1 APiE Excercise: FEM for Trusses

Consider a 2D truss as shown in Fig.1. The horizontal and vertical truss elements have an initial length L_0= 1m and the diagonal elements have an initial length of $\sqrt{2}L_0$. All members have E = $210GPa$, $A = 0.0031m^2$ and $\rho = 7800kg/m^3$. Constrain all the bottom nodes in vertical direction only (free along X) and constrain all the left side nodes in horizontal direction (free along Y),

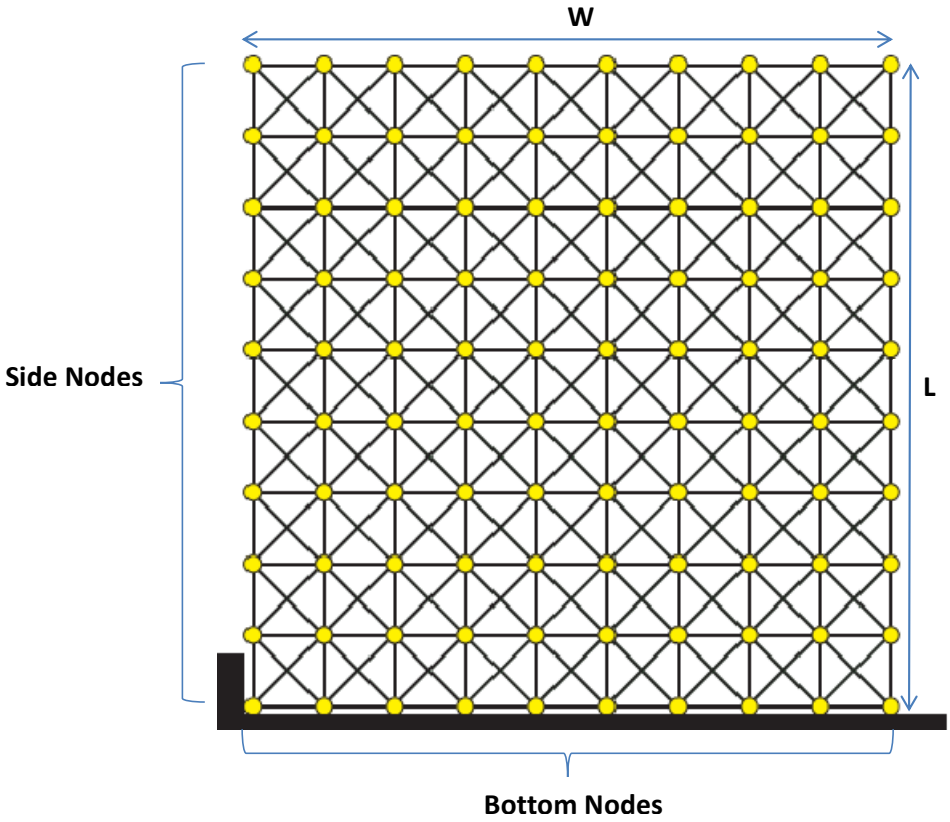

Figure 1: 2D truss

1. Apply a load 100 KN to the top right node +Y direction and use a finite element method to compute the displacement of this node. Compute the lowest 6 eigenpairs (eigenvalues and eigenvectors (also called mode shapes) of the stiffness matrix, check Matlab eig function for this). Plot the mode shapes and discuss your plots.

2. Considering the system as a solid structure, compute the Young's Modulus (E_{struc}), Shear Modulus (G_{struc}) and Poisson ratio (ν_{struc}) of the bulk structure using the following formula's ,

   (a) Apply a 10 KN vertical load (+Y direction) on all the top nodes of the structure measure the elongation (ΔL)

$$E_{struc} = \frac{LongitudinalStress}{LongitudinalStrain} = \frac{TotalLoad/d * W}{\Delta LL}$$

Where, $TotalLoad$ is sum of all the loads on the top nodes, and d is the cross-sectional diameter of the truss members (Compute using A).

(b) Also measure the change in width (ΔW) of the structure and compute Poisson ratio as,

$$\nu = \frac{Changeintransverselength}{Changeinlongitudinallength} = \frac{\Delta W}{\Delta L}$$

(c) To measure shear modulus first remove all the constraints on side nodes and fix the bottom nodes in horizontal direction as well. Now, apply a shear load (+X direction) of 10 KN on all the top nodes and compute G_{struc} as follows,

$$G_{struc} = \frac{ShearStress}{ShearStrain} = \frac{TotalLoad/d * W}{\Delta W L}$$

here, ΔW is the displacement of the top left node.

3. {Optional: For extra points only} Solve the dynamic/transient case of this problem by the finite element method. To do this formulate mass-matrix of the structure and solve $[Mglobal]\{\ddot{U}\} + [Kglobal]\{U\} = \{f\}$. Assign to the top node on the right side an initial horizontal velocity $V_0 = 0.02m/sec$ Hint: This implies $\{f\}$ is zero, only initial condition drives the system. Visualize the motion of the truss in a movie. Use Newmark scheme and lumped mass matrix approach. Compute the lowest 6 eigenpairs (Solve the generalized eigenvalue problem). Plot the mode shapes and discuss your plots.

   (a) Estimate the speed of sound (roughly) in the structure by observing the time t_s it takes for the top-displacement to become visible at the bottom. The speed can then be computed as $V_s = L/t_s$.