

MQSN – 2016 - First set for Part Boucherie

1. Consider a system consisting of a CPU with $J - 1$ clients that send jobs to the CPU. The CPU equally divides its computing time over the jobs present at the CPU. Each client has a Poisson arrival process of new jobs with rate λ_i for client i , $i = 1, \dots, J - 1$. A job that arrives at a client first needs to be handled by the client (e.g. obtains a time stamp). To this end, the client has a single server handling the jobs, and jobs wait for their turn in a queue of unlimited capacity (are treated First In First Out). The service time of a job at the client is exponential with rate ν_i for client i , $i = 1, \dots, J - 1$. Upon completion of this activity, the job is sent to the CPU. Here the required amount of service is exponential with rate μ . Upon completion of the job at the CPU, the job returns to a client to be handled again. The routing probabilities from CPU to clients are such that the fraction of jobs treated at the clients in the first and second visit are equal. At a new visit at a client, the job joins the tail of the queue, waits for its turn (FIFO) and has again exponential service time with rate ν_i to e.g. obtain a second time stamp. Upon this second service completion the job leaves the system.
 - (a) Make assumptions to model this system as a Jackson network of queues and give a complete description of the Markov chain that records the number of customers in each queue for the tandem network, i.e., state space, routing probabilities, service rates and the resulting transition rates.
 - (b) Give the stability condition for this network.
 - (c) Give the equilibrium distribution for this network and explicitly prove correctness using partial balance (you MUST also show that these equations hold at the boundary of the state space).
 - (d) Give the mean sojourn time in the system of a job that arrives at client 1.

2. Consider an open Kelly Whittle network of J queues with Markov routing. With n recording the number of customers at the queues, the transition rates of this network are

$$q(n, n - e_i + e_j) = \frac{\psi(n - e_i)}{\phi(n)} p_{ij}$$

where $\psi() \geq 0$, $\phi() > 0$, see slides of lecture 2 for details. Assume that the routing matrix $[p_{ij}, i, j = 0, \dots, J]$ is irreducible.

- (a) Give a complete description of the Markov chain that records the number of customers in each queue for the Kelly-Whittle network, i.e., state space, transition rates.
- (b) Formulate the traffic equations and show that the traffic equations have a unique non-negative solution.
- (c) Give the stability condition for the Kelly Whittle network and the equilibrium distribution.
- (d) Prove correctness of the equilibrium distribution via partial balance (again: you must also consider the boundary of the state space).