APiE Exercise 10 - Finite Volume Methods (Scalar Eqn's)

Part (i) is compulsory and is required to pass the assignment. The other parts are all optional and are only required for the higher grades. It should be noted that part $(ii)-(iv)$ are independent i.e. it is possible to complete (i) and (iii) or (i) and (iv) only etc...

(i) Write a simple finite volume code to solve the one dimensional linear advection equation (without limiters)

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial x} = 0.$$

Implement inflow on the left and outflow conditions on the right. Choose a numerical flux. Consider both

(a) square wave initial conditions

$$\rho(x) = \begin{cases} 0 & 0 \leq x \leq 1 \\ 1 & 1 < x \leq 3 \\ 0 & 3 < x \leq 10 \end{cases}$$

and (b) triangular wave initial conditions

$$\rho(x) = \begin{cases} x & 0 \leq x \leq 1 \\ (2-x) & 1 < x \leq 2 \\ 0 & 2 < x \leq 10 \end{cases}$$

In the report please show the time evolution of the solution and explain how you implemented the boundary conditions.

(ii) Optional: Repeat question(i) for the one dimensional inviscid burgers equation

$$\frac{\partial \rho}{\partial t} + \frac{1}{2}\frac{\partial \rho^2}{\partial x} = 0.$$

(iii) Optional: Investigate the effect of adding a limiter, try minmod, superbee and Woodward. Comment on which gives the best results for each problem.

(iv) **Voluntary +1 points**: For the problem in (i) find the exact solution. For different grid resolutions consider the difference between the exact solution and your numerical solution. Hence comment on the error in your numerical method as a function of the grid size.