Modeling Dynamics of Flexible Multi-body Systems for Control

An Application Example

"Mechatronic design of mechanism with flexible joints"

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Overview

- **Aspects of mechatronic design**
  - **Exact constraint design (SPACAR)**
  - **Influence of parasitic dynamics**
- **Problem background Active Encoder head**
- **Mechatronic design procedure for example**
  - **Conceptual design procedure**
  - **Controller design**
- **Parasitic dynamics and robust stability**
  - **Influence of manufacturing tolerances on stability**
  - **Co-location in 3D space ?**
  - **Using the outlined FEM-theory and SPACAR**

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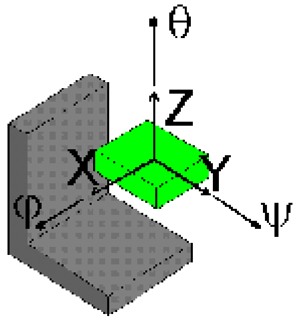
Aspects of Mechatronic design
Exact constraint design

- **A free body in 3D space has 6 degrees of freedom**



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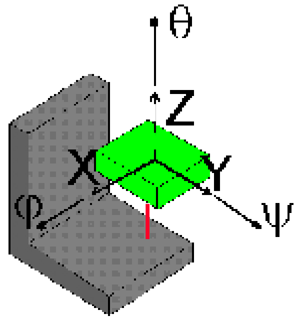
Exact constraint design

- **Wire spring application: 1 DOF is suppressed**



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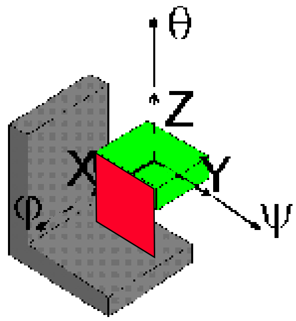
Exact constraint design

- **Leaf spring application 3 DOF's are suppressed**



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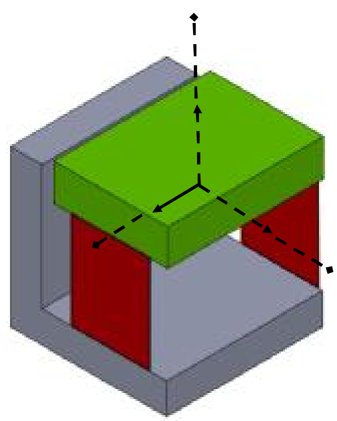
Exact constraint design



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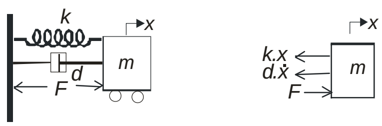
Control

- **Nominal Model of electro-mechanical system**



(a) (b)

$$F = \frac{U \cdot k_m}{R}$$

$$d = \frac{k_m^2}{R}$$

$$\frac{x(s)}{U(s)} = \frac{\frac{k_m}{R \cdot m}}{s^2 + \frac{k_m^2}{R \cdot m} s + \frac{k}{m}}$$

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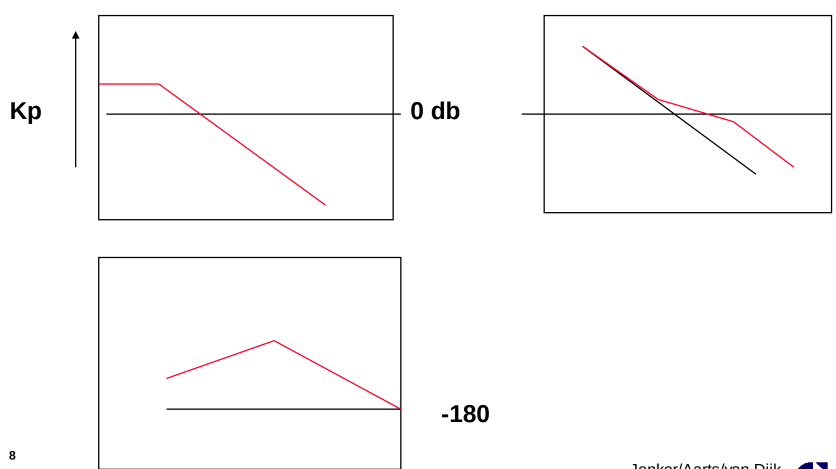
Controller



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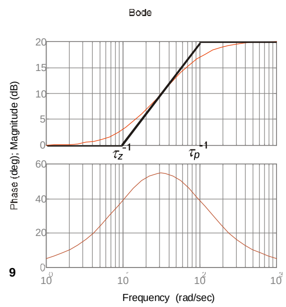
PD

$$K(s) = k_p \frac{st_z + 1}{st_p + 1}$$

•Put max phase lead at desired cross-over frequency



Observations:
Max phase is logarithmic half way between location-zero and location-pole.

$$w_{ph\max} = \log^{-1}\left(\frac{\log\frac{1}{t_z} + \log\frac{1}{t_p}}{2}\right) = \sqrt{\frac{1}{t_z \cdot t_p}}$$

Gain of PD-controller at w_{phmax}=

$$|K|_{ph\max} = k_p\left(\sqrt{\frac{t_z}{t_p}}\right) = k_p\left(\sqrt{\frac{1}{a}}\right)$$

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PD

$$w_c = \sqrt{\frac{1}{t_z \cdot t_p}}$$

$$t_p = a \cdot t_z \rightarrow$$

$$= \frac{1}{t_z} \cdot \sqrt{\frac{1}{a}} \rightarrow$$

$$t_z = \frac{\sqrt{\frac{1}{a}}}{w_c}$$

$$t_p = \frac{1}{\sqrt{\frac{1}{a}} w_c}$$

$$\|K \cdot G\|_{w_c} = \left\|\frac{K(jw_c)}{m_{eq} w_c^2}\right\| = \frac{k_p \cdot \sqrt{\frac{1}{a}}}{m_{eq} w_c^2} = 1 \rightarrow$$

$$k_p = \frac{m_{eq} w_c^2}{\sqrt{\frac{1}{a}}}$$

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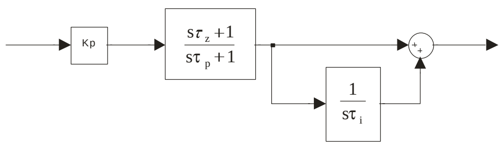
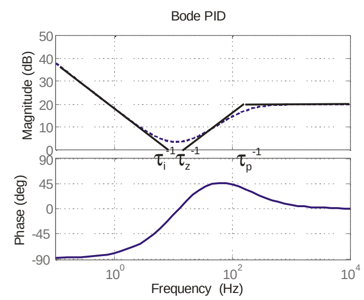
PID



$$K(s)=k_p\frac{(st_z+1)}{(st_p+1)}\left(1+\frac{1}{st_i}\right)=k_p\frac{(st_z+1)(st_i+1)}{st_i(st_p+1)}$$

$$K(s)=k_p\frac{(st_z+1)(st_i+1)}{st_i\left(s^2t_p^2+2zt_p s+1\right)}$$



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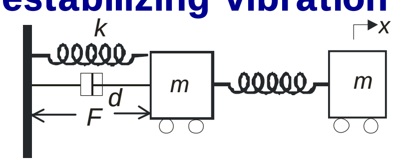
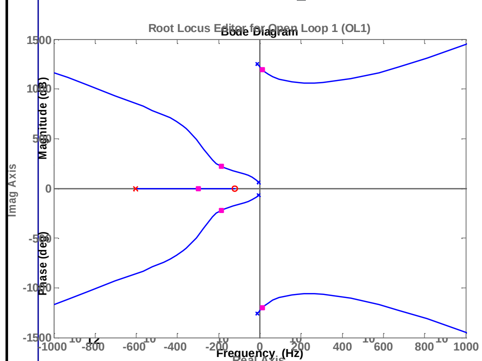
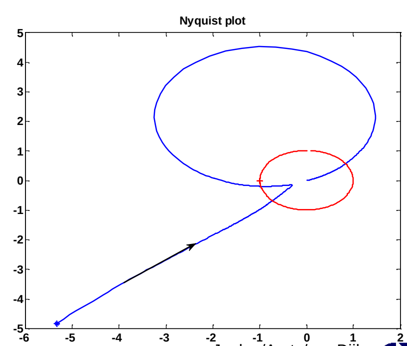
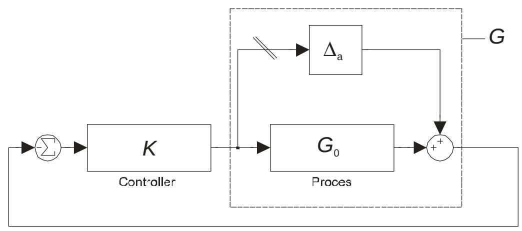
Relation between cross-over and frequency of destabilizing vibration modes







Relation between cross-over and frequency of destabilizing vibration modes



$$G(s) = G_0(s) + \Delta_a(s)$$
$$\Delta_a(s) = W_a \tilde{\Delta}_a(s)$$
$$\left\| \tilde{\Delta}_a(s) \right\|_\infty \leq 1$$

$$Q = -W_a (I + G_0 K)^{-1} K$$
$$r\left(Q\tilde{\Delta}_a\right) \leq \bar{s}\left(Q\tilde{\Delta}_a\right)$$

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Relation between cross-over and frequency of destabilizing vibration modes

$$\bar{s}\left(Q\tilde{\Delta}_a\right) < 1$$
$$\bar{s}\left(Q\right) < 1$$

Structure of perturbation known:

$$m\left(Q\right) < 1$$

We rely on: $\bar{s}\left(Q\right) < 1$

$$\bar{s}\left(W_a (I + G_0 K)^{-1} K\right) < 1$$
$$\bar{s}\left(W_a\right) < \bar{s}(K)^{-1} \quad \forall w > w_c$$
$$\bar{s}\left(W_a\right) \approx \bar{s}\left(G\right)$$
$$\bar{s}\left(G\right) < \bar{s}(K)^{-1}$$

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Problem Background

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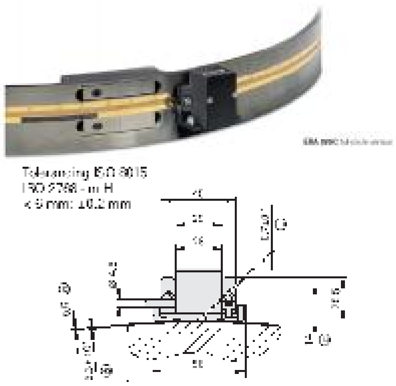
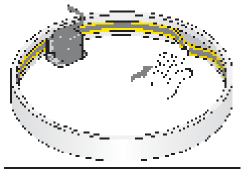
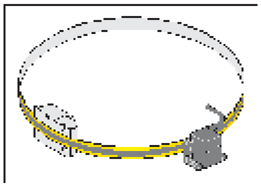
Encoding







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Specifications

- **High angular resolution (< 0.03 mrad) requires:**

  Distance-variation between encoder-strip and -head <0.2 mm

- **Disturbance at 1Hz 0.3 mm due to eccentricity**

- **Disturbance of 0.4 mm at 10 Hz due to deformation mode**

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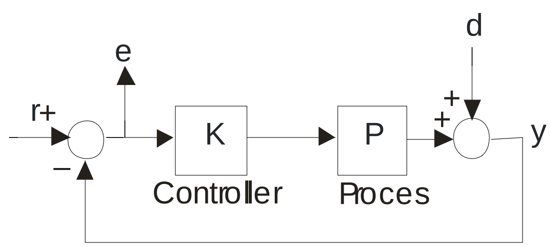
Specifications



$$\frac{e(s)}{d(s)} = \frac{-1}{1+K(s)P(s)} = S(s)$$

$$\left|S(j2p \cdot 10)\right| = \frac{1}{7}$$

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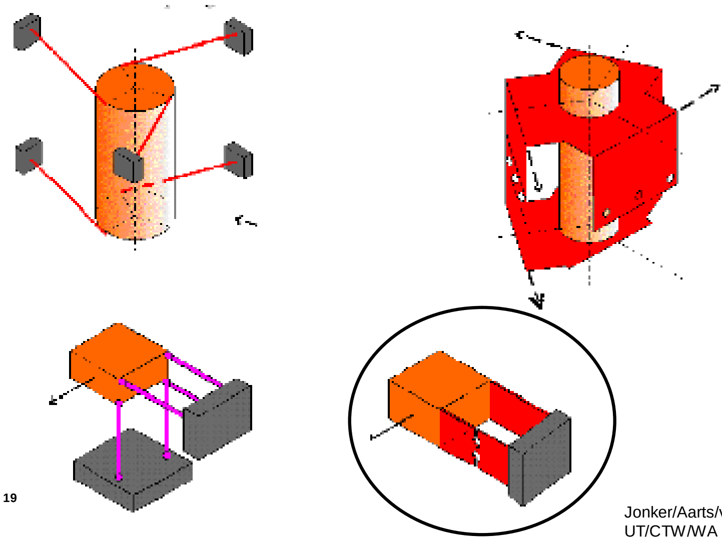
Concepts



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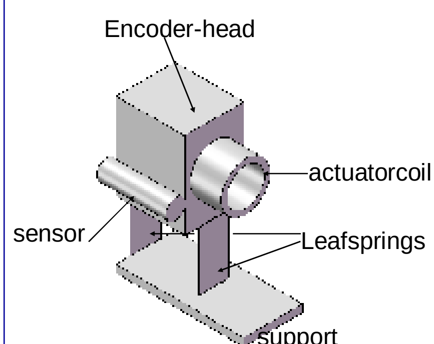
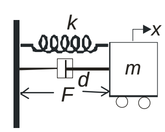
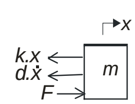
Nominal Model





(a)



(b)

$$F = \frac{U \cdot k_m}{R}$$

$$d = \frac{k_m^2}{R}$$

$$\frac{x(s)}{U(s)} = \frac{\frac{k_m}{R \cdot m}}{s^2 + \frac{k_m^2}{R \cdot m} + \frac{k}{m}}$$

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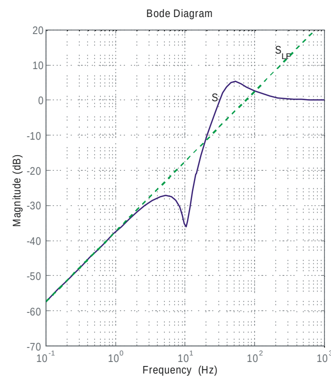
Required cross-over frequency

$$P(s) = \frac{\frac{k_m}{R\cdot m}}{s^2 + \frac{k_m^2}{R\cdot m} + \frac{k}{m}} \qquad K(s) = k_p \frac{(st_z + 1)(st_i + 1)}{st_i(st_p + 1)}$$

$$S(s) = \frac{st_i(st_p+1)(s^2 + \frac{k_m}{m_{eq}}s + w_1^2)}{st_i(st_p+1)(s^2 + \frac{k_m}{m_{eq}}s + w_1^2) + \frac{k_p}{m_{eq}}(st_z+1)(st_i+1)} \rightarrow$$

$$S_{LF}(s) = \frac{st_i w_1^2}{\frac{k_p}{m_{eq}}} = \frac{2w_1^2}{w_c^3 a} jw_d = \frac{1}{7} \rightarrow$$

$$w_c = \sqrt[3]{\frac{7\cdot 2w_1^2 w_d}{a}}$$



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Leaf-springs, actuator en sensor

length 0.030 m
width 0.015 m
d (thickn.) 0.0002 m
k 1866 N/m
ω_1 65 rad/sec.
ω_c 270 rad/sec.
α=0.2

Type	VM 2618	
Length	18	mm
diameter	26	mm
K_m	3.2	N/A
R	9.6	Ohm
Stroke	4	mm
Moving mass	0.006	kg.
Stator mass	0.1	kg.
Cont. current	1	A
stator-rotor play	0.2	mm

IWRM 08U9501 stroke 2mm res. 5 micron

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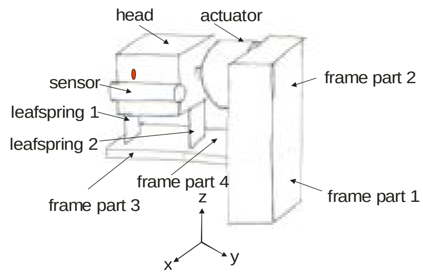
Excitation of Parasitic dynamics
actuation not in c.o.m.
sensor not in c.o.m.

- **Optics in c.o.m., sensor at x=-20 mm,z=-8mm**
- **Actuator not in c.o.m of combination**
- **Flexible frame connecting head to base**
- **Modeling goal**
  - **analyze influence on robust stability**



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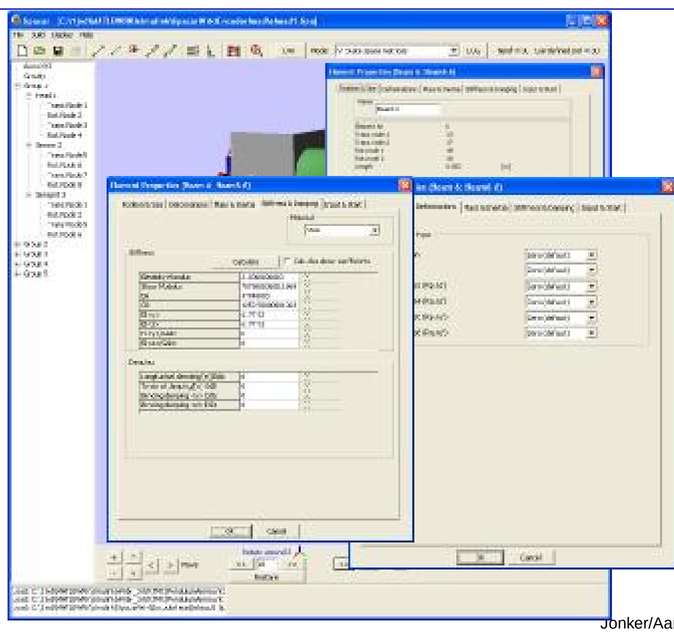

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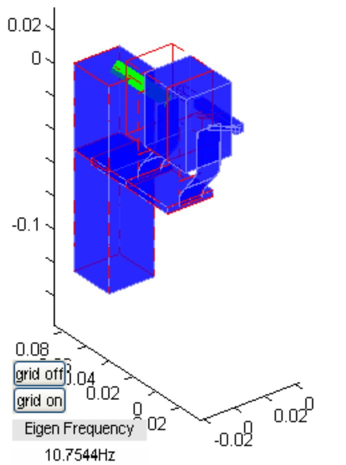
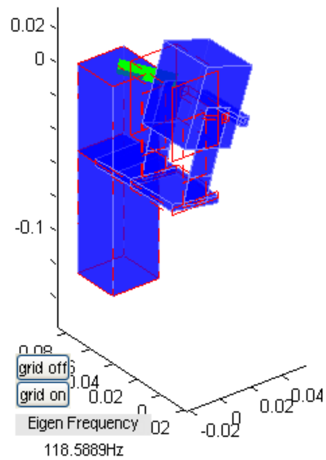
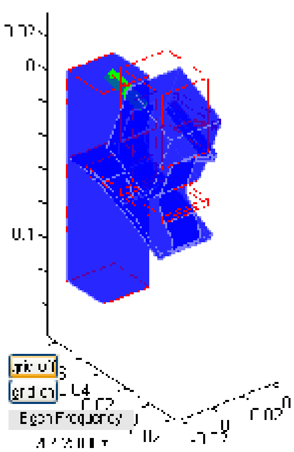
Resultaten
Mode-shapes

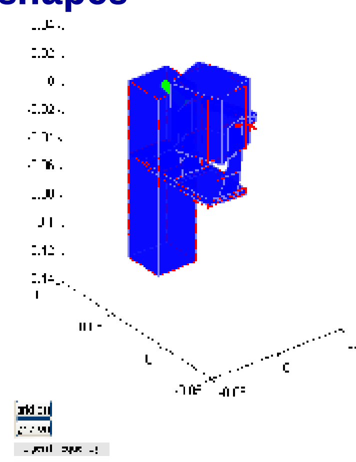




ı Dijk

Resultaten
Mode-shapes





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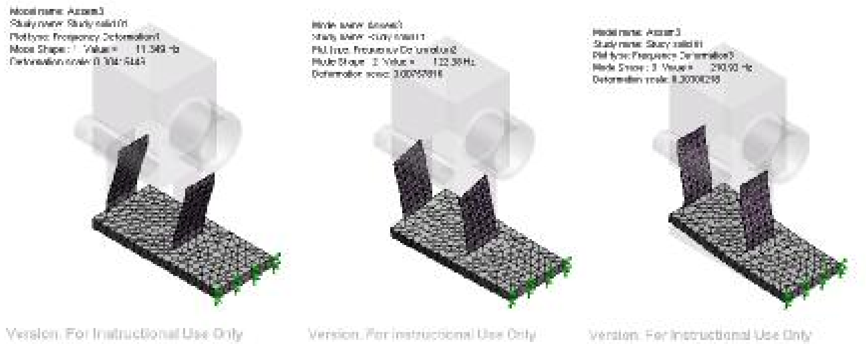
Modelvorming met CosmosWorks



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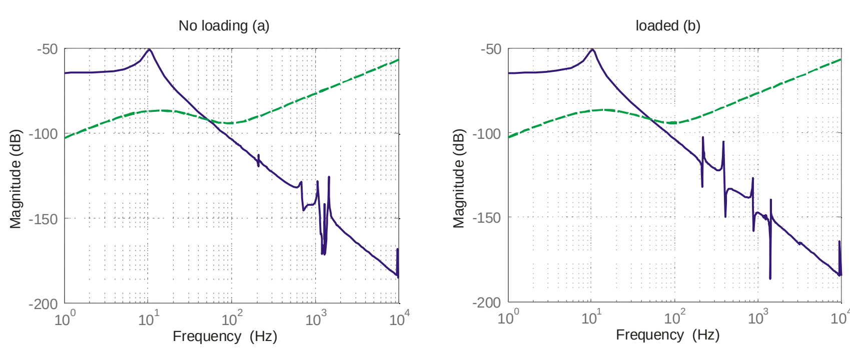
Small gain test results



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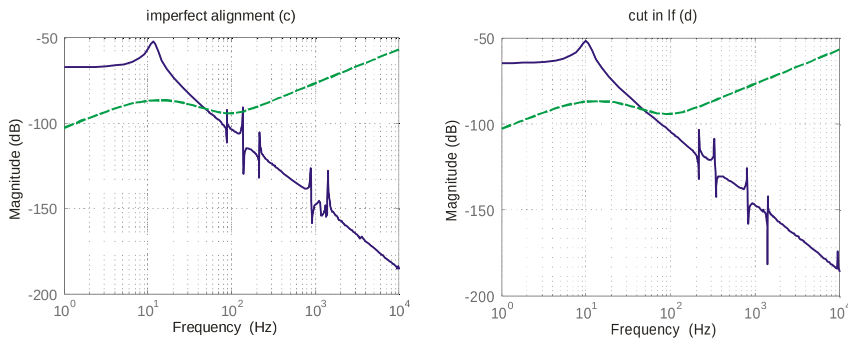

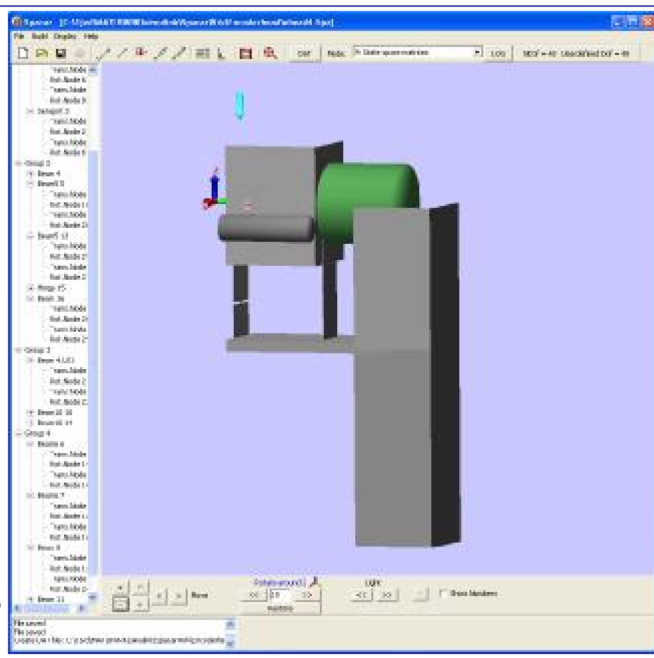

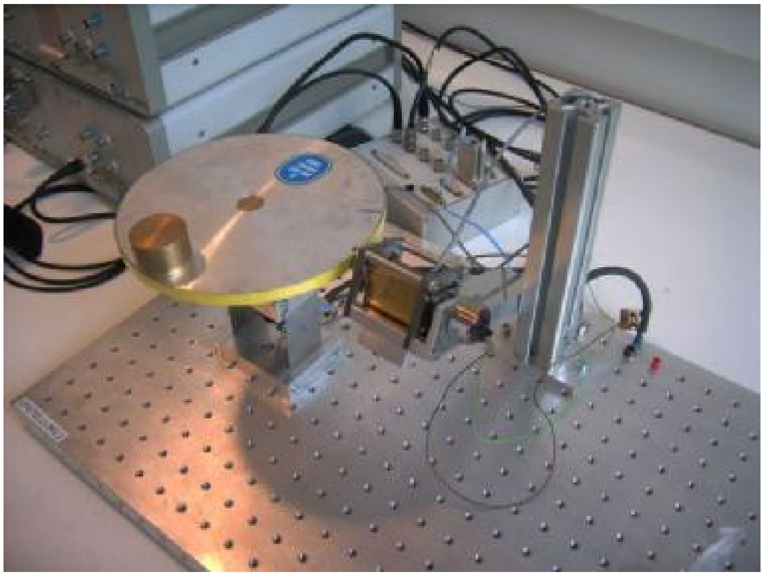

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Conclusions

- Be aware of co-location lost in 3D systems. Due to limited manufacturing tolerances
- Flexible frames something to worry about
- Have a tool at hand to model quickly 3D flexible multi-body systems for control purposes

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