APiE Exercise 11 - Finite Volume (System of Equations)

Exercise 1

Part (i) and (ii) is compulsory and is required to pass the assignment. The other parts are all optional and are only required for the higher grades.

Extend your finite volume code to solve the isothermal Euler equations, which are of the form

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + \frac{\partial \boldsymbol{f}(\boldsymbol{\omega})}{\partial x} = 0,$$

with

$$\boldsymbol{\omega} = \begin{pmatrix} \rho \\ \rho u \end{pmatrix} \text{ and } \boldsymbol{f} = \begin{pmatrix} \rho u \\ \rho u^2 + a^2 \rho \end{pmatrix}$$

(i) Initially solve the equations in a unit domain with inflow on the left and outflow on right side, with uniform density $\rho = 1$ and velocity $u = 1$. Prescribe as inflow $\rho = 0.5$ and $u = 1$. Plot the evolution of the solution until it reaches steady state.

(ii) Place a partition wall at $x = 1$ and give the fluid a uniform density and zero velocity to the left of this wall, place no material to the right of the wall (you may need to place a small density to the right). Start you code at $t = 0$ with the wall already removed. Plot the evolution of the density profile with time. You should see a sharp front (shock) propagating into the empty space.

(iii) Optional: Change to solid walls at $x = 0$ and $x = 2$.

(iv) Optional: Investigate the effect of implementation a limited for case (ii).

(v) Optional: Add a Hancock predictor step, i.e. change the scheme into a Modified Total Variation Diminishing Scheme (MTVDLF). Comment of the effect of adding this step. You may want to also use the problem from the previous examples sheet, were you could write down the exact time-dependent solution (Note: The wave speeds for the Euler equation are $u \pm |a|$).

Exercise 2 (Voluntary +3 points)

ADVANCED : Use dimensional splitting to extend your code to two dimensions. Consider a square domain with all solid walls and the initial conditions

$$\rho(x, y) = \begin{cases} 0.1 & 0.5 > (x^2 + y^2) \\ 1 & 0.5 \leq (x^2 + y^2) \end{cases}$$

and $u = 0$ and $v = 0$ everywhere.

This problem is a circular explosion trapped in a square room.