Assignment class 6 of MQSN (LNMB, October 2014): Sojourn times in queueing networks

Main reference:
O.J Boxma, H. Daduna, Sojourn times in queueing networks (1990). In: Stochastic analysis of Computer and Communication Systems. Editor H. Takagi, Elsevier, Amsterdam.

Also available electronically (pdf) from CWI repository as CWI report BS-R8916 (1989).
Link: http://oai.cwi.nl/oai/asset/5854/5854A.pdf

Spend a reasonable amount of time for this assignment, say 8 hours at most.

1. In an open Jackson tandem network, show that successive waiting times are *not* independent. Hint: try to mimick the proof of Theorem 2.1 for waiting times. Is $\text{Cov}(W_1, W_2)$ positive or negative (here the W_i are the stationary waiting times in nodes 1 and 2, experienced by the same customer)?
2. Check the overtake-free property for the eight paths mentioned in Examples A, B and C (below Definition 2.1), by identifying the relevant sets $P(i,j)$ and $P(i,k,j)$.
3. In a closed Jackson network as in Figure 1.1, can you find an expression for $\text{Cov}(T_i, T_j)$ or for the correlation coefficient $\rho(T_i, T_j)$? How does it behave?
   (If the general problem proves too cumbersome, focus on a special case of your choice, e.g. $J = 2$, or $j = i + 1$, or $\mu_j \equiv \mu$, or $N = 2$, or any combination of these).

Ad 3: Use that for two random variables X, Y we have: $E[XY] = \frac{\partial^2}{\partial s\,\partial u} E[e^{-sX-uY}]\Big|_{s=u=0}$.