FACTORIAL ANALYSIS FOR BEST STIRRING STRATEGY ASSESSMENT IN THE SMALL VIRC

Abstract. This report describes the definition, application and assessment of a factorial plan with the aim of gaining insight on what kind of stirring strategy could work the best in the small VIRC. Three different factors at three working frequencies were defined, and the overall stirring performance was assessed according to three different performance indicators. With the help of the main effect plots and the interaction plots it was possible to extract general guidelines on how the best stirring process should be. Some ideas to practically implement what was found are also addressed.

1. Stirring Strategies

An efficient stirring process produces changes in the cavity eigenmodes, in both amplitude and frequency. The vibration of the walls in the VIRC, as an example, causes the excitation field to become frequency modulated. These nonlinear effects are a result of the changes in the system parameters of the cavity (time-varying boundary conditions). The particular details of the dynamics and effectiveness of the stirring process affect the different performance indicators in RCs. Unfortunately, due to the intrinsic complexity involved in the reverberation process, it is not possible to acknowledge in a close form the exact impact of a particular stirring process into each performance indicator.

The aim of the present work is to link some of these effects to some actual practical performance measurements. The approach is empirical and intends to assess different stirring strategies. It was accomplished by means of the design of experiments technique, a statistical technique for factorial assessment.

2. Design of Experiments

The Design of Experiments (DoE) is a mathematical discipline widely used in industry to systematically investigate the processes or product variables that influence product quality. Since some of the means of experimental design are independent of the nature of the experiment, the DoE technique has very broad application across all the natural and social sciences.

The DoE technique is by no means an established methodology in the study of RCs, but it was found that some of their main findings can be applied to problems related to RCs with an optimal success. Especially the phase of planning where the correct definition of factorial plans (instead of one-factor-at-a-time methods) results in a successful way to assess factors influence and interaction.

3. Factorial Design

Several factors can have a main effect on the stirring efficiency. Some rule-of-thumb guidelines on these factors can be intuited, i.e. the vibration of the walls should be large, the movement should be as complex as possible, etc. Nevertheless, other factors (mainly practical ones) find a not so clear definition, i.e. should the tent be fixed in a tight or loose fashion? Should we excite the walls vibration on the corners or on the walls of the tent? Should the mechanical excitation be very complex and random-like or just any kind of movement is enough?

With the aim of trying to give some approximated answers to these kind of questions, a proper factorial design was planned and used. In the following, we describe the different factors and performance indicators studied.

3.1. **Factors of Interest.** Three different factors of interest were defined:

(1) Tightness
(2) Harmonic-like Movements
(3) Diversity

Each one of these factors will be described in the following.

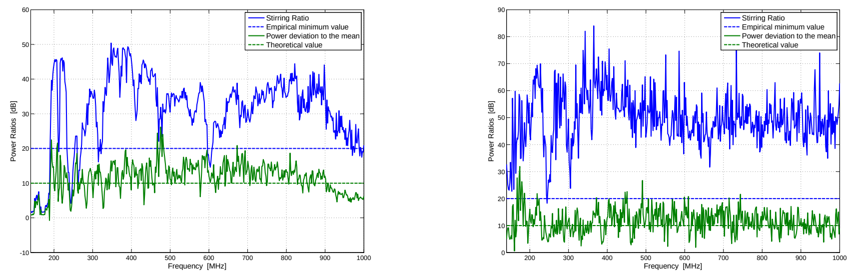
3.1.1. *Tightness.* The question about the tent being fixed in a tight or loose fashion is not trivial. On one hand, a tight configuration would maximize the total working volume of the VIRC for the same inner surface and thus increase the number of excited modes at the same frequency, improve the modal density and allow larger EUTs and/or antennas inside. There is an extra advantage and is that in the tight configuration the tent would really "vibrate", and any movement impinged over the structure would last longer. On the other hand, the loose configuration would allow a larger number of different configurations of the tent (since the walls would be more flexible), meaning a larger number of stir states or boundary conditions. Furthermore, the loose configuration allows for larger amplitude movements. There is also the fact that even though the volume is smaller than in the tight configuration, some modes at very low frequencies show, mainly due to some "rounding corners" effect (explained in a paper by Arnaut). In other words, the chamber would be more cylinder-like and that is known to be a major advantage for the modal density.

The advantages (and, by opposition) the disadvantages of each configuration can very quickly be seen in some figures of the last report: "Preliminary Measurements and Performance Assessment on a Small VIRC".

Figure 1 recalls the Stirring Ratio and Power Deviation to the Mean for both configurations. It can clearly be seen how the loose configuration largely outperforms the tight one. It is to be pointed out however, that since the stirring in both cases of Fig. 1 was done manually, the loose configuration allowed for unrealistic large variations of the walls.

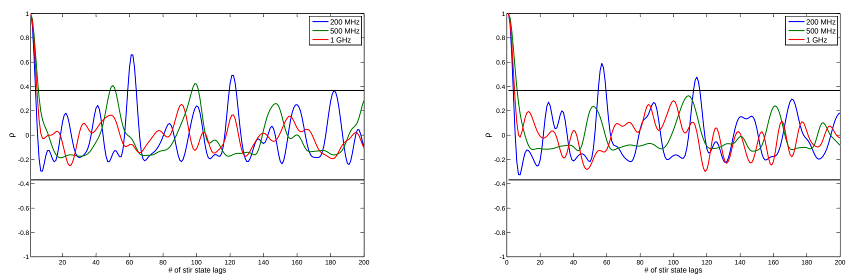
Figure 2 recalls the stirring efficiency measurements based on the autocorrelation of the power received by an antenna for 400 stir states. In this case it is not so clear that the loose configuration outperforms the tight one, as the differences in ρ seem close to be subtle.

Further tests were performed, measuring the received power by an antenna when the VIRC is stirred up by an impulsive "hit". No shaking or continued vibrations were done, just a punch in one of the walls and then the power as a function of time was registered until the oscillations die out. This kind of "Impulse Response" of the chamber was performed five times for three different working frequencies. The five strokes were intended to be as similar to one another as possible (same point, same strength, etc). Even though it was done manually, some interesting comparisons can be made. Figure 3 shows the different impulse responses for three



(a) Tight configuration (b) Loose configuration

FIGURE 1. Power deviation to the mean and stirring ratio values as a function of frequency for two configurations of the small VIRC.



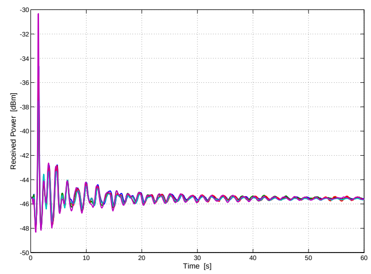
(a) Tight configuration (b) Loose configuration

FIGURE 2. Stirring efficiency measurements based on the autocorrelation of the power received by an antenna. The horizontal lines define the uncorrelated region $|\rho| \leq \frac{1}{e}$

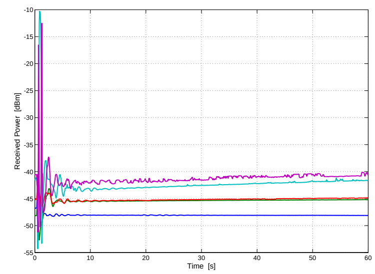
working frequencies, viz. 200 MHz, 500 MHz and 1 GHz measured during a time lapse of 1 minute.

It can be seen how the impulse responses for the tight configuration last longer than for the loose one. The VIRC is actually “vibrating” more. On the other hand, these vibrations are pretty much alike one to another, giving as a result that the resulting signals resemble also one another. This situation changes when considering the loose configuration, where the signals die out faster but each time in a rather different manner. It can be seen how the loose configuration signals converge into different resting conditions. This is due to the fact that being the walls more flexible, after each hit, they can assume a different final shape or position. By contrast, the opposite happens for the tight configuration, where the resting condition remains more or less the same.

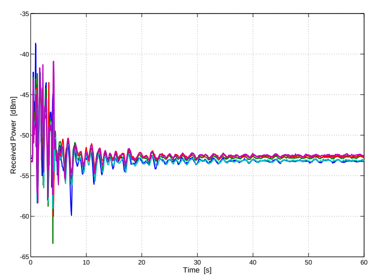
For each one of the six experiments of Fig. 3 the correlation matrix between the five signals was calculated. The correlation matrix is a measure of the similarity between two signals and is defined as $\bar{\bar{P}}_{ij} = \rho_{S_i S_j}$. More explicitly:



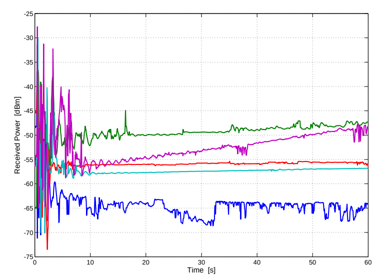
(a) Tight configuration at 200 MHz.



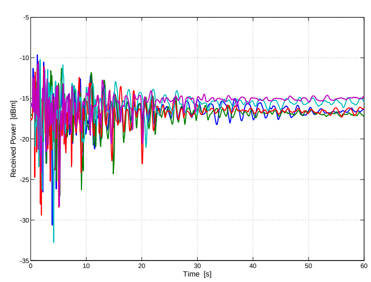
(b) Loose configuration at 200 MHz.



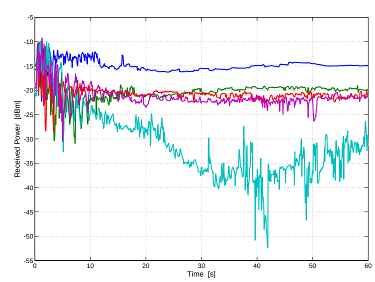
(c) Tight configuration at 500 MHz.



(d) Loose configuration at 500 MHz.



(e) Tight configuration at 1 GHz.



(f) Loose configuration at 1GHz.

FIGURE 3. Received power by an antenna inside the VIRC when hit by an impulsive stroke in one of the walls. At each of the three test frequencies, five different impulse responses were measured.

$$\bar{\bar{P}} = \begin{bmatrix} \rho_{S_1S_1} & \rho_{S_1S_2} & \rho_{S_1S_3} & \rho_{S_1S_4} & \rho_{S_1S_5} \\ \rho_{S_2S_1} & \rho_{S_2S_2} & \rho_{S_2S_3} & \rho_{S_2S_4} & \rho_{S_2S_5} \\ \rho_{S_3S_1} & \rho_{S_3S_2} & \rho_{S_3S_3} & \rho_{S_3S_4} & \rho_{S_3S_5} \\ \rho_{S_4S_1} & \rho_{S_4S_2} & \rho_{S_4S_3} & \rho_{S_4S_4} & \rho_{S_4S_5} \\ \rho_{S_5S_1} & \rho_{S_5S_2} & \rho_{S_5S_3} & \rho_{S_5S_4} & \rho_{S_5S_5} \end{bmatrix}, \tag{1}$$

	Tight	Loose
200 MHz	$\bar{\bar{P}} = \begin{bmatrix} 1 & 0.93 & 0.99 & 0.97 & 0.88 \\ 0.93 & 1 & 0.96 & 0.96 & 0.98 \\ 0.99 & 0.96 & 1 & 0.99 & 0.92 \\ 0.97 & 0.96 & 0.99 & 1 & 0.93 \\ 0.88 & 0.98 & 0.92 & 0.93 & 1 \end{bmatrix}$	$\bar{\bar{P}} = \begin{bmatrix} 1 & -0.36 & 0.59 & 0.08 & -0.04 \\ -0.36 & 1 & -0.11 & -0.57 & 0.01 \\ 0.59 & -0.11 & 1 & 0.18 & 0.19 \\ 0.08 & -0.57 & 0.18 & 1-0.12 & \\ -0.04 & 0.01 & 0.19 & -0.12 & 1 \end{bmatrix}$
500 MHz	$\bar{\bar{P}} = \begin{bmatrix} 1 & 0.72 & 0.63 & 0.69 & 0.73 \\ 0.72 & 1 & 0.77 & 0.78 & 0.71 \\ 0.63 & 0.77 & 1 & 0.84 & 0.64 \\ 0.69 & 0.78 & 0.84 & 1 & 0.81 \\ 0.73 & 0.71 & 0.64 & 0.81 & 1 \end{bmatrix}$	$\bar{\bar{P}} = \begin{bmatrix} 1 & -0.47 & 0.15 & 0.24 & -0.07 \\ -0.47 & 1 & 0.03 & 0.1 & 0.29 \\ 0.15 & 0.03 & 1 & 0.06 & 0.11 \\ 0.24 & 0.1 & 0.06 & 1 & 0.18 \\ -0.07 & 0.29 & 0.11 & 0.18 & 1 \end{bmatrix}$
1 GHz	$\bar{\bar{P}} = \begin{bmatrix} 1 & 0.52 & 0.43 & 0.18 & 0.3 \\ 0.52 & 1 & 0.17 & 0.3 & 0.13 \\ 0.43 & 0.17 & 1 & 0.03 & 0.59 \\ 0.18 & 0.3 & 0.03 & 1 & 0.2 \\ 0.3 & 0.13 & 0.59 & 0.2 & 1 \end{bmatrix}$	$\bar{\bar{P}} = \begin{bmatrix} 1 & -0.11 & 0.05 & 0.29 & 0.11 \\ -0.11 & 1 & -0.04 & -0.13 & 0.11 \\ 0.05 & -0.04 & 1 & 0.45 & 0.44 \\ 0.29 & -0.13 & 0.45 & 1 & 0.61 \\ 0.11 & 0.11 & 0.44 & 0.61 & 1 \end{bmatrix}$

Table 1. Correlation matrices for the experiments of Fig. 3.

where S_i is the i-th signal, with index $i = 1 \dots 5$ and ρ_{xy} is the correlation coefficient between signals x and y. The correlation matrix $\bar{\bar{P}}$ is Hermitian ($\bar{\bar{P}} = \bar{\bar{P}}^{\dagger}$) and with unitary diagonal. Table 1 shows the different correlation matrices for the six tests of Fig. 3.

It is to be noticed how in the loose configuration, the correlation matrices show a better response than in the tight configuration.

The experiments presented in this section serve to gain some initial insight about the complex dynamics of the "tightness" factor. The use of the DoE technique and the inclusion of tightness as a factor in the factorial plan will provide us with more uniform and consistent experiments.

3.1.2. *Harmonic-like Movements.* This factor is of particular interest, mainly for practical reasons. It is known that the most used means for stirring the VIRC is to attach an eccentrical load to an AC motor and mechanically connect it to the chamber. Combinations of more than one motor would help improving the stirring efficiency. Nevertheless, the motors provide with a harmonic-like movement. Even when placing more than one at different rotation frequencies, some harmonic situation can always be found. On one hand it is true that the intrinsic complexity of the tent will transform this harmonic motion into a quasi-random behavior of fields inside the cavity but on the other hand this is not entirely true at, i.e. low frequencies, when the dynamics of the chamber are not complex enough. One can then think that with a more random-like movement the use of the VIRC could be extended to lower frequencies. This effect is only seen when assessing statistical indicators like i.e. the autocorrelation ρ or some Goodness-of-Fit (GoF) tests, while can be transparent when assessing other types of indicators, mainly based on the performance rather than on the statistics, i.e. field uniformity or stirring ratio.

If it is found that producing a more random stirring could help in significantly lowering the LUF, it might be considered as an alternative stirring strategy. If, on the contrary, harmonic-like movements result as well in reasonably good stirring effectiveness, it is not worth to spend energy optimizing this factor. By means of the DoE technique, a glance at this trade-off situation will be done.

3.1.3. *Diversity.* We gave the name "Diversity" to the ability of a particular stirring process to produce movements in different orientations. As an example, if we place the rotating motor in one corner of the tent, this movement will mainly propagate into three walls of the chamber (the adjacent ones). This kind of movement will be more "3D" than rather move one wall at the center. On the other hand, the cost of this kind of movement is that the its amplitude would be smaller than if we place the motor at the center of a wall, and thus concentrating all the motion in one direction only.

It is known that moving a wall in only one direction is theoretically not able to produce adequate mode-stirring. But it is also true that this kind of movement in practice, even if it favors only one wall, propagates also to the rest of the walls. Taking this fact in mind, and being aware that in this case larger variations in amplitude can be achieved, we are facing again another trade-off situation where diversity vs. amplitude are in competition.

More three-dimensional movements even if smaller in amplitude, or larger movements even if they are done in only one direction mainly? The use of the DoE technique is aimed at trying to answer this kind of question.

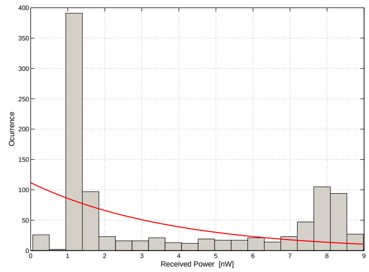
3.2. **Performance Indicators.** There is not only one, but a relatively large number of RC performance indicators. Most of the tools used to characterize and validate a RC lay into two main groups: the statistical indexes and the performance-based descriptors. Generally speaking, the statistical tools are more robust (i.e. autocorrelation coefficient, GoF tests, etc.), but can be of a great inconvenience when applied in industrial or commercial applications. Less robust yet practically more convenient indicators are the performance-based ones (i.e. field uniformity, stirring ratio, power deviation to the mean, etc.).

Each one of these indicators converge identically towards a well performing RC if we consider ideal reverberation conditions. Unfortunately, ideal conditions are hardly found in real RCs and thus the rate of convergence of the different indicators differ considerably. In other words, different looks at a particular RC performance can give different results. A chamber can at the same time, i.e. meet a satisfactory field uniformity but produce questionable results when assessed via the GoF tests. In this case the chamber is still acceptable for some EMC applications. On the other hand, a chamber could pass every kind of statistical test and still not be acceptable for some kind of application.

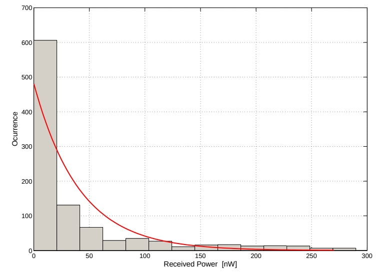
That is why is so critical to be aware that the choice we make on what kind of indicator to look, has an important impact on our final assessment. Three performance indicators were chosen for our factorial plan:

(1) Stirring Ratio
(2) χ^2 Goodness-of-Fit test p-value
(3) Autocorrelation coefficient

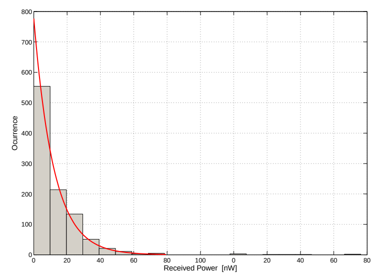
The Stirring Ratio is a performance-based indicator, while the other two are statistical tools. Indicators (1) and (2) are defined as "the higher (in value) the better (the performance)", while the third one is the opposite. Each one of this indicators will be briefly explained in the following.



(a) Low frequency (200 MHz) measurements.



(b) Mid-frequency (500 MHz) measurements.



(c) High frequency (1 GHz) measurements.

FIGURE 4. Histograms of the measured power for one cartesian axis component for three different frequencies at the same position confronted with the theoretical Exponential distribution. The data was measured performing 1000 samples while shaking the VIRC. The identification of each curve is obvious.

3.2.1. *Stirring Ratio.* The stirring ratio is defined as the relation between the maximum and the minimum field value measured in different stir states. It is an empirically defined index and even though it should be dependent on the number of samples, it is usually taken to meet satisfactory stirring performance when is above 20 dB. There is not upper bound for good performance, and the higher the stirring ratio, the better the performance. It provides consequently with a good comparative indicator.

3.2.2. χ^2 *Goodness-of-Fit test p-value.* The best and most reliable way to asses how well a RC is approaching to ideal reverberant conditions is to look at the distributions directly. If the measured data follows the theoretical PDF for a field magnitude, it means that the chamber is performing in good reverberant conditions, and viceversa.

As an example, Fig. 4 shows the histograms of the measured power (in watts) for one electric field cartesian component for three different frequencies at the same position. Together with the histograms, the theoretical PDF is also presented. For the case of a component power density, the expected distribution is the χ_2^2 also known as the "Exponential" distribution. It can be clearly seen that for the low frequency (undermoded regime) the measured data does not follow the Exponential distribution (Fig. 4(a)) while at the high frequency (overmoded regime) it does (Fig. 4(c)). A difficult situation is experienced in the case of Fig. 4(b), when at mid-point frequencies, it is not so clear to decide whether the data follows or not the theoretical PDF.

Unfortunately, making such a judgement by eye, not only requires a lot of time analyzing data, but it does not provide an objective, rigorous valuation. A "friend" or conservative eye will tend to see more fitting than a more stringent one. Furthermore, class (bin) width in histograms can greatly influence the visual appearance of a graph and the outcome of a visual valuation.

Statistical Goodness-of-Fit (GoF) tests are one of the most powerful ways to properly characterize a RC performance and helping in avoiding subjectivity. Goodness-of-Fit tests serve to examine whether a chosen distribution fits the experimental data well. They provide a systematic and rigorous comparison between measured and theoretical distributions. The most common result from a GoF test is the so-called "p-value". Very often, the resulting p-value is compared to a predefined level of significance or "α-level" (typically taken to be $\alpha = 0.05$). A p-value greater than or equal to α suggests that the distribution is a good fit. On the other hand, a p-value less than α suggests that the distribution is not a good fit.

A large variety of GoF tests exists, generally spanning two opposite extremes, i.e. the most stringent GoF tests where the rejection is high, and the laxest ones where the opposite occurs. A low-power (viz. closer to the relaxed extreme) GoF test is desired for this factorial design, and the χ^2 GoF was chosen. This is due to the basic reason that the used antenna did not perfectly capture the power density of only one component (a monopole was used). On the other hand, the drawbacks of using a low-power GoF test is somehow counteracted by the large number of measured data samples that the VIRC is able to provide us with (for the experiments 1000 data samples were measured).

The p-value provides also a comparative ability. So, the higher the p-value the better the fit, and viceversa.

3.2.3. *Autocorrelation coefficient.* In an ideal RC the field distribution between one stir state and the following one would be expected to change drastically and keep no similarities between them. When a quantitative and objective statement must be done, then this "similarity" is measured by means of the autocorrelation coefficient.

For a given frequency the change in the boundary conditions (i.e. the vibration of the walls in the VIRC) must be sufficiently large so as to produce the more number of independent samples as possible. The autocorrelation coefficient applied to RCs relates one field magnitude (i.e. the power received by an antenna) measured at a fixed spatial position for many different stir states with and same measurements but shifted a variable number of stir states. If the stirring process is efficient at a given frequency, then the autocorrelation should be low $|\rho| \approx 0$ with relatively low stir state lags.

Since it is in practice very unlikely to obtain exactly $|\rho| = 0$ for a finite number of samples $N < \infty$ out of finitely long ensembles, it is defined that $|\rho| \leq \frac{1}{e} \approx 0.37$ are the values of ρ that would indicate fairly reasonable uncorrelation.

Typical autocorrelation coefficient measurements are shown in Fig. 2. Two main effects are to be noticed:

(1) As frequency increases, the autocorrelation coefficient "enters" into the uncorrelated region sooner. This basically means that less stir state lags are necessary in order to obtain the first uncorrelated field distribution. Therefore, the stirring process is able to provide a larger quantity of independent field samples.
(2) Despite of having found a stir state lag providing the first uncorrelated sample, nothing assures that all the samples thereafter will continue to be independent. As seen in Fig. 2 for 200 MHz, there are some "violations" to correlation. These violations become less as frequency increases till they disappear as for f = 1 GHz in Fig. 2.

Even though the autocorrelation coefficient provides a consistent and reliable way for RC characterization, it makes it difficult to compare between two different performances. Several values can provide us with a comparison, i.e.: the number of stir states necessary to find the first uncorrelated sample, the number of violations to correlation, etc. We chose an overall indicator which is the total area under the whole ρ curve. The absolute value of ρ was taken in order to avoid cancellations due to sign changes. That is how the third performance indicator becomes $\int |\rho| dt$. In this case the indicator as it is defined, works in a "the lower the better" manner. Unfortunately, it is not possible to define an upper bound for satisfactory performance, as in the previous cases.

4. Factorial Plan.

The factorial plan was defined using the three factors defined in section 3.1 plus the frequency, which behaves as a pseudo-factor. It gives a good idea on how the chamber goes from the undermoded regime to the overmoded regime. Three levels of variation were defined for the frequency.

Factor *Tightness* was simply realized by fixing the tent as close as a perfect rectangular shape as possible for the *tight* configuration; and relaxing the corners of the tent and leaving the tent more movable and flexible for the *loose* configuration.

Factors *Harmonic-like Movements* and *Diversity* were implemented using three fans, each one pointing at one of the lateral walls of the tent. By using the oscillation function of the fans, the factor accounting for harmonic-like movements was achieved. When the oscillation is not present, the chamber is excited just by the wind produced by the fans, which is a more random-like movement. By using different number of fans exciting different numbers of walls, the diversity/amplitude factor was achieved.

Two factors have three levels of variation (*Frequency* and *Diversity*) and two factors have two levels of variation (*Tightness* and *Harmonic-like Movements*). That defines a total of $3^2 \cdot 2^2 = 36$ experiments for the factorial plan.

5. Test Setup.

- **Screened enclosure**. Fabric used: Kassel Copper-Silver SHIELDEX Fabric. Joined using sewing. Internal dimensions (tight): 1.5 m × 1.2 m × 1 m (height). Volume (tight): 1.8 m^3. One access panel, originally thought to hold the samples for shielding effectiveness measurements in the dual VIRC configuration. Holding structure.
- **TX/RX Antennas**. Two monopoles of 7 cm were used as the transmitting and receiving antennas. The antennas were directly plugged to feed-through

SMA connectors in the panel's handle holes. The distance from the wall to the antennas was more than 13 cm.

- **Spectrum Analyzer**. A Rohde & Schwarz FSL spectrum analyzer (f=9 kHz ... 18 GHz) was used. The tracking generator featured in this instrument was used to analyze the signals inside the VIRC.
- **Fans**. Three fans each one pointing at a different lateral wall of the chamber were positioned. The fans were also mechanically attached to the walls, with the aim of producing a harmonic motion of the walls when they are set in the oscillation mode.

Test #	Frequency	# of Fans	Oscillation	Tightness	Stirring Ratio	p-value	$\int \|\rho\| dt$
1	200	1	No	Loose	0.9	0.19	313.78
2	200	1	No	Tight	2.26	1.05e-10	105.5
3	200	1	Yes	Loose	14.76	2.75e-15	316.1
4	200	1	Yes	Tight	1.46	1.11e-06	155.27
5	200	2	No	Loose	6.75	2.52e-05	91.1
6	200	2	No	Tight	7.95	3.96e-06	46.9
7	200	2	Yes	Loose	46.14	8.3e-04	44.24
8	200	2	Yes	Tight	33.37	1.94e-11	207.15
9	200	3	No	Loose	11.11	8.3e-03	366.49
10	200	3	No	Tight	19.33	9.55e-04	38.06
11	200	3	Yes	Loose	27.77	39.5e-03	82.21
12	200	3	Yes	Tight	24.58	79.2e-03	277.28
13	500	1	No	Loose	10.64	0.28e-03	27.5
14	500	1	No	Tight	10.46	3.75e-09	57.2
15	500	1	Yes	Loose	53.61	66.07e-05	85.19
16	500	1	Yes	Tight	34.95	1.56e-07	53.4
17	500	2	No	Loose	18.8	3.86e-11	32.7
18	500	2	No	Tight	8.52	2.91e-05	62.28
19	500	2	Yes	Loose	49.99	0.09	46.56
20	500	2	Yes	Tight	31.69	0.55	160.74
21	500	3	No	Loose	11.69	2.63e-10	67.65
22	500	3	No	Tight	7.44	5.19e-07	32.36
23	500	3	Yes	Loose	60.3	0.01	49.44
24	500	3	Yes	Tight	38.49	30.57e-05	32.96
25	1000	1	No	Loose	27.82	0.1	41.9
26	1000	1	No	Tight	26.02	45.73e-05	44.75
27	1000	1	Yes	Loose	26.04	0.81	89.19
28	1000	1	Yes	Tight	31.95	0.14	213.1
29	1000	2	No	Loose	25.74	52e-04	364.57
30	1000	2	No	Tight	15.23	21e-04	55.4
31	1000	2	Yes	Loose	24.96	14.2e-03	86.6
32	1000	2	Yes	Tight	19.74	19.8e-03	152.12
33	1000	3	No	Loose	13.72	2.48-05	76.95
34	1000	3	No	Tight	11.43	3.98e-10	40.49
35	1000	3	Yes	Loose	32.39	0.7	96.47
36	1000	3	Yes	Tight	31.6	0.44	104.85

TABLE 2. The complete $3^2 \cdot 2^2$ factorial plan of section 4. The frequency is measured in MHz and the Stirring Ratio in dB.

6. RESULTS.

The 36 experiments defined in the factorial plan were carried out and results for the three performance indicators described in section 3.2 were measured and calculated. Table 2 shows the complete list of experiments with their correspondent results.

The assessment of each factors influence on every indicator and the interaction between them can be done by performing very simple operations over the response, i.e. calculating the mean value of the response for different combinations of the 36 experiments. A graphical summary can be done by means of the factorial plots. We will analyze two families of factorial plots: the main effect plots and the interaction plots. Both of them provide with a clearer understanding of the factors' influence than the one that can be done just by looking at table 2.

6.1. **Main Effect Plots.** Figures 5-7 show the main effect plots for the three performance indicators. The slopes indicate that a factor has a main effect. We can see that almost all four of the involved factors have a main effect on the responses, since in almost all cases a change in their levels means a significant change in the indicators. A factor without any influence would result in an horizontal line in graphs like Figs. 5-7 (i.e. the difference between placing 2 or 3 fans when the Stirring Ratio indicator is considered, in Fig. 5).

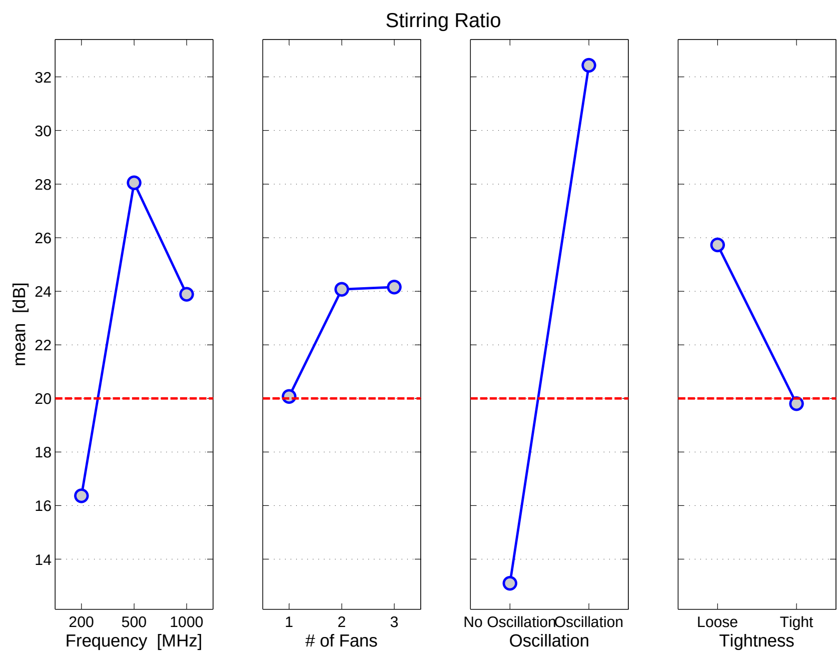


FIGURE 5. Main effect plot for the Stirring Ratio. The dashed red horizontal line indicates the empirical minimum value considered as good reverberation performance, SR_{min}=20dB.

For the cases where the slope is positive, it means that increasing one of these factors level imply increasing the response level. A negative slope meaning that increasing a factor level imply decreasing the response level. Even if almost all of them have a main effect, they can be compared by comparing the slopes. The higher the slope the larger the effect on a particular indicator.

6.1.1. *Discussion.* It can be seen that for frequency f = 200 MHz, the performance of the chamber is considerable worse than for the other frequencies. It is to be noticed that only for the p-value indicator, the performance at f = 1 GHz does better than at f = 500 MHz.

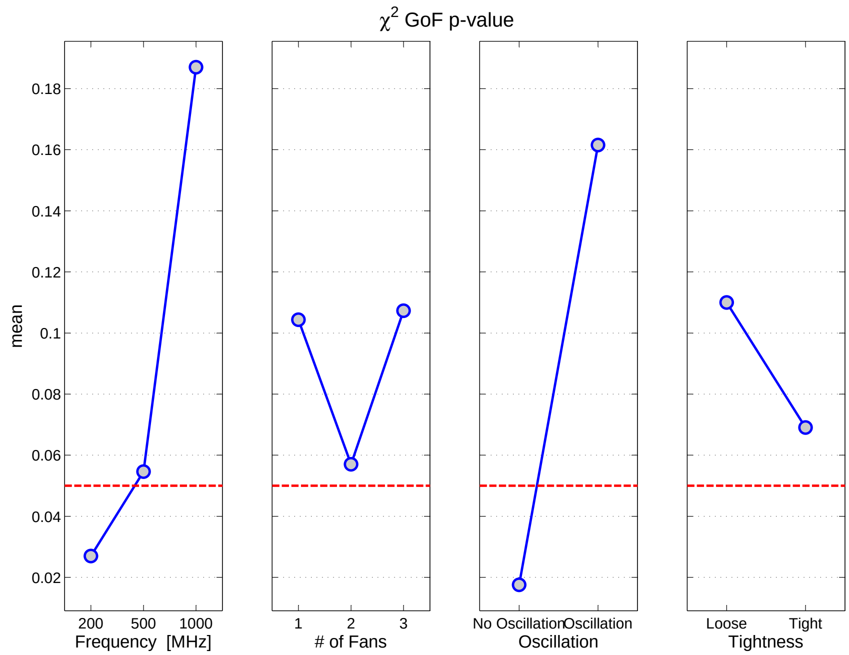


FIGURE 6. Main effect plot for the χ^2 GoF test p-value. The dashed red horizontal line indicates the normal level of significance ($\alpha = 0.05$).

The number of fans (diversity) has a relative lower effect than the other factors. Furthermore, the main effect of the diversity is well different for each type of indicator. If we focuss on the stirring ratio, we can see that moving two walls is significantly better than moving just one, but there is not a significant change in moving three walls instead of two. For the p-value, the effect is rather unusual: moving one or three walls give a better performance than moving two walls. This behavior is, however, above the significance level of $\alpha = 0.05$, assumed as the value of good reverberation performance. It is only for the $\int |\rho| dt$ indicator that a clear "the larger the diversity, the better the performance" is obtained.

Harmonic-like movements seem to favor the performance if assessed via the stirring ratio and the GoF test, but not if using the $\int |\rho| dt$ indicator. Oscillations, indeed, can produce larger variation of the chamber's walls, resulting in larger stirring ratios and better statistics. But they also provide with data that tend to be more correlated. Unfortunately, it is not possible to define a performance limit for $\int |\rho| dt$, in order to see whether with or without oscillation is in and/or out the good reverberation performance.

Exactly the contrary case as for the harmonic-like movements happens for the tightness factor. A tight VIRC will perform better than a loose one, only if we look at the $\int |\rho| dt$ indicator.

In summary, if we look at the stirring ratio or at the p-value, the best configuration for the VIRC would be a loose tent, excited with a harmonic-like movement

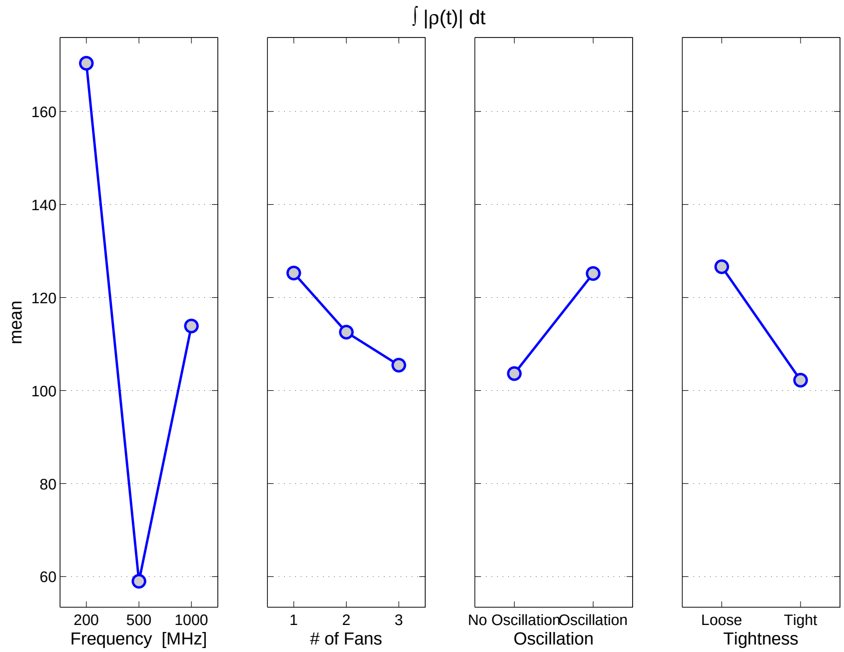


FIGURE 7. Main effect plot for the $\int|\rho|dt$ indicator. No limits for acceptable reverberation conditions can be established.

where at least two walls are moved. On the other hand, if we take the $\int|\rho|dt$ indicator into account, the best configuration would be a tight tent, with no harmonic-like movements (as random as possible) and with the maximum number of walls moved.

6.2. **Interaction Plots.** Interaction between factors means that the influence of one factor over a response is conditioned by the other factor level. The interaction plot is a matrix plot, with the number of rows and columns both equal to the number of factors. The factors names are printed on the diagonal of the plot matrix. The plot at off-diagonal position (i,j) is the interaction of the two variables whose names are given at row diagonal (i,i) and column diagonal (j,j), respectively. Figures 8-10 show the interaction plots for the factorial plan of table 2. It can be seen that for many cases, the way each factor influences the response strongly depends on the level of the other factors very often.

Usually, the interactions are not taken into account, but they behave as a key aspect of the experimental analysis. The presence of strong interaction between factors account for problems with a considerable level of complexity in them.

6.2.1. *Discussion.* Assessment and discussion of all the different implications that can be deduced from Figs. 8-10 is a complex, time-consuming and somehow bothersome task. What is clear is that the problem of optimizing the stirring process is not trivial at all, and we face a very complex problem. We will stress what we believe are the main remarks that can be figured out from Figs. 8-10.

Firstly, it is pretty clear that the diversity factor (modelled with the number of fans activated) present a great interaction with the other factors, regardless of

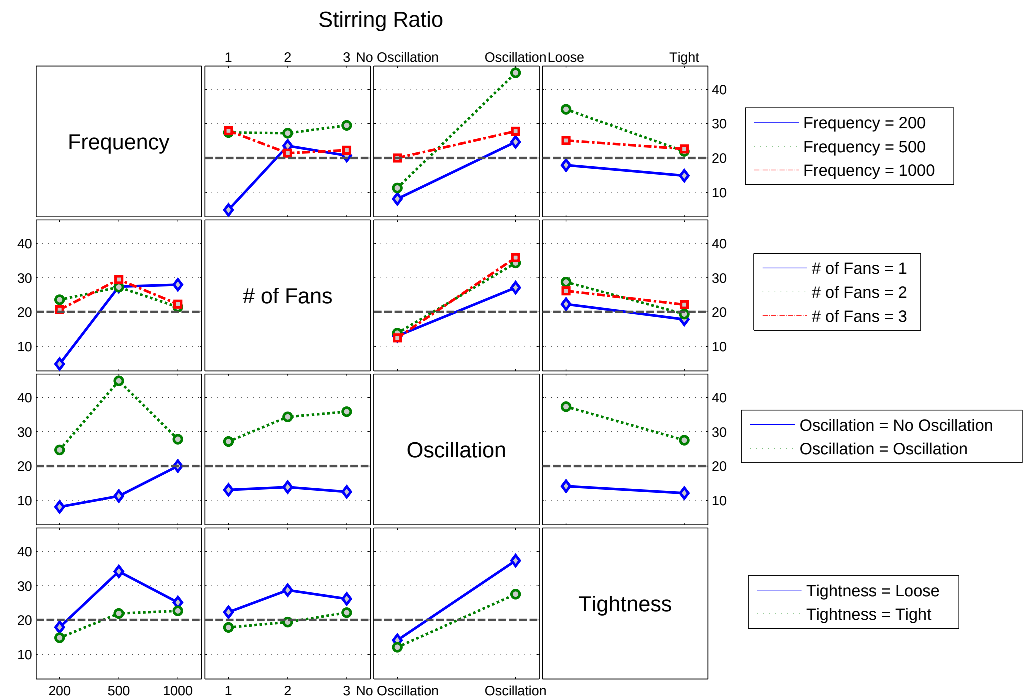


FIGURE 8. Interaction plot for the Stirring Ratio.

the indicator of interest. Especially with the frequency and with the tightness. It seems that the question about placing the motor(s) in the corner(s) or in the center of the walls cannot be answered in a close manner.

Secondly, it is very interesting to notice the behavior of factors *Tightness* and *Oscillation* when looking at the $\int |\rho| dt$ indicator in Fig. 10. Harmonic-like movements would be a vantage provided that the chamber is fixed in a loose fashion. On the contrary, if the tent is fixed in a tight configuration, the best strategy is to generate more random movements. This conclusion adds some extra knowledge, because looking at the other two indicators in Figs. 8 and 9, oscillation is always a vantage, no matter in what fashion the tent is fixed.

Finally, it has to be acknowledged that it is not possible to isolate one factor's effect from the others. Even when two factors seem to function independently, without any interaction between them, that is only true for some performance indicator and not for the other ones, like i.e. the frequency and oscillation in Fig. 10 show a typical plot of absence of interaction: the lines are almost perfectly parallel with each other. But if we look at these same two factors for the other two performance indicators the situation changes drastically.

7. CONCLUSIONS

The definition, application and assessment of a factorial plan was reported. Three different factors, named *Diversity*, *Harmonic-like Movements* and *Tightness* at three working frequencies were defined. The overall stirring performance was assessed according to three different performance indicators, viz. the *Stirring Ratio*,

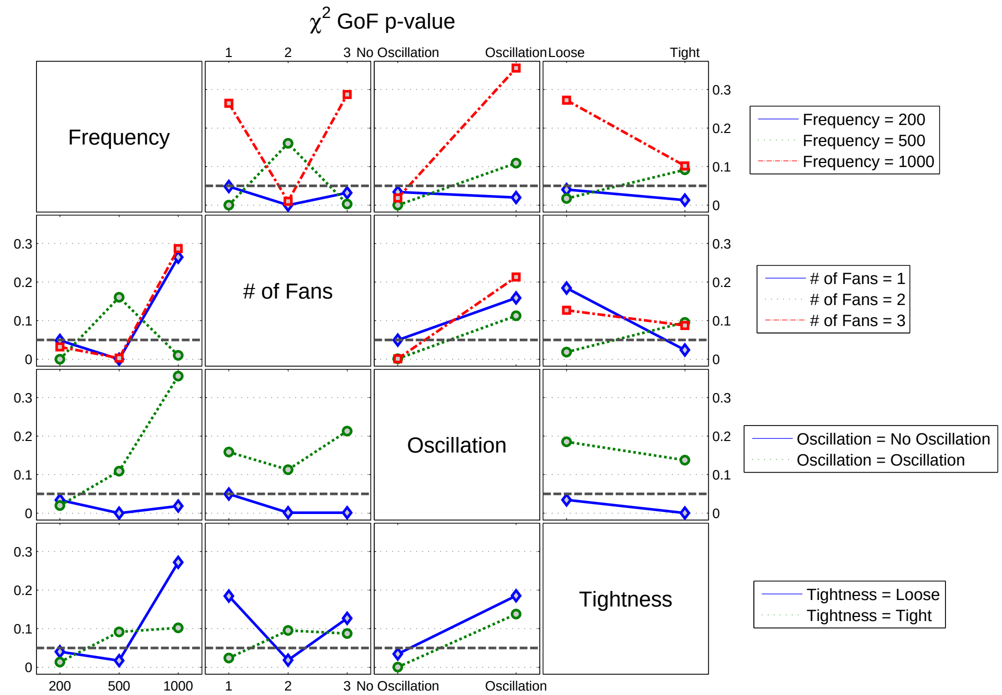


FIGURE 9. Interaction plot for the χ^2 GoF test p-value.

the χ^2 *GoF test p-value* and the absolute area under the autocorrelation coefficient, the $\int |\rho| dt$ indicator.

The DoE technique was used and it helped in gaining insight on what kind of stirring strategy could work the best in the small VIRC. Some results confirm the traditional means of stirring that are as well intuitive and clear to understand.

With the help of the main effect plots and the interaction plots it was possible to extract general guidelines on how the best stirring process should be. These general guidelines can be summed up as:

(1) It seems to be an advantage to provoke large diversity (meaning more "3-dimensional" movements or more walls excited), provided that these movements can achieve a large amplitude. Combinations of diversity and amplitude, i.e. placing one motor at a corner and another motor at the center of a wall, weren't tested, unfortunately. But perhaps this kind of combined effects could be implemented.

(2) If the tent has to be fixed in a tight fashion, then no harmonic-like movements are advisable.

(3) If the use of motors, and therefore harmonic-like movements, for the implementation of the stirring is inevitable, then the loose configuration of the tent is advisable.

The tent will be fixed to the supporting structure by means of springs attached to its corners. Some of these springs will be mechanically connected to some motors (initially two). The chosen motors are the ones for the car wipers, that provide an oscillating motion of about 85°. They should move the springs connected to the

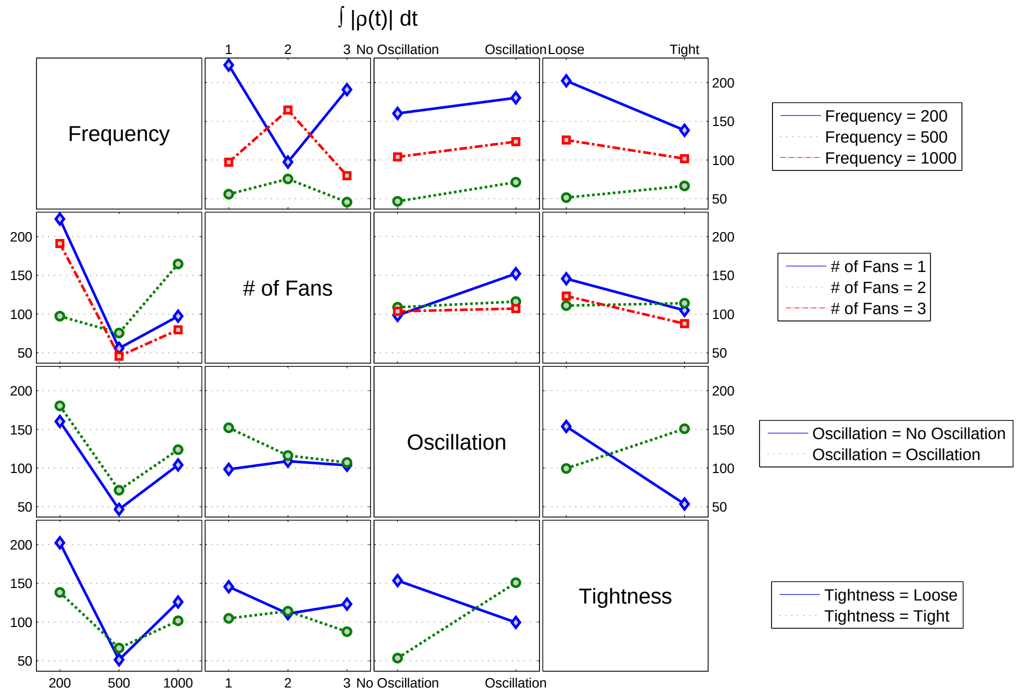


FIGURE 10. Interaction plot for the $\int |\rho| dt$ indicator

corners of the chamber in a way to translate that movement, as far as it is possible, to the larger number of walls. A movement in the same direction as the diagonal of the chamber should work for this.

A sufficiently large amplitude must also be provided. As a first, rough guideline, the relative displacement Δx of the walls should be approximately a quarter of a wavelength at the desired lowest useable frequency. If the f_{LUF} is meant to be ≈ 500 MHz, then the $\Delta x \approx 15$ cm.

The configuration should tend to fix the tent in a relatively loose fashion. If, for space reasons or other criteria, the tight configuration must be chosen, then an extra movement, more random-like might be considered as well.