UNIVERSITY OF TWENTE.

Master Thesis

Improving railway maintenance schedules by including hindrance and

capacity constraints in the design process of schedules

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Abstract

The availability of railway networks is important for society and the economy. In order to keep infrastructure in good conditions, maintenance is needed. By means of maintenance schedules, safe work zones are created in time and space for work crews to execute preventive maintenance activities. The research presented in this thesis aimed at optimising nightly maintenance schedules for both train operators and maintenance contractors, by considering hindrance for parked passenger trains and planned freight trains, and the workload for track workers. Meanwhile maintenance operations are distinguished into different engineering fields since this has an influence on the amount of hindrance. The method presented for designing maintenance schedules is a novel mixed integer programming (MIP) model that considers these aspects. A case study on part of the Dutch railway network assessed the new scheduling model on its performance and shows that large improvements can be made in terms of mean workload for work crews and total hindrance for train operators.

Keywords: Railway; Maintenance; Scheduling; Work zones.

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Arnhem, May 11, 2020

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II. Preface

This thesis is the result of my graduation research to obtain a Master Degree in Civil Engineering & Management at the University of Twente. During the final phase of my studies, I have been engaged in researching and writing this thesis during my internship at ProRail. From the start, I have felt very welcome and every one within the organisation and at external parties I have contacted, took the time to explain everything I needed to know on maintenance scheduling and related matters.

This thesis could not have been written without the support of my supervisors and initiators of the research problem, Harmen Zandman and Jaco Koster. They gave me the contacts I needed and gave me the thoughts, ideas, remarks and input I needed for my thesis. I would also like to thank all my colleagues for the sociability at the office, something I missed during the past months in which we all have to work from home due to the COVID-19 virus.

Furthermore, I would like to thank Eric van Berkum and Konstantinos Gkiotsalitis for helping me with your comments related to the theoretical and scientific parts of the research, and for being part of my graduation committee.

A special thank to my girlfriend Aline Rijks, who was there to discuss my research with, gave comments on my thesis and gave me the mental support I needed to finish my thesis.

During the final stage of my internship, I have been given the opportunity to apply for a job at ProRail which also concerns maintenance scheduling. After an exciting application procedure I got the job and I am really looking forward to start as a designer of maintenance models.

I hope to provide the readers of my thesis an insight into a new way of modelling maintenance schedules for railway networks.

Floris Nijland

Arnhem, May 11, 2020

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III. Samenvatting

Om het spoorwegennet in goede staat te houden en verstoringen in de treindienst te voorkomen, moet er preventief onderhoud worden uitgevoerd. Ter bescherming van baanwerkers vinden deze activiteiten plaats in treinvrije periodes die gepland zijn in een onderhoudsrooster. Bij het opstellen van een onderhoudsrooster moet er een afweging worden gemaakt tussen twee tegengestelde doelstellingen. Enerzijds moet het treinverkeer zo min mogelijk worden gestoord, anderzijders moet voorkomen worden dat onderhoudsploegen overwerkt raken. Daarnaast zijn er praktische beperkingen bij de inzet van werkoploegen, zoals hun capacitieit.

In de afgelopen jaren is er onderzoek gedaan naar verschillende aspecten van onderhoudsplanning van spoorwegen, zoals het gezamenlijk plannen van de dienstregeling en onderhoud, of het optimaliseren van de grote van onderhoudsperioden. Het modelleren van een onderhoudsrooster dat zowel hinder voor vervoerders als de beperkingen van werkploegen omvat, is echter nog niet veel onderzocht. Het onderzoek in dit proefschrift ontwikkelde daarom een nieuw geheeltallig programmerings (MIP) model om onderhoudsroosters te maken voor preventief spoorwegonderhoud. Het nieuwe model optimaliseert voor zowel spoorwegvervoerders als voor onderhoudsaannemers, terwijl het onderscheid maakt tussen verschillende techniekvelden in het onderhoud, om zo een betere balans te bieden tussen treinverkeer en onderhoudsbeheer.

Door een casestudy uit te voeren op een deel van het Nederlandse spoorwegnet, is het nieuwe planningsmodel beoordeeld op zijn prestaties. Er zijn verschillende roosters met elkaar vergeleken om te zien waar verbeteringen kunnen worden aangebracht met betrekking tot het huidige rooster van het gebied. Verder is er een gevoeligheidsanalyse uitgevoerd op de gewichtsparameters van het model, om inzicht te krijgen in hun invloed op de uitkomsten. In het casestudy gebied kunnen grote verbeteringen worden behaald door gebruik te maken van het nieuwe model om het onderhoudsrooster te maken. Zowel de hinder voor vervoerders als de werkdruk voor werkploegen kan worden verlaagd, maar dit gaat wel ten koste van een toename in het aantal nachten dat voor onderhoud wordt gebruikt.

IV. Summary

To keep railway networks in proper conditions and prevent disruptions in the operation of train services, preventive maintenance needs to be performed. To protect track workers, these activities take place during train-free periods which are scheduled in a maintenance schedule. When creating a maintenance schedule, a trade-off has to be made between two contrary objectives. On the one hand, train traffic should be disturbed as little as possible, while on the other hand the maintenance crews should not become overworked. Furthermore, there are practical restrictions in the deployment of work crews, such as their capacity.

The past years, studies investigated several aspects of railway maintenance planning, like combining the scheduling of the timetable with maintenance work, or optimising the dimension of maintenance windows. The modelling of a maintenance schedule that includes both hindrance for train operators and constraints of work crews is however not researched much yet. The research presented in this thesis therefore developed a new mixed integer programming (MIP) model to create maintenance schedules for preventive railway maintenance. The new model optimises for both train operators and maintenance contractors, whilst distinguishing maintenance engineering fields to provide a better balance between train traffic and maintenance management.

By executing a case study on part of the Dutch railway network, the new scheduling model is assessed on its performance. Several different schedules are compared to see where improvements can be made regarding the current schedule of the area. Furthermore, a sensitivity analysis is performed on the weight parameters of the model to provide insights on their influence. For the case study area, large improvements can be made when using the new model to create the maintenance schedule. Both the hindrance for train operators and the workload for maintenance contractors can be lowered, this is however at the expense of an increase in the number of nights that will be used for maintenance.

V. List of Abbreviations

CP	constraint programming.
KPI	key performance indicator.
MILP	mixed integer linear programming.
MIP	mixed integer programming.
STGs	single-track grids.

VI. List of Mathematical Definitions

- $\in \quad \text{ in } (a \in A: \text{ element } a \text{ in set } A).$
- \forall for all ($\forall a$: for all elements a).
- |A| cardinality (|A|: number of elements in set A).
- \sum summation.
- \Rightarrow implies.
- $\mathbb{Z}^* = \{0, 1, 2, 3, ...\},$ positive integers including 0.
- $\mathbb{R}^+ = \{ x \in \mathbb{R} \mid x \ge 0 \}, \text{ positive real numbers including } 0.$

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1. Introduction

Railway networks are of great importance for both society and the economy and for the next decades, a significant growth in transportation demand via railways is forecast (Bešinović et al., 2019). The railway network of The Netherlands is the busiest of Europe. More than 3.3 million train trips were made on the Dutch railways in 2015, with on average 1.1 million travellers per day (ProRail, 2019). Looking at the modal split of travellers, only 2% of journeys of Dutch people were made by train in 2014, but these people travelled a total distance of more than 16 billion kilometers on the railway network (CBS, 2016). More recent numbers show that the usage of trains for transportation is steadily increasing over the past decade and is expected to do so in the future, to 22 billion traveler-kilometers in 2023 (KiM, 2018).

With that many users, it is important that the railway infrastructure is kept in reliable conditions in order to prevent major disruptions in the operation of public and freight transportation services. When the capacity of railway infrastructure is heavily used, maintenance activities need to be performed in short and fragmented time slots or during nights (Odolinski & Boysen, 2019). This makes it difficult to find efficient possessions. Since railway tracks that are used very frequently are more sensitive to delays (Lindfeldt, 2015), there is reason to carry out preventive maintenance in time (Odolinski & Boysen, 2019). To increase the possibilities for suitable time slots for maintenance, maintenance windows can be given as input to construct the train timetable around them (Lidén & Joborn, 2016).

Due to serious accidents involving track workers, the Dutch Government decided to protect track workers in time and space by treating work crews as trains (den Hertog et al., 2005). This means that the track on which workers are working, is blocked for all other trains as if there is a train present on that section. In other words, to guarantee the safety of track workers, maintenance activities are only allowed during train-free periods. To reduce traffic disturbances due to these periods, these are therefore planned mostly at night. The schedule with train-free periods for maintenance activities is called the maintenance schedule.

Nowadays, Dutch railway manager ProRail gives a maintenance schedule to its maintenance contractors in which every railway section is planned train-free for maintenance at regular moments every week (or every two weeks). These periods are however not always used by contractors, thus traffic undergoes unnecessary disturbances. Furthermore, the increase in the amount of rolling stock results in parking problems when yards are planned train-free. Trains that are parked for the night on such yards, or sometimes even on station platforms due to a lack of parking places on yards, sometimes need to be parked elsewhere to allow workers to perform maintenance activities on that specific railway section. Also, contractors are limited in the deployment of their work crews. These practical issues should all be considered when creating maintenance schedules. As discussed by Lidén (2015), managing train traffic and maintenance activities on railway infrastructure are two main problems for railway managers. These two are however often treated separately, despite the fact that the two issues are strongly interconnected. Therefore, the research presented here adjusted an existing scheduling model to improve maintenance schedules by providing a better balance between train traffic and maintenance management. This is achieved by considering the workload of maintenance crews and the hindrance for train operators caused by train free periods. Maintenance activities are thereby distinguished in three different engineering fields: switches, straight tracks, and overhead wiring. In 2022, the Dutch timetable will change majorly. Therefore, there is an urgency to know whether the currently used maintenance scheduling method is still sufficient or if it can be further improved.

1.1. Railway Maintenance

Several different types of maintenance activities need to be performed on railway systems to keep them at required performance levels. When assets are below the required performance levels, the chance that a failure occurs rises and this may cause major disruptions in train services. van Noortwijk & Frangopol (2004) researched different models to maintain infrastructure. The model in Figure 1 shows a maintenance scenario that is commonly applied to railway systems. Preventive maintenance activities are regularly performed to upgrade the reliability of the infrastructure with small steps before a complete renewal is done.



Figure 1: Reliability profile assets (van Noortwijk & Frangopol, 2004).

Preventive maintenance activities include e.g. visual inspections, replacing sleepers, re-railing, rail grinding, ballast cleaning, and tamping (Higgins, 1998). These preventive maintenance works can be divided into two categories (Budai et al., 2006). Firstly, routine maintenance activities such as inspections of rails, switches, and signaling systems and small repairs (e.g. switch and track revision, and switch lubrication). Secondly, maintenance activities with larger works carried out less frequently (once or twice every few years) such as ballast cleaning, tamping, and rail grinding. Besides preventive maintenance works, there are unplanned corrective maintenance works (e.g. after incidents). The maintenance scheduling model developed in this research is intended to be used for the first type of preventive maintenance activities.

1.2. Thesis Structure

The thesis continues with a literature review and the research goals in Chapter 2. In Chapter 3 the problem description and the basis model are discussed. In Chapter 4 the developed mathematical model is presented and in Chapter 5 the results of the application of the model are given. The thesis concludes with a conclusion in Chapter 6 and a discussion in Chapter 7.

2. Literature Review & Research Goals

In this chapter, the reviewed literature on maintenance scheduling is discussed and the research goals are presented.

2.1. Maintenance Scheduling

An optimization model for the allocation of railway maintenance activities and crews developed by Higgins (1998) tried to minimize disruptions for scheduled trains. By scheduling activities as early as possible, it minimized the time that a given railway section is below a given performance threshold level. Some constraints to the problem were available budget, priority of maintenance activity, availability of tracks, and minimum travel time between track sections (Higgins, 1998). To find a solution to the problem, the method used a tabu search heuristic. One assumption made by Higgins (1998) is that work crews may perform maintenance activities simultaneously with train services. This is however not allowed in The Netherlands.

Cheung et al. (1999) developed a method to schedule enough preventive maintenance activities to avoid disruptions in the service operation of the subway system of Hong Kong. It scheduled maintenance during the five hours per night the tracks are not used by the operator, and followed certain rules and procedures e.g. safety rules. The method thus assumes that each day there are several hours during which there are no trains planned. In high-density railway networks this is however not the case.

More recently, van Zante-de Fokkert et al. (2007) used a two-step solution method to draw up an maintenance schedule in which track sections are blocked. First, they specified single-track grids (STGs), which are sets of working zones that can be blocked simultaneously. Then, the STGs were assigned to nights to create the actual maintenance schedule, making use of a MIP model.

Heinicke et al. (2015) developed an approach to create a tamping maintenance schedule. Instead of prioritization rules, they introduced penalty costs for maintenance tasks that need to be performed. The longer the performance level of a railway section is below the threshold level, the higher the penalty. The problem was presented as a vehicle routing problem with customer costs and was solved as a MILP problem.

To reduce computational time for large-scale railway infrastructure maintenance planning, Faris et al. (2018) proposed three distributed optimization methods. 'Parallel augmented Lagrangian relaxation', 'alternating direction method of multipliers' and 'distributed robust safe but knowledgeable'. The latter two differ from the first, but do use Lagrangian equations. Their track maintenance planning was also formulated as a MILP problem. The developed distributed approaches can be seen as heuristic methods to solve the problem.

Zhang et al. (2019b) developed an integrated model and algorithm which included train timetabling and track maintenance task scheduling on a microscopic level. Their MILP based approach used block sections as basic modeling units. By enforcing border constraints between sub-areas, global feasibility and optimality could be guaranteed (Corman et al., 2012). Meng et al. (2018) also considered integrating maintenance scheduling and train timetabling. In their MILP model their focus included speed limitations e.g. on a double track line due to maintenance activities on one of the tracks. Maintenance operations were modelled as virtual trains occupying sections and the model aimed to minimize the total run time of all trains. By means of numerical experiments on a fictional network, their method was assessed. To schedule track maintenance and create

timetables simultaneously for large scale railway networks, Albrecht et al. (2013) described how the problem space search meta-heuristic of Storer et al. (1992) can be used. They aimed at minimizing train delays and assessed their method on the North Coast Line (Queensland, Australia). They however note that the timetables generated are to be used as a starting point for refinement by train controllers. On a tactical level, D'Ariano et al. (2019) researched the integration of train scheduling and maintenance activities through optimization techniques. They modelled the problem in a MILP formulation, integrating traffic flow and track maintenance, constraints and objectives stochastically. Their bi-objective optimization problem aimed to minimize deviations from the original timetable and to maximize the number of aggregated maintenance works. A design strategy of an integrated system that automatically schedules maintenance jobs, combining asset condition monitoring, planning and scheduling of maintenance jobs and cost was presented by Durazo-Cardenas et al. (2018). In their process, railway infrastructure experts were consulted for the validation of the different components of the strategy.

Lidén & Joborn (2017) also addressed the capacity planning problem, integrating train services planning and maintenance windows scheduling. They developed a MILP model with the aim to find a long term tactical plan to optimally plan train free periods for the needed maintenance activities. As an extension to this MILP, Lidén et al. (2018) included maintenance resource constraints and costs to ensure that work crews could cover the scheduled maintenance windows. It included work bases, maximum working hours per day and minimum offtime between working days. Later, Lidén (2018) presented model reformulations on the earlier developed MILP to improve the solving performance by using a tighter formulation for maintenance window start variables and aggregating coupling constraints. Assessing the reformulations on the same data as the original model showed that optimal solutions are reached quicker.

In China, high-speed railway infrastructure is maintained every night which influences night trains. Zhang et al. (2019a) also formulated their problem as a MILP model, which integrated maintenance window selection and the timetable of the night trains, minimizing the total travel time of night trains. In practice, three operation modes were used to deal with regular maintenance tasks, which all come down to route selection between the high-speed railway and the normal-speed railway. Real data of the Beijing-Guangzhou (China) railway lines was used to asses the model with numerical experiments.

To deal with a new signalling system in Denmark, Pour et al. (2018a) developed a new approach for the maintenance scheduling process. A decentralized structure was used where workers start from home locations in stead of from a depot in a region. Workers were assigned to sub-regions around there living place and allocation of tasks were based on two key considerations, namely a balanced workload among all workers and a minimised distances between tasks to ensure quick responses when unexpected failures occur. A hyper-heuristic framework was used to improve initial solutions. To asses the approach, twelve problem instances were created on the peninsula of Jutland (Danmark). Pour et al. (2018b) developed a MIP model to schedule the preventive maintenance crews for the new signalling system containing practical constraints, e.g. dependencies between crew schedules and crew competence requirements. They used constraint programming (CP) to generate initial solutions which were used as input for the MIP solver. With this hybrid approach, they generate good results for problem instances with an eight-week planning horizon.

To address uncertainties in maintenance activities, Bababeik et al. (2019) provided a mathematical pro-

gramming model which aimed at rearranging timetables of trains in a single track considering maintenance operations. By adding buffer times to maintenance activities, delays in the initial maintenance plan which overlap the train scheduling were limited. Results of a small case study showed that the inclusion of buffer times decreases the propagation of delay through the timetable. Arenas et al. (2018) proposed a MILP formulation that adjusts a timetable to deal with the capacity taken by maintenance activities when such activities are unplanned due to incidents. They included maintenance trains and other constraints (e.g. temporary speed limits) in the problem and assessed three algorithms (a constrained formulation, a two phase algorithm where the output of the constrained formulation was used as an initial solution for the original one, and a two phase algorithm that used a greedy heuristic to find an initial solution) on their original formulation on a case study in the French railway network.

A switch maintenance scheduling problem considering the reliability of switches was addressed by Sun et al. (2017). The problem was again mathematically formulated as a MILP problem considering time windows for maintenance and the assignment and routing of maintenance teams. The method was based on a multiple traveling salesman problem with time windows, but it had multiple time windows available per switch of which only one could be selected, and each switch had a reliability constraint (a switch may not fail). The method was assessed on a network of twenty switches in front of a station.

Su et al. (2019) developed a method which integrated condition-based track maintenance planning and crew scheduling. A chance-constrained model predictive control controller determined the long-term maintenance plan at a higher level and minimised maintenance costs and condition deterioration, while making sure that the infrastructure stayed above the maintenance threshold. At a lower level, the maintenance activities suggested by the higher level controller were optimally scheduled and maintenance crews were optimally routed by a model that formulated the problem as a capacitated arc routing problem. Their approach was demonstrated by a numerical case study on a part of the Dutch railway network, treating squats in rails optimally. Su & de Schutter (2018) considered optimally scheduling track maintenance activities with the goal to find a time schedule and route for a maintenance crew to minimize total setup costs and travel costs. The routing problem was formulated as a capacitated arc routing problem with fixed cost. Three main settings (homogeneous, heterogeneous and flexible maintenance time periods) were evaluated of which flexible maintenance time windows showed the best results from a case study on a part of the Dutch railway network. With the aim to establish quantitative measures for comparing conflicting capacity requests from track maintenance and train traffic, Lidén & Joborn (2016) developed a model to dimension and assess maintenance windows. It considered effects on both maintenance costs and expected traffic demand of the timetable. A case study on the Northern Main Line (Sweden), a single track line, demonstrated their method.

2.2. Research Gap

The past years, many studies investigated aspects of railway maintenance planning, like combining the scheduling of the timetable together with maintenance works to reduce delays, and advanced maintenance scheduling to improve maintenance efficiency. Table 1 shows the topics of the literature reviewed together with their model objective. All researches that aimed at minimising traffic hindrance did this based on train travel times and delays. The issue of hindrance for parked trains is however not included in any of the researches whilst this is a rising issue due to the increasing amount of rolling stock that needs to be parked over night.

		Model Objective				
Authon	Train Timetable	Maintenance	Maintenance	Computational	Traffic	Maintenance
Author	Scheduling	Scheduling	Windows	Time	Delay	Efficiency
Higgins (1998)		Х			Х	
Cheung et al. (1999)		Х				Х
van Zante-de Fokkert		v	v			v
et al. (2007)		Λ	Λ			Λ
Albrecht et al. (2013)	Х	Х			Х	
Heinicke et al. (2015)		Х				Х
Lidén & Joborn			v			v
(2016)			Λ			Λ
Lidén & Joborn	x		x		x	x
(2017)	A		A			Λ
Sun et al. (2017)		Х				Х
Arenas et al. (2018)	X	Х			Х	
Durazo-Cardenas		x				v
et al. (2018)		A				Λ
Faris et al. (2018)				Х		
Lidén (2018)				Х		
Lidén et al. (2018)	X		Х		Х	Х
Meng et al. (2018)	Х	Х			Х	
Pour et al. (2018a)		X				Х
Pour et al. (2018b)		X				Х
Su & de Schutter		x				v
(2018)		Λ				Λ
Bababeik et al. (2019)	X		Х		Х	Х
D'Ariano et al. (2019)	X	Х			Х	Х
Su et al. (2019)		Х				Х
Zhang et al. (2019a)	X		Х		Х	
Zhang et al. (2019b)	X	Х			Х	Х

Table 1: Literature topics and model objectives

Regarding maintenance scheduling, different limitations were included in the reviewed literature. van Zantede Fokkert et al. (2007) considered that the capacity of work crews is limited at night and during weekends, Lidén et al. (2018) considered work time regulations, and Pour et al. (2018b) considered transportation time between task locations. Using these kind of constraints in a model, that besides efficient maintenance also aims at minimising hindrance for train operators and their parked trains, is however not researched.

The current methods used to create working zones for safe maintenance works are limited. Most studies only considered block sections when creating work zones (Arenas et al., 2018; D'Ariano et al., 2019; Meng et al., 2018; Zhang et al., 2019b) and the method of den Hertog et al. (2005) is also only based on the track layout of switches. These methods do not take into account the layout of the overhead wiring, whilst this is an important factor to take into account, since the power of the overhead wiring should be shut off when track workers are

working on some element of the infrastructure. Especially on yards this is an issue. Due to the necessity of switching off the power, parked trains need to be moved from the zone which is shut off, even if the particular tracks they are standing on do not need to be maintained at that moment. The reason for this is that service operators want their trains to have power at all times, also when parked to preheat passenger carriages in winter, pre-cool them in summer, and provide power for cleaning activities.

2.3. Research Questions

This thesis its research aims for an better balance between train traffic and maintenance management of railway networks, by optimising the design of the maintenance schedule by including traffic issues and limitations of maintenance contractors while distinguishing engineering fields. This aim was translated into the following research question:

What is the optimal design of the maintenance schedule when including hindrance for train operators and capacity constraints of maintenance crews in the design process of schedules?

To guide the research, several sub questions were formulated which together answer the research question. To answer the research questions, a new scheduling model is developed.

Train-free periods for maintenance have an impact on parked trains, since the power supply for trains via the overhead wiring system needs to be shut off during maintenance activities on power supply assets. Next to this, the maintenance schedule, also has impact on freight trains. To gain knowledge on the impact on the maintenance schedule when optimising for train traffic, the first sub question was stated.

(1) What is the impact on the performance of the maintenance schedule when optimising for train traffic?

An efficient maintenance schedule in the eyes of train operators and railway managers is not necessarily efficient for maintenance contractors. One could say this is not important for the asset owner as long as the railway system is kept on a satisfying performance level, but e.g. due to a lack of track workers, the constraints of maintenance contractors should be considered when creating a maintenance schedule. To gain knowledge on the impact on the maintenance schedule when optimising for maintenance contractors, the second sub question was stated.

(2) What is the impact on the performance of the maintenance schedule when optimising for maintenance contractors?

2.4. Contribution of the Thesis

This research of this thesis is close to the research of van Zante-de Fokkert et al. (2007), but improves it by considering hindrance for parked rolling stock, and by including more constraints that represent the limitations of work crews more realistic. Hereby, the impact on parked rolling stock can be better considered in maintenance schedules in the future and maintaining the assets of the railway system can become more efficient for work crews. The research also shows the benefits of distinguishing maintenance in the different engineering fields in railway maintenance, since this is strongly related to the hindrance caused by maintenance. All together, this research will complement the state-of-the-art on railway maintenance planning.

3. Problem Description

The problem investigated in this research is the creation of a maintenance schedule for regular, preventive maintenance activities that can be performed during nights, which is done to prevent large disturbances for train traffic. Since interviews can fill knowledge gaps (Wang et al., 2019; Whittle et al., 2019), several conversations with experts in the railway sector gave a broader idea of the problem. The insights given by the experts are presented in this chapter. This chapter also presents a basis model (van Zante-de Fokkert et al., 2007) that is used as a starting point to model the problem.

3.1. Interviews

In the Netherlands, the railway manager has performance-based contracts with maintenance contractors for infrastructure maintenance. Odolinski (2019) showed that these type of contracts, with incentives for contractors lead to less failures in the infrastructure. During the tender, Dutch maintenance contractors state in their bids how often they will maintain every infrastructure component. Whether or not they act conform this is not checked actively by the railway manager, but if a failure occurs, the contractor needs to prove that they actually executed the maintenance conform their maintenance plans. The local asset managers of the railway manager do not check what maintenance contractors do, but they do inspect the infrastructure to measure the performance levels. This is done based on three criteria: safety, availability and reliability, and durability. The inspections of the asset managers either take place outside the loading gauge during train services or in existing train free periods. Hence, these inspections are not considered in the problem.

Capacity managers of the railway manager are responsible for the distribution of the available capacity of the railway network. Capacity can be assigned to either train traffic or asset management (maintenance, replacements, and other projects). In principle, train traffic has a higher priority than asset management in the Netherlands, although maintenance is also important. Capacity managers would rather put contractors in difficult situations to perform their activities with train traffic possible, than make it easy for them which comes in hand with traffic blockades. Hindrance for train services should be minimized at all times. With a nightly maintenance schedule, hindrance for train service operators is still caused by train free periods on yards where trains are parked for the night. When the capacity of yards is assigned for maintenance, train service operators sometimes need to move their trains to other places or the power on overhead wiring is shut off. When the passage of a yard, or tracks between yards are blocked for all trains to perform maintenance operations this causes hindrance for freight trains. These freight trains either have to wait until the train free period has ended, or have to take a detour.

Just like for railway managers, safety is also important for maintenance contractors. Track workers must be able do their jobs without concerns on their safety. When maintaining components of the power supply infrastructure, power should be turned off and pantographs of parked trains need to be put down to avoid electrocution hazards. There are three different type of engineering fields in which work crews are active. These correspond to infrastructure parts of railway systems, namely switches, straight tracks, and overhead wiring.

3.2. Basis Model

As a starting point for the new scheduling model, the model of van Zante-de Fokkert et al. (2007) was used. Their problem was modelled as a MIP, as done by others in the past (Heinicke et al., 2015; Pour et al., 2018b; Zhang et al., 2019b). New parameters, variables and constraints are added to this basis model to compose the new maintenance scheduling model, which is explained in Chapter 4.

In the following, the model of van Zante-de Fokkert et al. (2007) is presented. An explanation to the objective function and constraints is given in Paragraph 3.2.5.

3.2.1. Sets

C contractors.

S single-track grids (STGs).

 $T = \{ \text{left, right} \}$ track sides.

N nights.

 $P \subset S \times S$ set of permitted STG combinations, i.e., $s_1, s_2 \in P$, $s_1 \neq s_2$ when STGs s_1 and s_2 can be combined during one night.

3.2.2. Parameters

 Λ weight factor that indicates whether the workload is measured by the number of switches ($\Lambda = 1$), by the number of kilometers ($\Lambda = 0$), or by a weighted combination of the two ($0 < \Lambda < 1$). binary parameter that indicates whether STG s can be assigned to night n ($R_{sn} = 1$), or not R_{sn}

$$(R_{sn}=0), s \in S, n \in N.$$

number of switches to be maintained on track side t of STG s by contractor $c, s \in S, t \in T, c \in C.$

number of kilometers to be maintained on track side t of STG s by contractor
$$c, s \in S, t \in T, c \in C.$$

 Q_c^{max} maximum number of switches that can be maintained per night by contractor $c, c \in C$.

M a large number.

3.2.3. Variables

binary variable that indicates whether night n is used in the schedule $(w_n = 1)$, or not $(w_n = 0)$, w_n $n \in N$.

binary variable that indicates whether track side t of STG s is assigned to night $n (x_{stn} = 1)$, or x_{stn} not $x_{stn} = 0, s \in S, t \in T, n \in N$.

- y_n maximum number of switches to be maintained per night by contractor c over all nights, $c \in C$.
- z_c maximum number of kilometers to be maintained per night by contractor c over all nights, $c \in C$.

3.2.4. Objective Function & Constraints

(P)	$\operatorname{Min}\sum_{c\in C}(\Lambda y_c + (1-\Lambda)z_c) + M\sum_{n\in N}w_n$		(3.2.0)
s.t.	$\sum_{n \in N} x_{stn} = 1$	$\forall s \in S, t \in T,$	(3.2.1)
	$\sum_{t \in T} x_{stn} \le R_{sn}$	$\forall s \in S, n \in N,$	(3.2.2)
	$\sum_{t \in T} x_{s_1 tn} + \sum_{t \in T} x_{s_2 tn} \le 1$	$\forall (s_1, s_2) \notin P, n \in N,$	(3.2.3)
	$\sum_{s \in S} \sum_{t \in T} x_{stn} Q_{stc} \le y_c$	$\forall c \in C, n \in N,$	(3.2.4)
	$\sum_{s \in S} \sum_{t \in T} x_{stn} V_{stc} \le z_c$	$\forall c \in C, n \in N,$	(3.2.5)
	$y_c \leq Q_c^{\max}$	$\forall c \in C,$	(3.2.6)
	$\frac{1}{2 S } \sum_{s \in S} \sum_{t \in T} x_{stn} \le w_n$	$\forall n \in N,$	(3.2.7)
	$w_n \in \{0, 1\}$	$\forall n \in N.$	(3.2.8)
	$x_{stn} \in \{0, 1\}$	$\forall s \in S, t \in T, n \in N,$	(3.2.9)

3.2.5. Explanation

The objective function (3.2.0) consists of two parts and minimises both the sum of the maximum scheduled workload of contractors and the number of nights with planned maintenance. Constraints (3.2.1) and (3.2.2) ensure that both the left and right tracks of all STGs are assigned to one allowed night and these must be different nights. Constraint (3.2.3) ensures that only STGs that are combinable are assigned to the same night. Constraints (3.2.4) and (3.2.5) determine respectively the maximum number of switches and the maximum number of kilometers to be maintained per night by a contractor. Constraint (3.2.6) limits the number of switches that a contractor can maintain in one night. Constraint (3.2.7) determines whether or not STGs are assigned in a particular night in the schedule. The binary variable w_n is forced to be one when at least one STG is assigned to night n. By dividing the left side of this equation by the total number of STGs, this side is either one or zero. Constraints (3.2.8) and (3.2.9) ensure that the used nights and the decision variable for the assignment are binary.

4. Mathematical Model

The newly developed model formulation in this research is based on that of van Zante-de Fokkert et al. (2007)(see Section 3.2) and is also a MIP. In the following sections the new model is presented and explained. A complete overview of the model can be found in Appendix A. This chapter also gives a description of the validation of an implementation of the model, the results of computational tests, and the potential of indicator constraints.

4.1. Sets

Like van Zante-de Fokkert et al. (2007), the problem consists of work zones (similar to STGs and track sides) in which maintenance operations need to be performed by maintenance contractors during train free periods at night. To make the problem more specific, maintenance crews are introduced and also train operators, since the hindrance considered is caused to their trains. The used sets are listed below.

- Z work zones.
- N nights.
- C maintenance crews.
- O train operators.

4.2. Parameters

Different than the model of van Zante-de Fokkert et al. (2007), the problem now distinguishes maintenance activities on switches, straight tracks and overhead wiring. It is predefined how many maintenance operations need to be performed in the available nights. Like in the model of van Zante-de Fokkert et al. (2007), for every zone it is known how many infrastructure is present and the amount of operations a maintenance crew can perform in a night is limited.

- Q_z total number of switches present in work zone $z, z \in Z$.
- S_z total length of straight tracks present in work zone $z, z \in Z$.
- B_z total length of overhead wiring present in work zone $z, z \in Z$.
- DQ_z total number of switch maintenance operations to be performed in work zone $z, z \in Z$.
- DS_z total length of straight track maintenance operations to be performed in work zone $z, z \in Z$.
- DB_z total length of overhead wiring maintenance operations to be performed in work zone $z, z \in Z$.
- Q_c^{\max} maximum number of switches that can be maintained in a night by crew $c, c \in C$.
- S_c^{\max} maximum length of straight tracks that can be maintained in a night by crew $c, c \in C$.
- B_c^{\max} maximum length of overhead wiring that can be maintained in a night by crew $c, c \in C$.

A work crew needs to be specialized in a specific engineering field to be able to maintain those parts of the infrastructure. Also, a network can be split up into different regions. In such a case, not all zones are necessarily being maintained by every crew. Therefore, it needs to be indicated for every zone which crews can be scheduled to perform maintenance operations on its infrastructure. Furthermore, not all work zones are necessarily available every night and combining multiple work zones in one night may be prohibited for certain combinations. The latter can also be used to represent a limitation of work crews, namely that they can only work in multiple zones when these are e.g. adjacent zones. binary parameter that indicates whether the switches in work zone z can be maintained by crew c F_{zc}^{γ} $(F\gamma_{zc} = 1)$, or not $(F\gamma_{zc} = 0)$, $z \in Z$, $c \in C$.

- F_{zc}^{μ} binary parameter that indicates whether the straight tracks in work zone z can be maintained by crew c ($F\mu_{zc} = 1$), or not ($F\mu_{zc} = 0$), $z \in Z$, $c \in C$.
- $F_{zc}^{\delta} \quad \begin{array}{l} \text{binary parameter that indicates whether the overhead wiring in work zone z can be maintained by \\ \text{crew c} (F\delta_{zc}=1), \text{ or not } (F\delta_{zc}=0), \ z\in Z, \ c\in C. \end{array}$

$$\begin{array}{l} \mbox{binary parameter that indicates whether work zone z can be assigned to night n ($R_{zn}=1$), or not$ $($R_{zn}=0$), $z\in Z$, $n\in N$. \end{array}$$

binary parameter that indicates whether work zones
$$i$$
 and j may be combined in night n ($P_{nij} = 0$),
 P_{nij} or not ($P_{nij} = 1$), when $i = j$ ($P_{nij} = 0$), $n \in N$, $i \in Z$, $j \in Z$.

New is the amount of hindrance for train operators, that may differ per zone and per night depending on what infrastructure part is being maintained. There are three options in the amount of hindrance: direct hindrance (=1), indirect hindrance (=0.5), and no hindrance (=0) for train operators. Indirect hindrance is hindrance caused to an adjacent zone of the zone which is being maintained. Also, a parameter is added to limit the number of nights that can be used by the model. To determine the weight of the workload for maintenance crews and the hindrance for train operators in the objective function, there are weight parameters included which are 1 by default.

$$\begin{split} & \underset{ozn}{\text{hindrance for train operator } o \text{ when work zone } z \text{ is maintained on switches during night } n, o \in O, \\ & z \in Z, \, n \in N. \end{split}$$

 $\begin{aligned} & \text{hindrance for train operator } o \text{ when work zone } z \text{ is maintained on straight tracks during night } n, \\ & o \in O, \, z \in Z, \, n \in N. \end{aligned}$

 $H_{ozn}^{\delta} \qquad \mbox{hindrance for train operator } o \mbox{ when work zone } z \mbox{ is maintained on overhead wiring during night} \\ n, \ o \in O, \ z \in Z, \ n \in N.$

 N^{\max} maximum number of nights that may be used for maintenance in the schedule.

- Λ^{γ} weight parameter for the workload of switches.
- Λ^{μ} weight parameter for the workload of straight tracks.
- Λ^{δ} weight parameter for the workload of overhead wiring.
- Λ^{ϕ} weight parameter for the hindrance for operators.
- M a large number.

4.3. Variables

van Zante-de Fokkert et al. (2007) only used variables to indicate the usage of nights for maintenance, the assignment of tracks to nights, and to keep track of the highest workloads for contractors. Now, various binary variables are needed to indicate whether or not infrastructure in a work zone is maintained, whether crews are working on a type of infrastructure part, and which contractor maintains which zone in which night.

binary variable that indicates whether work zone z is assigned to night n for any maintenance x_{zn} $(x_{zn} = 1)$, or not $(x_{zn} = 0)$, $z \in Z$, $n \in N$.

$$\begin{array}{l} \begin{array}{l} \text{binary variable that indicates whether work zone z is assigned to night n for maintenance on x_{zn}^{γ} \\ \end{array} \\ \begin{array}{l} \text{switches } (x_{zn}^{\gamma}=1), \, \text{or not } (x_{zn}^{\gamma}=0), \, z \in Z, \, n \in N. \end{array} \end{array}$$

 $\begin{array}{l} \underset{x_{zn}^{\mu}}{\overset{\mu}{x_{zn}}} & \text{binary variable that indicates whether work zone z is assigned to night n for maintenance on straight tracks $(x_{zn}^{\mu}=1)$, or not $(x_{zn}^{\mu}=0)$, $z \in Z$, $n \in N$. } \end{array}$

- x_{zn}^{δ} binary variable that indicates whether work zone z is assigned to night n for maintenance on overhead wiring $(x_{zn}^{\delta} = 1)$, or not $(x_{zn}^{\delta} = 0)$, $z \in Z$, $n \in N$.
- binary variable that indicates whether crew c maintains switches in night n ($w_{cn}^{\gamma} = 1$), or not $(w_{cn}^{\gamma} = 0), n \in N, c \in C$.
- $\begin{aligned} & \underset{cn}{\text{binary variable that indicates whether crew } c \text{ maintains straight tracks in night } n \; (w_{cn}^{\mu} = 1), \text{ or not} \\ & (w_{cn}^{\mu} = 0), \; n \in N, \; c \in C. \end{aligned}$
- $w_{cn}^{\delta} \quad \mbox{binary variable that indicates whether crew c maintains overhead wiring in night n ($w_{cn}^{\delta}=1$), or $not ($w_{cn}^{\delta}=0$), $n \in N$, $c \in C$. }$
- $\begin{array}{l} \underset{czn}{\text{binary variable that indicates whether crew c maintains the switches of zone z in night n ($v_{czn}^{\gamma}=1$), v_{czn}^{γ} or not ($v_{czn}^{\gamma}=0$), $c\in C$, $z\in Z$, $n\in N$.} \end{array}$
- $\begin{array}{l} \text{binary variable that indicates whether crew c maintains the straight tracks of zone z in night n}\\ v_{czn}^{\mu} & (v_{czn}^{\mu}=1), \, \text{or not} \; (v_{czn}^{\mu}=0), \, c \in C, \, z \in Z, \, n \in N. \end{array}$
- $\begin{array}{l} \underset{czn}{\text{binary variable that indicates whether crew c maintains the overhead wiring of zone z in night n}\\ (v_{czn}^{\delta}=1), \, \text{or not } (v_{czn}^{\delta}=0), \, c\in C, \, z\in Z, \, n\in N. \end{array}$

Also, there are three variables that indicate how much infrastructure is maintained in a night and three other variables are used like in the model of van Zante-de Fokkert et al. (2007) to keep track of the highest workload in a night. Furthermore, a variable is needed to keep track of the highest hindrance caused by maintenance operations and another one to keep track of the usage of nights.

variable that indicates the number of switches maintained in zone z night n by crew $c, z \in Z$, q_{czn} $n \in N$.

variable that indicates the length of straight tracks maintained in zone z in night n by crew c, s_{czn} $z \in Z, n \in N.$

variable that indicates the length of overhead wiring maintained in zone z in night n by crew c, $z \in Z, n \in N.$

- y_c largest number of switches to be maintained in one night by crew c over all nights, $c \in C$.
- largest number of kilometers of straight tracks to be maintained in one night by crew c over all u_c nights, $c \in C$.
- $\begin{array}{l} \text{largest number of kilometers of overhead wiring to be maintained in one night by crew c over all d_c $ nights, $c \in C$. \\ \end{array}$
- h_{ozn} largest hindrance for operator o in night n caused by maintenance in zone $z, o \in O, z \in Z, n \in N$. variable that indicates whether night n is assigned for any maintenance $(t_n = 1)$ or not $(t_n = 0)$, $n \in N$.

4.4. Objective Function

The objective function (4.0) aims to minimise the workload for work crews and the hindrance for train operators by minimising the maximum workload of all crews combined and the sum of the hindrance for train operators over all zones and nights. To determine the maximum workload for crews working e.g. on switches, their largest number of switches to be maintained in one night is divided by the maximum number of switches they are able to maintain in a night. This way, the maximum workload of a work crew is measured relatively to its capacity, to ensure equality among work crews working in different engineering fields or having different capacities. The weight parameters can be used to determine the balance between the workload of maintenance crews and the hindrance for train operators.

$$(P) \quad \text{Minimise} \quad \sum_{c \in C} (\Lambda^{\gamma} \frac{y_c}{Q_c^{\max}} + \Lambda^{\mu} \frac{u_c}{S_c^{\max}} + \Lambda^{\delta} \frac{d_c}{B_c^{\max}}) + \Lambda^{\phi} \sum_{o \in O} \sum_{z \in Z} \sum_{n \in N} (h_{ozn})$$
(4.0)

4.5. Constraints

All maintenance operations that are demanded should be performed within the available nights. Constraints (4.1) to (4.3) ensure this by setting the sum of performed maintenance operations over all nights equal to the demand. Obviously, the amount of infrastructure maintained in one night in a zone cannot be more than is present in that zone. This is ensured by constraints (4.4) to (4.6). Constraints (4.7) to (4.9) ensure that the variables are restricted to their allowed values. Switches can only be entirely maintained in a night, therefore q_{czn} may only take integer values.

$$\sum_{c \in C} \sum_{n \in N} (q_{czn}) = DQ_z \qquad \qquad \forall z \in Z,$$
(4.1)

$$\sum_{c \in C} \sum_{n \in N} (s_{czn}) = DS_z \qquad \forall z \in Z,$$

$$\sum_{c \in C} \sum_{n \in N} (b_{czn}) = DB_z \qquad \forall z \in Z,$$
(4.2)
(4.3)

$$q_{czn} \leq Q_z$$
 $\forall c \in C, z \in Z, n \in N,$ (4.4) $s_{czn} \leq S_z$ $\forall c \in C, z \in Z, n \in N,$ (4.5) $b_{czn} \leq B_z$ $\forall c \in C, z \in Z, n \in N,$ (4.6) $q_{czn} \in \mathbb{Z}^*$ $\forall c \in C, z \in Z, n \in N,$ (4.7) $s_{czn} \in \mathbb{R}^+$ $\forall c \in C, z \in Z, n \in N,$ (4.8)

$$b_{czn} \in \mathbb{R}^+ \qquad \qquad \forall c \in C, \ z \in Z, \ n \in N,$$
(4.9)

When maintenance operations are performed in a zone during a night, the zone should be noted as used. Also, when no maintenance activities take place, the work zone should not be noted as used. These two statements can logically be expressed by: $x_{zn}^{\gamma} = 0 \Leftrightarrow q_{czn} = 0$ for switch maintenance, by: $x_{zn}^{\mu} = 0 \Leftrightarrow s_{czn} = 0$ for straight track maintenance, and by : $x_{zn}^{\delta} = 0 \Leftrightarrow b_{czn} = 0$ for overhead wiring maintenance. Constraints (4.10) and (4.13) ensure this for switches, constraints (4.11) and (4.14) ensure this for straight tracks, and constraints (4.12) and (4.15) ensure this for overhead wiring. Constraints (4.16) to (4.18) restrict the variables to be binary.

$$\begin{split} \sum_{c \in C} (q_{czn}) &\leq x_{zn}^{\gamma} \cdot M & \forall z \in Z, n \in N, \\ \sum_{c \in C} (s_{czn}) &\leq x_{zn}^{\mu} \cdot M & \forall z \in Z, n \in N, \\ \sum_{c \in C} (b_{czn}) &\leq x_{zn}^{\delta} \cdot M & \forall z \in Z, n \in N, \\ \sum_{c \in C} (b_{czn}) &\leq x_{zn}^{\delta} \cdot M & \forall z \in Z, n \in N, \\ x_{zn}^{\gamma} &\leq \sum_{c \in C} (q_{czn}) & \forall z \in Z, n \in N, \\ x_{zn}^{\mu} &\leq \sum_{c \in C} (s_{czn}) \cdot M & \forall z \in Z, n \in N, \\ x_{zn}^{\delta} &\leq \sum_{c \in C} (b_{czn}) \cdot M & \forall z \in Z, n \in N, \\ x_{zn}^{\gamma} &\in \{0,1\} & \forall z \in Z, n \in N, \\ x_{zn}^{\delta} &\in \{0,1\} & \forall x_{zn}^{\delta} &\in \{0,1\} & \forall x_{zn}^{\delta} &\in \{0,1\} & \forall x_{zn}^{$$

Work zones can only be used for maintenance activities when they are allowed to be planned train free in that night. This can be logically expressed by: $R_{zn} = 0 \Rightarrow x_{zn}^{\gamma} = 0$, $R_{zn} = 0 \Rightarrow x_{zn}^{\mu} = 0$, and $R_{zn} = 0 \Rightarrow x_{zn}^{\delta} = 0$. Constraints (4.19) to (4.21) ensure this for respectively switch maintenance, straight track maintenance, and overhead wiring maintenance.

$$\chi_{zn}^{\gamma} \le R_{zn} \qquad \qquad \forall z \in Z, \, n \in N, \tag{4.19}$$

$$\forall z \in Z, \, n \in N, \tag{4.20}$$

$$\forall z \in Z, \, n \in N,$$

$$(4.21)$$

When a work zone is used for one or more types of maintenance in one night, variable x_{zn} should be set to 1. If a zone is not used for any type of maintenance it should be set to 0. This can be logically expressed by: $x_{zn}^{\gamma} + x_{zn}^{\mu} + x_{zn}^{\delta} = 0 \Leftrightarrow x_{zn} = 0$. Constraints (4.22) and (4.23) ensure this.

 $x_{zn}^{\gamma} + x_{zn}^{\mu} + x_{zn}^{\delta} \le x_{zn} \cdot M \qquad \qquad \forall z \in \mathbb{Z}, n \in \mathbb{N},$ (4.22)

$$x_{zn} \le x_{zn}^{\gamma} + x_{zn}^{\mu} + x_{zn}^{\delta} \qquad \qquad \forall z \in \mathbb{Z}, \, n \in \mathbb{N},$$

$$(4.23)$$

In a work zone, each type of infrastructure parts can only be maintained by one crew a night. Constraints (4.24) to (4.26) ensure this by setting the sum of variables v_{czn}^{γ} , v_{czn}^{μ} , and v_{czn}^{δ} over all contractors equal to variables x_{zn}^{γ} , x_{zn}^{μ} , and x_{zn}^{δ} per zone and night. Only crews that are allowed to maintain that type of infrastructure parts in a work zone can maintain it. This can be logically expressed by: $F_{zc}^{\gamma} = 0 \Rightarrow v_{czn}^{\gamma} = 0$, $F_{zc}^{\mu} = 0 \Rightarrow v_{czn}^{\ell} = 0 \Rightarrow v_{czn}^{\delta} = 0 \Rightarrow v_{czn}^{\delta} = 0$. Constraints (4.27) to (4.29) ensure this for respectively switch maintenance, straight track maintenance, and overhead wiring maintenance. Constraints (4.30) to (4.32) restrict the variables to be binary.

$$\sum_{c \in C} (v_{czn}^{\gamma}) = x_{zn}^{\gamma} \qquad \forall z \in Z, \ n \in N,$$

$$(4.24)$$

$$\sum_{c \in C} (v_{czn}^{\mu}) = x_{zn}^{\mu} \qquad \forall z \in Z, \ n \in N,$$
(4.25)

$$\sum_{c \in C} (v_{czn}^{\delta}) = x_{zn}^{\delta} \qquad \forall z \in Z, n \in N,$$

$$v_{crm}^{\gamma} \leq F_{\gamma}^{\gamma} \qquad \forall c \in C, z \in Z, n \in N,$$

$$(4.26)$$

$$\forall c \in C, z \in Z, n \in N,$$

$$(4.27)$$

$$v_{czn}^{\mu} \le F_{zc}^{\mu} \qquad \qquad \forall c \in C, \ z \in Z, \ n \in N,$$

$$(4.28)$$

$$\forall c \in C, \ z \in Z, \ n \in N, \tag{4.29}$$

$$v_{czn}^{\gamma} \in \{0,1\} \qquad \forall c \in C, \ z \in Z, \ n \in N,$$

$$v_{czn}^{\mu} \in \{0,1\} \qquad \forall c \in C, \ z \in Z, \ n \in N,$$

$$\forall c \in C, \ z \in Z, \ n \in N.$$

$$(4.30)$$

 $v_{czn}^{\delta} \le F_{zc}^{\delta}$

$$v_{czn}^{\delta} \in \{0,1\} \qquad \qquad \forall c \in C, \ z \in Z, \ n \in N,$$

$$(4.31)$$

$$\forall c \in C, \ z \in Z, \ n \in N,$$

$$(4.32)$$

When a maintenance crew performs any maintenance operations in a zone during a night, it should be noted that the crew is working in that zone during that night. Also, when a maintenance crew does not work in a zone during a night, it should be noted that the crew is not working in that zone during that night. These two statements can logically be expressed by: $v_c^{\gamma} = 0 \Leftrightarrow q_{czn} = 0$ for switch maintenance, by: $v_{czn}^{\mu} = 0 \Leftrightarrow s_{czn} = 0$ for straight track maintenance, and by : $v_{czn}^{\delta} = 0 \Leftrightarrow b_{czn} = 0$ for overhead wiring maintenance. Constraints (4.33) and (4.36) ensure this for switches, constraints (4.34) and (4.37) ensure this for straight tracks, and constraints (4.35) and (4.38) ensure this for overhead wiring.

$$q_{czn} \le v_{czn}^{\gamma} \cdot M \qquad \qquad \forall c \in C, \ z \in Z, \ n \in N,$$

$$(4.33)$$

$$s_{czn} \le v_{czn}^{\mu} \cdot M \qquad \qquad \forall c \in C, \ z \in Z, \ n \in N,$$

$$b_{czn} \le v_{czn}^{\delta} \cdot M \qquad \qquad \forall c \in C, \ z \in Z, \ n \in N,$$

$$(4.34)$$

$$\forall c \in C, \ z \in Z, \ n \in N,$$

$$(4.35)$$

$$v_{czn}^{\gamma} \le q_{czn} \qquad \qquad \forall c \in C, \ z \in Z, \ n \in N,$$
(4.36)

- $v_{czn}^{\mu} \le s_{czn} \cdot M \qquad \qquad \forall c \in C, \ z \in Z, \ n \in N,$ (4.37)
- $v_{czn}^{\delta} \le b_{czn} \cdot M \qquad \qquad \forall c \in C, \ z \in Z, \ n \in N,$ (4.38)

When a maintenance crew is not maintaining infrastructure in any zone during a night, that night should not be noted as a work night for that crew. This statement can logically be expressed by: $\sum_{z \in Z} (v_{czn}^{\gamma}) =$ $0 \Leftrightarrow w_{cn}^{\gamma} = 0$ for switch maintenance, by: $\sum_{z \in Z} (v_{czn}^{\mu}) = 0 \Leftrightarrow w_{cn}^{\mu} = 0$ for straight track maintenance, and by : $\sum_{z \in Z} (v_{czn}^{\delta}) = 0 \Leftrightarrow w_{cn}^{\delta} = 0$ for overhead wiring maintenance. Furthermore, when a maintenance crew is maintaining infrastructure in at least one zone during a night, that night should be noted as a work night. This statement can logically be expressed by: $\sum_{z \in Z} (v_{czn}^{\gamma}) > 0 \Leftrightarrow w_{cn}^{\gamma} = 1$ for switch maintenance, by: $\sum_{z \in Z} (v_{czn}^{\mu}) > 0 \Leftrightarrow w_{cn}^{\mu} = 1$ for straight track maintenance, and by: $\sum_{z \in Z} (v_{czn}^{\delta}) > 0 \Leftrightarrow w_{cn}^{\delta} = 1$ for overhead wiring maintenance. Constraints (4.39) to (4.44) ensure these logical expressions. Also, constraint (4.45) excludes that crews work on more than one type of infrastructure part in one night. Constraints (4.46) to (4.48) restrict the variables to be binary.

$$\sum_{z \in Z} (v_{czn}^{\gamma}) \le w_{cn}^{\gamma} \cdot M \qquad \qquad \forall c \in C, \ n \in N,$$
(4.39)

$$\begin{split} \sum_{z \in Z} (v_{czn}^{\mu}) &\leq w_{cn}^{\mu} \cdot M & \forall c \in C, n \in N, \end{split} \tag{4.40} \\ \sum_{z \in Z} (v_{czn}^{\delta}) &\leq w_{cn}^{\delta} \cdot M & \forall c \in C, n \in N, \end{aligned} \tag{4.41} \\ w_{cn}^{\gamma} &\leq \sum_{z \in Z} (v_{czn}^{\gamma}) & \forall c \in C, n \in N, \end{aligned} \tag{4.42} \\ w_{cn}^{\mu} &\leq \sum_{z \in Z} (v_{czn}^{\delta}) & \forall c \in C, n \in N, \end{aligned} \tag{4.43} \\ w_{cn}^{\delta} &\leq \sum_{z \in Z} (v_{czn}^{\delta}) & \forall c \in C, n \in N, \end{aligned} \tag{4.44} \\ w_{cn}^{\gamma} + w_{cn}^{\mu} + w_{cn}^{\delta} \leq 1 & \forall c \in C, n \in N, \end{aligned} \tag{4.45} \\ w_{cn}^{\gamma} &\in \{0, 1\} & \forall c \in C, n \in N, \end{aligned} \tag{4.46} \\ w_{cn}^{\mu} &\in \{0, 1\} & \forall c \in C, n \in N, \end{aligned} \tag{4.47} \\ w_{cn}^{\delta} &\in \{0, 1\} & \forall c \in C, n \in N, \end{aligned}$$

It is possible that some combinations of work zones are not allowed in the same night. Since $P_{nij} = 1$ when a combination is not allowed, constraint (4.49) ensures that in this case only one of the two zones can be used for maintenance that night. Note that it is possible to combine three or more zones when all individual combinations are allowed.

$$P_{nij}(x_{in} + x_{jn}) \le 1 \qquad \qquad \forall n \in \mathbb{N}, \ i \in \mathbb{Z}, \ j \in \mathbb{Z},$$

$$(4.49)$$

In order to determine the highest workload in a night for a crew, constraints (4.50) to (4.52) sum all maintenance operations performed in a night per crew. Constraints (4.53) to (4.55) ensure that the variables are restricted to their allowed values, the same as q_{czn} , s_{czn} , and b_{czn} .

$\sum_{z \in Z} (q_{czn}) \le y_c$	$\forall c \in C, n \in N,$	(4.50)
$\sum_{z \in Z} (s_{czn}) \le u_c$	$\forall c \in C, n \in N,$	(4.51)
$\sum_{z \in Z} (b_{czn}) \le d_c$	$\forall c \in C, n \in N,$	(4.52)
$y_c \in \mathbb{Z}^*$	$\forall c \in C,$	(4.53)
$u_c \in \mathbb{R}^+$	$\forall c \in C,$	(4.54)
$d_c \in \mathbb{R}^+$	$\forall c \in C,$	(4.55)

In order to prevent that a work crew has to perform more maintenance operations in a night than possible, y_c , u_c , and d_c are restricted to the given maximum workload per crew by constraints (4.56) to (4.58).

$$y_c \le Q_c^{\max} \qquad \qquad \forall c \in C, \tag{4.56}$$

$$u_c \le S_c^{\max} \qquad \qquad \forall c \in C, \tag{4.57}$$

$$d_c \le B_c^{\max} \qquad \qquad \forall c \in C, \tag{4.58}$$

To avoid addition of hindrance when in a zone multiple infrastructure parts are maintained in a night, only the highest hindrance should be considered. Constraints (4.59) to (4.61) ensure this and constraint (4.62) restricts the variable to its allowed values.

$$x_{zn}^{\gamma} \cdot H_{ozn}^{\gamma} \le h_{ozn} \qquad \qquad \forall o \in O, \ z \in Z, \ n \in N,$$

$$(4.59)$$

$$\begin{aligned} x_{zn}^{\mu} \cdot H_{ozn}^{\mu} &\leq h_{ozn} & \forall o \in O, \ z \in Z, \ n \in N, \\ x_{zn}^{\delta} \cdot H_{ozn}^{\delta} &\leq h_{ozn} & \forall o \in O, \ z \in Z, \ n \in N, \end{aligned}$$

$$(4.60)$$

$$h_{ozn} \in \mathbb{R}^+ \qquad \qquad \forall o \in O, \ z \in Z, \ n \in N,$$

$$(4.62)$$

To avoid that all available nights will be scheduled, the number of nights with scheduled maintenance needs to be limited. For this, it is needed to keep track of the usage of nights. This means that if any maintenance is scheduled to a zone in a night, that night should be noted as being used. This statement can logically be expressed by: $\sum_{z \in Z} (x_{zn}) \ge 1 \Rightarrow t_n = 1$. Constraint 4.63 ensures this. The final constraint, constraint 4.64 ensures that the total number of nights used in the schedule does not exceed the maximum.

$$\sum_{z \in Z} (x_{zn}) \le t_n \qquad \qquad \forall n \in N, \tag{4.63}$$

$$\sum_{n \in N} (t_n) \le N^{\max}. \tag{4.64}$$

4.6. Implementation Validation

To be able to create new maintenance schedules with the new model, it is implemented in AIMMS (2019) and solved using CPLEX 12.9, an acknowledged solver. A test scenario is optimised by hand to validate the implementation of the model. This process is demonstrated in the following. The input for this small test scenario is given in Table 2.

To find an optimal solution for a case, certain steps are followed. Since the objective function 4.0 aims at minimising workload and hindrance, these are the two parts to focus on. To start, all maintenance operations

Table 2: Input parameters test scenario																	
Z	4	2		$c \rightarrow$	1	2	3		R_{zn}	$z\downarrow n\rightarrow$	1	2	3	4	5	6	7
N	-	7		Q_c^{\max}	3	3	3			1	1	1	1	1	1	0	1
C		3		S_c^{\max}	4.0	4.0	4.0			2	1	1	1	1	1	0	1
O	-	2		B_c^{\max}	5.0	5.0	5.0	H_{ozn}^{γ}	$o\downarrow$	$z\downarrow n\rightarrow$	1	2	3	4	5	6	7
$z \rightarrow$	1	2	F_{zc}^{γ}	$z\downarrow c\rightarrow$	1	2	3		1	1	0	0	0	0	0	0	0
Q_z	2	2		1	1	0	0			2	0	0	0	0	0	0	0
S_z	2.0	2.0		2	1	0	0		2	1	0	1	1	1	1	0	0
B_z	2.5	2.5	F^{μ}_{zc}	$z\downarrow c\rightarrow$	1	2	3			2	0	0	0	0	0	0	0
$z \rightarrow$	1	2		1	0	1	0	H^{μ}_{ozn}	$o\downarrow$	$z\downarrow n\rightarrow$	1	2	3	4	5	6	7
DQ_z	2	2		2	0	1	0		1	1	0	0	0	0	0	0	0
DS_z	2.0	2.0	F_{zc}^{δ}	$z\downarrow c\rightarrow$	1	2	3			2	1	1	1	1	1	1	1
DB_z	2.5	2.5		1	0	0	1		2	1	0	1	1	1	1	0	0
Λ^{γ}	1	.0		2	0	0	1			2	0	0	0	0	0	0	0
Λ^{μ}	1	.0	$P_{n,i}$	$P_{n,i,j(\forall n \in N)} i \downarrow j \rightarrow$		1	2	H_{ozn}^{δ}	$o\downarrow$	$z\downarrow n\rightarrow$	1	2	3	4	5	6	7
Λ^{δ}	1	.0			1	0	0		1	1	1	1	1	1	1	1	1
Λ^{ϕ}	1	.0			2	0	0			2	1	1	1	1	1	1	1
N^{\max}	į	5	L						2	1	0	1	1	1	1	0	0
										2	0	0	0	0	0	0	0

able 2: Input parameters test scenar	able	2: Input	parameters	test	scenari	0
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demanded are planned in the least number of nights possible, considering the nights which cause the lowest hindrance. Then, the workload is lowered by planning maintenance operations in extra nights, considering the lowest increase in hindrance. At a certain step, the increase in hindrance is higher than the decrease in workload. At this point, the optimal solution is found in the previous step and exploring more options is not necessary since these would all yield worse results.

When following these steps to find the optimal solution for the test scenario, this means that the maintenance operations on switches are planned in nights 1 and 7 (the capacity of crew 1 is not high enough to perform all demanded operations in a single night) and the maintenance operations on straight track and overhead wiring are planned in night 1 or 7 (these nights have equal hindrance for the train operators). This way, all operations are planned in nights causing the least amount of hindrance. The result of this first step is shown in Table 3.

			Ta	ble 3: Test scenario: s	tep 1.	
	Oper	ations	czn	Workload	Hindrance	
q_{111}	2	q_{127}	2	$\sum_{c \in C} \left(\Lambda^{\gamma} \frac{y_c}{Q_c^{\max}} \right)$	$ \Lambda^{\phi} \sum_{o \in O} \sum_{z \in Z} \sum_{n \in N} (h_{ozn}) $	
				= 0.67	= 2	
s ₂₁₁	2.0	s_{221}	2.0	$\sum_{c \in C} \left(\Lambda^{\mu} \frac{u_c}{S_c^{\max}} \right)$		
				= 1		
b ₃₁₁	2.5	b_{321}	2.5	$\sum_{c \in C} \left(\Lambda^{\delta} \frac{d_c}{B_c^{\max}} \right)$		
				= 1		
Result Objective Function			Function	0.67 + 1 + 1 + 2 = 4.67		

Table 3: Test scenario: step

The next step is to take extra nights to plan maintenance in, but with a minimal increase in hindrance. All maintenance operations on switches can be planned in individual nights without causing any hindrance. Also, the maintenance operations on straight tracks and overhead wiring in zone two can be moved to another night without causing an increase in hindrance. The result of this step is shown in Table 4.

			Ta	ble 4: Test scenario: s	tep 2.
	Oper	ations	czn	Workload	Hindrance
q_{111}	1	q_{122}	1	$\sum_{c \in C} \left(\Lambda^{\gamma} \frac{y_c}{Q_c^{\max}} \right)$	$ \Lambda^{\phi} \sum_{o \in O} \sum_{z \in Z} \sum_{n \in N} (h_{ozn}) $
q_{117}	1	q_{123}	1	= 0.33	= 2
s ₂₁₁	2.0	s_{222}	2.0	$\sum_{c \in C} \left(\Lambda^{\mu} \frac{u_c}{S_c^{\max}} \right)$	
				= 0.5	
b ₂₁₁	2.5	b_{222}	2.5	$\sum_{c \in C} \left(\Lambda^{\delta} \frac{d_c}{B_c^{\max}} \right)$	
				= 0.5	
Resul	lt Obj	ective	Function	0.33 +	0.5 + 0.5 + 2 = 3.33

The next step to lower the workload at a minimal increase in hindrance, is to plan the maintenance operations for straight tracks and overhead wiring in extra nights. If one extra night would be used, the workload would decrease with 0.33 and hindrance would increase with 2. If two extra nights would be used, the workload would decrease with 0.5 and the hindrance would still increase with 2. The latter option is preferred in this case. The result of this third step is shown in Table 5.

As can be seen, the result of step 3 is worse then the result of step 2. This means that it is not needed to

Table 5: Test scenario: step 3.													
	Opera	tions $_{cz}$	n	Workload	Hindrance								
q_{111}	1	q_{122}	1	$\sum_{c \in C} \left(\Lambda^{\gamma} \frac{y_c}{Q_c^{\max}} \right)$	$ \Lambda^{\phi} \sum_{o \in O} \sum_{z \in Z} \sum_{n \in N} (h_{ozn}) $								
q_{117}	1	q_{123}	1	= 0.33	= 4								
s_{211}	1.0	s_{222}	1.0	$\sum_{c \in C} \left(\Lambda^{\mu} \frac{u_c}{S_c^{\max}} \right)$									
s_{217}	1.0	s_{223}	1.0	= 0.25									
b_{311}	1.25	b_{322}	1.25	$\sum_{c \in C} \left(\Lambda^{\delta} \frac{d_c}{B_c^{\max}} \right)$									
b_{317}	1.25	b_{323}	1.25	= 0.25									
Resul	lt Obje	ctive F	unction	0.33 + 0	0.25 + 0.25 + 4 = 4.83								

proceed with an extra step, since that would also yield a worse result. To show this, an extra step is executed of which the results can be found in Table 6. As can be seen, the result of step 4 is even worse.

			Ta	ble 6: Test scenario: s	tep 4.					
	Oper	ations	czn	Workload	Hindrance					
<i>q</i> ₁₁₁	1	q_{122}	1	$\sum_{c \in C} \left(\Lambda^{\gamma} \frac{y_c}{Q_c^{\max}} \right)$	$ \Lambda^{\phi} \sum_{o \in O} \sum_{z \in Z} \sum_{n \in N} (h_{ozn}) $					
q_{117}	1	q_{123}	1	= 0.33	=7					
s ₂₁₁	0.8	s_{222}	0.8	$\sum_{c \in C} \left(\Lambda^{\mu} \frac{u_c}{S_c^{\max}} \right)$						
s ₂₁₄	0.4	s_{223}	0.8	= 0.2						
s ₂₁₇	0.8	s_{224}	0.4							
b ₃₁₁	1.0	b_{322}	1.0	$\sum_{c \in C} \left(\Lambda^{\delta} \frac{d_c}{B_c^{\max}} \right)$						
b ₃₁₄	0.5	b_{323}	1.0	= 0.2						
b ₃₁₇	1.0	b_{324}	0.5							
Resul	lt Obj	ective	Function	0.33 +	0.2 + 0.2 + 7 = 7.73					

Table 6: Te	t scenario:	step
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The result of the schedule found at step 2 is the lowest and this is equal to the result of the optimal schedule found by the solver. This means that the model is correctly implemented in AIMMS (2019). Note that the solver can return a slightly different schedule with other nights used, since the hindrance for operators is the same in nights 1, 6, and 7 and nights 2, 3, 4, and 5 (see Table 2). Varying the weight parameters will lead to different optimal solutions, as can be seen in Table 7.

Weight Parameter	Value	Weight Parameter	Value	Weight Parameter	Value
Λ^γ	1.0	Λ^{γ}	3.0	Λ^{γ}	10.0
Λ^{μ}	1.0	Λ^{μ}	3.0	Λ^{μ}	10.0
Λ^δ	1.0	Λ^{δ}	3.0	Λ^{δ}	10.0
Λ^{ϕ}	1.0	Λ^{ϕ}	0.5	Λ^{ϕ}	0.25
Result Obj. Function	3.33	Result Obj. Function	4.5	Result Obj. Function	9.083
Schedule	Step 2	Schedule	Step 3	Schedule	Step 4

Table 7: Results test scenario with different weight parameters.

4.7. Computational Performance

To test the computational performance of the model, the model solved different problems with varying set sizes in AIMMS (2019) using CPLEX 12.9 as solver and an Intel[®] CoreTMi3-8145U dual core processor with 4 GB RAM. In Table 8, the results of the computational test are shown. As can be seen, the computational time increases when the sizes of the problems get bigger. Case 6 cannot be solved optimally within six hours, but the best solution found after six hours, is the same one found after one hour of solving. The gap between the optimal solution of the linear relaxation of the MIP and the best solution found yet is only 4.58%. Since the optimal solution of the linear relaxation of the MIP can never be reached, the actual gap between the best solution found after one hour and the optimal solution is even smaller. Therefore, the best solution found yet is expected to be a descent solution.

Case	Problem Size	Time [sec.]
1	Z = 1, N = 7, C = 3, O = 3	0.02
2	Z = 2, N = 7, C = 3, O = 3	0.03
3	Z = 2, N = 14, C = 3, O = 3	0.09
4	Z = 5, N = 14, C = 3, O = 3	0.56
5	Z = 10, N = 21, C = 3, O = 3	25.31
6	Z = 10, N = 28, C = 3, O = 3	-

Table 8: Computational results

4.8. Big-M Formulations

When using CPLEX as a solver, it is possible to replace constraints with big-M formulations with indicator constraints (AIMMS, 2020). This could improve the computational performance of a model. Consider the following constraint:

 $x_1 + x_2 \leq y \cdot M$ where y is binary and $x \in \mathbb{R}^+$.

This can be reformulated to:

if y = 0 then $x_1 + x_2 = 0$

The constraint $x_1 + x_2 = 0$ now only becomes active when y takes value 0. In the model, this method can be applied to the constraints in which a binary variable is multiplied by M (4.10 to 4.12, 4.32 to 4.34, and 4.38 to 4.40).

Indicator constraints are used to optimise Case 5 to see the impact on the computational time. In contrast to what was expected, the computational time increases massively from 25.31 to 92.39 seconds. An explanation for this is that in this model, the big-M factor is eliminated in many constraints by the presolver (IBM, 2018). Due to these eliminations, there is no need for indicator constraints when solving the new model in (AIMMS, 2020) using CPLEX as a solver.

5. Case Study

To asses a model properly, numerical experiments with real data need to be performed (Albrecht et al., 2013; Arenas et al., 2018; Lidén & Joborn, 2016; Pour et al., 2018a; Zhang et al., 2019a). In this research, this is done by applying the model to a small area of the Dutch railway network, see Figure 2. This area is particularly interesting since the lack of parking places for rolling stock is becoming an issue here. Furthermore, the boundaries to the area are clear and straight forward, which makes it a suitable case study area. This chapter consecutively discusses the infrastructure of the area, the current maintenance schedule, the creation of work zones in the area, the model input parameters, the results of comparisons, and a sensitivity analysis.



Figure 2: A map of the Dutch railway network, with a circle around the case study area (South Limburg) (Dennistw, 2018).

5.1. Infrastructure

Not all infrastructure in the area is managed by ProRail, see Figure 3. Only the main railway network and the connection between industrial yards to the main network is managed by ProRail. E.g. the heritage line, on which the South Limburg Steam Train Company operates, is not managed by ProRail. The infrastructure that ProRail does manage in the area are 226 switches and around 200 km of railway tracks (including double tracks and yards). There are three main yards in the area: Heerlen, Maastricht, and Sittard. The infrastructure is maintained by a maintenance contractor commissioned by ProRail. Depending on the classification, important switches need to be maintained four times a year while infrequently used switches on yards are only maintained once a year. On average, the straight tracks and overhead wiring need to be maintained three times a year.



Figure 3: The study area with in red the railway infrastructure managed by ProRail (Esri Nederland, Community Map Contributors | Copyright ProRail).

5.2. Current Schedule

In the maintenance schedule of 2019, the area is split up into eight different parts containing multiple zones that can be planned train free for a night once a week. In 2019, there were 1667 available slots in the schedule over all nights. Of these slots, 329 were used by the maintenance contractor in 168 different nights. On average this is two zones per work night. Besides for maintenance activities, 164 slots were used for other activities e.g. cleaning yards, or preparation works for large projects. Also, 256 slots could not be used due to other reasons e.g. events in the area, or a planned inspection train. The unused slots were given back for train operation two weeks ahead, but most train operators have already planned their train movements around the slots by then.

Nightly freight trains are nowadays only planned in the area during week nights (Monday-Friday), with six to nine trains a night. Most of these freight trains find their destination at the industrial area near Geleen (Lutterade DSM). Only a few trains cross the area from North (Sittard) to South (Eijsden Grens) continuing their journey in Belgium. When the number of nights with maintenance on this freight corridor is reduced, an increase in the number of freight trains is expected.

Currently, some parts of the yards in the case study area are only available for maintenance during train free periods in the daytime. Since the scheduling problem focuses on a nightly maintenance schedule, these parts of the network are not considered in the problem. For this reason, part of the tracks at Heerlen (zone 12) are not considered (see Figure 12, Appendix B), and at Maastricht the tracks of service company Nedtrain (track 47) and the tracks of Lijnwerkplaats (tracks 45 and 46) (see Figure 13, Appendix B).

5.3. Work Zones

In order to execute maintenance activities safely, so called work zones are used (van Zante-de Fokkert et al., 2007). Such a work zone can be made safe by blocking a block section (section of railway tracks in-between signals) (Arenas et al., 2018).

In the Netherlands, the railway system was divided into work zones (van Zante-de Fokkert et al., 2007) which can be blocked for trains when maintenance activities need to be performed. den Hertog et al. (2005) managed to handle the conflicting interests of the many parties involved and divided the network into working zones based on the layout of switches and placed boundaries at the middle of switches and between switches as in Figure 4. When mirroring switches on the horizontal axis in situations 2, 3 and 4, one can always end up in situation 1. Looking at track layouts, these are the only situations in which a boundary is needed between switches when considering train movements.



Figure 4: Boundary location between working zones in four track layout situations.

The yards in the study area are divided into work zones using the method of den Hertog et al. (2005) and by analysing the layouts of the overhead wiring system. Figure 5 shows how part of the yard of Sittard is divided. Boundaries are based on a combination of the method of den Hertog et al. (2005) and the layout of overhead wiring. Since most hindrance for operators is caused by shutting of the power of overhead wiring, boundaries



Figure 5: Zone boundaries (©SporenplanOnline).

are always based on these layouts. Figures 12 to 14 in Appendix B show how the three yards and the connecting tracks of case study area are divided into 25 work zones.

5.4. Model Input

To be able to create a new maintenance schedule for the case study area with the model, the known data of the area is converted into model input parameters. The cardinalities of the sets of the model are: 25 work zones, 364 nights (52 weeks), 3 work crews (one for each engineering field), and 3 train operators (operator main lines, operator regional lines, operators freight lines), see Table 9. The specific amount of infrastructure present in each zone and the demand for maintenance operations can be found in Appendix C and the lengths are given in km. The maximum amount of maintenance operations that the work crews active in the case study area can perform in a night are in the same units, but are confidential and therefore not given. Each of the three work crews is specialised in one maintenance engineering field, meaning that crew 1 maintains the switches in all zones, crew 2 the straight tracks in all zones, and crew 3 the overhead wiring in all zones. This results in the parameter tables F_{zc}^{γ} , F_{zc}^{μ} , and F_{zc}^{δ} which can be found in Appendix C. Besides Saturday nights, all nights of the week are available for maintenance in every zone. This results in a parameter table R_{zn} which can found in Appendix C. To ensure that crews do not have to travel from zone to zone in a night, only adjacent zones are allowed to be combined. Since this holds for every night, the parameter table P_{nij} , which can be found in Appendix C, applies for all nights. The hindrance for train operators when maintenance operations are performed in a zone is determined for train operators at 1 when parked trains in that zone are hindered, at 0.5 when parked trains are indirectly hindered in another zone, and at 0 when no parked trains are hindered. For the freight operators, this is determined per zone by whether or not the freight corridor is accessible or not. The hindrance caused by maintenance in a zone is equal for all nights for passenger operators, but for freight operators hindrance is only caused in nights in which freight trains are planned through the study area. This

		<u> </u>	
Cardinality Set	Value	Weight Parameter	Value
Z	25	Λ^{γ}	1.0
N	364	Λ^{μ}	1.0
C	3	Λ^{δ}	1.0
O	3	Λ^{ϕ}	0.04

Table 9: Case study input parameters

(Tables of all parameters can be found in Appendix C)

all results in the parameter tables H_{ozn}^{γ} , H_{ozn}^{μ} , and H_{ozn}^{δ} which can be found in Appendix C. The maximum number of nights that may be used for maintenance is 260 nights, an average of five nights per week. The weight parameters of the share of the workloads in the objective function are kept at the default values of 1, but the weight parameter for the share of hindrance is set at 0.04 (see Table 9) to ensure a better balance between the workloads and the hindrance in the outcome of the objective function. If the latter weight parameter was also left at the default value of 1, hindrance would indirectly be made much more important since the share of workload in the objective function is at most 3, which is less than the minimum of the share of hindrance.

5.5. Results

To fairly compare different schedules, some general indicators are needed. The output of the objective function of the new model cannot be used directly, since the model optimises this function and therefore would show a good performance. Also, this function cannot be applied to the current schedule. Three key performance indicators (KPIs) are introduced in the following paragraph before the comparisons between different models and the current schedule are presented.

5.5.1. KPIs

For maintenance contractors, it is desirable that the maintenance operations are spread-out over the whole schedule, therefore the first KPI is the total mean workload for maintenance crews. This is determined by adding up the average workloads per infrastructure part. For operators, the total amount of times they are hindered is important, therefore the second KPI is the total hindrance for train operators. For the railway manager it is interesting to know how often work takes place during nights, therefore the third KPI is the total number of nights used in the schedule. The determination of the KPIs is given in the following.

(1) Total Mean Workload for crews:

Mean Workload Switches + Mean Workload Straight Tracks + Mean Workload Overhead Wiring

(1.1) Mean Workload Switches, for crews maintaining switches: $Average(\frac{\sum_{z \in Z} \sum_{n \in N} (q_{czn})}{\sum_{n \in N} (w_{cn}^{\gamma}) \cdot Q_{c}^{\max}}), \quad \forall c \in C \text{ where } \sum_{z \in Z} \sum_{n \in N} (q_{czn}) > 0$

 $\begin{array}{ll} (1.2) & \text{Mean Workload Straight Tracks, for crews maintaining straight tracks:} \\ & Average(\frac{\sum_{z\in Z}\sum_{n\in N}(s_{czn})}{\sum_{n\in N}(w_{cn}^{\mu})\cdot S_{c}^{\max}}), \quad \forall c\in C \text{ where } \sum_{z\in Z}\sum_{n\in N}(s_{czn})>0 \end{array}$

(1.3) Mean Workload Overhead Wiring, for crews maintaining overhead wiring: $Average(\frac{\sum_{z \in Z} \sum_{n \in N} (b_{czn})}{\sum_{n \in N} (w_{cn}^{\delta}) \cdot B_c^{\max}}), \quad \forall c \in C \text{ where } \sum_{z \in Z} \sum_{n \in N} (b_{czn}) > 0$

- (2) Total Hindrance for operators: $\sum_{o \in O} \sum_{z \in Z} \sum_{n \in N} (h_{ozn})$
- (3) Total Nights Used: $\sum_{i=1}^{n} (t_i)^{i}$

$$\sum_{n\in N} (t_n)$$

5.5.2. Comparisons

To be able to conclude whether or not the new model is an improvement, the KPIs of the new model are compared with the current schedule and the basis model from section 3.2. Also, the performance of partial optimisations of the new model are compared.

To determine the KPIs for the current schedule, the schedule and usage of the maintenance contractor in 2019 is used. To determine the KPIs for the basis model, the new model was modified to become more similar to the basis model. The basis model only contains a constraint that limits the maximum workload of switch maintenance, and not for straight track and overhead wiring maintenance. The workload of straight track and overhead wiring maintenance. The workload of straight track and overhead wiring maintenance constraints 4.57 and 4.58 are removed. Next to this, the objective function of the basis model differs from the new model. It aims to minimise the workload and used nights. Since overhead wiring is not present in the basis model, this is added to come to the objective function given in the following. For clarity, the objective function (4.0) of the new model is also given. The weight parameters used are given in Table 10.

Basis Model

$$\operatorname{Min} \sum_{c \in C} (\Lambda^{\gamma} y_c + \Lambda^{\mu} u_c + \Lambda^{\delta} d_c) + M \sum_{n \in N} (t_n)$$

New Model

$$\operatorname{Min} \sum_{c \in C} (\Lambda^{\gamma} \frac{y_c}{Q_c^{\max}} + \Lambda^{\mu} \frac{u_c}{S_c^{\max}} + \Lambda^{\delta} \frac{d_c}{B_c^{\max}}) + \Lambda^{\phi} \sum_{o \in O} \sum_{z \in Z} \sum_{n \in N} (h_{ozn})$$

Basis Model		New Model							
Weight Parameter	Value	Weight Parameter	Value						
Λ^{γ}	0.33	Λ^{γ}	1.0						
Λ^{μ}	0.33	Λ^{μ}	1.0						
Λ^{δ}	0.33	Λ^{δ}	1.0						
		Λ^{ϕ}	0.04						

Table 10: Weight parameters basis model and new model.

In Figure 6, the values of the KPIs of the current schedule, the basis model and the solution of the new model are depicted. As can be seen, the total mean workload for work crews and the total hindrance for operators are both lower in the new model solution compared with the current schedule and the basis model. However, the total number of nights used is a lot higher. The reason for this is that the new model tries to spread-out all maintenance as much as possible, thereby using the maximum number of nights allowed. The total mean workload of the basis model is higher than three, since work crew capacities on straight track and overhead wiring maintenance are not considered.

To see the effect on the performance of maintenance schedules when optimising only for either the maintenance contractor or the train operators and to discover the minimums of the total mean workload and the total hindrance, sub optimisations are performed by leaving out parts of the objective function of the new model.



Figure 6: KPIs of the current schedule, basis model, and new model solution.

To optimise only for the maintenance contractor, only the workload is minimised and hindrance is taken out of the objective function. To optimise only for the train operators, only the hindrance is minimised and workload is taken out of the objective function. Doing so leads to the following two objective functions. For clarity, the objective function (4.0) of the original new model optimising both workload and hindrance is also given. The same weight parameters from Table 10 are used.

New Model - Workload Only

$$\mathrm{Min}\,\sum_{c\in C} (\Lambda^{\gamma}\frac{y_c}{Q_c^{\mathrm{max}}} + \Lambda^{\mu}\frac{u_c}{S_c^{\mathrm{max}}} + \Lambda^{\delta}\frac{d_c}{B_c^{\mathrm{max}}})$$

New Model - Hindrance Only

$$\operatorname{Min} \Lambda^{\phi} \sum_{o \in O} \sum_{z \in Z} \sum_{n \in N} (h_{ozn})$$

$$\operatorname{Min} \sum_{c \in C} (\Lambda^{\gamma} \frac{y_c}{Q_c^{\max}} + \Lambda^{\mu} \frac{u_c}{S_c^{\max}} + \Lambda^{\delta} \frac{d_c}{B_c^{\max}}) + \Lambda^{\phi} \sum_{o \in O} \sum_{z \in Z} \sum_{n \in N} (h_{ozn})$$



Figure 7: KPIs of partial optimisation solutions and the original new model solution.

New Model

In Figure 7, the values of the KPIs of the sub optimisations are depicted next to that of the original new model which optimises both workload and hindrance. Note that the latter is the same as in the previous comparison. When optimising the problem for work crews only, the total mean workload is lower, but the total hindrance for operators increases drastically where more than half of the total hindrance is caused to freight trains. When optimising the problem for operators only, the total mean workload increases while the total hindrance decreases only a little. As said, these partial model optimisations give an indication on the minimum values of the total mean workload and total hindrance. With this in mind, the new model seems to be a good compromise with a total mean workload of 11% above the minimum and a total hindrance of 16% above the minimum. Especially when looking at the performance of the current schedule and basis model in Figure 6, the new model shows a large improvement on these two KPIs, coming close to the minimums. The schedule produced by the new model can be found in Appendix D.

5.5.3. Experts

The results of the comparisons were discussed with railway management experts and a railway maintenance expert. All are pleased to see the possible decreases in both the workload for maintenance crews and the hindrance for train operators. Despite the increase in the total number of nights used, the schedule created by the new model is favourable. The schedule planned maintenance operations on average five nights per week, but this is only on part of the network. This means at other locations there is still room for other activities. For the maintenance crews themselves, the average number of nights they are scheduled is with 4.2 nights per week lower, since not all crews are scheduled in every night used.

When determining the hindrance in the actual number of trains hindered, a detailed look on the schedule is needed since the hindrance points cannot be converted directly into these numbers. To determine the hindrance for the parked trains on the yards, the number of trains needed to start up the timetable on an average weekday is used to determine the number of trains that are hindered when the power of the overhead wiring is shut off for maintenance. Since passenger trains differ in length, the number of cars is used. To determine the hindrance for freight trains, the current timetable was used to determine the number of individual freight trains that are hindered. For the schedule produced by the new model, the total number of parked passenger cars that will hindered due to maintenance on the overhead wiring system for a night is 620 a year and no freight trains are hindered. With the current schedule, these are 864 passenger cars and no freight train, so the new model decreases the this hindrance with 28%.

It is important for maintenance that infrastructure elements on the boundaries between zones can be maintained. For this, it is necessary that both zones are scheduled train free in the same night. On the yard of Sittard two zone combinations do not occur in the schedule produced by the new model, but in practice these can be made without causing extra hindrance. When specifically looking at maintenance on overhead wiring, there are more combinations that do not occur in the schedule. In practice, these can be made without causing extra hindrance, except for one combination on the yard of Maastricht where six parked passenger cars hindered for an extra night.

5.6. Sensitivity Analysis

Since the new model makes use of weight parameters in the objective function, the sensitivity of the KPIs to these parameters are analysed for the case study problem. For this, one weight parameter is varied at a time while keeping the others at their original value. The upper and lower bound to these variations are respectively ten times larger and ten times smaller than the standard value, with in total eight different weight parameter values. In Figures 8 to 11, the relative changes in the KPIs are depicted for the variations of the weight parameters. As can be seen, the KPI total nights used is not sensitive for any weight parameter. Varying the weight parameter for switches does not affect the KPIs much. The same holds for varying the weight parameter for straight tracks, although total hindrance increases at the upper bound of the parameter. For the weight parameters for overhead wiring and hindrance, more clear trends can be seen. When the weight parameter for such a more holds for straight parameter for hindrance. When the total mean workload increases half as much. This is also the case when increasing the weight parameter for hindrance. When the weight parameter for hindrance moves the other way, total hindrance will increase drastically at the bound of the variation, but this only results in a minor decrease in total mean workload.



Sensitivity KPIs to change in weight parameter switches: λ^{γ}





Sensitivity KPIs to change in weight parameter straight tracks: λ^{μ}

Figure 9: Sensitivity of the three KPIs to the weight parameters for straight tracks.



Sensitivity KPIs to change in weight parameter overhead wiring: λ^{δ}

Figure 10: Sensitivity of the three KPIs to the weight parameters for overhead wiring.

45.00% 35.00% 25.00% 15.00% 5.00% -5.00% 0.004 0.01 0.02 0.03 0.04 0.06 0.08 0.4 0.16 -15.00% •••••• Total Mean Workload --- Total Hindrance - Total Nights Used

Sensitivity KPIs to change in weight parameter hindrance: λ^{ϕ}

Figure 11: Sensitivity of the three KPIs to the weight parameters for hindrance.

6. Conclusion

The aim of the research is to find a better balance between train traffic and maintenance management of railway networks, by optimising the design of the maintenance schedule. This is done by including hindrance for train operators, the practical limitations of maintenance contractors, and by distinguishing maintenance engineering fields in a new scheduling model. Before drawing the main conclusion, the sub questions are discussed shortly.

(1) What is the impact on the performance of the maintenance schedule when optimising for train traffic?

When only optimising the maintenance schedule for train traffic, the hindrance for train operators is brought to a minimum. The workload of maintenance crews is not considered in this case and thereby crews have to work at their maximum capacities regularly to perform all demanded maintenance operations within the schedule.

(2) What is the impact on the performance of the maintenance schedule when optimising for maintenance contractors?

When only optimising the maintenance schedule for maintenance contractors, the workload for maintenance contractors is brought to a minimum. Thereby, the maintenance operations demanded are spread out more equally over the schedule that uses all the available days. Through this, maintenance crews never have to work at their limits. This all is at the expense of a drastic increase in hindrance for train operators, since the hindrance is not considered in this case.

The knowledge acquired by answering the sub questions is combined to answer the main research question.

What is the optimal design of the maintenance schedule when including hindrance for train operators and capacity constraints of maintenance crews in the design process of schedules?

An optimal design of a maintenance schedule in terms of workload for maintenance crews and hindrance for train operators can be produced by the new developed model that considers all the different aspects and restrictions. It minimises both the hindrance for train operators and the workload of maintenance contractors, where the performance is near the minimums for workload and hindrance. Since the work is spread out over as many nights as possible to lower the workload, a limit needs to be set on the number of nights used in order to prevent the schedule using every available night. Optimising the maintenance schedule for the case study area by using the new model, shows that large improvements can be realised on the performance of the schedule. Both the mean workload for maintenance contractors and the total hindrance for train operators are lowered compared to the current schedule. This lowering is reached by including the capacities of work crews and by the distinguishing in maintenance engineering fields, since most hindrance for parked trains is only caused by maintenance on the overhead wiring system. Freight trains are not hindered at all in schedules when maintenance on the freight corridors can be scheduled at nights with no traffic. Based on the case study results, it may be concluded that with the new developed scheduling model, railway managers can make better considerations in the trade-off between train traffic and maintenance management to come to an optimal maintenance schedule.

7. Discussion

For further research regarding maintenance scheduling that includes hindrance, it is recommended to consider the following. As said, hindrance for parked trains is mainly caused by maintenance operations on overhead wiring. However, due to the layout of the system, shutting off power in one zone, may lead to a power shut off in another zone. E.g. when on Maastricht zone 4 is being maintained, power is also turned off in zone 2 since zone 4 is fed with power via zone 2. In this case, maintaining zone 4 indirectly causes hindrance for zone 2, but when both zones are maintained at the same time, hindrance for zone 2 should not be doubled. To solve this, a more complex method of determining hindrance is needed. When this is achieved, the number of passenger cars parked per zone could be used directly in the model, instead of working with hindrance points and determining the number of passenger cars hindered afterwards by analysing a produced schedule in detail.

As discussed in the case study, not all boundaries of adjacent zones can be maintained with the schedule created by the new model. Two adjacent zones need to be scheduled train free simultaneously to maintain the assets at the boundary. This means that the created schedule cannot be directly used, but has to be extended with extra train free periods to include the necessary combinations of zones that are missing. Further research should consider adding extra constraints to the new model to solve this issue in the optimisation process.

Since the zones used in the new model are smaller than the zones currently used, the time needed to secure a zone for a train free period will most likely increase. Train free periods should hereby be extended in order to prevent a decrease in the capacities of maintenance crews (maximum number of maintenance operations per night). In practice, extending time windows is not always possible, therefore more research is needed to determine the magnitude of the decrease in the capacities of crews in these cases.

The maintenance schedule created by the new model does meet all the constraints, but lacks of regularity, something that is desired by train operators. For them, it is easier to deal with train free periods that return on a regular basis. To meet this desire, the new model could be used to create a schedule for a four week time period, which can then be copied thirteen times to make an annual maintenance schedule.

For Maintenance contractors, a more regular schedule for individual crews is desired. To prevent track workers of becoming overtired, three or four nights of work in a row followed by a few nights off is desirable. Such circumstance could be created by extending the new model by giving penalties to schedules which violate this regularity.

The new model only considers hindrance for train operators in terms of parked passenger trains and delayed freight trains, other hindrance such as noise pollution to local residents living near railways is not considered. To be able to consider such an kind of hindrance, it is needed to determine what the cause of this type of hindrance is to be able to reduce it by extending the new scheduling model with extra variables and constraints.

The weight parameters used in the objective function of the new model are now used to create a better balance between train traffic and maintenance management, but they could also be translated into cost parameters to create cost efficient maintenance schedules.

The total number of nights used by the new model will, because of its objective function, always equal the maximum number allowed that is determined in the input. Using many nights is not by definition a big problem, since only parts of an area are planned train free for a night. If needed, the total number of nights used could

be lowered by including an extra term in the objective function of the model. Doing so will lead to an increase in the total mean workload of crews since the demanded maintenance operations have to be performed in less nights.

Further research could also make use of Pareto multi-objective optimisation to find optimal solutions to the scheduling problem, instead of using the additive objective function of the new model. When such an optimisation is performed, a clear image that depicts the trade-off curve of the contrary objectives can be produced. Railway managers could use this trade-off curve to make decisions related to railway maintenance.

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A. Appendix: Model Overview

Indices and Sets

- Z work zones.
- N nights.
- C maintenance crews.
- O train operators.

Parameters

 Q_z total number of switches present in work zone $z, z \in Z$.

- S_z total length of straight tracks present in work zone $z, z \in Z$.
- B_z total length of overhead wiring present in work zone $z, z \in Z$.
- DQ_z total number of switch maintenance operations to be performed in work zone $z, z \in Z$.
- DS_z total length of straight track maintenance operations to be performed in work zone $z, z \in Z$.
- DB_z total length of overhead wiring maintenance operations to be performed in work zone $z, z \in Z$.
- Q_c^{\max} maximum number of switches that can be maintained in a night by crew $c, c \in C$.
- S_c^{\max} maximum length of straight tracks that can be maintained in a night by crew $c, c \in C$.
- B_c^{\max} maximum length of overhead wiring that can be maintained in a night by crew $c, c \in C$.
 - $\begin{array}{l} F_{zc}^{\gamma} \\ c \ (F_{zc}^{\gamma}=1), \, {\rm or \ not} \ (F_{zc}^{\gamma}=0), \, z \in Z, \, c \in C. \end{array} \end{array}$
 - $F_{zc}^{\mu} \quad \mbox{binary parameter that indicates whether the straight tracks work zone z can be maintained by crew c (<math>F_{zc}^{\mu} = 1$), or not ($F_{zc}^{\mu} = 0$), $z \in Z$, $c \in C$.
 - $F_{zc}^{\delta} \qquad \text{binary parameter that indicates whether overhead wiring work zone } z \text{ can be maintained by} \\ \text{crew } c \ (F_{zc}^{\delta} = 1), \text{ or not } (F_{zc}^{\delta} = 0), \ z \in Z, \ c \in C.$
 - binary parameter that indicates whether work zone z can be assigned to night n $(R_{zn} = 1)$, or R_{zn} not $(R_{zn} = 0), z \in Z, n \in N$.
- $P_{nij} \qquad \qquad \text{binary parameter that indicates whether work zones } i \text{ and } j \text{ may be combined in night } n \ (P_{nij} = P_{nij} = 0), \text{ or not } (P_{nij} = 1), \text{ when } i = j \ (P_{nij} = 0), n \in N, i \in Z, j \in Z.$
- $\begin{array}{l} \text{hindrance for train operator } o \text{ when work zone } z \text{ is maintained on switches, } o \in O, \ z \in Z, \\ n \in N. \end{array}$

 $\begin{aligned} & \underset{ozn}{\text{hindrance for train operator } o \text{ when work zone } z \text{ is maintained on straight tracks, } o \in O, \, z \in Z, \\ & n \in N. \end{aligned}$

 $\begin{array}{l} \text{hindrance for train operator } o \text{ when work zone } z \text{ is maintained on overhead wiring, } o \in O, \, z \in Z, \\ H_{ozn}^{\delta} & n \in N. \end{array}$

- N^{\max} maximum number of nights that may be used in the schedule.
 - Λ^{γ} weight parameter for the workload of switches.
 - Λ^{μ} weight parameter for the workload of straight tracks.
 - Λ^{δ} weight parameter for the workload of straight tracks.
 - Λ^{ϕ} weight parameter for the hindrance for operators.
 - M a large number.

Variables

- binary variable that indicates whether work zone z is assigned to night n for any maintenance x_{zn} $(x_{zn} = 1)$, or not $(x_{zn} = 0)$, $z \in Z$, $n \in N$.
- x_{zn}^{γ} binary variable that indicates whether work zone z is assigned to night n for maintenance on switches $(x_{zn}^{\gamma} = 1)$, or not $(x_{zn}^{\gamma} = 0)$, $z \in Z$, $n \in N$.
- binary variable that indicates whether work zone z is assigned to night n for maintenance on x_{zn}^{μ} straight tracks $(x_{zn}^{\mu} = 1)$, or not $(x_{zn}^{\mu} = 0)$, $z \in Z$, $n \in N$.
- $\begin{array}{l} x_{zn}^{\delta} & \text{binary variable that indicates whether work zone z is assigned to night n for maintenance on overhead wiring (<math>x_{zn}^{\delta} = 1$), or not ($x_{zn}^{\delta} = 0$), $z \in Z$, $n \in N$.
- $\begin{array}{l} \underset{c_n}{\text{binary variable that indicates whether crew c maintains switches in night n ($w_{cn}^{\gamma}=1$), or not} \\ (w_{cn}^{\gamma}=0), \, n \in N, \, c \in C. \end{array}$
- w_{cn}^{μ} binary variable that indicates whether crew c maintains straight tracks in night n ($w_{cn}^{\mu} = 1$), or not ($w_{cn}^{\mu} = 0$), $n \in N$, $c \in C$.
- $$\begin{split} w_{cn}^{\delta} & \text{binary variable that indicates whether crew c maintains overhead wiring in night n ($w_{cn}^{\delta}=1$), or not ($w_{cn}^{\delta}=0$), $n \in N$, $c \in C$. \end{split}$$
- $\begin{array}{l} \underset{czn}{\text{binary variable that indicates whether crew c maintains the switches of zone z in night n ($v_{czn}^{\gamma} = v_{czn}^{γ}), or not ($v_{czn}^{\gamma} = 0$), $c \in C$, $z \in Z$, $n \in N$.} \end{array}$
- binary variable that indicates whether crew c maintains the straight tracks of zone z in night n $(v_{czn}^{\mu} = 1)$, or not $(v_{czn}^{\mu} = 0)$, $c \in C$, $z \in Z$, $n \in N$.
- $\begin{aligned} v_{czn}^{\delta} & \text{binary variable that indicates whether crew c maintains the overhead wiring of zone z in night n} \\ (v_{czn}^{\delta} = 1), \text{ or not } (v_{czn}^{\delta} = 0), \ c \in C, \ z \in Z, \ n \in N. \end{aligned}$
- q_{czn} variable that indicates the number of switches maintained in zone z night $n, z \in Z, n \in N$.
- s_{czn} variable that indicates the length of straight tracks maintained in zone z in night $n, z \in Z, n \in N$. $variable that indicates the length of overhead wiring maintained in zone z in night <math>n, z \in Z$, $n \in N$.
 - y_c largest number of switches to be maintained in one night by crew c over all nights, $c \in C$.
 - u_c largest number of kilometers to be maintained in one night by crew c over all nights, $c \in C$. d_c largest number of kilometers of overhead wiring to be maintained in one night by crew c over all nights, $c \in C$.
- h_{ozn} largest hindrance for operator o in night n caused by maintenance in zone $z, o \in O, z \in Z, n \in N$. variable that indicates whether night n is assigned for any maintenance $(t_n = 1)$ or not $(t_n = 0)$, $n \in N$.

Objective Function

(P) Minimise
$$\sum_{c \in C} (\Lambda^{\gamma} \frac{y_c}{Q_c^{\max}} + \Lambda^{\mu} \frac{u_c}{S_c^{\max}} + \Lambda^{\delta} \frac{d_c}{B_c^{\max}}) + \Lambda^{\phi} \sum_{o \in O} \sum_{z \in Z} \sum_{n \in N} (h_{ozn})$$
(A.0)

Constraints

- s.t. $\sum_{c \in C} \sum_{n \in N} (q_{czn}) = DQ_z$ $\forall z \in Z,$ (A.1)
 - $\sum_{c \in C} \sum_{n \in N} (s_{czn}) = DS_z \qquad \qquad \forall z \in Z,$ (A.2)
 - $\sum_{c \in C} \sum_{n \in N} (b_{czn}) = DB_z \qquad \forall z \in Z,$ $q_{czn} \leq Q_z \qquad \forall c \in C, \ z \in Z, \ n \in N,$ (A.3)
 (A.4)
 - $q_{czn} \le Q_z \qquad \qquad \forall c \in C, \ z \in Z, \ n \in N, \qquad (A.4)$ $s_{czn} \le S_z \qquad \qquad \forall c \in C, \ z \in Z, \ n \in N, \qquad (A.5)$

$b_{czn} \le B_z$	$\forall c \in C, z \in Z, n \in N,$	(A.6)
$\sum_{c \in C} (q_{czn}) \le x_{zn}^{\gamma} \cdot M$	$\forall z \in Z, n \in N,$	(A.7)
$\sum_{c \in C} (s_{czn}) \le x_{zn}^{\mu} \cdot M$	$\forall z\in Z,n\in N,$	(A.8)
$\sum_{c \in C} (b_{czn}) \le x_{zn}^{\delta} \cdot M$	$\forall z \in Z, n \in N,$	(A.9)
$x_{zn}^{\gamma} \le \sum_{c \in C} (q_{czn})$	$\forall z \in Z, n \in N,$	(A.10)
$x_{zn}^{\mu} \le \sum_{c \in C} (s_{czn}) \cdot M$	$\forall z \in Z, n \in N,$	(A.11)
$x_{zn}^{\delta} \le \sum_{c \in C} (b_{czn}) \cdot M$	$\forall z \in Z, n \in N,$	(A.12)
$x_{zn}^{\gamma} \le R_{zn}$	$\forall z \in Z, n \in N,$	(A.13)
$x_{zn}^{\mu} \le R_{zn}$	$\forall z \in Z, n \in N,$	(A.14)
$x_{zn}^{\delta} \le R_{zn}$	$\forall z \in Z, n \in N,$	(A.15)
$x_{zn}^{\gamma} + x_{zn}^{\mu} + x_{zn}^{\delta} \le x_{zn} \cdot M$	$\forall z \in Z, n \in N,$	(A.16)
$x_{zn} \le x_{zn}^{\gamma} + x_{zn}^{\mu} + x_{zn}^{\delta}$	$\forall z \in Z, n \in N,$	(A.17)
$\sum_{c \in C} (v_{czn}^{\gamma}) = x_{zn}^{\gamma}$	$\forall z \in Z, n \in N,$	(A.18)
$\sum_{c \in C} (v_{czn}^{\mu}) = x_{zn}^{\mu}$	$\forall z \in Z, n \in N,$	(A.19)
$\sum_{c \in C} (v_{czn}^{\delta}) = x_{zn}^{\delta}$	$\forall z \in Z, n \in N,$	(A.20)
$v_{czn}^{\gamma} \leq F_{zc}^{\gamma}$	$\forall c \in C, z \in Z, n \in N,$	(A.21)
$v^{\mu}_{czn} \le F^{\mu}_{zc}$	$\forall c \in C, z \in Z, n \in N,$	(A.22)
$v_{czn}^{\delta} \leq F_{zc}^{\delta}$	$\forall c \in C, z \in Z, n \in N,$	(A.23)
$q_{czn} \le v_{czn}^{\gamma} \cdot M$	$\forall c \in C, z \in Z, n \in N,$	(A.24)
$s_{czn} \le v_{czn}^{\mu} \cdot M$	$\forall c \in C, z \in Z, n \in N,$	(A.25)
$b_{czn} \le v_{czn}^{\delta} \cdot M$	$\forall c \in C, z \in Z, n \in N,$	(A.26)
$v_{czn}^{\gamma} \le q_{czn}$	$\forall c \in C, z \in Z, n \in N,$	(A.27)
$v_{czn}^{\mu} \le s_{czn} \cdot M$	$\forall c \in C, z \in Z, n \in N,$	(A.28)
$v_{czn}^{\delta} \le b_{czn} \cdot M$	$\forall c \in C, z \in Z, n \in N,$	(A.29)
$\sum_{z \in Z} (v_{czn}^{\gamma}) \leq w_{cn}^{\gamma} \cdot M$	$\forall c \in C, n \in N,$	(A.30)
$\sum_{z \in Z} (v_{czn}^{\mu}) \le w_{cn}^{\mu} \cdot M$	$\forall c \in C, \ n \in N,$	(A.31)
$\sum_{z \in Z} (v_{czn}^{\delta}) \leq w_{cn}^{\delta} \cdot M$	$\forall c \in C, \ n \in N,$	(A.32)
$w_{cn}^{\gamma} \leq \sum_{z \in Z} (v_{czn}^{\gamma})$	$\forall c \in C, n \in N,$	(A.33)
$w_{cn}^{\mu} \leq \sum_{z \in Z} (v_{czn}^{\mu})$	$\forall c \in C, n \in N,$	(A.34)
$w_{cn}^{\delta} \leq \sum_{z \in Z} (v_{czn}^{\delta})$	$\forall c \in C, n \in N,$	(A.35)
$w_{cn}^{\gamma} + w_{cn}^{\mu} + w_{cn}^{\delta} \le 1$	$\forall c \in C, n \in N,$	(A.36)
$P_{nij}(x_{in} + x_{jn}) \le 1$	$\forall n \in N, i \in Z, j \in Z,$	(A.37)
$\sum_{z \in Z} (q_{czn}) \le y_c$	$\forall c \in C, n \in N,$	(A.38)
$\sum_{z \in Z} (s_{czn}) \le u_c$	$\forall c \in C, n \in N,$	(A.39)
$\sum_{z \in Z} (b_{czn}) \le d_c$	$\forall c \in C, n \in N,$	(A.40)
$y_c \le Q_c^{\max}$	$\forall c \in C,$	(A.41)
$u_c \le S_c^{\max}$	$\forall c \in C,$	(A.42)
$d_c \le B_c^{\max}$	$\forall c \in C,$	(A.43)
$x_{zn}^{\gamma} \cdot H_{ozn}^{\gamma} \le h_{ozn}$	$\forall o \in O, z \in Z, n \in N,$	(A.44)
$x_{zn}^{\mu} \cdot H_{ozn}^{\mu} \le h_{ozn}$	$\forall o \in O, z \in Z, n \in N,$	(A.45)

$x_{zn}^{\delta} \cdot H_{ozn}^{\delta} \le h_{ozn}$	$\forall o \in O, z \in Z, n \in N,$	(A.46)
$x_{zn}^{\gamma} \in \{0,1\}$	$\forall z\in Z,n\in N,$	(A.47)
$x_{zn}^{\mu} \in \{0,1\}$	$\forall z \in Z, n \in N,$	(A.48)
$x_{zn}^{\delta} \in \{0,1\}$	$\forall z\in Z,n\in N,$	(A.49)
$v_{czn}^{\gamma} \in \{0,1\}$	$\forall c \in C, z \in Z, n \in N,$	(A.50)
$v_{czn}^{\mu} \in \{0,1\}$	$\forall c \in C, z \in Z, n \in N,$	(A.51)
$v_{czn}^{\delta} \in \{0,1\}$	$\forall c \in C, z \in Z, n \in N,$	(A.52)
$w_{cn}^{\gamma} \in \{0,1\}$	$\forall c \in C, n \in N,$	(A.53)
$w_{cn}^{\mu} \in \{0, 1\}$	$\forall c \in C, n \in N,$	(A.54)
$w_{cn}^{\delta} \in \{0,1\}$	$\forall c \in C, n \in N,$	(A.55)
$q_{czn} \in \mathbb{Z}^{\geq 0}$	$\forall z \in Z, n \in N,$	(A.56)
$s_{czn} \in \mathbb{R}^+$	$\forall z \in Z, n \in N,$	(A.57)
$b_{czn} \in \mathbb{R}^+$	$\forall z \in Z, n \in N,$	(A.58)
$y_c \in \mathbb{Z}^{\geq 0}$	$\forall c \in C,$	(A.59)
$u_c \in \mathbb{R}^+$	$\forall c \in C,$	(A.60)
$d_c \in \mathbb{R}^+$	$\forall c \in C,$	(A.61)
$h_{ozn} \in \mathbb{R}^+ \ge 0$	$\forall o \in O, z \in Z, n \in N,$	(A.62)
$\sum_{z \in Z} (x_{zn}) \le t_n$	$\forall n \in N,$	(A.63)
$\sum_{n \in N} (t_n) \le N^{\max}.$		(A.64)

B. Appendix: Zoning

Note: Dotted lines: tracks outside controlled area (e.g. part of zone 12 of Heerlen). Smaller lines: tracks without overhead wiring (e.g. zone 17, and track 214 in zone 16 of Heerlen).



Figure 12: Division of the yard of Heerlen (©SporenplanOnline).



Figure 13: Division of the yard of Maastricht (OSporenplanOnline).



Figure 14: Division of the yard of Sittard (©SporenplanOnline).

C. Appendix: Case Study Model Input

Z	25	1	Λ^{γ}	1.0			$c \rightarrow$	• :	L	2	3		1	V ^{max}	:	260)							
N	364		Λ^{μ}	1.0		Q	c^{\max}	: (Conf	ide	ntia	1												
C	3		Λ^{δ}	1.0		S	c^{\max}	: (Conf	ide	ntia	1												
O	3	1	Λ^{ϕ}	0.04	1	В	c_c^{\max} Confidential																	
$z\downarrow$	Q_z	ć	S_z		B_z		D	Q_z	DS_z			1	D	B_z]									
1	6	20.	752	20).752	2	2	24	62.256			65	2.	526										
2	12	4.5	206	3	3.727			33		2.618		1	1.	181										
3	4	1.0	075		0			4	3	.22	5		()										
4	11	2.	142	2	.142		4	1	6	.420	3	6	.4	26										
5	7	2.	130	2	.130		2	28	6	.390)	6	5.3	90										
6	16	4.	507	4	.507		6	54	1:	3.52	1	1	3.	521										
7	5	2.	757	2	2.757			2.757			20	8	.27	1	8	.2	71							
8	7	5.	341	1	.050		2	28	16	5.02	3	3	.1	50										
9	11	40.	845	40).305	5	3	88	12	2.53	35	12	0.	915										
10	9	37.	.673	36	6.330)	3	86	11	3.0	19	10	8	.990										
11	10	3.3	810		0		1	3	1	1.43	0		()										
12	15	5.2	296	4	.530			0		0			()										
13	5	1.	522	1	.522		1	7	4.566			4.566												
14	6	0.3	395	0	.395		2	21	1.185			1.185												
15	9	3.3	869	3	.869		7	6	11.607			11.607												
16	6	1.0	642	1	.366		1	.8	4	.920	3	4	.0	98										
17	6	1.'	752		0			6	4.346		3		()										
18	0	32.	.339	32	32.339		32.339		32.339			0	97	7.01	7	9'	7.0	017						
19	12	17.	385	14	1.487	7	2	27	55	2.15	5	4	3.4	461										
20	7	1.0	028		0			7	3	.084	1		()										
21	11	2.	366	1	.983		4	1	7	.098	8	5	.9	49										
22	6	2.	167	2	.167		2	24	6	.50	1	6	.5	01										
23	16	2.5	271	2	.271		6	52	6	.81	3	6	5.8	13										
24	9	0.	559	0	.559		3	86	1	.67′	7	1	3	577										
25	10	2.	385	0	.385		3	88	7	.15	5	7	.1	55										
F_{zc}^{γ}	$z\downarrow c$	\rightarrow	1	2	3	1	F_{zc}^{μ}	, <i>z</i>	$\downarrow c$	\rightarrow	1	2		3	I	$\frac{\delta}{zc}$	z	$\downarrow c$	\rightarrow					
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		:	:	:	:					:	:	:		:										
		$\begin{array}{c c c c c c c c c c c c c c c c c c c $						25	0	1		0					25	5						
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211		1	1	1	1		1	1	0	1	1													
							•																	
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		25	1	1	1		1	1	0	1														

repeats every 7 days

 $\mathbf{2}$ 3

 $\begin{array}{c} \vdots \\ 0 \end{array}$

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3	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	0	0	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
6	0	1	1	1	0	0	0	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	0	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
9	1	1	1	1	0	0	1	1	0	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
10	1	1	1	1	1	0	0	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1
11	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
13	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	1	1	1	1	1	1	1	1	1	1
14	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	1	1	1	1	1	1	1	1	1	1
15	1	1	1	1	1	1	1	1	0	1	1	1	0	0	0	0	1	0	0	1	1	1	1	1	1
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1
17	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1
18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	1	1	1	1	0	1	0
19	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1	1
20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1
21	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	1
22	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1
23	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	0	1	1	0	0	0	0	1
24	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0	0
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repeats every 7 days $% \left({{{\rm{T}}_{{\rm{T}}}}} \right)$

repeats every 7 days

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		2	1	1	1	1	1	1	1			2	0	0	0	0	0	0	0
		3	0	0	0	0	0	0	0			:	:	÷	:	:	÷	÷	÷
		4	0	0.5	0.5	0.5	0.5	0	0			5	0	0	0	0	0	0	0
		5	0	0.5	0.5	0.5	0.5	0	0			6	0	1	1	1	1	0	0
		6	0	0	0	0	0	0	0			7	0	0	0	0	0	0	0
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		11	0	0	0	0	0	0	0			9	0	0	0	0	0	0	0
		12	1	1	1	1	1	1	1			10	0	1	1	1	1	0	0
		13	1	1	1	1	1	1	1			11	0	0	0	0	0	0	0
		14	0	0	0	0	0	0	0			:	:	:	:	:	:	:	:
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		5	1	1	1	1	1	1	1										
		6	1	1	1	1	1	1	1										
		7	0	0	0	0	0	0	0										
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		14	0	0	0	0	0	0	0										
		15	1	1	1	1	1	1	1										
		16	1	1	1	1	1	1	1										
		17	0	0	0	0	0	0	0										
		÷	:	:	:	÷	÷	:	:										
		22	0	0	0	0	0	0	0										
		23	1	1	1	1	1	1	1										
		24	0	0	0	0	0	0	0										
		25	0	0	0	0	0	0	0										

D. Appendix: Maintenance Schedule New Model







