# **Inverse Analysis of Dynamic Systems**

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### Introduction

Due to the increasing demand for light weight constructions (e.g. in cars) material fatigue becomes a key design driver. The STW Project "Robust Inversion of Nonlinear Dynamic Systems" (INVERT) aims at improving speed and robustness of existing tools to predict life time and to limit expensive modifications to physical prototypes. It is a joint project of TU Delft and University of Twente.

## **Objective**

For entirely new designed cars full scale (physical) testing of a drivable prototype is necessary. To this end responses of a prototype are measured on a test track. By inversion of frequency response function (FRF) models, the actuator loads of a test rig are then adjusted such that laboratory responses closely match the outdoor responses (fig.1).

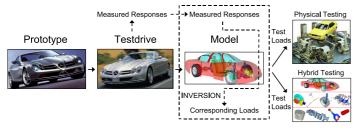


Figure 1 : Experimental life time estimation

Due to non-linearities this inversion is inaccurate and matching is a time-consuming iterative process: Reduction of this iteration time is one project objective. The second is to provide estimated responses of a virtual prototype with modified physical components (hardware in the loop) via inversion of a model of an older physical prototype, so called hybrid testing.

## **Methods**

The inverse analysis of dynamic systems to estimate forces from responses, so called force prediction, is categorized into two types <sup>1</sup>: The direct method and the optimization method. Both methods are applicable in time and frequency domain.

**Direct Inverse Method** The direct method is depicted in figure 2, where f(P,t) is a general force function in point P and  $\hat{w}(P,t)$  and  $\hat{W}(P,\omega)$  denote the measured time and frequency responses at P.

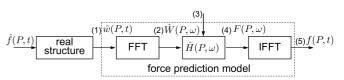


Figure 2 : Direct Inverse Method, frequency domain

The procedure is as follows:

- 1. measure response  $\hat{w}(P,t)$  to unknown  $\hat{f}(P,t)$
- 2. take FFT on  $\hat{w}(P,t)$  to get  $\hat{W}(P,\omega)$
- 3. measure structural FRF  $\hat{H}(P,\omega)$
- 4. obtain  $F(P,\omega) = [\hat{H}(P,\omega)]^{-1} \hat{W}(P,\omega)$
- 5. perform IFFT on  $F(P, \omega)$  to get f(P, t)

**Optimization Method** The optimization method proceeds as follows (fig.3):

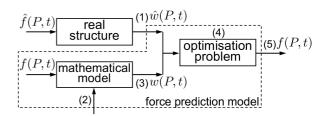


Figure 3 : Optimization Inverse Method, time domain

- 1. measure response  $\hat{w}(P,t)$  to unknown  $\hat{f}(P,t)$
- 2. create structural/mathematical model
- 3. predict model response w(P,t)
- 4. minimize the error between predicted and measured responses
- 5. solve this optimization problem for force f(P,t)

## **Discussion**

Non-linearities cause the FRF to be force dependent which makes the direct method tedious to apply. The optimization method seems therefore best suited for force prediction in non-linear dynamic systems <sup>2</sup>. The applicability of both methods will be investigated further.

## References

- 1. Wang, B.-T. (2002) Prediction of Impact and Harmonic Forces Acting on Arbitrary Structures, Mechanical Systems and Signal Processing 16(6).
- 2. Ma, C.-K. et al (2004) An Inverse Method for the Estimation of Input Forces Acting on Non-Linear Structural Systems, Journal of Sound and Vibration 275.

