1 APiE Excercise: 1D FEM code for bar

(Note: You are NOT required to hand in solutions to this exercise.)
Understand the various parts of the 1D Matlab code demonstrated in the lecture and carry out the following tasks.

1. Determine the analytical solution of the PDE below and compute the error between analytical solution and the Finite Element solution, plot the curve of this error w.r.t number of element in FEM.

$$\frac{d}{dx}\left(E(x)\frac{du}{dx}\right) = f, \quad x \in [0,1]$$

   and $u = 0$ at $x = 0$ and $f = x$ and assume $E(x) = E_0 = 1$.

2. Modify the 1D Matlab code provided during the lecture to accomodate a spatially varying Young's modulus of the bar such that,

$$E(x) = E_0(2 - x)$$

   Where E_0 is a constant value.

3. Check the Eigen values/shapes of the global stiffness matrix, How does it differ from the case with constant E_0?

4. Now modify the code provided such that Young's modulus is spatially constant but depends on the displacement 'u' itself, i.e. system is nonlinear,

$$E(u) = E_0(1 - du/dx)$$

   (Hint: To solve Non-linear problems you need iterative methods like Newton-Raphson)

Success!