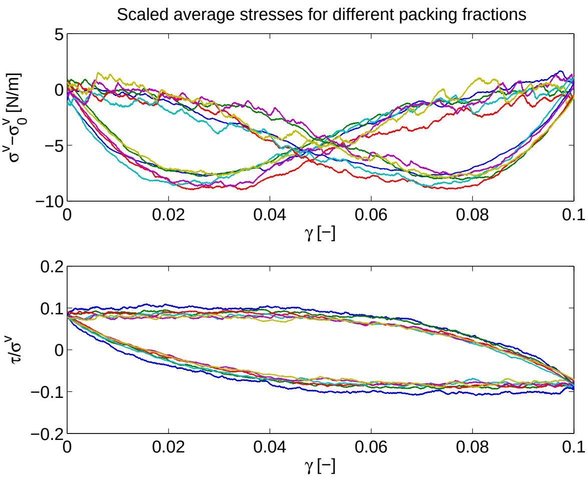


Results different packing fractions



Packing fractions vary from 0.85 (blue) till 0.90 (yellow).

Results different packing fractions



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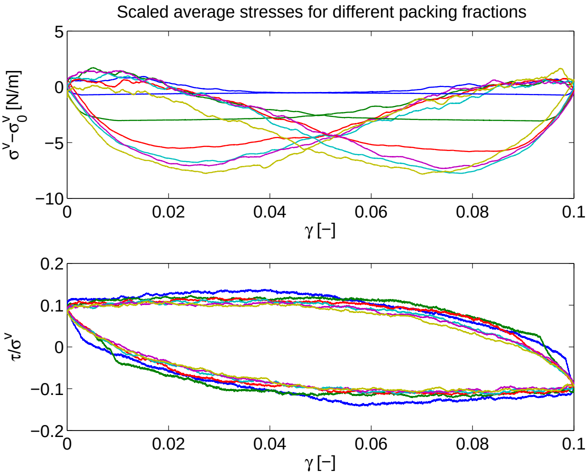
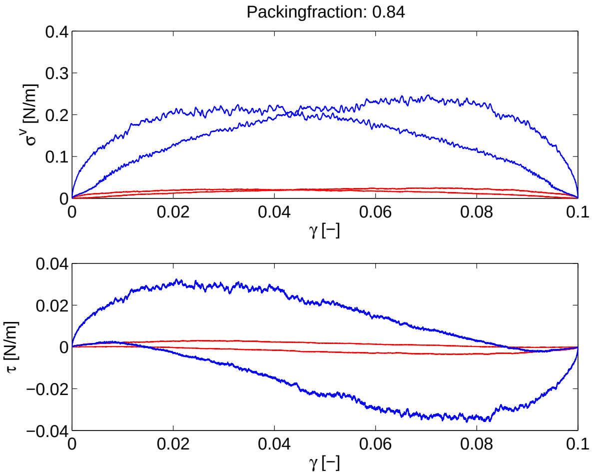
Results different packing fractions



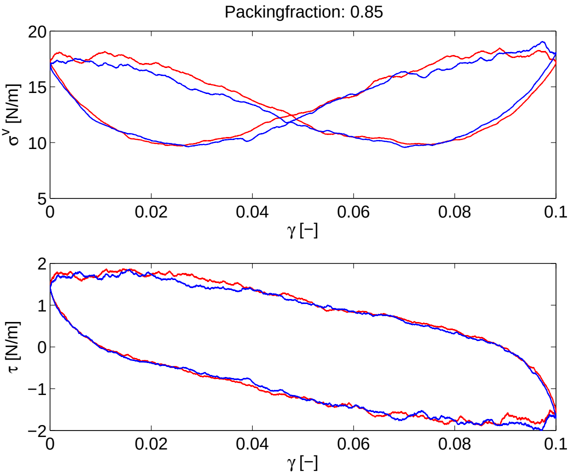
Packing fractions vary from 0.845 (blue) till 0.850 (yellow).

Rate dependency for low packing fractions



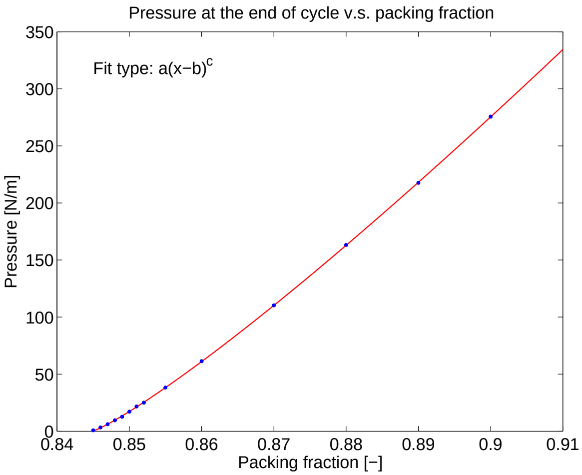
Blue line $8 \cdot 10^4\ \mathrm{t_c/cycle}$, red line $8 \cdot 10^5\ \mathrm{t_c/cycle}$

Independent on initial conditions



Blue line initial pressure 21.9 $\mathrm{N/m}$, red line initial pressure 7.1 $\mathrm{N/m}$

Pressure power law

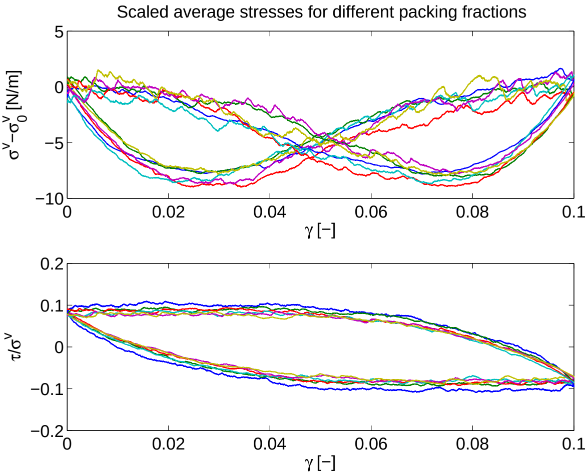


Critical packing fraction: 0.845, with exponent: 1.165

Conclusion

- Scaling works well for packing fractions of 0.848 and higher.
- For lower packing fractions results depend on speed of deformation (i.e. not quasi steady any more), so require further study
- Information about the initial state vanishes after a few cycles
- Pressures at the end of cycles nicely shows power law behaviour

Results different packing fractions



Packing fractions vary from 0.85 (blue) till 0.90 (yellow).

Model

Basic equation:

$$\begin{bmatrix} \delta\sigma^v \\ \delta\tau \end{bmatrix} = \begin{bmatrix} 2B & A \\ A & 2G \end{bmatrix} \begin{bmatrix} \delta\varepsilon^v \\ S\delta\gamma \end{bmatrix} \tag{1}$$

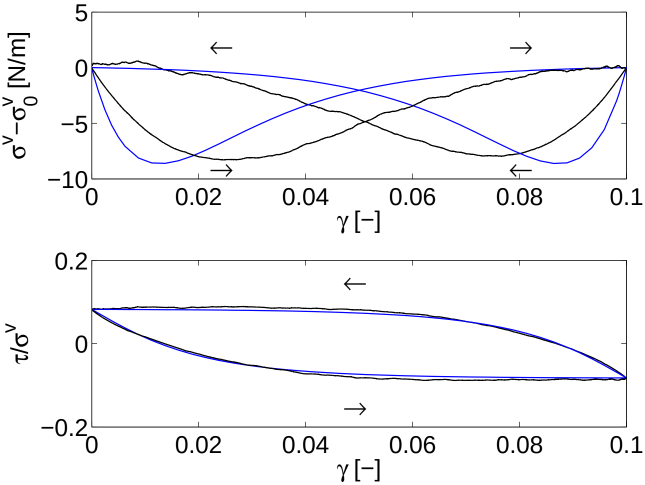
Anisotropy:

$$\frac{dA}{d\gamma} = \beta_A \left(A_{max} - \mathrm{sign}\left(\delta\gamma\right) A \right) \tag{2}$$

Stress saturation:

$$S = 1 - \frac{\tau}{\sigma^v} \frac{\mathrm{sign}\left(\delta\gamma\right)}{s^d_{max}} \tag{3}$$

Model fit



Blue line: model, black line: average results

Conclusion

- Model is able to predicts shear stresses well
- Location of minimum in pressure in not correctly predicted
- In model location of minimum in pressure is at the deformation where $\gamma = 0$, this is not true in the simulations

Model with non-constant parameters may produce better results.

Outlook

- Perform slower simulations for low packing fractions (and maybe observe shear jamming like Bob Behringer)
- Apply model with non-constant parameters
- Try different models (like the granular hydrodynamics model)
- Study influence of tangential forces
- Study influence of 3th dimension