Modeling Dynamics of Flexible Multi-body Systems for Control

An Application Example
“Mechatronic design of mechanism with flexible joints”

Overview

• Aspects of mechatronic design
  – Exact constraint design (SPACAR)
  – Influence of parasitic dynamics
• Problem background Active Encoder head
• Mechatronic design procedure for example
  – Conceptual design procedure
  – Controller design
• Parasitic dynamics and robust stability
  – Influence of manufacturing tolerances on stability
  – Co-location in 3D space?
  – Using the outlined FEM-theory and SPACAR
Aspects of Mechatronic design
Exact constraint design

- A free body in 3D space has 6 degrees of freedom

Exact constraint design

- Wire spring application: 1 DOF is suppressed
Exact constraint design

• Leaf spring application 3 DOF’s are suppressed
Control

• Nominal Model of electro-mechanical system

\[ F = \frac{U \cdot k_m}{R} \]
\[ d = \frac{k_m^2}{R} \]
\[ \alpha(s) = \frac{k_m}{R \cdot m} \frac{1}{s^2 + \frac{k_m^2}{R \cdot m} \cdot s + \frac{k}{m}} \]

Controller

Kp

0 db

-180
\[ K(s) = k_p \frac{s\tau_c + 1}{s\tau_p + 1} \]

**PD**

Observe:
Max phase is logarithmic half way between location-zero and location-pole.

\[ \omega_{\text{ph max}} = \log \left( \frac{1}{\tau_c} + \log \frac{1}{\tau_p} \frac{1}{2} \right) = \frac{1}{\sqrt{\tau_c \tau_p}} \]

Gain of PD-controller at \( \omega_{\text{ph max}} \):

\[ |K|_{\text{ph max}} = k_p \left( \frac{\tau_c}{\sqrt{\tau_p}} \right) = k_p \left( \frac{1}{\sqrt{\alpha}} \right) \]

**Observations:**
- Put max phase lead at desired cross-over frequency

\[ \omega_c = \frac{1}{\sqrt{\tau_c \cdot \tau_p}} \]

\[ \tau_p = \alpha \cdot \tau_c \Rightarrow \frac{1}{\tau_c} \cdot \frac{1}{\sqrt{\alpha}} \]

\[ \tau_c = \frac{\sqrt{1}{\omega_c}}{\omega_c} \]

\[ \tau_p = \frac{1}{\sqrt{\alpha} \cdot \omega_c} \]

\[ \| K \cdot G \|_{\omega_c} = \frac{K(j\omega_c)}{m_{eq} \omega_c^2} = \frac{k_p}{m_{eq} \omega_c^2} = 1 \Rightarrow \]

\[ k_p = m_{eq} \omega_c^2 \left( \frac{1}{\sqrt{\alpha}} \right) \]
**Relation between cross-over and frequency of destabilizing vibration modes**

\[
K(s) = K_p \left( \frac{s \tau_i + 1}{s \tau_p + 1} \right) \left( \frac{1}{s \tau_z} \right) = K_p \left( \frac{s \tau_i + 1}{s \tau_p + 1} \right) \frac{(s \tau_z + 1)}{s \tau_z (s^2 + 2\zeta \tau_p s + 1)}
\]

**Nyquist plot**

**Root Locus plot**
Relation between cross-over and frequency of destabilizing vibration modes

\[ G(s) = G_0(s) + \Delta_a(s) \]
\[ \Delta_a(s) = W_a \Delta_a(s) \]
\[ \| \Delta_a(s) \| \leq 1 \]

\[ Q = -W_a (I + G_0 K)^{-1} K \]
\[ \rho(Q \Delta_a) \leq \sigma(Q \Delta_a) \]

Relation between cross-over and frequency of destabilizing vibration modes

Structure of perturbation known:

\[ \sigma(Q \Delta_a) < 1 \]
\[ \sigma(Q) < 1 \]

We rely on:

\[ \sigma(W_a (I + G_0 K)^{-1} K) < 1 \]
\[ \sigma(W_a) < \sigma(K)^{-1} \quad \forall \omega > \omega_c \]
\[ \sigma(W_a) = \sigma(G) \]
\[ \sigma(G) < \sigma(K)^{-1} \]
Problem Background

Encoding
Specifications

- High angular resolution (< 0.03 mrad) requires:
  Distance-variation between encoder-strip and -head < 0.2 mm
- Disturbance at 1 Hz 0.3 mm due to eccentricity
- Disturbance of 0.4 mm at 10 Hz due to deformation mode

\[
\begin{align*}
\frac{e(s)}{d(s)} &= \frac{-1}{1 + K(s)P(s)} = S(s) \\
|S(j2\pi \cdot 10)| &= \frac{1}{7}
\end{align*}
\]
Concepts

Nominal Model

\[ F = \frac{U \cdot k_m}{R} \]
\[ d = \frac{k^2}{R} \]
\[ x(s) = \frac{k_m}{R \cdot m} \cdot \frac{k}{s^2 + \frac{k^2}{R \cdot m} + \frac{k}{m}} \]
Required cross-over frequency

\[ P(s) = \frac{k_m}{s^2 + \frac{k_m}{R \cdot m} s + \frac{k}{R \cdot m}} \]

\[ K(s) = \frac{k_p}{s} \frac{(s \tau_c + 1)(s \tau_p + 1)}{s \tau_p + 1} \]

\[ S(s) = \frac{s \tau_c (s \tau_p + 1)(s^2 + \frac{k}{m \omega_n^2} s + \omega_n^2)}{s \tau_c (s \tau_p + 1)(s^2 + \frac{k}{m \omega_n^2} s + \omega_n^2) + s \tau_p (s \tau_p + 1)(s \tau_c + 1)(s \tau_p + 1)} \]

\[ S_{sf}(s) = \frac{s \tau_c \omega_n^2}{k_p} \frac{2 \omega_n^2}{\omega_n^2} \frac{1}{\omega_n^2} \]

\[ \omega_n = \sqrt{\frac{7 \cdot 2 \alpha \omega_c}{\alpha}} \]

Leaf-springs, actuator en sensor

length 0.030 m
width 0.015 m
d (thickn.) 0.0002 m
k 1866 N/m
\( \omega_c \) 65 rad/sec.
\( \omega_p \) 270 rad/sec.
\( \alpha = 0.2 \)

<table>
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<tr>
<th>Type</th>
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<tr>
<td>Leaf</td>
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<td>Diameter</td>
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<td>( \omega_c )</td>
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<td>( R )</td>
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<tr>
<td>Stator current</td>
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IWRM 06U9501 stroke 2mm res. 5 micron
Excitation of Parasitic dynamics
actuation not in c.o.m.
sensor not in c.o.m.

- Optics in c.o.m., sensor at $x=-20$ mm, $z=-8$ mm
- Actuator not in c.o.m of combination
- Flexible frame connecting head to base
- Modeling goal
  - analyze influence on robust stability
Modelvorming met CosmosWorks

Small gain test results

No loading (a)

loaded (b)
Small gain test results

![Graph of imperfect alignment (c)]

![Graph of cut in lf (d)]
Conclusions

• Be aware of co-location lost in 3D systems. Due to limited manufacturing tolerances

• Flexible frames something to worry about

• Have a tool at hand to model quickly 3D flexible multi-body systems for control purposes