From micro to macro in granular matter

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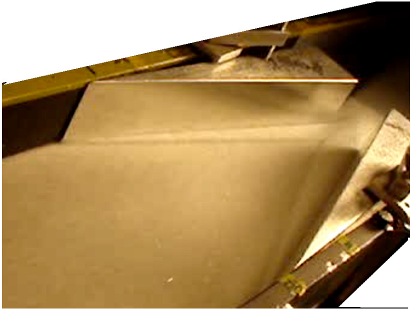
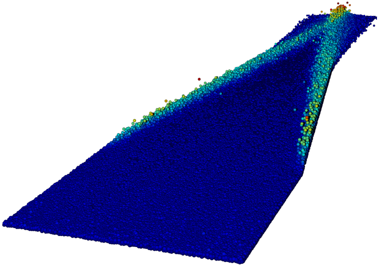
Objective: Micro-macro transition for shallow granular flow

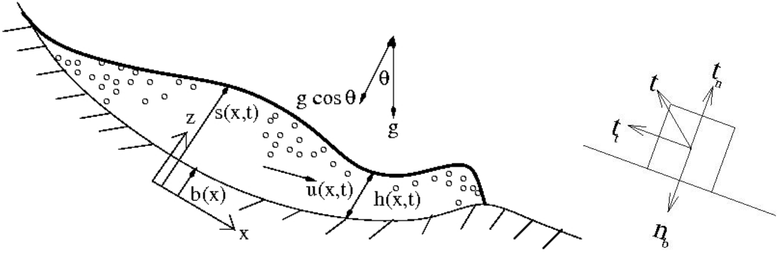




Experiment and DEM simulation of glass particles flowing through a contraction $d = 1mm$, $N \approx 400\,000$, $dt = 5\,\mu s$.

Experiments: Vreman *et al.*, *J. Fluid Mech.* **578** (2007) 233-269

The continuum model for shallow granular chute flow



Depth- and width-averaging mass- & momentum balance yields the lithostatic balance, $\sigma_{zz}(z) \approx \rho g cos\theta(h-z)$, and

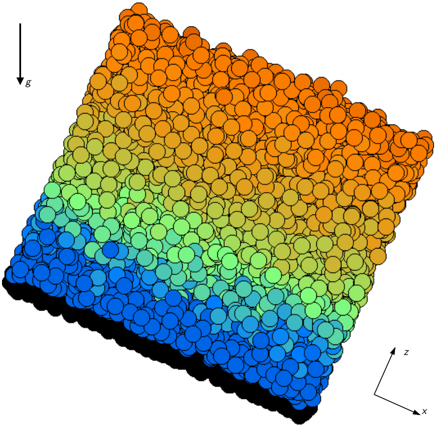
$$\frac{\partial h}{\partial t} + \frac{\partial h\bar{u}}{\partial x} = 0,$$

$$\frac{\partial}{\partial t}(h\bar{u}) + \frac{\partial}{\partial x}(h\alpha\bar{u}^2) + \frac{\partial}{\partial x}\left(\frac{K}{2}gh^2\cos\theta\right) = gh(\sin\theta - \mu\frac{\bar{u}}{|\bar{u}|}\cos\theta),$$

where closure relations are required for velocity shape factor $\alpha = \frac{\overline{u^2}}{\bar{u}^2}$, normal stress ratio $K = \frac{\bar{\sigma}_{xx}}{\bar{\sigma}_{zz}}$, and bed friction $\mu = \frac{|t_t|}{|t_n|}$ at $z = b$.

Objective

Find closure relations $\mu(h, \bar{u})$, $\alpha(h, \bar{u})$ and $K(h, \bar{u})$ using small DEM simulations of steady uniform flow.



DPM of steady uniform chute flow, periodic in x- and y-direction, fixed-particle layer at the base, unrestrained surface, inclination θ.

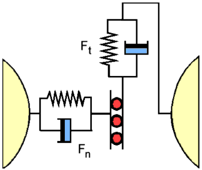
Parameters of the DPM

- scaled s.t. diameter $d = 1$, mass $m = 1$, gravity $g = 1$.
- linear elastic, dissipative and frictional contact forces:

$$\vec{f}_{ij} = f_{ij}^n \vec{n} + f_{ij}^t \vec{t},$$

$$f_{ij}^n = -k_n \delta_{ij} - \gamma_n v_{ij}^n,$$

$$f_{ij}^t = -min(|k_t \vec{\delta}_{ij}^t + \gamma_t \vec{v}_{ij}^t|, \mu_c f_{ij}^n),$$



- collision time $t_c = 5 \cdot 10^{-3} \sqrt{d/g}$, restitution $r = 0.88$, friction $\mu_c = 0.5$ ($k_n = 2 \cdot 10^5$, $\gamma_n = 25$, $k_t = 2/7 k_n$, $\gamma_t = \gamma_n$)
- integration with velocity verlet, $dt = t_c/50$ until $t = 2000$.

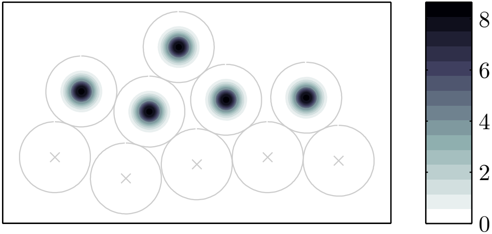
Statistics by coarse-graining

We define the macrosc. density using a coarse-graining function ϕ,

$$\rho(\vec{r}) = \sum_{i=1}^{n} m_i \phi(\vec{r} - \vec{r}_i).$$

We define the velocity field $\vec{V}$, s.t. it satisfies mass balance *exactly*,

$$\vec{V} = \vec{p}/\rho, \text{ where } \vec{p} = \sum_{i=1}^{n} m_i \vec{v}_i \phi(\vec{r} - \vec{r}_i).$$



Density ρ of a static system of 5 fixed and 5 free particles for a Gaussian coarse-graining function of width $w = d/8$.

Statistics by coarse-graining

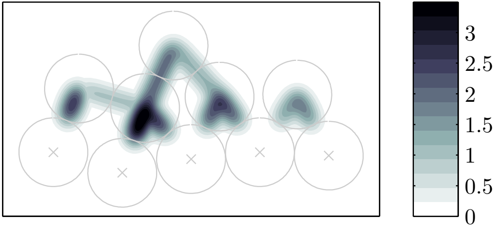
We define the stress tensor s.t. it satisfies momentum balance *exactly*. Then $\vec{\sigma} = \vec{\sigma}^c + \vec{\sigma}^k$, with contact and kinetic stress

$$\sigma^k_{\alpha\beta} = -\sum_{i=1}^{n} m_i v'_{i\alpha} v'_{i\beta} \phi(\vec{r} - \vec{r}_i),$$

$$\sigma^c_{\alpha\beta} = -\sum_{\text{contacts } \{i,j\}} f_{ij\alpha} r_{ij\beta} \int_0^1 \phi(\vec{r} - (\vec{r}_i + s\vec{r}_{ij}))\, ds$$

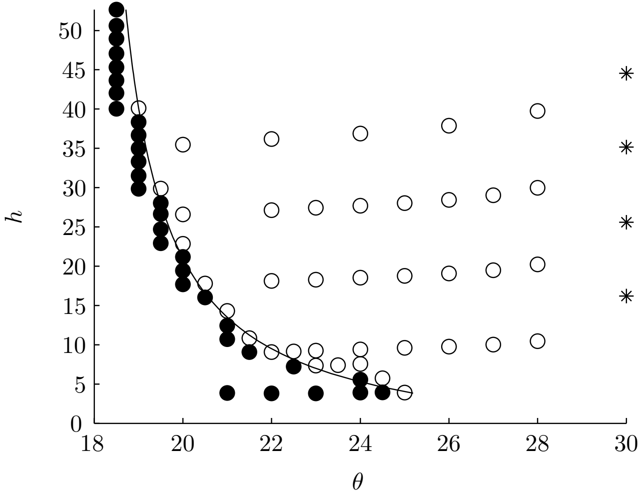
$$- \sum_{\text{wall contacts } \{i,k\}} f_{ik\alpha} a_{ik\beta} \int_0^1 \phi(\vec{r} - (\vec{r}_i + s\vec{a}_{ik}))\, ds,$$

with branch vector $\vec{a}_{ik}$ and fluctuation velocity $\vec{v}'_i = \vec{v}_i - \vec{V}$.



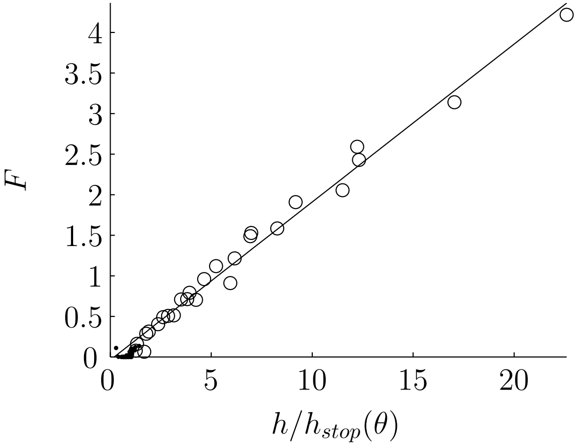
Stress norm $|\boldsymbol{\sigma}^c|_2$, $w = d/8$.

Three regimes: arresting •, steady ○, accellerating ∗



Demarkation line $h_{stop}(\theta)$ between arrested and steady flow, fitted to $h_{stop}(\theta) = Ad\dfrac{\tan(\theta_2) - \tan(\theta)}{\tan(\theta) - \tan(\theta_1)}$ (fit from Pouliquen, Forterre, 2003).

Closure for the friction $\mu = \frac{|t_t|}{|t_n|} = \tan\theta$

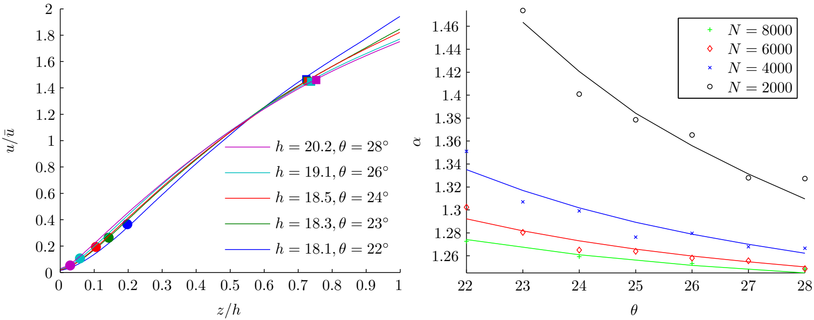


Steady flows fitted to $F = \beta \dfrac{h}{h_{stop}(\theta)} - \gamma$, where $F = \frac{\bar{u}}{\sqrt{g\cos\theta h}}$.

Substituting $\mu = \tan(\theta)$ yields

$$\mu(h, \bar{u}) = \tan(\theta_1) + (\tan(\theta_2) - \tan(\theta_1)) \left(\frac{\beta}{Ad} \frac{h}{F+\gamma} + 1 \right)^{-1}.$$

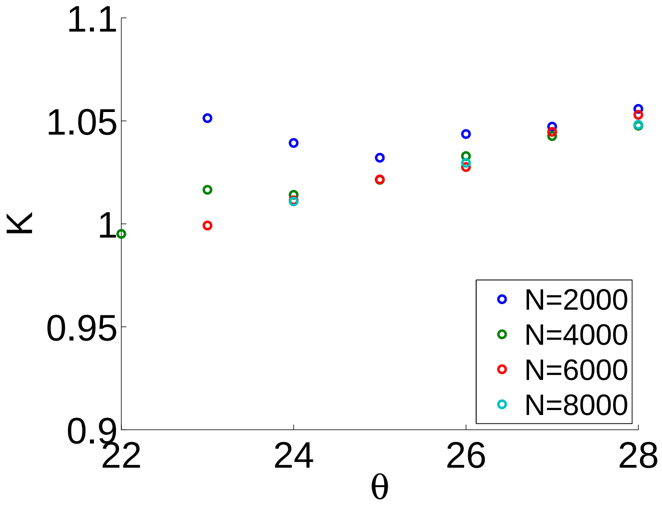
Closure for the velocity shape factor $\alpha = \frac{\overline{u^2}}{\bar{u}^2}$



Left: We see Bagnold velocity profile in bulk, quadratic near base ($z < b\, h_{stop}(\theta)$), linear near surface ($z > h - 5$).

Right: Shape factor $\alpha(h, \theta)$ from simulations (markers) and fit (lines).

Closure for the normal stress ratio $K = \frac{\bar{\sigma}_{xx}}{\bar{\sigma}_{zz}}$



The normal stress ratio $K \approx 1$.

Dependence of the basal smoothness

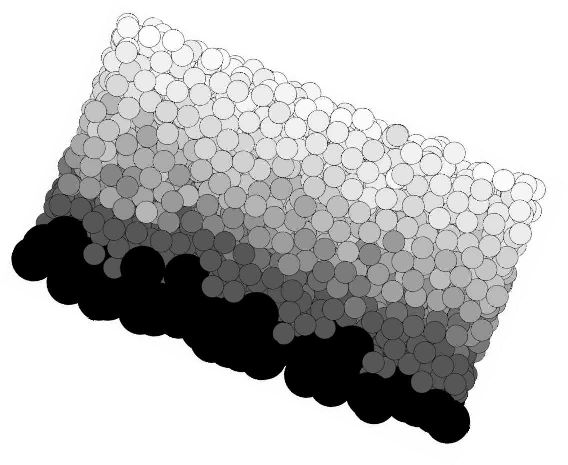


Figure: We change the base roughness by varying the base particle diameter $0 < \lambda \leq 2$. Figure shows $\lambda = 2$.

Dependence of the basal smoothness

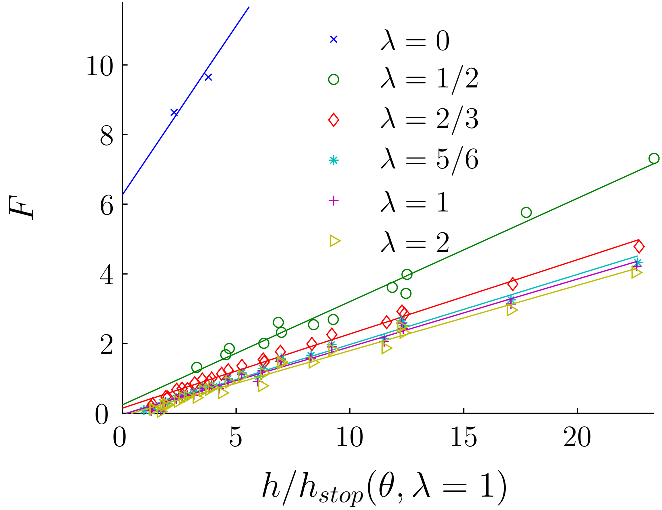


Figure: Fit of steady disordered flows to $F = \beta \frac{h}{h_{stop}(\theta; \lambda=1)} - \gamma$.
Froude number increases for smoother base.

Conclusions:

- Density, velocity and stress definition satisfies mass and momentum balance exactly, even at external boundaries.
  - W, Thornton, Luding, Bokhove, *From discrete particles to continuum fields near a boundary*, Granular Matter
- Discrete particle simulations were used to obtain closure rules

$$\mu(h, \bar{u}),\ \alpha(h, \bar{u}),\ K \approx 1.$$

Micro-macro transition: Closure depends on the particle and contact properties, f.e. bed roughness λ, bed friction μ_b:

  - W, Thornton, Luding, Bokhove, *Closure Relations for Shallow Granular Flows from Particle Simulations*, Granular Matter, submitted,
  - Thornton, W, Luding, Bokhove, *Frictional dependence of shallow granular flows from discrete particle simulations*, EPL, submitted.

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Future work:

- Apply the new stress definition to bi-dispersed flows (segregation effects).
- Validate the closures for nonuniform flow (contraction, obstacles)