Prototype modelling of mechanic systems

An introduction for Project F

Course number: 110325

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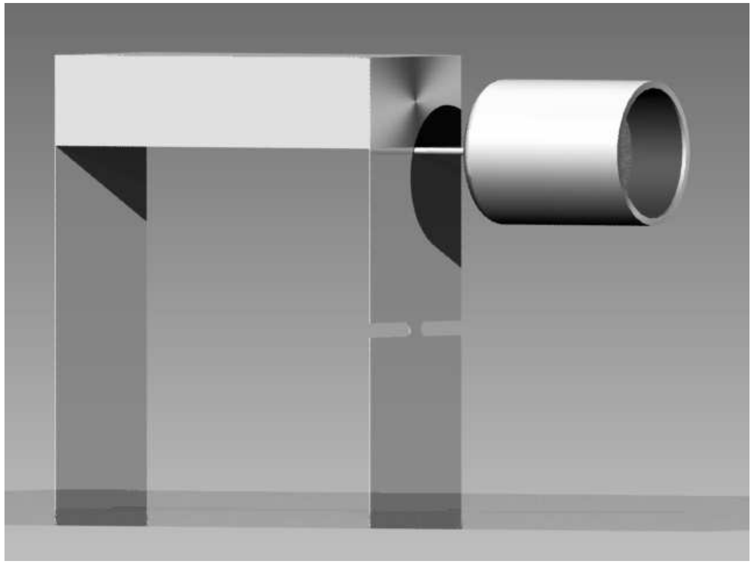
TeleTOP: `http://teletop.utwente.nl/07110325.nsf`

Overview



- Introduction prototype modelling.
  $\longrightarrow$ Example system.
  $\longrightarrow$ SPACAR software package
- Simple 1-DOF mass-spring model
  $\longrightarrow$ Non-linear elements, nodal coordinates and deformation parameters.
  $\longrightarrow$ Selection of degrees-of-freedom (DOF's).
- Two-dimensional model: 1-DOF and more-DOF.
- Three-dimensional model: 1-DOF and more-DOF.

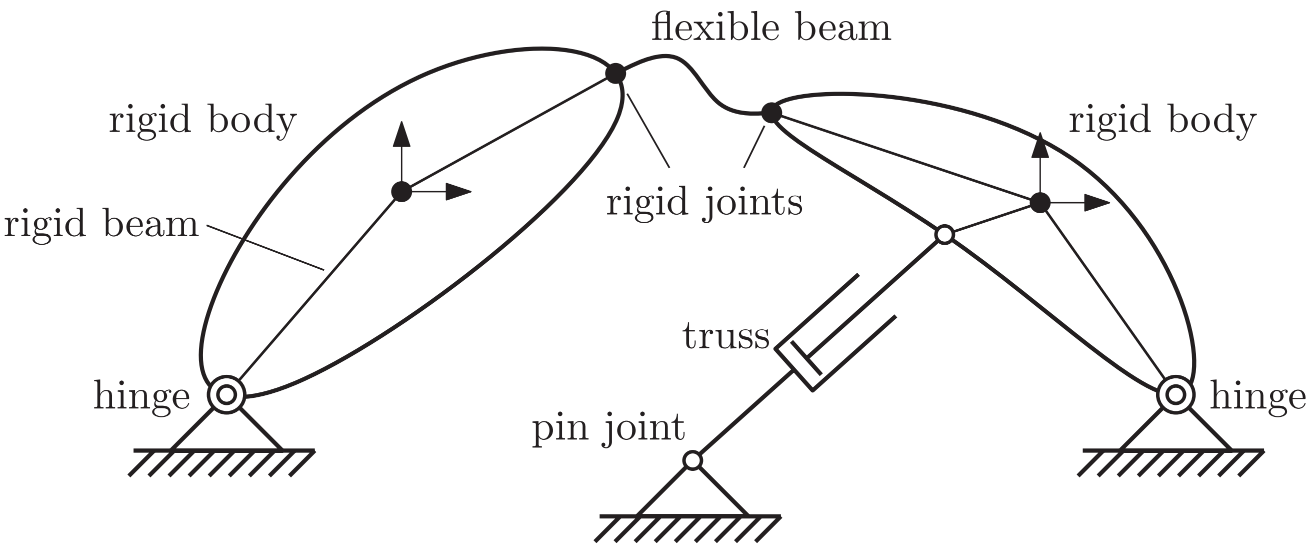
- Defining systems with a GUI
- Visualisation

Introduction

- Modelling and analysis enable designers to test whether design specifications are met
  $\longrightarrow$ with varying level of detail.
- In the early, conceptual stage: high level analysis when only a few design details are known.
- Simple prototype models with a few degrees of freedom:
  - Capture only the relevant systems dynamics
    $\longrightarrow$ offer insight.
  - Quick to evaluate, quick to change
    $\longrightarrow$ immediate feedback on design decisions.
  - Comprehensive exploration of design alternatives
    $\longrightarrow$ well-considered selection of “best” design concept
    $\longrightarrow$ to be analysed in more detail (e.g. with ANSYS).

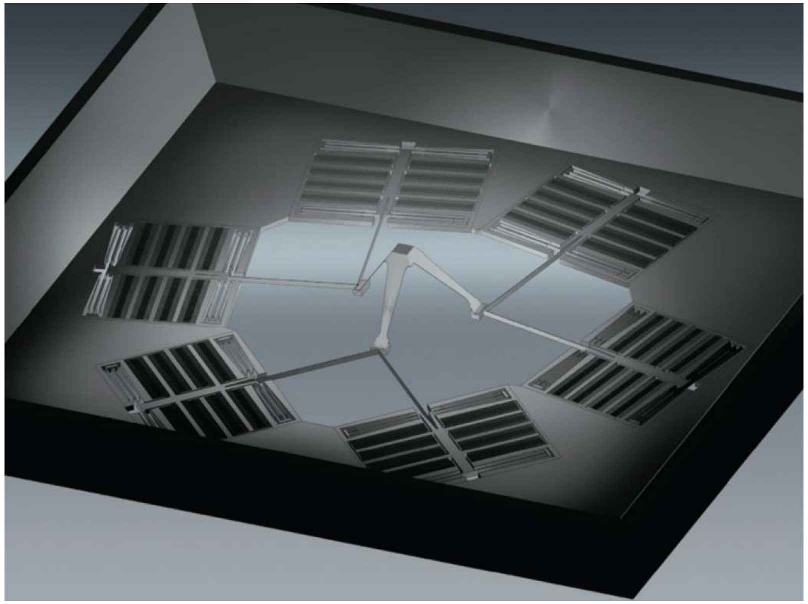
Multibody system approach

- Equations of motion expressed in terms of system's degrees of freedom (DOF's) $\longrightarrow$ Lagrange equations.



- Linearised equations of motion $\longrightarrow$ State-space equations.

Software package SPACAR



- Kinematic and dynamic analysis of
  - flexible multibody systems,
  - flexible structures.
- Based on the finite element method
- MATLAB user interface for the analysis of (linear) systems including visualisation.

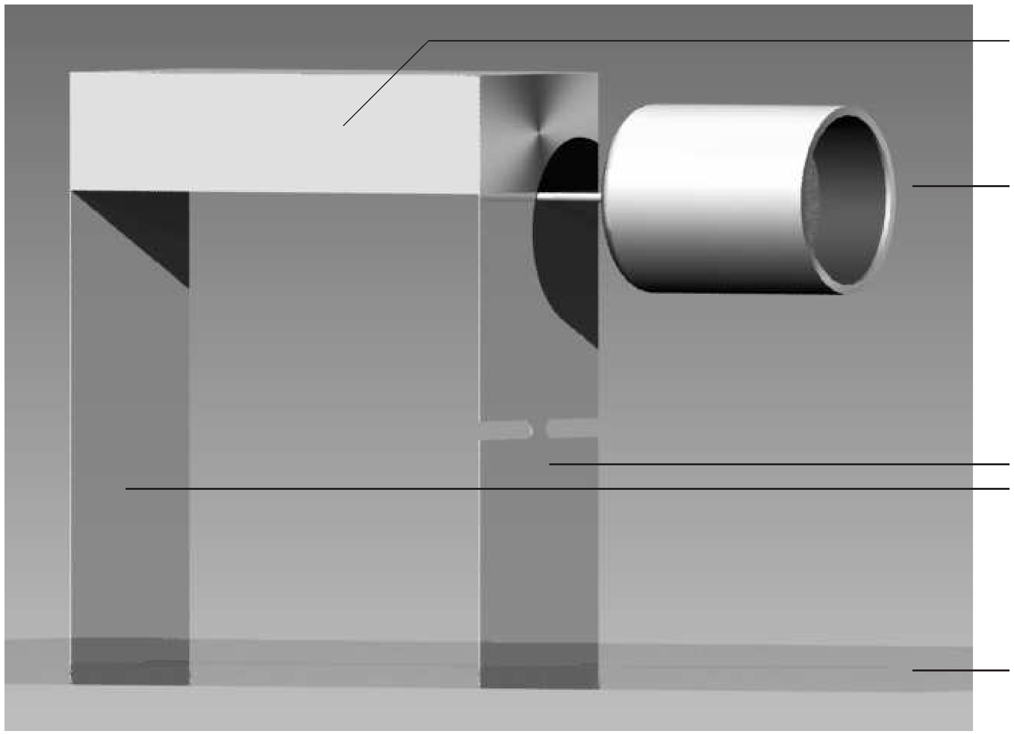
Available for download and installation, see TeleTOP.

Mechatronic system design

- Conceptual design
  - Kinematic analysis.
- Dimensioning the concepts
  - Natural frequencies and mode shapes
  - Static stability (buckling)
  - State space input output formulations (SISO or MIMO)
  - Simulation of the dynamic behaviour.
- Computer aided prototyping.
- Final design (fine tuning, e.g. with ANSYS).

Example system

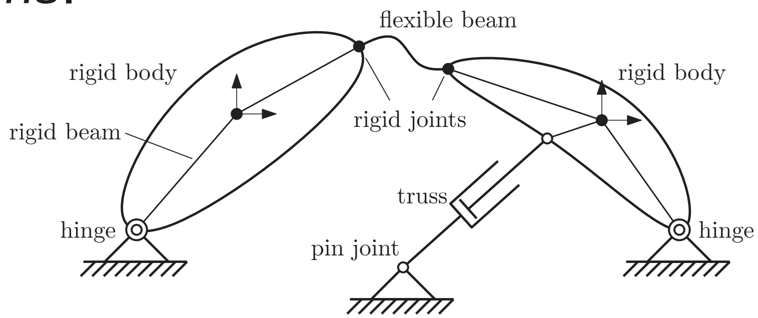
- One degree of freedom (1-DOF) VCM-driven support mechanism with elastic leaf springs. Both springs are fixed at the bottom (clamped support).



- This system will be analysed with an increasing degree of complexity using SPACAR.

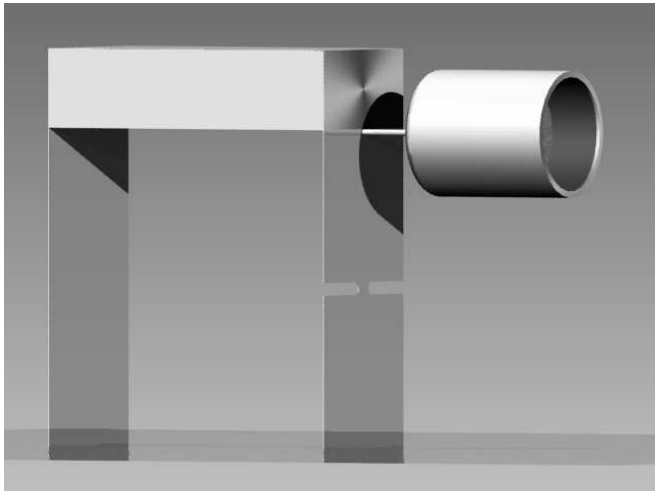
Analysis of example system with SPACAR

- Introduction of the finite element concept with *nodal coordinates* and *element deformations*.



- Two-dimensional (planar) and three-dimensional (spatial) models with (a small number) truss and beam elements.
- Each element has nodal points: The coordinates of translational and rotational nodal points describe the element's position and orientation.
- For each element a fixed number of independent (discrete) deformation modes are defined as functions of the nodal coordinates. Deformation modes are always invariant for rigid body movements of the element.
- Systems are defined in SPACAR input files and after the call to `spacar` the results are available in MATLAB variables and stored in output files.

Mass-spring model



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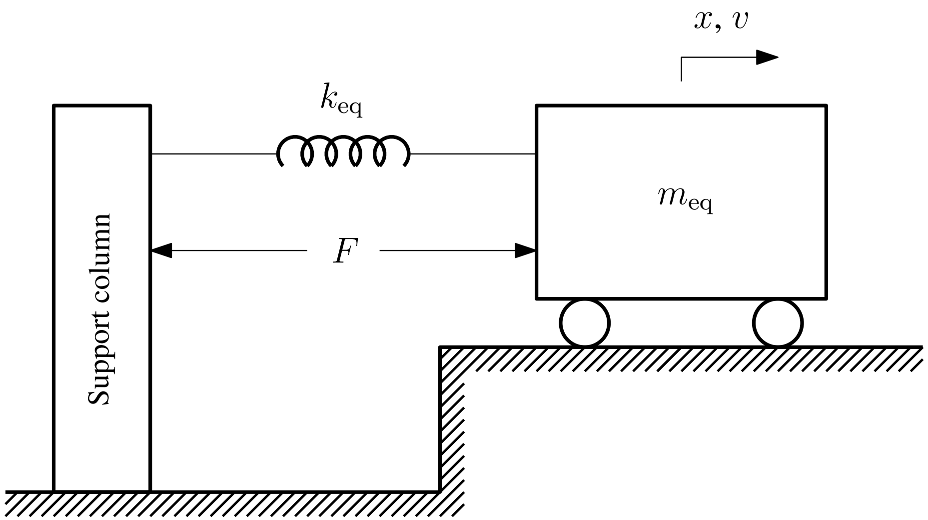
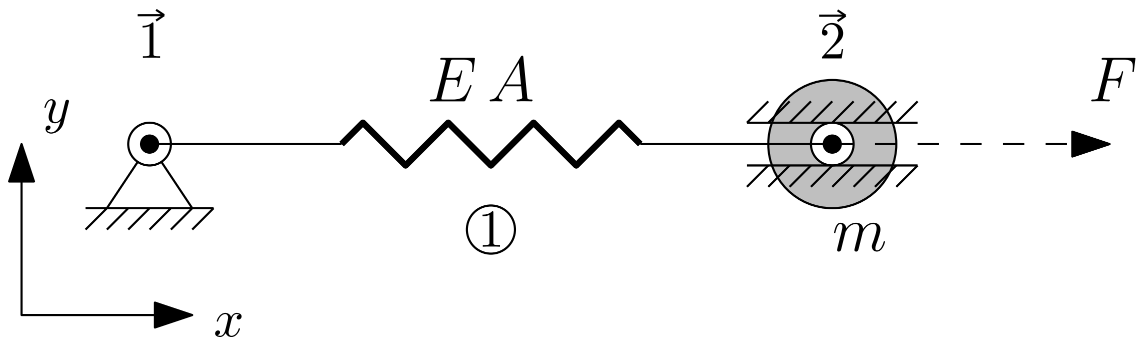


- Model system as a simple one degree-of-freedom mass-spring system.
- All mass is lumped in a single equivalent mass m_{eq} and the equivalent stiffness k_{eq} represents all elastic components.
- Input VCM force F and output position x:

$$G(s) = \frac{x}{F} = \frac{1}{m_{\text{eq}}\, s^2 + k_{\text{eq}}}.$$

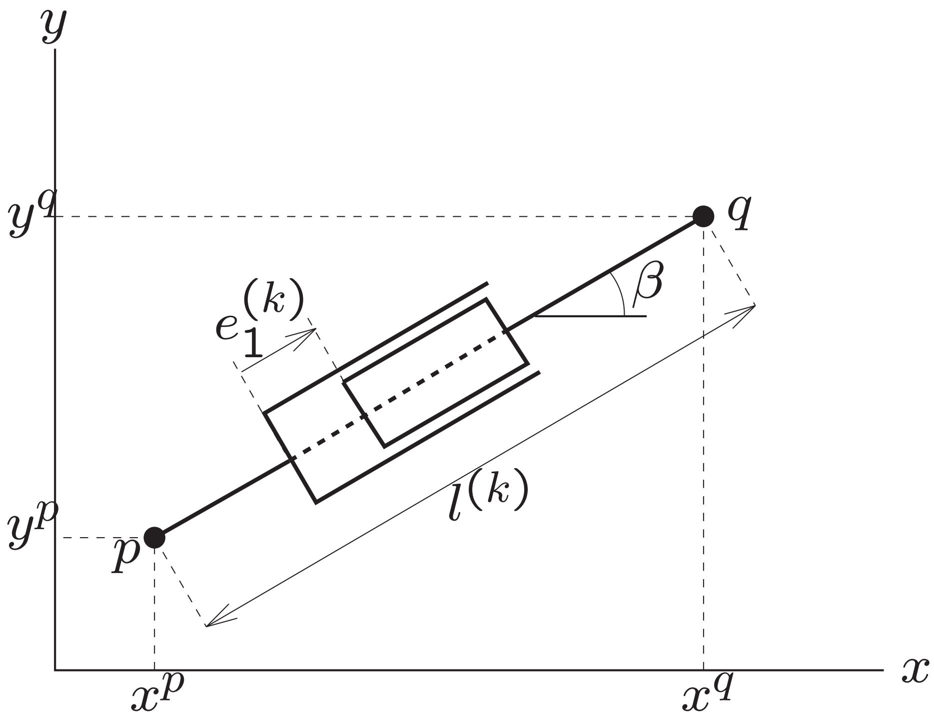
First SPACAR model: x degree-of-freedom

- The mass-spring system can be modelled as a one-dimensional SPACAR model: Identify elements with their coordinates and deformations.



- Two translational nodal points: $\vec{1}$ and $\vec{2}$.
- One element for the spring: a (two-dimensional) *truss* element ①.

Planar truss element (PLTRUSS)



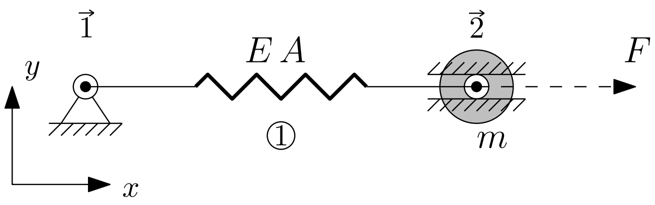
- Two translational nodal points p and q with two coordinates each:

$$\boldsymbol{x}_{\text{truss}}^{(k)} = \begin{bmatrix} \boldsymbol{x}^p \\ \boldsymbol{x}^q \end{bmatrix} = [x^p, y^p, x^q, y^q]^T .$$

- A single deformation mode e_1 of this element represents the elongation:

$$e_1^{(k)} = l^{(k)} - l_0^{(k)}, \text{ with } l^{(k)} = \sqrt{(x^p - x^q)^2 + (y^p - y^q)^2}.$$

Input file `pf1dtrx.dat` – Kinematic section




```
PLTRUSS   1   1   2
```

Define a planar truss: Element number `1` and two translational end nodes with numbers `1` and `2`.

```
X   1   0.0   0.0
X   2   0.1   0.0
```

Specify the (initial) Cartesian nodal positions. The keyword `X` is followed by the nodal point number and (for planar translational nodes) the x and y coordinates.

Input file `pf1dtrx.dat` – Kinematic section (2)

Next discriminating the fixed and varying nodal point coordinates and element deformation parameters.

```
FIX  1
FIX  2  2
```

Nodal point coordinates are free unless defined otherwise.
Keyword `FIX` specifies which nodal coordinates are fixed to the support. It is followed by the nodal point number and (optionally) the Cartesian coordinate direction.

```
RLSE  1  1
```

Element deformations are prescribed zero unless defined otherwise.
Keyword `RLSE` defines a released deformation parameter. It is followed by the element number and the deformation parameter to be released ($e_1^{(1)}$ in this case).

Input file `pf1dtrx.dat` – Kinematic section (3)

The next crucial step is the specification of the degree(s) of freedom.
The number of degrees of freedom `NDOF` has to satisfy

$$\texttt{NDOF} = \texttt{NX} - \texttt{NXO} - \texttt{NEO},$$

`NX` $= 4$ is the number of nodal coordinates,
`NXO` $= 3$ the number of *absolute constraints*: Keyword `FIX`,
`NEO` $= 0$ the number of *relative constraints*: The remaining unreleased element deformation parameters.

So `NDOF` $= 4 - 3 - 0 = 1$ degree of freedom has to be defined. E.g.:

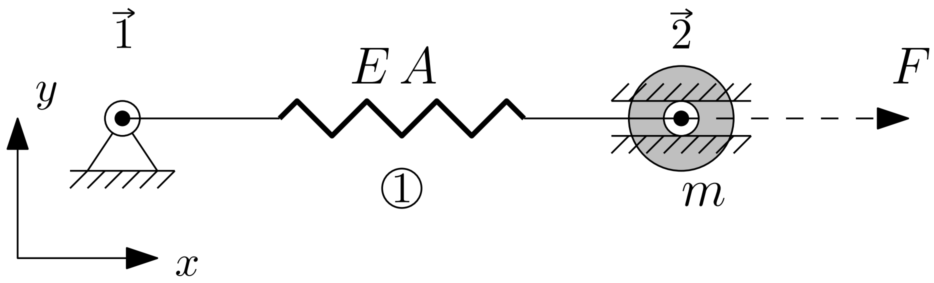
```
DYNX  2  1
```

Keyword `DYNX` defines a so-called absolute degree of freedom. It is followed by the nodal point number and coordinate number.

Input file `pf1dtrx.dat` – Kinematic section (4)

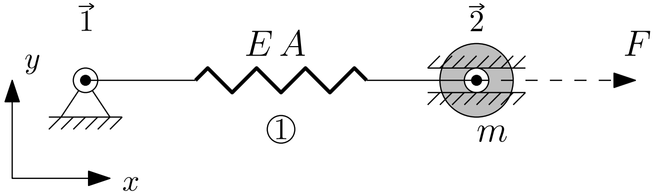
```
END
HALT
```

The kinematic section of the input file has to be ended with the two keywords `END` and `HALT`.



Input file `pf1dtrx.dat` – Dynamic section

```
XM   2   0.206
EM   1   0.1413
```

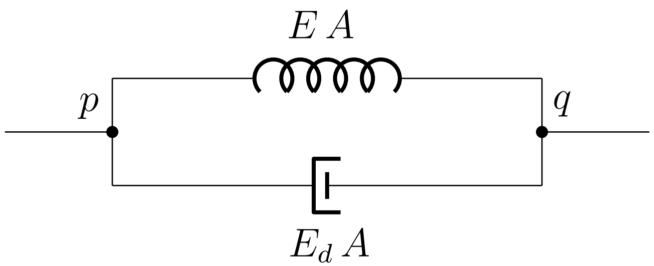


Keyword `XM` specifies a *lumped* mass attached at a nodal point. It is followed by the nodal number and the mass or inertia parameter(s). For a translational nodal point, the latter parameter(s) is/are the mass ([kg]) attached to the nodal point.

Keyword `EM` specifies a *distributed* mass along an element. It is followed by the element number and the mass per length ([kg/m]) of the element.

Input file `pf1dtrx.dat` – Dynamic section (2)

```
ESTIFF 1  94.5
EDAMP  1   0.00365
```

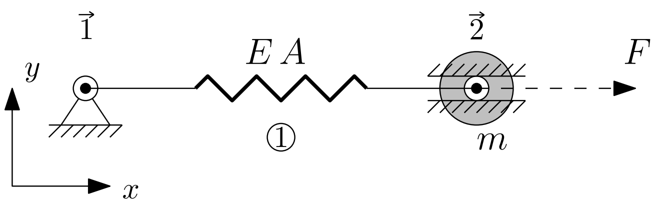


Keyword `ESTIFF` specifies the axial rigidity EA of the truss element.
It is followed by the element number and knowing the equivalent longitudinal stiffness k_{eq} we write $(EA)_{\text{eq}} = k_{\text{eq}} l_0$.
Keyword `EDAMP` specifies the elements internal viscous damping.

For the considered spring leaves the damping is computed assuming a *relative damping* ζ in the range of 0.01 to 0.001 and knowing that the damping $d_{\text{eq}} = 2\zeta\sqrt{k_{\text{eq}} m_{\text{eq}}}$.

Next $(E_d A)_{\text{eq}} = d_{\text{eq}} l_0$.

Input file `pf1dtrx.dat` – Dynamic section (3)




```
XF   2   1.   0.
```

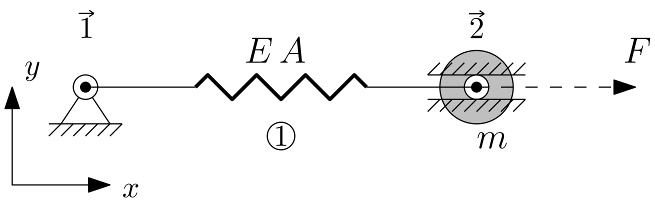
Keyword `XF` is used to specify external static forces. It is followed by the node number and (for a planar translational node) the force components in x and y direction applied at the node.

```
END
HALT
```

The dynamic section of the input file has to be ended with the two keywords `END` and `HALT`.

Input file `pf1dtrx.dat` – Input-output section

```
INPUTF   1  2  1
OUTX     1  2  1
```



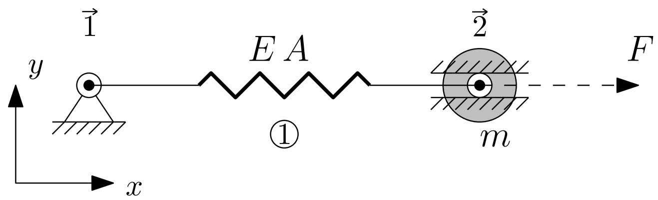
The keywords `INPUTF` and `OUTX` define a single input force F and a single output coordinate X, respectively.
The first parameter of both keywords is the input or output number that corresponds with its position in the input vector $\boldsymbol{u}$ or output vector $\boldsymbol{y}$.
The other two parameters define the input's or output's nodal point number and the corresponding Cartesian nodal coordinate.

```
END
END
```

The final section of the input file has to be ended using twice the keyword `END`.

Input file `pf1dtrx.dat` – SPACAR results

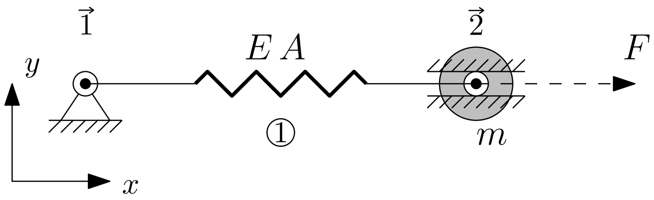
```
>> spacar(7,'pf1dtrx')
```



Calling SPACAR from the MATLAB command prompt in mode "7": Static analysis and the computation of the natural frequencies and mode shapes of the system. Output:

- MATLAB plot window with the system in its equilibrium configuration.
- Log file `pf1dtrx.log`.
- Binary data files `pf1dtrx.sbd` and `pf1dtrx.sbm`.
- MATLAB variables with results read from these files.

Input file `pf1dtrx.dat` – SPACAR results (2)



Horizontal displacement of node 2:

```
>> x(lnp(2,1))              =   0.1011
```

The normal force in the truss element:

```
>> sig(le(1,1))             =   1.0000
```

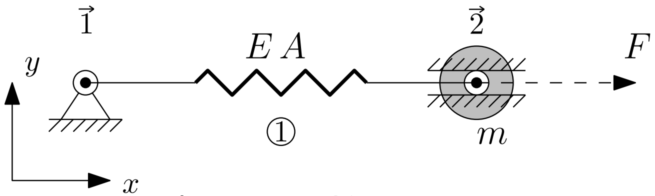
Mass, stiffness, and (first and only) natural frequency:

```
>> m0                       =   0.2107
>> k0                       = 945
>> sqrt(eig(k0,m0))/2/pi =  10.6584
```

Visualisation:

```
>> spavisual('pf1dtrx');
```

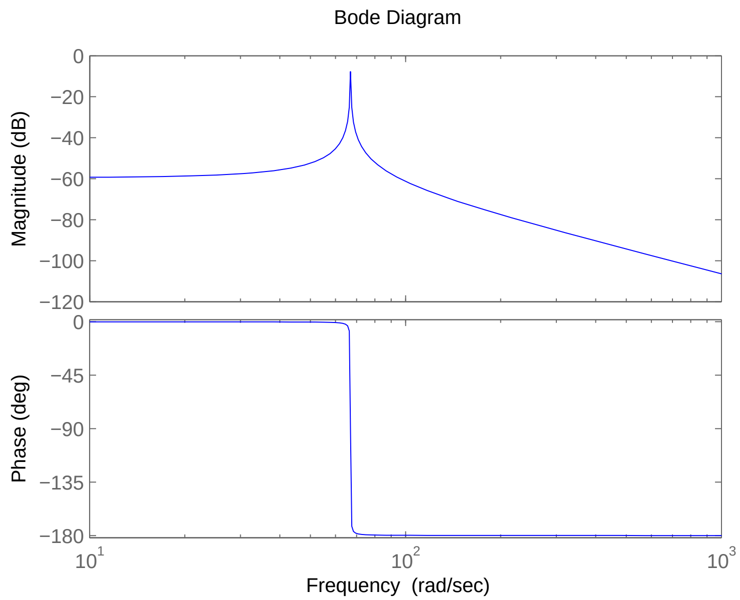
Input file `pf1dtrx.dat` – SPACAR results (3)




```
>> spacar(9,'pf1dtrx')
```

With a mode “9” run the state space matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, and $\boldsymbol{D}$ are computed and stored in binary data file `pf1dtrx.ltv`. They can be read with the `getss` command.

```
>> [A,B,C,D] = getss(pf1dtrx);
>> G=tf(getss(pf1dtrx))
Transfer function:
        4.746
----------------------
s^2 + 0.1732 s + 4485
>> bode(G)
```

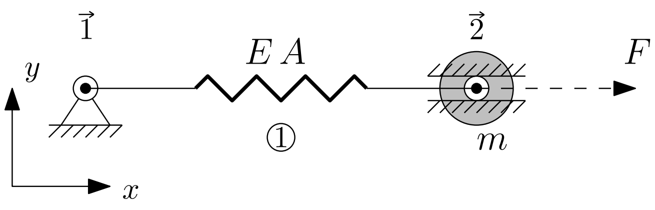


The latter command is used to create a Bode plot.

Second SPACAR model: e degree-of freedom

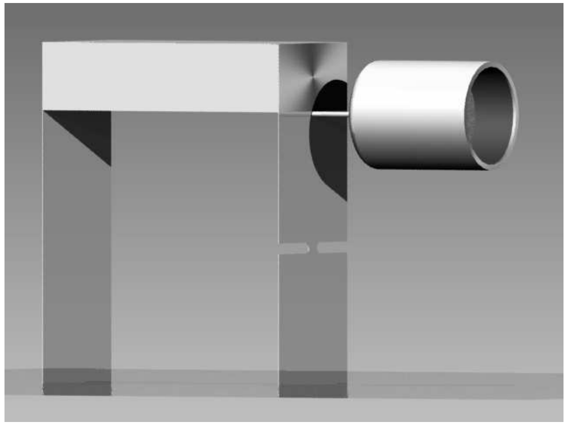
Alternatively, a *relative* degree of freedom can be defined with the `DYNE` keyword, as long as `NDOF` = `NX` − `NXO` − `NEO` is satisfied.

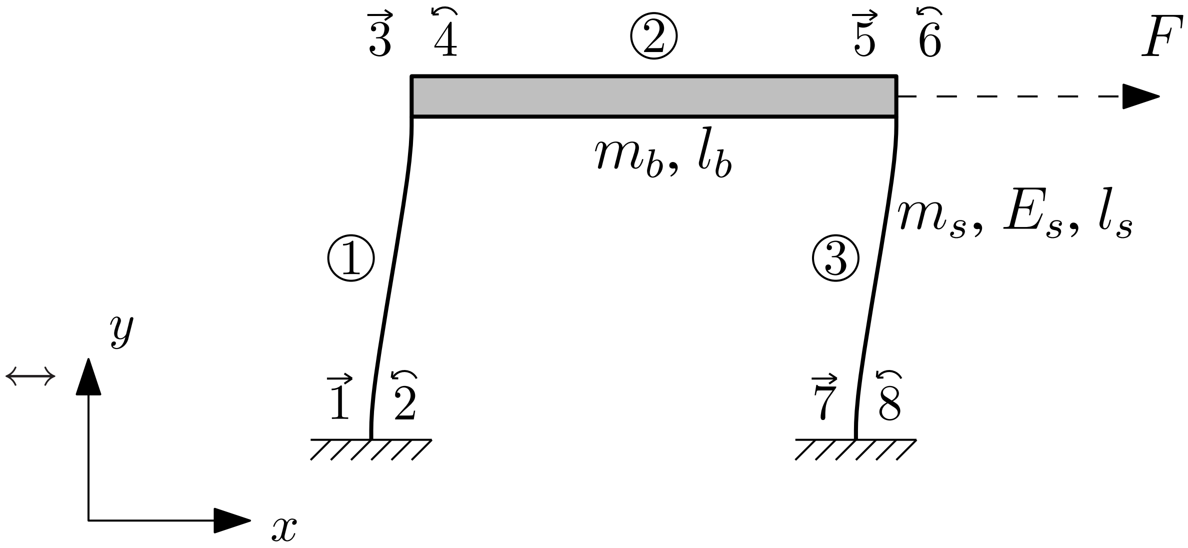
```
PLTRUSS  1  1  2
X     1  0.0  0.0
X     2  0.1  0.0
FIX   1
FIX   2  2
DYNE 1  1
END
HALT
```



Outcome as before ...

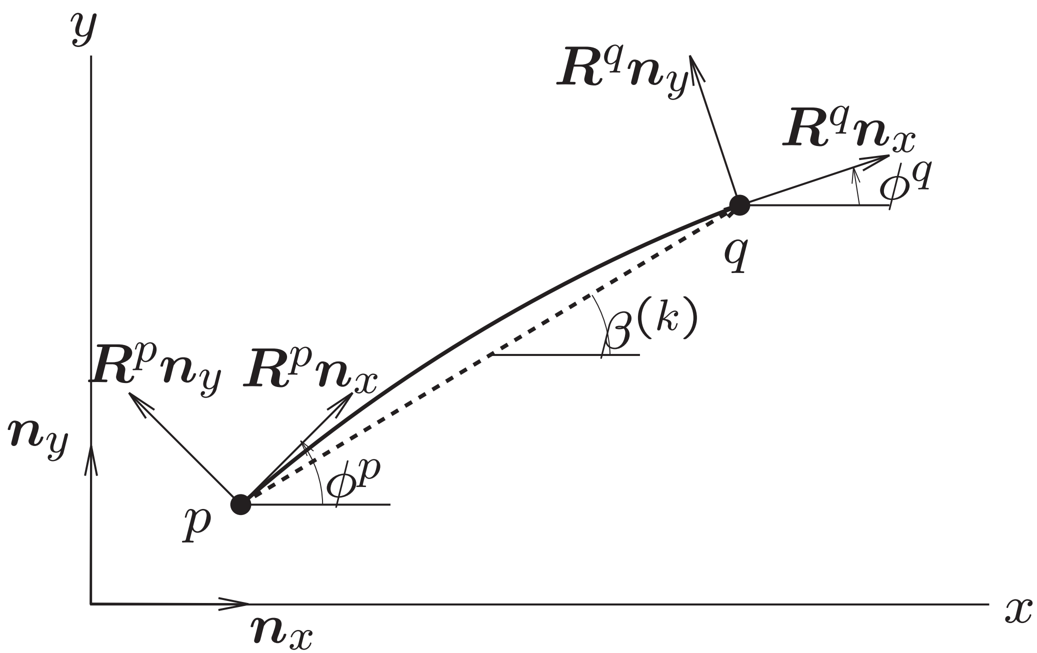
Two-dimensional model





- Bending of the leaf springs modelled more accurately using planar *beam* elements.
- Translational and *rotational* planar nodal points.

Planar beam element (PLBEAM)

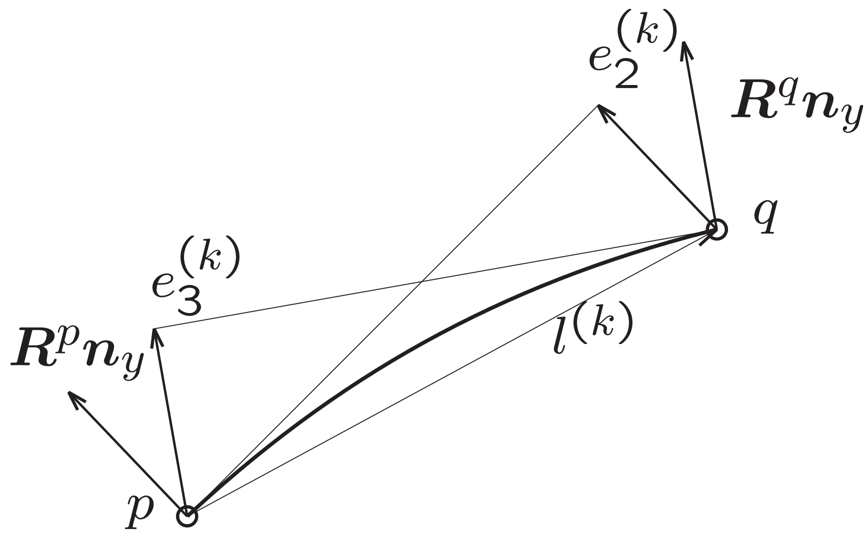


- Four Cartesian coordinates (x^p, y^p), (x^q, y^q) describing the position of the beam in the (x, y)-coordinate system.
- Two rotation angles ϕ^p and ϕ^q representing the angular orientation of the triads $(\boldsymbol{R}^p\boldsymbol{n}_x, \boldsymbol{R}^p\boldsymbol{n}_y)$ and $(\boldsymbol{R}^q\boldsymbol{n}_x, \boldsymbol{R}^q\boldsymbol{n}_y)$ at the nodes p and q respectively.

Planar beam element (PLBEAM) (2)

Nodal coordinates: $\boldsymbol{x}^{(k)}_{\text{beam}} = \begin{bmatrix} \boldsymbol{x}^p \\ \phi^p \\ \boldsymbol{x}^q \\ \phi^q \end{bmatrix} = [x^p, y^p, \phi^p, x^q, y^q, \phi^q]^T.$

Deformation parameters:



elongation: $e_1^{(k)} = l^{(k)} - l_0^{(k)},$

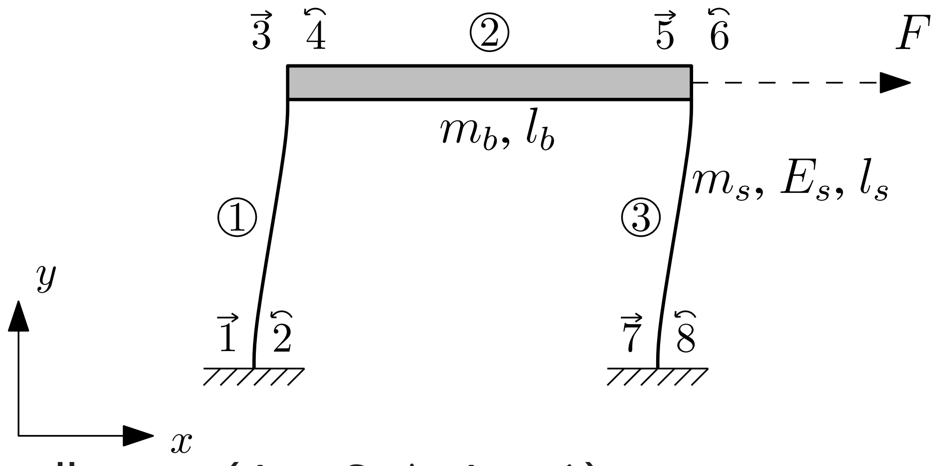
bending: $e_2^{(k)} = -(\boldsymbol{R}^p\boldsymbol{n}_y, \boldsymbol{l}^{(k)}),$

$e_3^{(k)} = \ \ (\boldsymbol{R}^q\boldsymbol{n}_y, \boldsymbol{l}^{(k)}),$

The deformations are independent for rigid body movements of the element and have a clear physical meaning.

1-DOF model

$$\begin{aligned} \texttt{NDOF} &= \texttt{NX} - \texttt{NXO} - \texttt{NEO} \\ &= 12 - 6 - 5 \\ &= 1, \end{aligned}$$



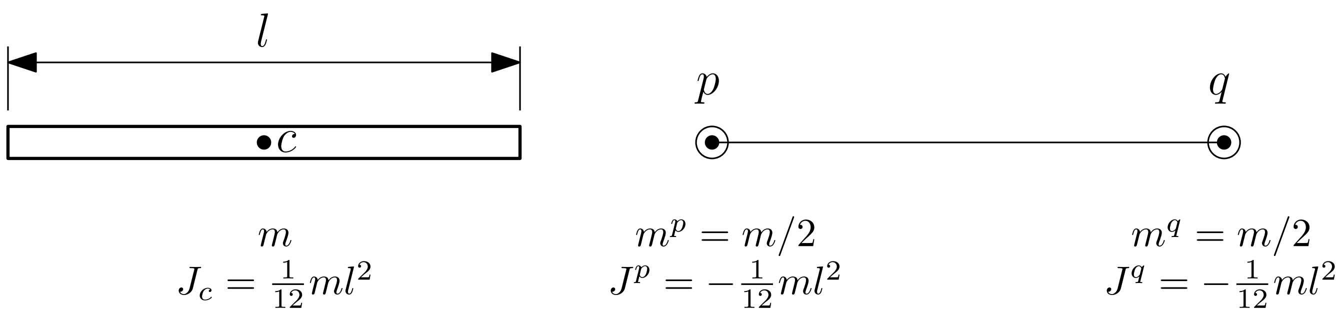
NX $=$ 12 is the number of nodal coordinates ($4 \times 2 + 4 \times 1$),
NXO $=$ 6 the number of absolute constraints (FIX $2 \times 2 + 2 \times 1$),
NEO $=$ 5 the number of relative constraints, so $3 \times 3 - 5 = 4$ deformation parameters have to be released,
NDOF $=$ 1: Horizontal position of node 3.

```
PLBEAM  1  1  2  3  4   ...
X  1  0.0  0.0  ...
FIX   1       FIX   2       FIX   7       FIX   8
RLSE  1  2 3                RLSE  3  2 3
DYNX  3  1
END  HALT
```

1-DOF model (2)

Distributed inertia of the solid bar ② in a lumped mass representation:

1. The mass of the element should be equal to the sum of the lumped masses.
2. The center of mass of the element and that of the discrete mass model should coincide.
3. The rotational inertia of the element and that of the lumped system should be equal.



Solid bar — Discrete mass model

Solid bar with mass m and length l: the rotational inertia relative to the centre of mass $J_c = \frac{1}{12}ml^2$. Equivalent lumped masses and rotational inertias are then:

$$m^p = m^q = \frac{1}{2}m \quad \text{and} \quad J^p = J^q = -\frac{1}{12}ml^2.$$

1-DOF model (3)

```
XM   3   0.103     XM   4   -0.0001716
XM   5   0.103     XM   6   -0.0001716

EM   1   0.07065   EM   3   0.07065

ESTIFF  1  1890000.   0.039375    ESTIFF  3  ...
EDAMP   1     0.3654  0.0000053   EDAMP   3  ...
END HALT
```

ESTIFF parameters for the planar beam element:
Axial rigidity EA (related to longitudinal stiffness EA/l) and
flexural rigidity EI (related to bending stiffness EI/l^3).

1-DOF model (4)

SPACAR analysis mode 7 and 9 as before: Still 1 natural frequency!

SPACAR analysis mode 8 for buckling analysis:
The critical loading parameter λ_i is computed, such that the buckling load $f_i = \lambda_i f_0$, where f_0 represents a static reference loading vector of nodal forces and torques.

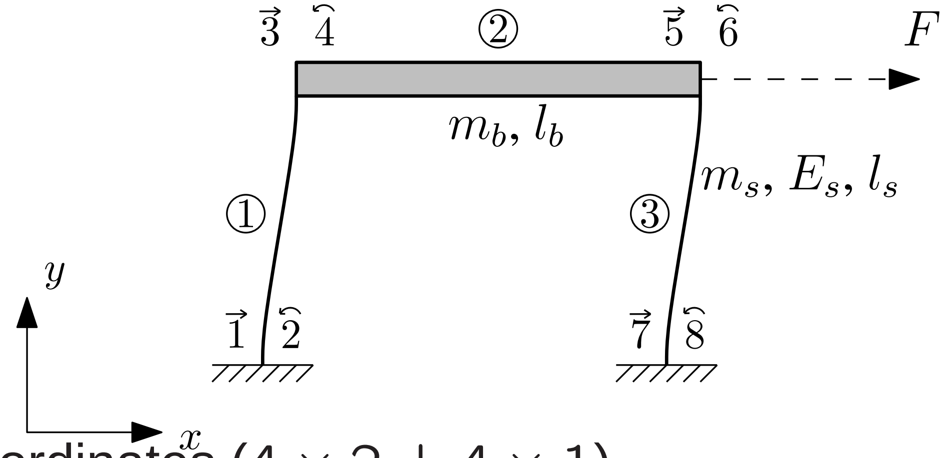
Vertical force with a magnitude of 1 N is applied at node 3 in the negative y direction in input file:

```
XF 3 0. -1.
```

To determine the equilibrium configuration and the critical loading parameter λ_1:

```
>> spacar(8,pf2d)
>> eig(-k0,g0)         = 78.7500
```

3-DOF model



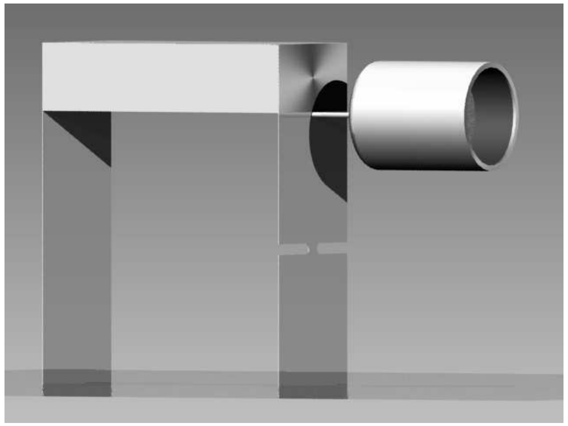
$$\begin{aligned}\texttt{NDOF} &= \texttt{NX} - \texttt{NXO} - \texttt{NEO}\\ &= 12 - 6 - 3\\ &= 3,\end{aligned}$$

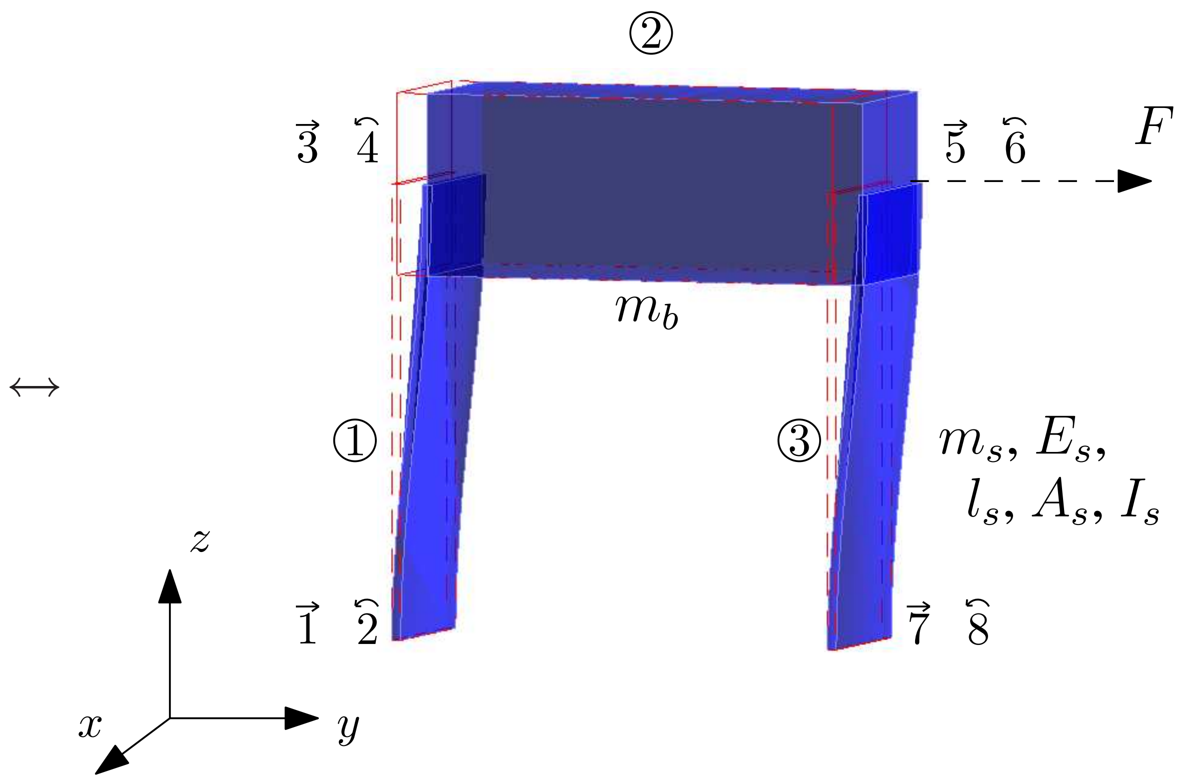
NX $= 12$ is the number of nodal coordinates ($4 \times 2 + 4 \times 1$),
NXO $= 6$ the number of absolute constraints (FIX $2 \times 2 + 2 \times 1$),
NEO $= 3$, so $3 \times 3 - 3 = 6$ deformation parameters have to be released,
NDOF $= 3$: Horizontal and vertical position of node 3, rotation of node 4.

```
PLBEAM  1  1  2  3  4   ...
X  1  0.0  0.0  ...
FIX   1          FIX   2          FIX   7          FIX   8
RLSE  1  1 2 3   RLSE  3  1 2 3
DYNX  3          DYNX  4
END  HALT
```

Next to the first natural frequency (10.6 Hz), now also higher natural frequencies are found (2129 Hz and 3585 Hz), justifying the previous 1-DOF approximation.

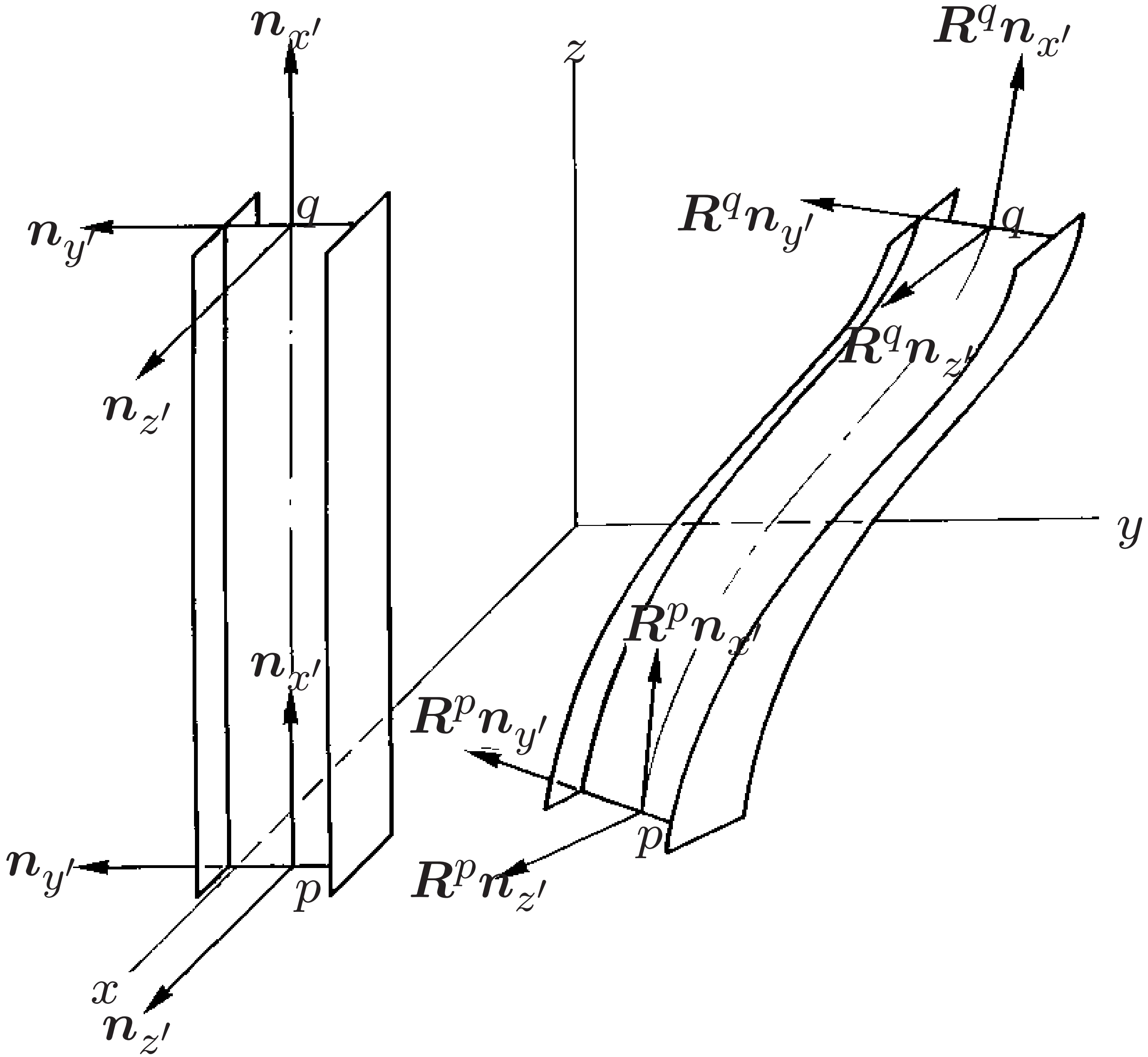
Three-dimensional model





- Modelled with *spatial* elements.
- Translational and rotational *spatial* nodal points.

Spatial beam element (BEAM)



Spatial beam element (BEAM) (2)

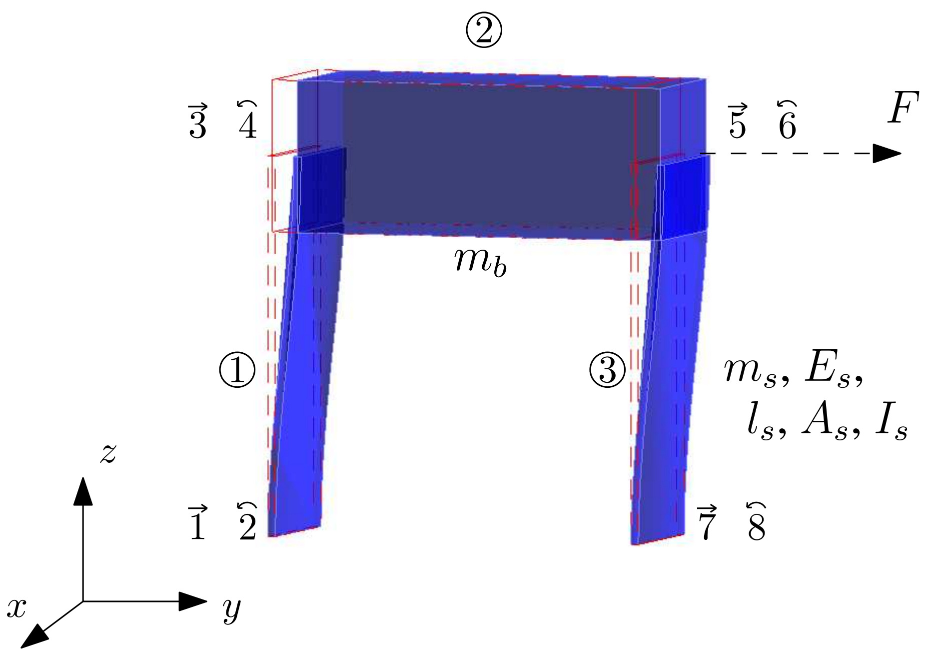
- Six translational coordinates are from two position vectors $\boldsymbol{x}^p$ and $\boldsymbol{x}^q$ describing the position of the beam in the fixed inertial coordinate system.
- Six independent rotational coordinates as the orientation of each rotational node in three dimensions is given by three independent rotation coordinates collected in the vectors $\boldsymbol{\lambda}^p$ and $\boldsymbol{\lambda}^q$ respectively.
  $\longrightarrow$ orientation of the triads $(\boldsymbol{n}_{x'}, \boldsymbol{n}_{y'}, \boldsymbol{n}_{z'})$ at the nodes p and q.
- Spatial beam: 12 (independent) nodal coordinates.
  As a rigid body: 6 degrees of freedom.
  So $12 - 6 = 6$ independent deformation parameters:

$$
\begin{aligned}
&\text{elongation:} && e_1^{(k)} = l^{(k)} - l_0^{(k)}, \\
&\text{torsion:} && e_2^{(k)} = \frac{1}{2} l_0^{(k)} \left[(\boldsymbol{R}^p \boldsymbol{n}_{z'}, \boldsymbol{R}^q \boldsymbol{n}_{y'}) - (\boldsymbol{R}^p \boldsymbol{n}_{y'}, \boldsymbol{R}^q \boldsymbol{n}_{z'}) \right], \\
&\text{bending:} && e_3^{(k)} = -(\boldsymbol{R}^p \boldsymbol{n}_{z'}, \boldsymbol{l}^{(k)}), \quad e_4^{(k)} = (\boldsymbol{R}^q \boldsymbol{n}_{z'}, \boldsymbol{l}^{(k)}), \\
& && e_5^{(k)} = (\boldsymbol{R}^p \boldsymbol{n}_{y'}, \boldsymbol{l}^{(k)}), \quad e_6^{(k)} = -(\boldsymbol{R}^q \boldsymbol{n}_{y'}, \boldsymbol{l}^{(k)}).
\end{aligned}
$$

1-DOF model

$$\begin{aligned} \texttt{NDOF} &= \texttt{NX} - \texttt{NXO} - \texttt{NEO} \\ &= 24 - 12 - 11 \\ &= 1, \end{aligned}$$

Note: Internally in SPACAR *rotational nodal points are described by so-called* Euler parameters *(4 in each node)*
$\Rightarrow$ NX *and* NEO *are counted differently.*



NX $= 24$ is the number of nodal coordinates ($4 \times 3 + 4 \times 3$),

NXO $= 12$ the number of absolute constraints (FIX $2 \times 3 + 2 \times 3$),

NEO $= 11$ the number of relative constraints, so $3 \times 6 - 11 = 7$ deformation parameters have to be released,

NDOF $= 1$: Horizontal position of node 3.

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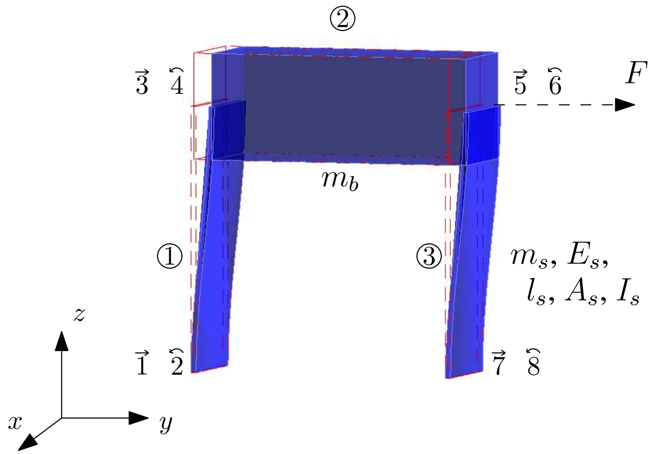
1-DOF model (2)




```
BEAM  1  1  2  3  4  0.0  1.0  0.0
BEAM  2  3  4  5  6  0.0  0.0  1.0
BEAM  3  5  6  7  8  0.0  1.0  0.0
```

The definition of the spatial beam includes a specification of the initial direction of the principal y' axis of the beams cross-section

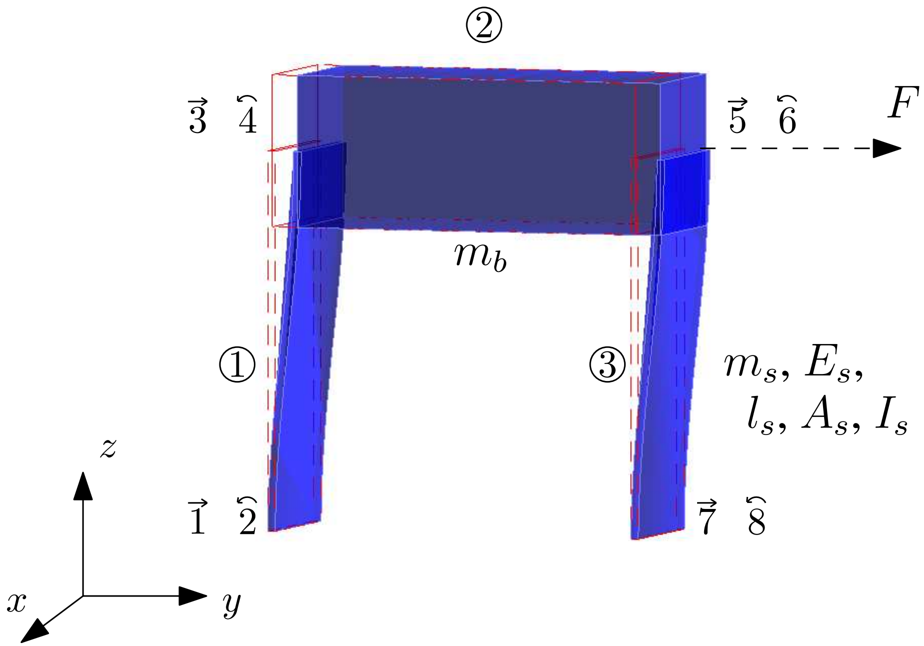
```
X  1  0.0  0.0  0.0    X  3  0.0  0.0  0.1
X  5  0.0  0.1  0.1    X  7  0.0  0.1  0.0

FIX  1    FIX  2    FIX  7    FIX  8
```

1-DOF model (3)

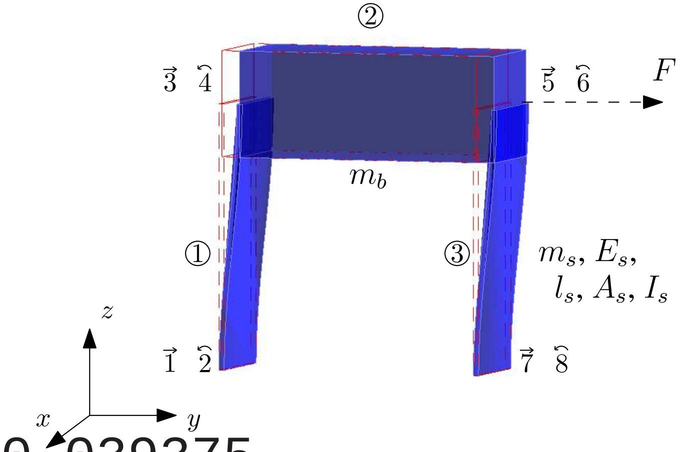
```
RLSE  1  5 6
RLSE  3  2 3 4 5 6

DYNX  3  2
END HALT
```



Two RLSE keywords release the bending modes of the leaf springs associated with the expected bending $e_5^{(1)}$, $e_6^{(1)}$, $e_5^{(3)}$ and $e_6^{(3)}$.
In addition the torsion and remaining bending modes of element 3 are released in accordance with the local cut in the leaf spring.

1-DOF model (4)




```
XM   3   0.103       XM   5   0.103
EM   1   0.07065     EM   3   0.07065

ESTIFF 1   1890000.   0.05895   51.030   0.039375
ESTIFF 3   1890000.   0.        0.       0.039375

EDAMP   1    0.3654   0.0000065   0.00019   0.0000053
EDAMP   3    0.3654   0.          0.        0.0000053
END HALT
```

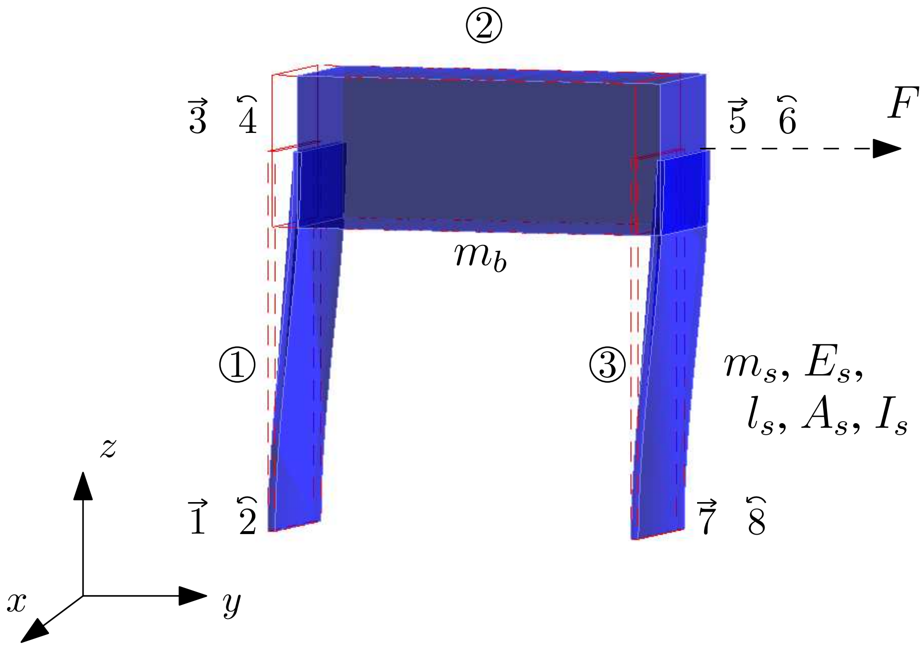
Keyword `ESTIFF` has four real parameters for the spatial beam element:
Axial rigidity EA (related to longitudinal stiffness EA/l),
torsional rigidity GI_t (related to the (equivalent) torsional stiffness GI_t/l^3), and
flexural rigidities $EI_{y'}$ and $EI_{z'}/l^3$ (related to bending stiffnesses $EI_{y'}/l^3$ and
$EI_{z'}/l^3$).

Analysis in MATLAB as before ...

6-DOF model

Releasing all 12 deformations of the spring leaves:

$$\begin{aligned} \mathtt{NDOF} &= \mathtt{NX} - \mathtt{NXO} - \mathtt{NEO} \\ &= 24 - 12 - 6 \\ &= 6, \end{aligned}$$



NX $=$ 24 is the number of nodal coordinates ($4 \times 3 + 4 \times 3$),

NXO $=$ 12 the number of absolute constraints (FIX $2 \times 3 + 2 \times 3$),

NEO $=$ 6 the number of relative constraints (only beam 2),

NDOF $=$ 6: All coordinates of node 3 + ???

(consider relative DOF's, see tutorial and the rest is left as an exercise to the reader ....)