Modeling Dynamics of Flexible Multi-body Systems for Control

An Application Example

“Mechatronic design of mechanism with flexible joints”

Tilting Mirror

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Overview

- **Problem background**
- **Mechatronic design procedure for example**
  - **Conceptual design procedure**
  - **Controller design**
- **Parasitic dynamics and robust stability**
  - **Influence of manufacturing tolerances on stability**
  - **Using the outlined FEM-theory and** SPACAR

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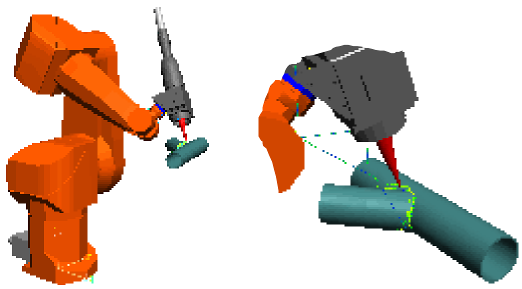
Problem background



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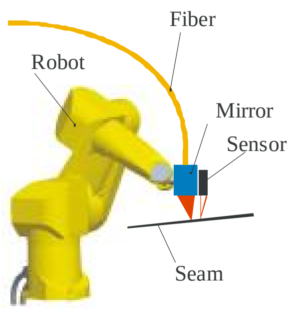
2-axis tilting mirror

- **Required accuracy: 0.1 mm at 250 mm/sec**
- **Problem of industrial robots is in corners of the seam: vibrations 1 mm amplitude at 11-18 Hz.**



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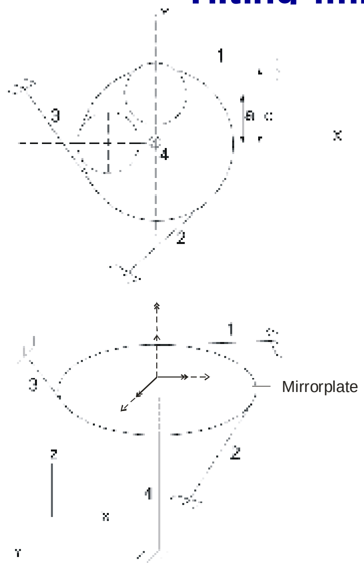
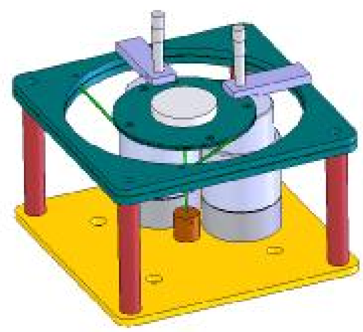
Tilting mirror concept





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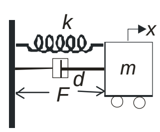
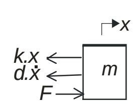
Control

- **Nominal Model of electro-mechanical system**



(a)



(b)

$$F = \frac{U \cdot k_m}{R}$$

$$d = \frac{k_m^2}{R}$$

$$\frac{x(s)}{U(s)} = \frac{\frac{k_m}{R \cdot m}}{s^2 + \frac{k_m^2}{R \cdot m} s + \frac{k}{m}}$$

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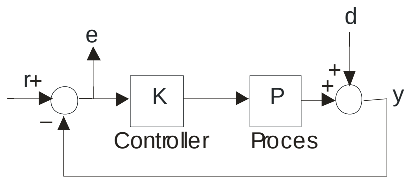
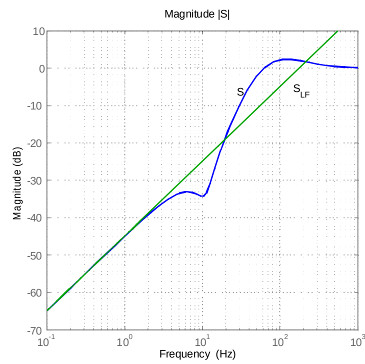
Specifications





$$\frac{e(s)}{d(s)} = \frac{-1}{1+K(s)P(s)} = S(s)$$

$$\left|S(w_d = j2p \cdot 18)\right| = \frac{1}{10} = -20\text{dB}$$

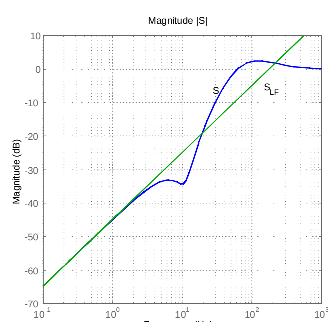
Required cross-over frequency

$$P(s) = \frac{\dfrac{k_m}{R \cdot m}}{s^2 + \dfrac{k_m^2}{R \cdot m} + \dfrac{k}{m}} \qquad K(s) = k_p \frac{(st_z+1)(st_i+1)}{st_i(st_p+1)}$$

$$S(s) = \frac{st_i(st_p+1)(s^2 + \frac{k_m}{m_{eq}}s + w_1^2)}{st_i(st_p+1)(s^2 + \frac{k_m}{m_{eq}}s + w_1^2) + \frac{k_p}{m_{eq}}(st_z+1)(st_i+1)}$$

$$S_{LF}(s) = \frac{st_i w_1^2}{\dfrac{k_p}{m_{eq}}} = \frac{2w_1^2}{w_c^3 a} jw_d = \frac{1}{10} \rightarrow$$

$$w_c = \sqrt[3]{\frac{10 \cdot 2w_1^2 w_d}{a}}$$



PID$^+$ Controller parameters

- α=0.2
- Cross-over frequency 60 Hz.
- $K(s)=\begin{bmatrix}\text{PID}^+ & 0\\ 0 & \text{PID}^+\end{bmatrix}$

Relation between cross-over and frequency of destabilizing vibration modes

$\bar{s}\left(Q\tilde{\Delta}_a\right)<1$

$\bar{s}\left(Q\right)<1$

Structure of perturbation known:

$m\left(Q\right)<1$

We rely on: $\bar{s}\left(Q\right)<1$

$\bar{s}\left(W_a\left(I+G_0K\right)^{-1}K\right)<1$

$\bar{s}\left(W_a\right)<\bar{s}\left(K\right)^{-1}\quad\forall w>w_c$

$\bar{s}\left(W_a\right)\approx\bar{s}\left(G\right)$

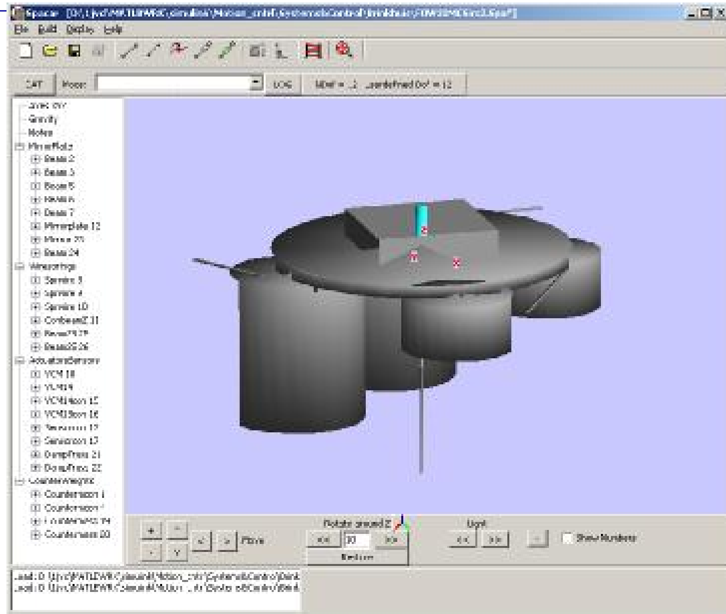
$\bar{s}\left(G\right)<\bar{s}\left(K\right)^{-1}$

$G<(K)^{-1}$

$\left|G\left(jw_r\right)\right|=\left|\dfrac{1}{m_{eq}2zw_r^2}\right|$

$\left|K_{HF}\left(jw_r\right)\right|=\left|\dfrac{k_p}{s\cdot a^2\cdot t_z}\right|=\left|\dfrac{m_{eq}w_c^3}{s\cdot a}\right|$

$w_r>w_c\sqrt[3]{\dfrac{1}{2za}}$



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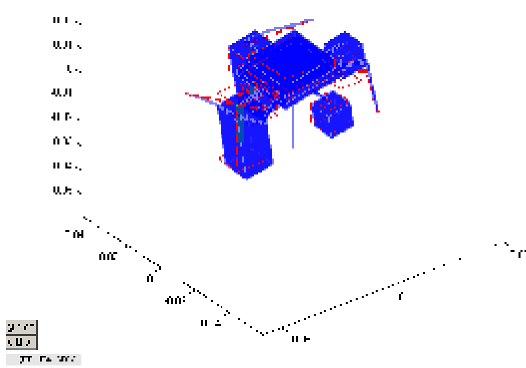
Analyses:Mode-shapes



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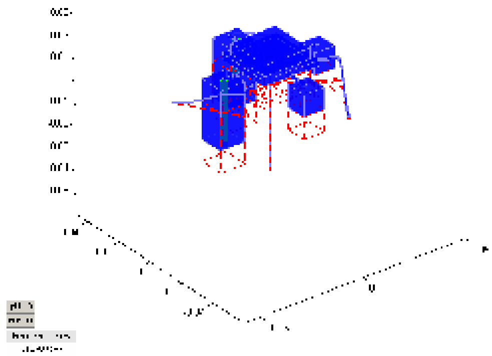
Analyses:Mode-shapes



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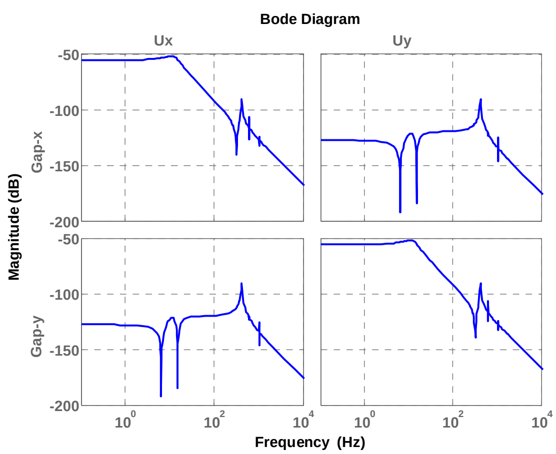
Bodemagnitude plot



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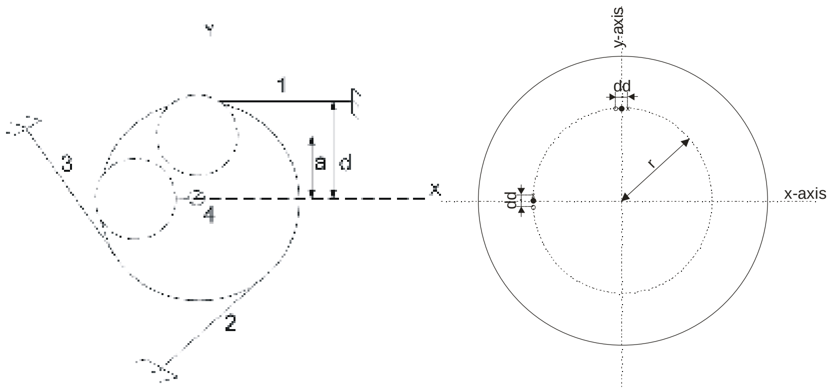
Misplacement of actuators and sensors



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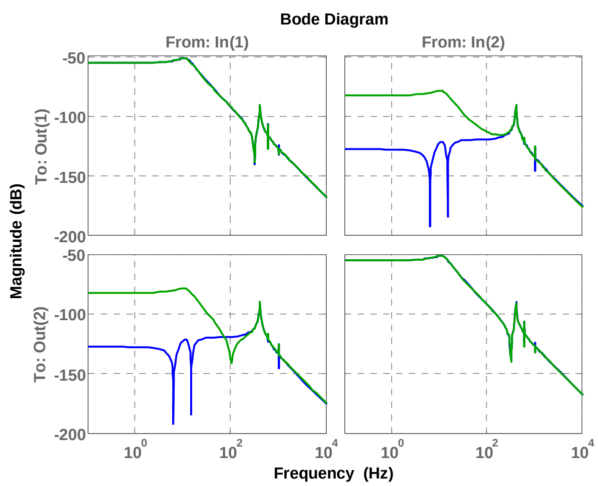
Bode-magnitude plot manufacturing errors



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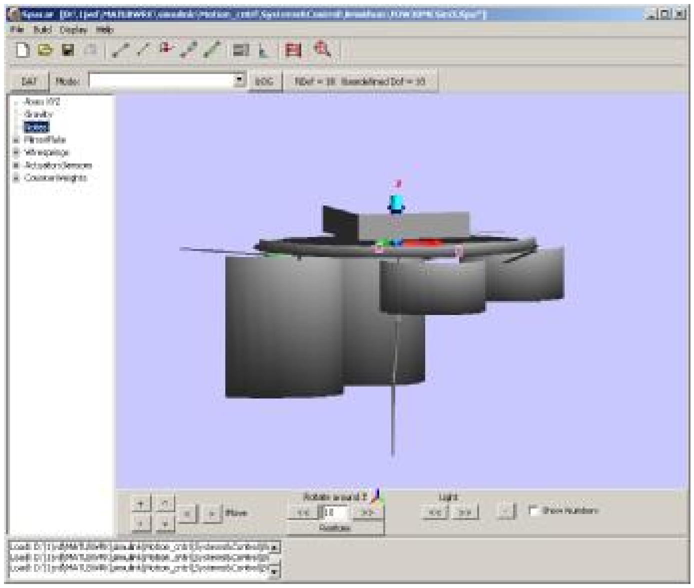

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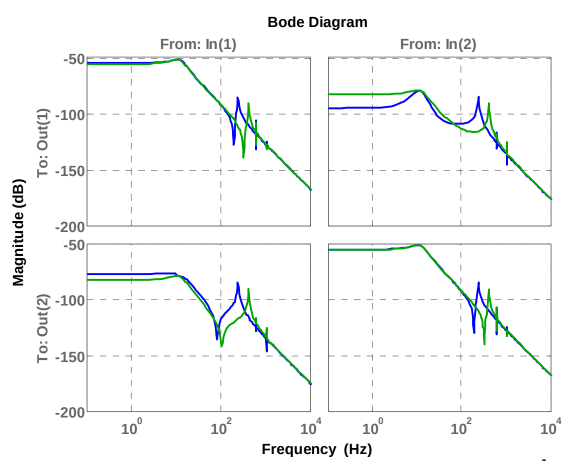
Bode-magnitude plot z-wirespring misplacement



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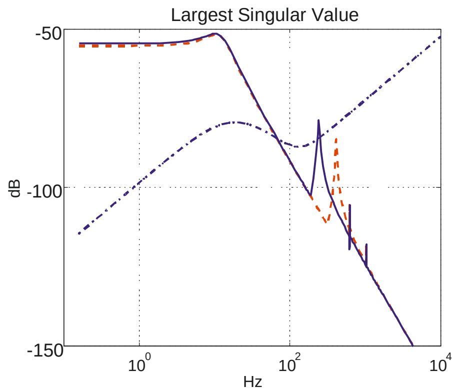
Largest singular value plot



Solid line: z-wirespring misplaced
Dashed line: with manufacturing errors
Dash-dot: $K(s)^{-1}$

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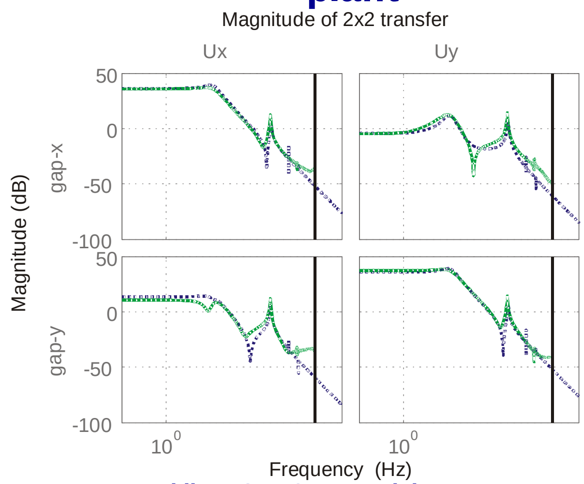
Comparison Identified plant and modeled plant



Dotted line: SPACAR model
Solid line: identified plant

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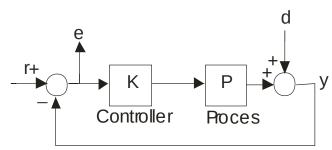
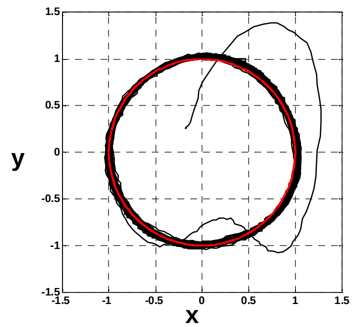
Experimental results

- **Changed to servo-problem**



- $\mathbf{r=[r_x,r_y]^T}$
- $\mathbf{r_x= a.sin(18.2\pi.t)}$
- $\mathbf{r_y= a.cos(18.2\pi.t)}$



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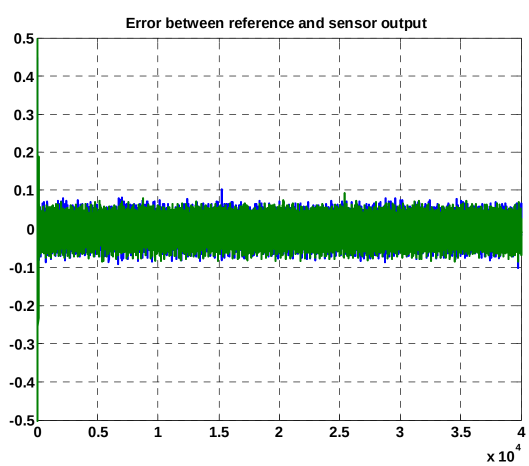
Experimental results



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Conclusions

- **Small gain theorem links cross-over frequency and frequency of first stability endangering deformation mode**
- **To check this, input-output models or necessary**
- **Have a tool at hand to model quickly 3D flexible multi-body systems for control purposes**

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