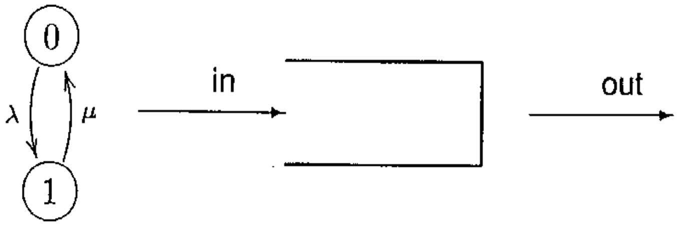
Standard Fluid Queue



- Continuous-time Markov process $X(t) \in \{1, 2, \ldots, N\}$; generator Q
- Content process $C(t) \in [0, \infty)$ or $[0, B]$
- Net input rate r_i when $X(t) = i$, $R = \text{diag}(r_1, r_2, \ldots, r_N)$
- Interest:
  - $F_i(y, t) = \mathbb{P}\{X(t) = i, C(t) \leq y\}$
  - $F_i(y) = \lim_{t \to \infty} F_i(y, t)$

Motivation

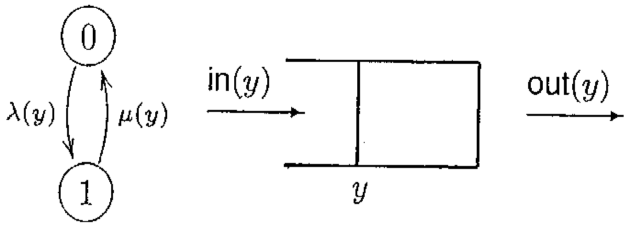
Fluid models often used in teletraffic modeling

- $X(t)$: State of one or more traffic sources / arrival streams
- $C(t)$: Buffer content of some bottleneck queue / router in the network

Problem: source behaviour often depends on state of the network due to feedback

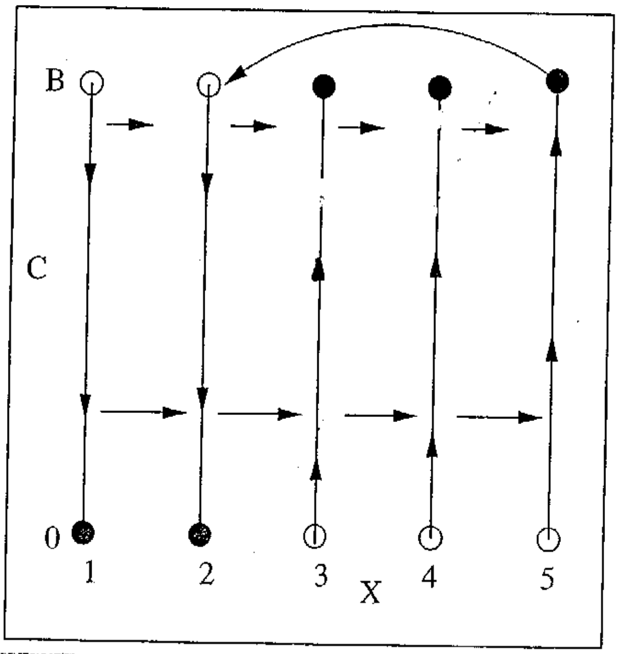
Solution: Feedback fluid queues

Feedback Fluid Queue



- Modulating process $X(t) \in \{1, 2, \ldots, N\}$
- 'Generator' $Q(y)$ when $C(t) = y$
- Content process $C(t) \in [0, \infty)$ or $[0, B]$
- Net input rate matrix $R(y)$ when $C(t) = y$

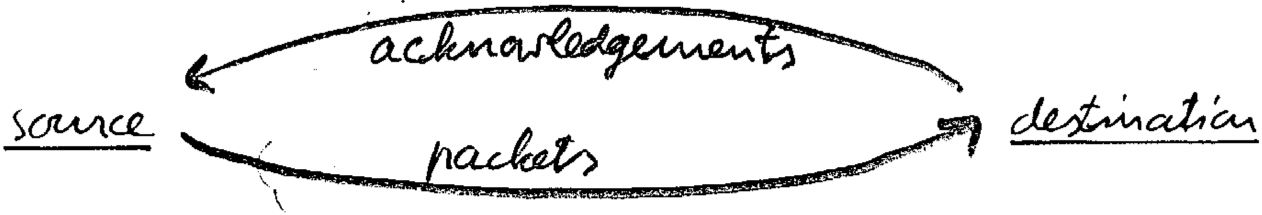
A TCP Source



Feedback Fluid Queues

Often, feedback from the network (buffers) to the sources

e.g. TCP:



RTT = Round-trip-time: time between sending a packet and receipt of corresponding acknowledgement (ack)

Linear increase:

Each RTT the source increases its rate by 1 packet per RTT due to positive ack until
- some maximum rate is achieved

or
- some packet is lost

Multiplicative decrease:

When packet is lost → sender receives negative ack
→ sending rate divided by 2 !

Followed by linear increase, etc.

Fluid model for single source:

- Source state $X(t) \in \{1, \dots, N\}$
- Sending rate $i \cdot r$ when $X(t) = i$
- Buffer state $C(t) \in [0, B]$
- Capacity c
- Assume $RTT \sim \exp(\lambda)$ (rather than constant!)

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$\{X(t)\}$ is <u>not</u> a markov chain!

Stationary distribution $p_i = \lim_{t\to\infty} P(X(t)=i)$ unknown.

Let $P_i(t,x) = P(X(t)=i,\ C(t)\le x)$ $\quad x<B$ as before

$D_i(t) = P(X(t)=i,\ C(t)=B)$

For state i with $\underline{i\cdot r > c}$ we have

$$D_i(t) = (1-\lambda\Delta t)\,D_i(t-\Delta t) + \lambda\Delta t\big(D_{2i}(t-\Delta t) + D_{2i+1}(t-\Delta t)\big)$$
$$+ (1-\lambda\Delta t)\Big(P_i(t-\Delta t, B-) - P_i(t-\Delta t, B-(ir-c)\Delta t)\Big) + o(\Delta t)$$

Rearranging terms, divide by Δt, <u>let $\Delta t\to 0$</u>:

$$D_i'(t) - (ir-c)\frac{\partial}{\partial x}P_i(t,x)\Big|_{x=B-} = -\lambda D_i(t) + \lambda D_{2i}(t) + \lambda D_{2i+1}(t)$$

<u>let $t\to\infty$,</u> $D_i = \lim_{t\to\infty} D_i(t)$, $F_i(x) = \lim_{t\to\infty} P_i(t,x)$:

$$\underline{-(ir-c)\,F_i'(B-) = -\lambda D_i + \lambda D_{2i} + \lambda D_{2i+1}}$$

Now let $R = \mathrm{diag}(r, 2r, \dots, Nr)$ and

for $y<B$: $\qquad\qquad$ for $y=B$:

$$Q = \begin{pmatrix} -\lambda & \lambda & & & \\ & -\lambda & \lambda & & \\ & & \ddots & \ddots & \\ & & & \ddots & \lambda \\ & & & -\lambda & \lambda \\ & & & & 0 \end{pmatrix} \qquad \tilde{Q} = \begin{pmatrix} 0 & & & & \\ \lambda & -\lambda & & & \\ \lambda & & -\lambda & & \\ & \lambda & & \ddots & \\ & \lambda & & & \\ & \vdots & & & -\lambda \end{pmatrix}$$

Then
$$\left\| \begin{array}{ll} -(R-cI)\,F'(B-) = \tilde{Q}^T D & \\ (R-cI)\,F'(x) = Q^T F(x) & x<B \quad \text{as before} \end{array}\right.$$

where $F(x) = (F_1(x), \dots, F_N(x))^T$

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$$(*) \qquad -(R - cI)\, F'(B-) = \tilde{Q}^T D$$

$$(**) \qquad (R - cI)\, F'(x) = Q^T F(x) \qquad x < B$$

Holds for <u>any feedback fluid queue</u> with $\tilde{Q}$ at level B
Q below level B

<u>2N unknowns:</u>
- D_i, $i = 1, \dots, N$
- Coefficients in solution of (**) (spectral expansion as before)

<u>Boundary conditions:</u>
- equations (*)
- $F_i(0) = 0$ when $i \cdot r > c$
- $D_i = 0$ when $i \cdot r < c$ (rather than $F_i(B-) = p_i$ as earlier)
- normalization $\sum_{i=1}^{N} F_i(B-) + D_i = 1$

In <u>total 2N + 1</u>, but one is dependent.

Extensions:

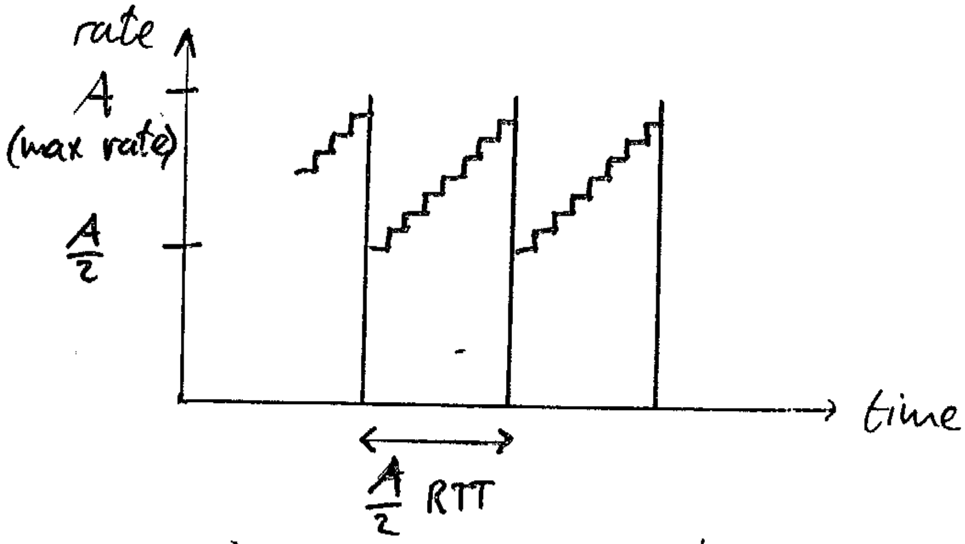
- More sources
- More buffer regimes, ? even continuous $Q(y)$
- More buffers

"Root p law" for TCP

Throughput in packets per unit time $= \frac{\sqrt{3/2}}{RTT \cdot \sqrt{p}}$

p = packet loss probability

<u>"proof"</u>:



$$\text{Throughput} = \frac{\left(\frac{3}{2}\right)\left(\frac{A}{2}\right)^2}{RTT \cdot \frac{A}{2}} = \frac{3}{4}\frac{A}{RTT} = \frac{\sqrt{3/2}}{RTT\sqrt{p}}$$

$$p = \frac{1}{\frac{3}{2}\left(\frac{A}{2}\right)^2} \Rightarrow A = \sqrt{\frac{8}{3p}}$$

Note:
- RTT is constant
- loss is periodic and exogeneous

The feedback fluid model with
- stochastic RTT ($\sim exp(\lambda)$)
- endogeneous loss

yields similar result (v Foreest, Mandjes, Scheinhardt)