Managing the overflow of Intensive Care patients

A mathematical study on the allocation of IC beds for emergency patients within a region.

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Abstract

Intensive Care Units (ICU) in the Netherlands often cope with capacity problems. This is a serious problem, because patients requiring immediate intensive care can not wait for a bed. The Erasmus Medical Centre (Erasmus MC) also copes with these capacity problems. Consequently they occasionally have to send regional emergency patients to a hospital outside the region. Regional emergency patients are patients not yet placed in the care of a hospital. They are the joint responsibility of the hospitals in the region where the patient comes from. Hospitals, however, tend to give priority to patients already hospitalised. These are patients needing intensive care after an elective (planned) operation (elective patients), or coming from a ward after deteriorating (internal emergency patients). Consequently, the Erasmus MC receives a large part of the regional patients and therefore, besides not being able to help all the regional patients, has to cancel elective operations or create so called “over beds” for the internal emergency patients. An over bed is a bed that is actually not staffed but in case of emergency can be used. These beds can be created as the constructional bed capacity is larger than the operational (staffed) beds. The research documented in this report aims to find a solution to the problem of a region not being able to take care of the regional patients requiring immediate intensive care. The research question is, whether jointly reserving IC beds in the region for regional emergency patients, will considerably reduce the number of regional patients sent outside the region. We want to determine how many regional beds are needed.

To answer the research questions, we develop a mathematical method, based on the Equivalent Random Method (ERM). The ERM is a method that deals with overflow and is widely used in the telecommunications industry. In the situation of the ICUs, the regional beds are the overflow. We distinguish three patient types: regional emergency patients, elective patients and internal emergency patients. We assume they all arrive according to a Poisson Process, their Length of Stay (LOS) is exponentially distributed and that all patients have the same mean LOS. Regional emergency patients who find all operational beds at the ICU occupied are transported to an overflow ICU. Elective operations are cancelled when no bed is available, and the internal emergency patients are placed on an over bed. The adjustment to the ERM concerns the over beds. Insofar we know, no version of the ERM has been developed with a flexible capacity. Using the ERM adjusted to ICUs, an approximation can be given of how many beds in a region should be allocated for regional emergency patients, in order to guarantee a certain probability of not having to send such a patient to a hospital outside the region.

To verify the analytic method, we develop a simulation model using the object oriented simulation software package eM-Plant. In the simulation we use data concerning the arrival process and the LOS of the Erasmus MC, and estimated data concerning the arrival process of several hospitals in the region Rijnmond (the region where the Erasmus MC is located). First of all, the results show that correct and sufficient data is important for a good approximation. To determine how many regional beds should be appointed in the region Rijnmond, the necessary data of the cooperating hospitals should be analysed accurately. We explore the different outcomes when the exponential distribution is used and when the more plausible Log Normal distribution is used for the LOS. The simulation model shows no significant difference to this matter. The different hospitals, however, do seem to have a different mean LOS. The results of the method when used solely with data of the Erasmus MC let expect that the method can provide a good approximation of the number of regional beds required in the region. Finally the results show that jointly reserving several regional IC beds will yield a profit compared to the situation where ICUs will try to solve the problem self-handedly.

The allocation of regional beds for the overflow of regional emergency patients appears to be a good solution. We do, however, not know if it is the best solution and therefore recommend, investigating other options and comparing the different options. If the management of the region decides to implement regional beds, the ERM for ICUs is a good method to approximate the number of regional beds required. The management of the region will first need to consider what boundary is acceptable for the probability of having to refuse a regional patient in the region. Subsequently, using data of the region in the method, an approximation can be made of the number of regional beds needed. Finally, after having made a decision on the number of regional beds needed, the management of the region will have to consider how to distribute the beds over the different ICUs in the region, and how these beds can be introduced such that they are used only for regional patients.

Preface

Before you lies the report of my final thesis, accomplished for the Master Program of the studies Applied Mathematics at the University of Twente in Enschede. The final thesis was carried out at cluster 17 of the Erasmus Medical Centre in Rotterdam, with as subject the Intensive Care capacity. My supervisor from the Erasmus Medical Centre is Mark Van Houdenhoven; my supervisors from the university are Richard Boucherie and Nelly Litvak. Erwin Hans supported me by commenting on my report.

I have had a great and instructive time at cluster 17. It was a good experience to apply my theoretical knowledge to a real life problem, and to work in a hospital environment. I am very grateful to Mark Van Houdenhoven who gave me this opportunity. Also I am grateful to Jan Bakker, head of the Intensive Care department, who taught me much about the department. My final thesis was not only instructive because of the practical experience. Thanks to Richard Boucherie and Nelly Litvak I learnt much about doing mathematical research. I am very grateful to them for the good supervision they have given me. Also I want to thank Erwin Hans, for the support and advice he has given me.

The great time during my graduation was for a large part owing to the employees and other students at the cluster office. I am very grateful to them for the enjoyable time and the support they have given me. With joy I will remember the lunches, the conversations and the fun I have had with them.

As most people know, graduating can give moments of stress. I am grateful to Michiel who kept listening patiently to my accounts and never lost his faith in me. Last but not least I would like to thank my parents for the emotional and financial support they gave me during the 8 years of my studies and thereby giving me the opportunity of having a fantastic time as a student!

Marleen van Rijsbergen

Contents

Chapter 1

Introduction

1.1 Rationale

"Each year, hundreds of patients die unnecessarily." This was the announcement in the NOVA[1] broadcast on November 6th, 2001 in which the capacity shortage on Intensive Care Units (ICUs) in Dutch hospitals was discussed with several specialists. Although the Dutch minister of Health, Welfare and Sports (HWS) denied that patients decease because of bed shortages, she did admit there were problems concerning the ICUs. At that time, some enquiries had been made at the request of the minister. In April 2001, the Julius Centre for Health Sciences and Primary Care presented a report [27] in which they concluded that there was an extensive problem with regard to the admission and release of Intensive Care (IC) patients. Their research was done for a selected group of 18 general IC departments of large non-academic hospitals. They discovered that almost 10% of the severely ill patients who needed intensive care was refused, 4% was admitted even though there was actually no space, and 3% was released earlier to make place. The most important reason for the refusal of a patient or the stop of admissions was the unavailability of operational (staffed) beds. In most of the cases, when beds were closed, this was caused by a shortage of nurses. The results of an earlier enquiry of the Julius Centre [26] had already shown that at the 113 cooperating IC departments, every six days a patient was refused or an admission stop was announced.

Short after the Julius Centre had presented their second report, the minister of HWS requested the Dutch Board for Hospital Facilities to picture the current capacity and use of intensive care for adults. The research that followed this request is documented in two reports ([5] and [6]). All Dutch IC departments for adults participated in the enquiry. From these two reports it appears that a large part of the IC capacity in the Netherlands is unused due to a shortage of nurses. Insufficient finances for new personal and scarcity on the labour market cause the shortage of nurses. In addition, many ICUs have an occupation degree higher than 80% (mainly the academic hospitals). According to [5] there is, with such a high occupancy rate, a large probability of having to refuse a patient.

[1] Nova is a Dutch current affairs programme

In 2001, the minister of HWS founded an Intensive Care group that had to propose solutions for the problems concerning the adult ICUs. Based on the aforementioned enquiries, the group proposed three objectives to resolve the capacity problem [31]:

- Bring the available IC capacity that is not yet operational, in use.
- Increase the accessibility of the available capacity.
- Implement a system for the safe transportation of IC patients.

A program that has to realise these objectives follows up the group. The program is coordinated by the Dutch society of care insurers, the society for hospitals and the society for academic hospitals. Its main target is to offer patients controllable high qualitative intensive care. Much has been done since the start of the program, but the main target has not been achieved up to now.

The Erasmus Medical Centre (Erasmus MC) in Rotterdam also has IC capacity problems. Here too, scarcity of nurses restricts the number of operational beds. In practice an operational bed is always found for a patient needing intensive care, but this does not mean that there is no problem. First of all, it may take hours to locate an operational bed for a patient needing immediate care. Furthermore, a patient may have to be transferred to another hospital, an operation may have to be postponed, an over bed[2] may have to be created or another patient may have to be released earlier. All these situations are not preferable.

Since 1999, the Erasmus MC (at that time still the Academic Hospital Rotterdam) is appointed a trauma centre[3] for the region Southwest Netherlands. Therefore the Erasmus MC is specialised in the treatment of trauma patients (victims of accidents). According to a strategic analysis of cluster 17 of the Erasmus MC [30], the number of trauma patients offered to the IC department of the Erasmus MC has increased since the acknowledgement of the trauma centre. The further extension of the trauma centre and the expected 24 hours trauma helicopter will most likely increase this demand more.

Some of the capacity problems at the IC department of the Erasmus MC are presumably caused by some other hospitals in the region, which are not willing to cancel elective (planned) operation for emergency patients in the region. Consequently, many of these patients are sent to the Erasmus MC, which is obliged to cancel elective operations on patients that require an IC bed afterwards, in case of emergency. As mentioned in [30], the impression exists that the operational IC capacity in the region Rijnmond reasonably approaches the demand for IC beds. At present however, emergency patients are occasionally sent outside the region Rijnmond because no operational bed can be found in

[2] An over bed is a bed that originally is not staffed. An over bed can be created when no operational beds are available. In such a case the available staff will have to take care of an extra patient, which decreases the level of care at the ICU.

[3] A trauma centre is a facility for integral care for patients from severe accidents. In the Netherlands the minister has appointed ten hospitals to fit out a trauma centre. For more information, see the National Atlas of Public Health[23].

the region. If all hospitals in the region allocate several IC beds as emergency beds, the region can most likely take care of all emergency patients in the region Rijnmond [30]. This does imply that sometimes hospitals might have to cancel elective operations while having an empty operational bed.

During the project documented in this report we used mathematical methods to examine whether the region Rijnmond is able to take care of all the emergency patients in the region.

1.2 Problem formulation

1.2.1 Objective

The research documented in this report has the following objective:

To minimise the number of emergency patients that are transferred from the region Rijnmond to an ICU outside the region Rijnmond because of a shortage of operational IC beds, and to distribute the emergency patients more evenly among the hospitals in the region taking into account the size and location of the ICUs.

The number of cancelled operations due to a full ICU and the number of times an over bed is created should stay under a certain threshold.

1.2.2 Research questions

We want to answer the following questions:

1. *Will the number of emergency patients needing intensive care in the region Rijnmond that are sent outside the region because of capacity shortages be reduced, if all ICUs in the region jointly appoint several IC beds for regional emergency patients?*
2. *How many beds does each ICU need to appoint as regional emergency bed in case the number of patients transferred outside the region is minimised?*

1.2.3 Research demarcation

The models developed for this research, are based on the patient streams of the ICUs of cluster 17 of the Erasmus MC. Examining other ICUs is beyond the scope of this graduation project. We assume, however, that similar patient streams exist in other ICUs. Consequently the outcomes of our research can be used for different regions with a different number of ICUs. Further we have taken the streams of arriving patients as input for our models and we have not tried to optimise these.

1.3 Research approach

Analytical models can give a good understanding of the system they represent. Furthermore analytical models can be accurate and can give concrete solutions.

Simulation models can model more complex systems than analytic models can, but are less accurate. Simulation models demonstrate the behaviour of the system rather than that they give an exact solution. The two approaches therefore form a good combination for research. Often simulation models are used to validate an analytical model or to determine how an approximation approach performs. For these reasons an analytic model and a simulation model are developed. The analytic model is developed in order to assist the management of a region to decide how many regional beds to reserve. The simulation model is developed to validate the analytic model and to demonstrate the patient flows in the ICUs of a region. The IC department of the Erasmus MC is studied to model the patient flows concerning an ICU.

1.4 Overview

In Chapter 2 the Intensive Care Unit and its patient streams are discussed. First the Intensive Care department of the Erasmus MC is considered. Consequently information on the patient flows through this Intensive Care department is translated into a model. Finally a model is given for a region of ICUs with regional beds. Then, in Chapter 3 an analytical method for the approximation of the number of regional IC beds needed in a region is developed. Next, in Chapter 4 the simulation model developed to validate the analytic method is discussed. Subsequently, in Chapter 5 the results generated by the analytical method and the simulation model are given. Finally, in Chapter 6 the conclusions and recommendations are presented.

Chapter 2

The Intensive Care Unit: present and future

In this Chapter we discuss the current situation and a future scenario of the Intensive Care Unit. First, in Section 2.1 we describe the Intensive Care department of the Erasmus Medical Centre. Subesequently, in Section 2.2 we shortly discuss two models of ICUs. The first model describes the current situation, the second model describes a future scenario of ICUs with regional beds.

2.1 The Intensive Care department of the Erasmus MC

The Erasmus MC [10] originated in 2002 from a merger of the Academic Hospital Rotterdam and the faculty of medicine and health science of the Erasmus University Rotterdam. The Erasmus MC is the largest and most diverse academic hospital of Europe, with more than 10,000 employees. The mission of the Erasmus MC is to create new knowledge in the area of disease and health, to pass this knowledge on to future professionals and to provide health care. The main tasks of the Erasmus MC are patient care, education and research. The hospital consists of 17 clusters (see Appendix B.1 for the organisational chart of the hospital). Each cluster consists of several departments connected by a shared patient population or shared logistic. Each department contains several units. A unit is a recognisable part of a medical department. For example, an outpatient clinic and a nursing department are units. The supporting activities of a cluster, e.g. finance, human resource management and logistics, are accommodated in the cluster office. Cluster 17 has a facilitating service for the other clusters. It consists of three departments: the Operation Theatre Department, the department of Intensive Care and the department of Anaesthesiology (see Appendix B.2 for the organisational chart of cluster 17). In Section 2.1 we discuss the Intensive Care department of the Erasmus MC. In Section 2.2.1, we use the information from this section to develop the model of an Intensive Care Unit.

2.1.1 Description of the Intensive Care department

Intensive Care (IC) is specific medical treatment and nursing to severely ill patients of whom one or more vital organ functions are disrupted or have failed. Patients who are indicated as IC patients require intensive monitoring, mostly elaborate pharmacological treatment and in many cases support with artificial ventilation. Cluster 17 of the Erasmus MC has three Intensive Care Units (ICUs) (located at the south side of floor 3, 6 and 10). Besides the ICUs of cluster 17 the Erasmus MC has an ICU specialised for children and neonates and an ICU for cardiac surgery patients. These two ICUs are not part of cluster 17 and are not considered here. Each ICU of cluster 17 has its own specialisation. Unit 3 south receives patients from internal medicine, unit 6 south from neurosurgery and unit 10 south from other surgical specialisations. Currently, patients are only admitted to one of the other units in case an operational bed is unavailable at the original unit. This policy is due to the staff's high specialisation, which makes exchange of patients and staff difficult. The intention is to broaden the expertise of the three units such that the exchange of patients will be easier and the capacity of the ICUs will be utilised better. The current bed capacity of the ICUs is represented in Table 2.1.

ICU	**Number of constructional beds**	**Number of operational beds**	**Specialisation**
3 south	18	9 - 11	Internal medicine
6 south	16	15	Neurosurgery
10 south	18	10 - 12	Surgery
Total	**52**	**34-38**	

Table 2.1: The number if IC beds

The IC department has a head of department and a manager. Each ICU has a nursing head, a medical coordinator, intensivists, fellow specialists in training and nurses. An intensivist is a physician specialised in the care of critically ill patients and is the one who decides whether a patient will be admitted to the ICU (depending on the condition of the patient and the availability of beds).

In the near future some changes will be realised at the IC department of cluster 17. One of the changes is the introduction of special care beds. The other change is the realisation of a Post Operation Care Unit (POCU). In the POCU patients will be treated who require, for at most 24 hours, some kind of monitoring after an operation. Currently these patients occupy an IC bed because they can only spent a few hours in the recovery room[1].

The IC department holds patients with different care weights (need for care). Based on these care weights, the patients at the ICU can be categorised into

[1] A recovery room is a hospital ward where the vital organ functions of the patient are strictly guarded after surgery.

Intensive Care patients, High Care patients and Medium Care patients. Intensive Care patients have a threatening or existing disorder at more than one of the vital organ functions and therefore need guarding, nursing and treatment by approximately one nurse per patient. High Care patients have a threatening or existing disorder at one vital organ function and therefore need guarding, nursing and treatment by approximately 0.5 nurse per patient. Medium Care patients do not have a threatening or existing disorder at a vital organ function but need intensive guarding because of their critical situation. For these patients approximately 0.25 nurses per patient is needed. Some hospitals have separate high care or medium care units but at the Erasmus MC these patients are all taken care of at an ICU. Until now these patients have been treated with the same care weight (the bed-nurse ratio is the same for each bed). On average there are 0.9 nurses per bed during daytime, 0.6 during the evening and 0.5 during the night. Plans are being made to fit out several special care beds. These beds will be used for step up patients. Step up patients are patients who at arrival need high care or medium care treatment. An example is a patient who has undergone major surgery, and does not require mechanical ventilation. Currently these patients are unnecessarily treated like IC patients. The patients in a special care bed will not require mechanical ventilation and there will be 0.25 nurse per bed instead of 0.5 up to 1. Step down patients (patients who are recovering) will not be transferred to a special care bed. When they no longer need mechanical ventilation, step down patients usually still require more attention than high care or medium care patients. As soon as these step down patients require less care, they can most often be discharged from the ICU and transferred to the ward. In case of emergency, the special care beds can be used for IC patients. In this case two special care beds become one Intensive Care bed, because of the difference in nurse-bed ratio.

2.1.2 Patient flows through the Intensive Care department

The arrival and admission of patients at the IC department are not straightforward. The patients arrive at random, any time of the day and usually need to be admitted immediately. The Dutch Society for Intensive Care has formulated directives for the admission and release of IC patients [3]. There are, however, no unambiguous agreements on how to deal with an arriving patient when no operational IC bed is available. Figure 2.1 gives an overview of the most likely made decisions and carried out actions when a patient arrives at the IC department of the Erasmus MC.

There are two kinds of patient arrivals: elective arrivals, and emergency arrivals. Elective arrivals come from the operating theatre. These patients have undergone a planned operation and require intensive care after the operation. Their arrival is announced the morning of the operation. Emergency arrivals can come from outside the hospital or from inside the hospital. These patients arrive unexpectedly and require immediate intensive care.

A surgion plans an operation two weeks in advance. The surgeon does not consult the intensivist but does use his knowledge on the occupation of the ICU. For the utilisation of the ICUs it would be better if specialists would consult

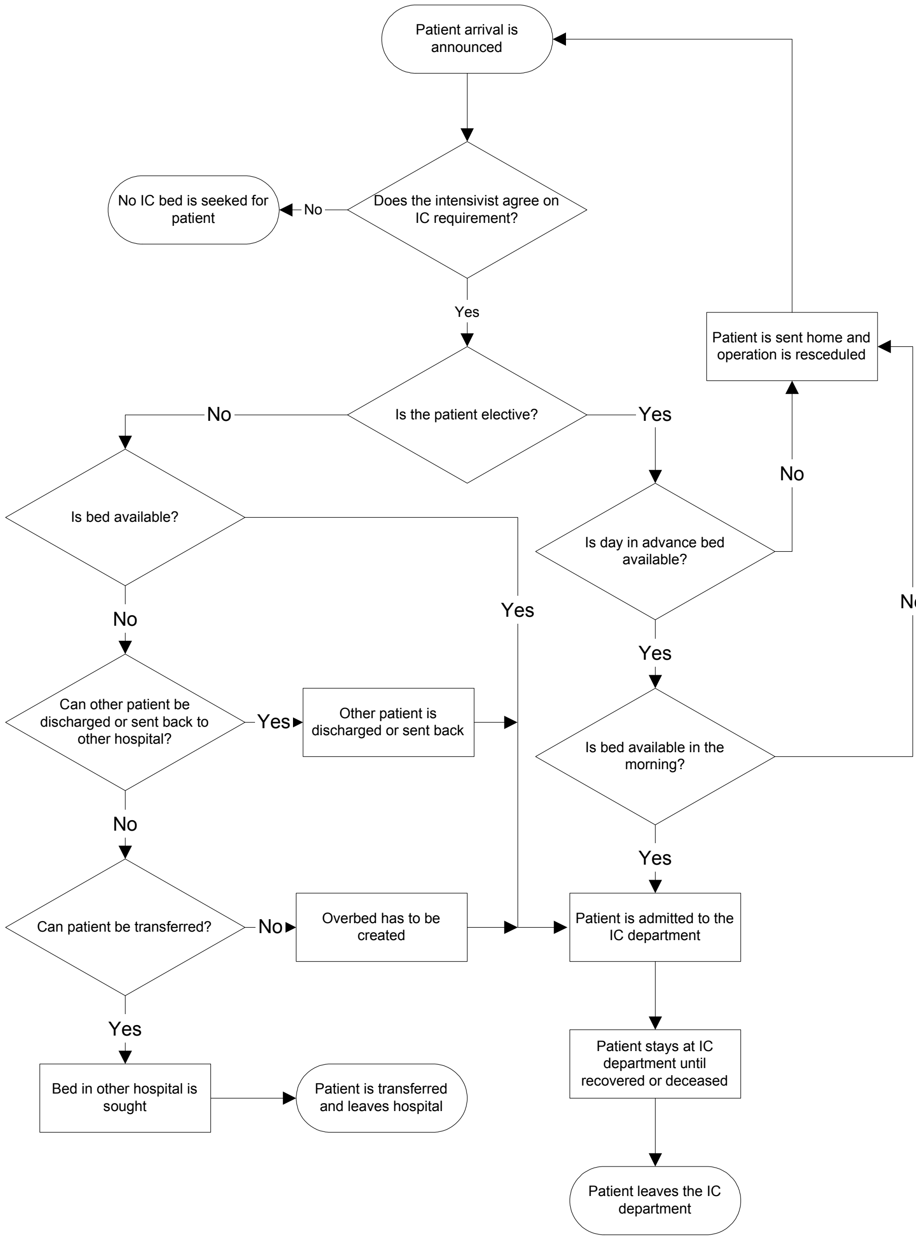

Figure 2.1: Flow chart of patients arriving at the IC department

an intensivist while planning the operations. The day before the operation the elective patient is hospitalised, most often at the ward, and the surgeon contacts the intensivist to check the availability of an operational bed for the next day. If there is no operational bed available in any of the three units the operation is cancelled and the patient is sent home. If a bed is available, the surgeon contacts the intensivist a second time in the morning before the operation (if the patient is scheduled for the first operation, this happens at 7.30 a.m., otherwise it happens at 9.00 a.m.). If the bed is still available, the operation is carried out and the patient is certain of a bed at an ICU after the operation. If the bed is taken by an emergency patient, the operation is cancelled and the patient is sent home. This patient has priority the next time he / she is scheduled for the operation. Again however, the operational bed capacity is not considered when planning the operation. Some elective operations can not be cancelled. These are operations that involve many people (staff and patients), for example a liver transplantation with a living donor. For such patients, beds are reserved that will not be taken by another patient.

Emergency patients arrive unexpectedly and require immediate care and consequently their admission can not be postponed. They come in through the emergency room (regional emergency patients) or from a nursing ward (internal emergency patients). The patients that come in through the emergency room are in most cases brought by an ambulance. The ambulance nurse, in cooperation with the central ambulance post, decides to which hospital the patient is taken. In case of an emergency the patient is taken to the nearest hospital. In the region where the Erasmus MC is located, the ambulance nurse does not have information on the availability of beds.

Occasionally patients are transferred from other hospitals. One cause is that the patient immediately requires an IC bed and the hospital he / she came from did not have an IC bed available. Another cause is that the patient requires treatment that can not be offered in the hospital of origin. Before sending the patient, the other hospital phones to the IC department to check the availability. Therefore these patients only arrive if there is a bed available.

After the specialist has informed the intensivist on the patient who needs intensive care, the intensivist decides whether the patient is admitted to the ICU. The specialist usually informs the intensivist by phone. Only in case of doubt will the intensivist see the patient before admitting him / her to the ICU. If a patient arrives at night, the acting representative of the intensivist makes the decisions on new arriving patients. In case of doubt, an intensivist can be contacted at home.

If an emergency patient should be admitted to an ICU but there is no bed available, the intensivist contacts the other units to check their operational bed availability. If no bed is available at all, another solution has to be found. In these cases it might happen that a patient already at an ICU is predischarged from the ICU. This can only be done if the discharge of the patient was already at hand. In addition, a patient who originates from a different hospital because a special procedure could not be carried out in that hospital, can be sent back if the special procedure is finished off. If no operational bed can be made available,

Type of arrival	Mean	Std. Deviation
Elective	0.58	0.92
Elective excluding weekend days	0.42	0.79
Internal emergency	0.62	0.74
Regional emergency	0.46	0.60
Total	0.18	0.39

Table 2.2: Interarrival times in days

the decision taken depends on the type of patient. If the patient is a regional emergency patient, generally an operational bed in another hospital is sought. If the patient is an internal emergency patient, it is preferable to keep the patient in the hospital. Juridically a patient can only be transferred if the transfer has benefits for the patient. Moreover, it is not preferable to transport a critically ill patient. Therefore it might happen that an over bed is created for an internal emergency patient. This denotes that the patient is placed in an IC bed that was not staffed. In such a case the physicians and nurses have to work harder as they have a patient extra to take care of. Such a solution requires flexible staff. Also for the patients at the ICU it is not preferable to create an over bed. The quality of care of the patients at the ICU decreases because the patients have to share the care with the patient in the over bed. One of the reasons that for internal emergency patients over beds are created and for regional patients not, is that hospitals tend to give priority to patients already admitted to their hospital. Another reason is that a patient not yet admitted to the hospital can jurdically be sent to another hospital, whereas a patient already admitted can not always be sent to another hospital.

When a patient is admitted to an ICU he stays there for a certain amount of time, we call this the length of stay (LOS). During the stay at the ICU a patient's health can improve or deteriorate. When a patient requires less than high care / medium care he is discharged from the ICU and transferred to the ward. Occasionally a patient stays longer at the ICU because there is no bed available at the ward. Unfortunately patients do not only leave the ward because their need for care has decreased, but also because of mortality.

2.1.3 Data concerning the patient flows

This report is part of a larger project. As another part of this project, Lesscher, Meutstege and Rouhof [19] analyse data concerning the patient arrivals and the Length of Stay at the IC department of cluster 17 of the Erasmus MC. In this section we discuss the relevant outcomes of their research.

Lesscher, Meutstege and Rouhof [19] find the average time interval between two arrivals at the IC department to be 0.18 days. If a distinction is made between the three different types of arrivals (elective arrivals, internal emergency arrivals and regional emergency arrivals) a significant difference between the interarrival times of these patients is found. Table 2.2.gives these interarrival times.

As the elective patients never arrive on weekend days, we also calculate the mean

Type of arrival	Mean	Std. Deviation
Elective	3.88	6.44
Internal emergency	8.15	12.69
Regional emergency	7.95	13.78
Total	6.93	11.90

Table 2.3: Mean Length of Stay in days

and standard deviation of the interarrival times for elective patients leaving out the weekend days. This mean and standard deviation is given in Table 2.2. From the data analysis it appears that there also is a difference in mean interarrival times between patient groups from different specialisms. For our research, however, we do not make this distinction. We test the hypothesis that the patients arrive according to a Poisson process. This hypothesis is not rejected for internal and regional emergencies, using the Kolmogorov-Smirnov Test in SPSS (statistical software for data management and analysis). For elective patients, however, this hypothesis is rejected.

Lesscher, Meutstege and Rouhof [19] also examine the LOS of the patients. The total mean LOS is 6.93 days. If again distinction is made between the three different types of arrivals it can be concluded that the mean LOS of internal emergencies and regional emergencies do not significantly differ. The mean LOS of the elective patients, however, does differ significantly from the two types of emergency patients. The mean LOS of the three patient types can be found in Table 2.3

From the data analysis it follows that there is a significant difference in the mean LOS of the patients situated at the different units. Most likely this difference results from a different mean LOS of patient from the different specialisms. For our research we leave this difference out of consideration and only make a distinction between elective patients, internal emergency patients and regional emergency patients. Lesscher, Meustege and Rouhof [19] try to fit a distribution to the data of the LOS of these three patient types. They find the Lognormal distribution, with mean and standard deviation as given in Table 2.3, to fit well.

A note has to be made on the LOS. The LOS as registered is larger than the actual LOS. Each calendar day the patient is at an ICU is counted as a whole day, even if the patient is only there a part of the day. For example, if a patient arrives one day at 11:00 p.m. and leaves the ICU at 3 p.m. the next afternoon, the LOS of the patient is two days. In reality the patient is there for less than 24 hours.

2.2 Models of a region of Intensive Care Units

In the previous section we discussed the Intensive Care Units (ICUs) of the Erasmus MC and the patient flows through these ICUs. In this Section we will first use this information to describe a model of the current situation of the ICUs. Subsequently we will describe a future scenario of a region of ICUs with regional beds.

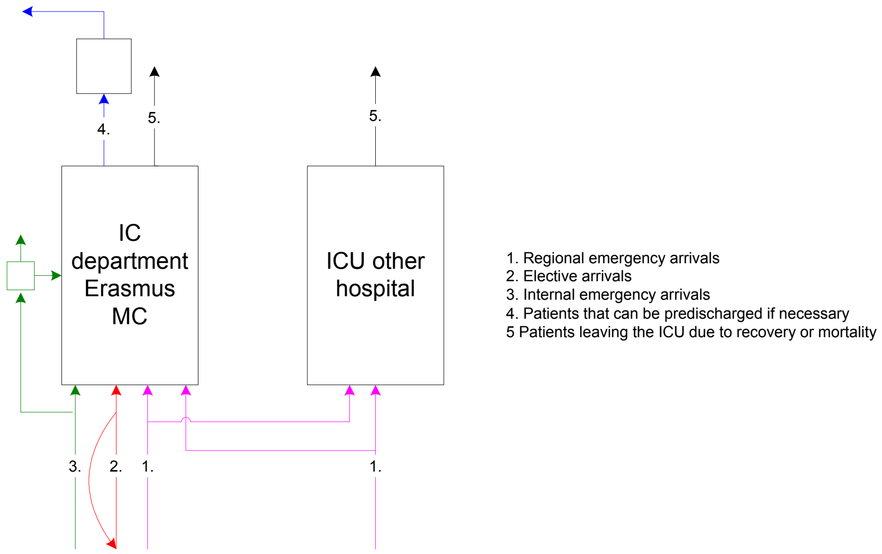

Figure 2.2: Patient flows through the ICU

2.2.1 Model of the current situation

Figure 2.2 gives a schematic representation of the patient flows as discussed in Section 2.1.2. In this figure we only depict one patient flow for the other hospitals. The reason for this is that we only know the number of patients that overflow the other hospitals and are taken to the Erasmus MC. We do not know anything about the patient flows in these hospitals or between these hospitals. Although the IC department of cluster 17 of the Erasmus MC consists of three separate units with different specialisms, the units take patients actually meant for another unit in case of operational bed shortages. Therefore, as long as an operational bed is available in one of the units, there are no consequences for patients who arrive at any of the three ICUs. In addition, we assume that the unit the patient is situated at does not influence the Length of Stay (LOS) of the patient, in spite of the different specialisms of the units. For these reasons we model the IC department of cluster 17 of the Erasmus MC as one unit, as can be seen in Figure 2.2. From now on, when we talk about an ICU, we intend a group of one or more ICUs that act as one Unit.

In Figure 2.2, patient flow 1 reflects the flow of regional emergency patients who are transferred to or from another hospital in case all beds are occupied. Patient flow 2 reflects the flow of elective patients. If no operational bed is available at their arrival, they are sent home to return later. Patient flow 3 reflects the flow of internal emergency patients who are not transferred in case of a full IC department. If no operational bed is available an over bed is created for these patients. If another patient leaves the ICU while the patient is still in the over bed, the over bed becomes an operational bed. If none of the other patients leave during the stay of the patient in the over bed, the patient leaves the ICU directly from the over bed and the over bed disappears. Patient flow 4 reflects the patients whose discharge is at hand and who can be predischarged in case of an incoming emergency. As long as these patients are at the ICU they occupy a normal, staffed, IC bed. Patient flow 5 is the flow of patients who leave the ICU (because of recovery or mortality). Patients who leave the ICU because of recovery go to the ward of the treating specialism.

2.2.2 Model of the scenario with overflow

In the previous section we discussed the model of the current situation of the patient streams through a region with ICUs. Next we want to study the scenario where the ICUs in a region jointly reserve beds (regional beds) for regional emergency patients. In such a situation, regional emergency patients refused at an ICU in the region because of bed shortages, will be sent to a regional bed instead of to another ICU. Elective patients and internal emergency patients cannot occupy these beds. We study this scenario because we want to know if the reserving of regional beds can be a solution to the problem that regional patients are refused at the ICUs in the region. To model the regional beds, we will add a (virtual) overflow, containing the regional beds, to the model of the region. We model the overflow as an extra ICU that is intended for regional emergency patients that are refused at an original ICU. In practise such an overflow will not actually exist. The beds of our virtual overflow will in reality be distributed over the ICUs in the region, but will function like the virtual overflow ICU.

We assume all ICUs to have the same patient streams as the IC department of the Erasmus MC. Therefore our model is generic and can be used for any region with any number of ICUs. Figure 2.3 pictures the model of the the ICUs with overflow. Although the model developed in this section can be used for any number of ICUs, we only depict two ICUs. The patient streams are the same as those described in the previous section.

2.2.3 Assumptions about the arrivals and Length of Stay

In this section we make assumptions concerning the arrival process at the ICUs and the Length of Stay of patients for both models discussed.

Arrivals

As can be seen in Figure 2.2 and 2.3 , three flows of patients arrive at the ICU. Two flows consist of emergency patients and one flow consists of elective patients. We assume all arrivals are Poisson arrivals. For the emergency arrivals this is a good assumption. Emergency patients originate from a large population (all inhabitants of the region in which the ICU is situated) and the probability that someone from the population suddenly needs to be treated at an ICU is very small. For the elective arrivals, however, the assumption is questionable as their arrivals are actually planned. Nevertheless, if we assume that the surgeon does not use his knowledge on the occupation of the ICU when planning the operations and given that only a fraction of operated patients require intensive care after the operation, the assumption of Poisson arrivals is plausible. Furthermore we assume all ICUs to have a different mean arrival rate.

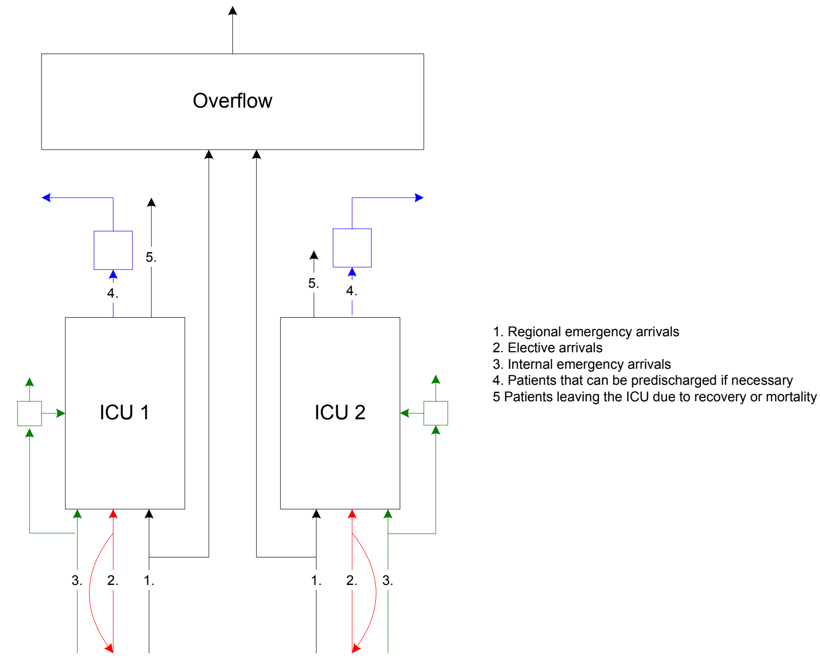

Figure 2.3: Overview of the ERM including all patient streams

Let λ_i denote the total arrival rate at ICU i, this is the average number of patients arriving per time unit at ICU i. For ICU i we denote the probability that an arriving patient is a regional emergency patient by $p_{1,i}$, the probability that an arriving patient is an elective patient by $p_{2,i}$ and the probability that an arriving patient is an internal emergency patient by $p_{3,i}$ $(p_{1,i} + p_{2,i} + p_{3,i} = 1)$. Accordingly, the rate of regional emergency arrivals at ICU i is $p_{1,i}\lambda_i$, the rate of elective arrivals at ICU i is $p_{2,i}\lambda_i$ and the rate of the internal emergency arrivals at ICU i is $p_{3,i}\lambda_i$.

We have to note that we model the return of elective patients after a cancelled operation as a new arrival. If we modelled the returning of arrivals as a separate arrival stream, we would have a retrial queue. A retrial queue is a queue in which customers who find all servers occupied may retry after a period of time. Falin and Templeton [11] consider retrial queues with Poisson arrivals in full detail. As we consider more than one IC bed, the ICU would involve a multiserver retrial queue. From [11] it can be concluded that for multiserver retrial queues it is difficult to find an analytical solution. Falin and Templeton, however, prove that as the retrial rate goes to zero, the number of servers has an Erlang loss distribution with the arrival rate equal to the sum of the primary arrival rate and the retrial rate. The retrial rate of the elective patients is very low. It is not preferable that an operation is postponed and much is done to prevent this from happening. Sometimes, however, it is inevitable and consequently there is a small number of cancelled operations. Because of the low retrial rate we can approximate the retrial queue with a loss system in which returning patients are modelled as new arrivals.

Length of stay

The LOS at an ICU is known to be highly variable. The expected LOS at an ICU is several days, but some patients stay for weeks. The exponential distribution does not model this high variability well. We, however, use the exponential distribution for the LOS. We do this for reasons of simplicity. Using the exponential distribution makes it easier to find explicit formulas for the expectation and variance of the overflow. Later on, the model can be adjusted for another distribution of the LOS. A phase type distribution can be used for this. Phase type distributions are a mixture of Erlang distributions and can approximate almost any distribution. Therefore it will most likely be easy to adjust the model with exponential LOS to a model with a phase type LOS.

We assume the mean LOS of patients with different disorders to be the same. Moreover we assume the three different patient types, regional emergencies, electives and external emergencies, to have the same mean LOS. For internal emergency patients and regional emergency patients this assumption is plausible. Elective patients, however, are usually more stable than emergency patients and their LOS is more predictable. Therefore, it is likely that the LOS has a smaller mean and variance than that of emergency patients. For now we assume the mean LOS of the three patient types to be equal. Furthermore we assume that the mean LOS of the overflow is the same as the mean LOS of the ICU the overflow originates from. As we take the same mean LOS for all three patient types this assumption is plausible. We do assume the mean LOS to be dependent of the ICU because some ICUs treat more severely ill patients than other ICUs. We denote the mean LOS for patients at ICU i by μ_i^{-1}.

Chapter 3

Analysis

3.1 Introduction

In Chapter 2 we have discussed a model of a region with regional beds. In this chapter we will use this model to develop a method to calculate how many regional beds the overflow of the region requires, in order to guarantee a certain minimum probability of admitting a regional patient to an IC bed in the region. We do make one simplification of the model. This simplification concerns the patients that are prereleased in case new patient arrives to the ICU. This patient flow is difficult to define as it is not clear if and when a patient can be prereleased; there are no distinct norms for this. Therefore, we have decided to leave this flow out of consideration and from now on, we only consider patient flows 1, 2, 3 and 5 of Section 2.2.2. The model of the ICUs with overflow that we will use from now on is depicted in Figure 3.1.

Although overflow at ICUs or other hospital departments has not been discussed in literature, overflow of other systems have been discussed. In the telecommunications overflow systems are more common and much research has been done on this. From the literature it appears that the Equivalent Random Method (ERM) is much used to analyse the overflow of telecommunication systems. The Equivalent Random Method (ERM) is a method for the approximate analysis of an overflow system. It has proven to give a good approximation of blocking probabilities of overflow systems. Furthermore, the client flows in a telecommunication overflow system resemble the patient flows in the overflow model of the ICUs. Therefore we believe that the ERM is a good method to analyse the patient streams through the ICUs in a region with regional beds. In Section 3.2 we discuss the ERM. In this section we talk about customers, servers and stations which can be translated to patients, beds and ICUs. In Section 3.3 we find formula for the expectation and variance of the overflow of an ICU. Finally, in Section 3.4 we give a method to determine how many regional beds a region requires in order to be able to guarantee regional emergency patients to be admitted with a certain minimum probability.

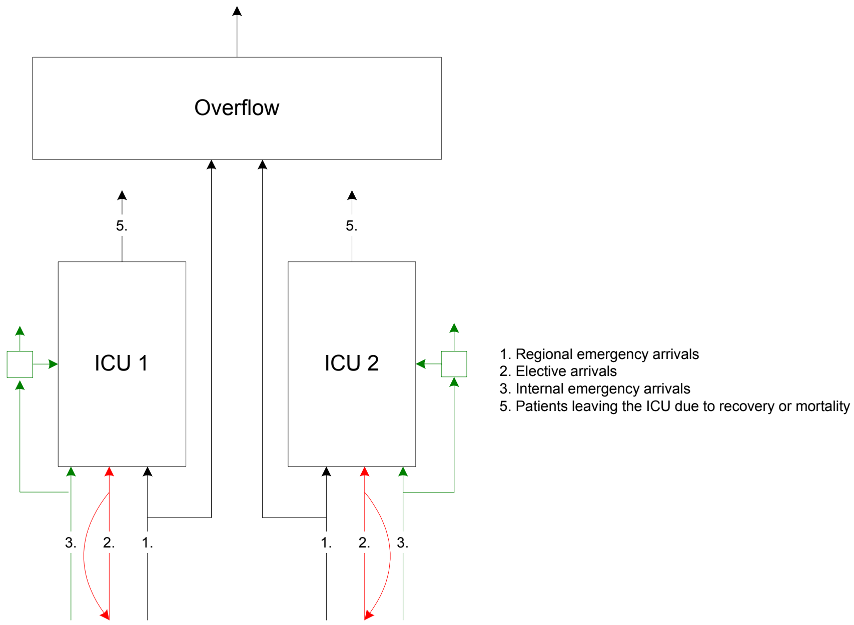

Figure 3.1: Model of the ERM including elective patients and overbeds

3.2 Equivalent Random Method

The ERM is developed by Wilkinson [34] and it is widely used in telecommunications where it has proven to be a good approximation method. Bretschneider [7] developed, independent of Wilkinson, a similar model. Consider a system consisting of n primary stations S. Customers arrive at station S_i according to a Poisson process with rate λ_i. Station S_i has c_i servers reserved for customers arriving at that station. Each server has a negative exponential service time with rate μ_i. The customers who find all servers busy at their primary station are directed toward a single overflow station S_0 that has c_0 servers. The service time of the overflow station is also assumed negative exponential with rate μ_0. The overflow load of station i is represented by its expectation E_i and variance V_i. Customers who are blocked at the overflow station are cleared from the system. This system is illustrated in Figure 3.2.

The expectation E_i and the variance V_i of the overflow load can be calculated through:

$$E_i = \rho_i B(c_i, \rho_i), \qquad i = 1...n, \tag{3.1}$$

$$V_i = E_i(1 - E_i) + \frac{\rho_i}{s_i + 1 + E_i - \rho_i} \qquad i = 1...n, \tag{3.2}$$

with $\rho_i = \lambda_i/\mu_i$ the offered load and $B(c_i, \rho_i)$ the Erlang loss formula:

$$B\left(c_i, \rho_i\right) = \frac{\left(\rho_i\right)^{c_i} c_i!}{\sum_{k=0}^{c_i} \left(\rho_i\right)^k / k!}.$$

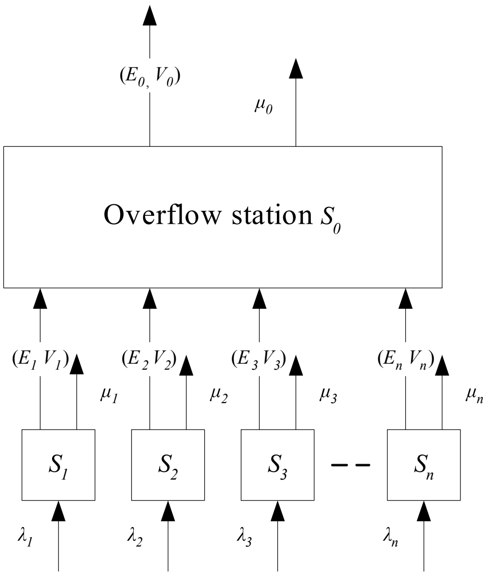

Figure 3.2: Overflow system

The objective is to calculate the amount of customers that find all servers busy at their primary station and also find all servers in the overflow station busy. Therefore the n primary stations with the Poisson loads are replaced by an "equivalent random" primary station S and a single "equivalent random" load such that the expectation and variance of the traffic that overflows the equivalent primary station are equal to the expectation and variance of the total overflow of the primary stations $S_1, S_2, .., S_n$:

$$
\begin{aligned}
E &= \sum_{i=i}^{n} E_i \\
V &= \sum_{i=1}^{n} V_i
\end{aligned}
$$

The equivalent random system is illustrated in Figure 3.3.The equivalent primary station has service rate μ_0 and the offered load ρ and the number of servers c can be determined from the following equations:

$$E = \rho B(c, \rho), \tag{3.3}$$

$$V = E\left(1 - E + \frac{\rho}{c + 1 + E - \rho}\right). \tag{3.4}$$

Then the expected load overflowing the overflow station can be approximated by :

$$\overline{E} = \rho B\left(c + c_0, \rho\right).$$

The probability that a customer arriving to primary station S_i is blocked is then

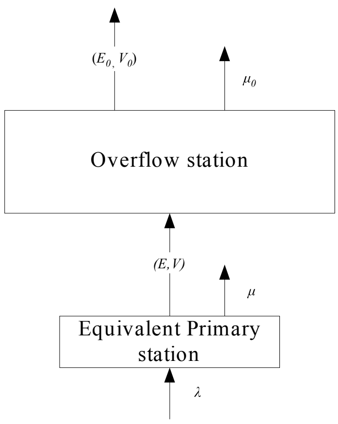

Figure 3.3: Equivalent overflow system

approximated as:

$$B_i \sim B(c_i, \rho_i)\frac{B(c+c_0,\rho)}{B(c,\rho)}.$$

Jagers and van Doorn [16] prove the uniqueness of the solution of the equations (3.3) and (3.4). Rapp [24] gives approximations for the offered load ρ and the number of servers c of the equivalent primary group:

$$\begin{aligned}
\rho &\approx V + 3\frac{V}{E}\left(\frac{V}{E}-1\right), \\
c &\approx \frac{\rho\left(E+\frac{V}{E}\right)}{\alpha+\frac{V}{E}-1} - E - 1.
\end{aligned}$$

Jagerman [15] presents practical formula and a computer program for the ERM. He uses Newton's method to solve equation (3.3) for ρ.

3.2.1 Modifications of the ERM

Fischer, Garbin and Swinsky [12] consider a system where primary customers can also arrive directly at the overflow station. In this case the ERM can not separately compute blocking probabilities for the calls first arriving at a primary station and calls directly arriving at the overflow station. In [12] an extension to the ERM is presented that can compute the individual blocking probabilities. Customers first arriving at the primary station are of type 1 and customers arriving directly at the overflow station are of type 2. Expressions are then given for the loss probabilities that type 1 customers see (B_1) and the loss probabilities that type 2 customers see (B_2):

$$\begin{aligned}
B_1 &= B(c,\rho)\frac{PB_1}{PB}\left(1+\left(\frac{V}{E}-\frac{V+\rho_2}{E+\rho_2}\right)k\right), \\
B_2 &= \frac{PB_1}{PB}\left(1+\left(1-\frac{V+\rho_2}{E+\rho_2}\right)k\right).
\end{aligned}$$

Hereby PB is the blocking probability of the primary station, PB_1 is the blocking probability of the primary and the overflow station, ρ_2 is the load offered directly to the overflow station and

$$k = (2.5459[(V + \rho_2)/(E + \rho_2)]^{-2.82}) \exp(\gamma_1 + \gamma_2),$$

with

$$\begin{aligned} \gamma_1 &= -0.0528c_0[(V + \rho_2)/(E + \rho_2)]^{-4.163}, \\ \gamma_2 &= -5.456PB_1/PB[(V + \rho_2)/(E + \rho_2)]^{-2.025}. \end{aligned}$$

Schehrer [25] also considers arrivals directly to the overflow station, but does this in combination with different mean holding times for the primary group and the overflow group. He defines the expectation and variance of the traffic overflowing primary server i as E_i and $V_{i,prim}$ and the expectation and variance of the traffic from server i offered to the overflow station as $E_{i,\text{sec}}$ and $V_{i,\text{sec}}$. The ratio of $E_{i,prim}$ and $E_{i,\text{sec}}$ is given by the ratio of the corresponding holding times: $E_{i,prim}/E_{i,\text{sec}} = \mu_2/\mu_1 = c$. In [25] calculations can be found for the variances $V_{i,\text{sec}}(c)$. When the expectation and variance of the traffic offered to the overflow station have been calculated, the ERM can be used to calculate the equivalent random total load offered to the overflow station. Figure 3.4 illustrates the overflow system, as discussed by Schehrer.

Borst, Boucherie and Boxma [4] extend Schehrers modified ERM for systems with repacking. Customers at the overflow station, originating from primary station S_i, are arbitrarily repacked into station S_i with rate γ_i. Let $EX_{0,i}$ be the expected number of customers in the overflow station that come from primary station S_i. In [4] the overflow system is approximated by saying that customers arrive at the primary station S_i with Poisson rate $\lambda_i + \gamma_i EX_{0,i}$ and that customers leave the overflow station with rate $\mu_{0,i} + \gamma_i$. Schehrers method is used to approximate the expected number of customers in the overflow station. An approximation is given for the probability that a customer arriving at primary station S_i is blocked.

Machihara [20, 21] considers the ERM in combination with hyper-exponential service distributions: $k_1 e^{-\mu_1 t} + k_2 e^{-\mu_2 t}$ with $k_1 + k_2 = 1$. In this case the service rate is $\mu = (k_1/\mu_1 + k_2/\mu_2)^{-1}$. In [20] an exact calculation is given for the load E_i/V_i of the overflow of the primary server groups with hyper-exponential service distributions. However, this calculation is slow. In [21] Machihara gives a quick approximation, but this approximation only applies for certain combinations of ρ (the load) and c (number of servers of the primary group). Shortle [29] modifies the approximation of Machihara so that it applies for all values of ρ and c. He uses this modified approximation to adjust the ERM for hyper-exponential service times.

Labetoulle [17] discusses the shortcomings of the ERM when customers have different mean holding times. Mixing customers with different holding times influences the overall blocking at the overflow group. The ERM does not take this into account as it computes load blocking (ratio of the blocked load to the total offered load) and not call blocking (ratio of the number of blocked calls to

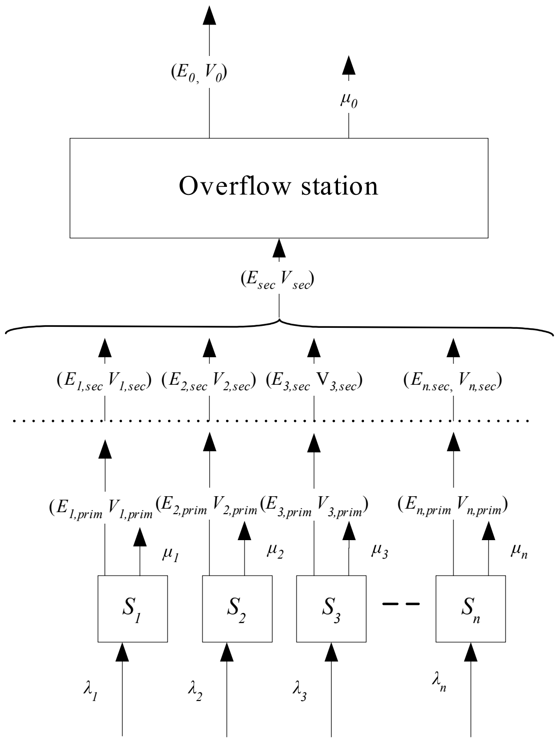

Figure 3.4: Overflow system with different holding times

the total number of call attempts). Fredericks [13] introduces a modified ERM that does take this into account. Assume that two groups of customers arrive at the same server group, with rate $\lambda_i, i = 1, 2$ and service rate μ_i. Then the average call blocking B_C and the average load blocking B_L are given by:

$$\begin{aligned} B_C &= \frac{\lambda_1 B_{C,1} + \lambda_2 B_{C,2}}{\lambda_1 + \lambda_2}, \\ B_L &= \frac{\rho_1 B_{C,1} + \rho_2 B_{C,2}}{\rho_1 + \rho_2}. \end{aligned}$$

Fredericks [13] found the following relationship between call blocking and load blocking:

$$\begin{aligned} B_C = &\left(B_L \left(1 + f(z_1, z_2)\right) + f(z_1, z_2) B_{C,1} \left(\tfrac{\mu_1}{\mu 2} - 1\right)\right) \\ &/ \left(1 + \tfrac{\mu_1}{\mu_2} f(z_1, z_2)\right), \end{aligned} \tag{3.5}$$

with z_i the peakedness (ratio of the variance and expectation) of the servers, z the overall peakedness defined by $z = (\rho_1 z_1 + \rho_2 z_2)/(\rho_1 + \rho_2)$ and $f(z_1, z_2) = (z_2 - z)/(z - z_1)$ for $z_1 \neq z_2$ and $f(z_1, z_2) = 1$ for $z_1 = z_2$. According to Fredericks [13] formula (3.5) can be used in addition to the ERM to predict the call blocking, which takes the different mean holding times into account.

3.3 Expectation and variance of the overflow

In the previous section we have discussed the ERM. We have seen that, to apply the ERM, we need to find formula for the expectation and variance of the number of patients that overflow an ICU. We can not use the formulas that are used for the classical ERM, as we have different patients flows. Particularly the flow of internal emergency patients causes differences as for these patients over beds can be created and thus the capacity of the ICU can be exceeded. In this Section we develop formula for the expectation and variance of the number of patients that overflow one of the ICUs in this model, let us say for ICU i. For this model we make the same assumptions as we make in Section 2.2.3. We will shortly repeat these assumptions. The patients arrive to the ICU according to a Poisson process with rate λ_i. Regional emergency patients arrive with rate $p_{1,i}\lambda_i$ and are transferred to the overflow ICU in case of a full ICU. Elective patients arrive with rate $p_{2,i}\lambda_i$ and are sent home in case of a full ICU. Internal emergency patients arrive with rate $p_{3,i}\lambda_i$ and are placed in an over bed in case of a full ICU. Now $p_{1,i} + p_{2,i} + p_{3,i} = 1$. The LOS for ICU i is assumed exponential with mean μ_i^{-1}. We assume the ICU to have an operational capacity of c_i and the overflow to have an infinite capacity. The ICU can be represented by an $M/M/c_i$ system. To find the expectation and the mean of the overflow of ICU i we assume the regional patients that are blocked at ICU i to be sent to an overflow ICU with infinite capacity. Figure 3.5, shows the patient flows used to calculate the expectation and variance of the number of patients that overflow ICU i.

Let $P_i\,(j, k)$ be the probability $P_i\,(J = j, K = k)$ that $J = j$ patients are at ICU i and $K = k$ patients are at the overflow ($j = 0, 1, ..; k = 0, 1, ..$). To find

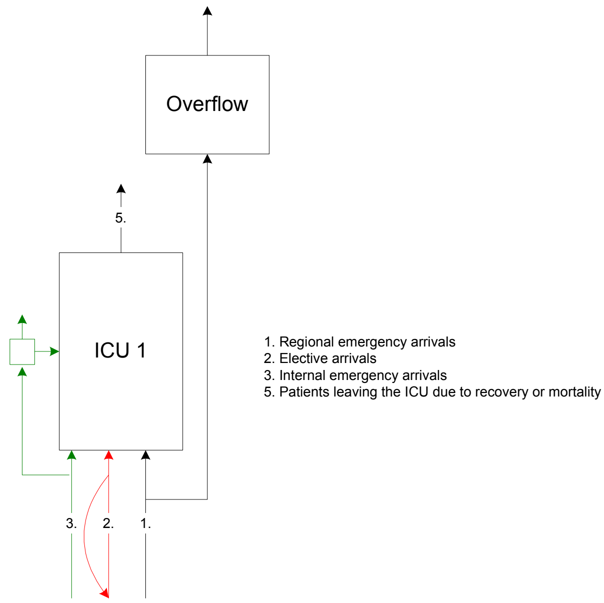

Figure 3.5: ICU i with overflow

the expectation and variance of the number of patients in the overflow we use balance equations. Balance equations reflect that in the long run, the rate at which transitions into state (j,k) occur must equal the rate at which transitions out of state (j,k) occur. For the ICU and its overflow the balance equations are:

$$\boxed{\begin{aligned}&(\lambda_i + j\mu_i + k\mu_i)\, P_i(j,k)\\ &\qquad = \lambda_i P_i(j-1,k) + (j+1)\,\mu_i P_i(j+1,k)\\ &\qquad\quad + (k+1)\,\mu_i P_i(j,k+1),\\ &P_i(-1,k)=0, \quad j=0,1,..,c_i-1, \quad k=0,1,...\end{aligned}} \tag{3.6}$$

$$\boxed{\begin{aligned}&((p_{1,i}+p_{3,i})\,\lambda_i + c_i\mu_i + k\mu_i)\, P_i(c_i,k)\\ &\qquad = \lambda_i P_i(c_i-1,k) + p_{1,i}\lambda_i P_i(c_i,k-1)\\ &\qquad\quad + (c_i+1)\,\mu_i P_i(c_i+1,k)\\ &\qquad\quad + (k+1)\,\mu_i P_i(c_i,k+1),\\ &P(c_i,-1)=0, \quad k=0,1,...\end{aligned}} \tag{3.7}$$

$$\boxed{\begin{aligned}&((p_{1,i}+p_{3,i})\,\lambda_i + j\mu_i + k\mu_i)\, P_i(j,k)\\ &\qquad = p_{3,i}\lambda_i P_i(j-1,k) + p_{1,i}\lambda_i P_i(j,k-1)\\ &\qquad\quad + (j+1)\,\mu_i P_i(j+1,k)\\ &\qquad\quad + (k+1)\,\mu_i P_i(j,k+1),\\ &P_i(j,-1)=0, \quad j=c_i+1,c_i+2,.., \quad k=0,1,...\end{aligned}} \tag{3.8}$$

Next we shall perform some manipulations on these balance equations. We multiply the balance equations (3.6), (3.7) and (3.8) by $z^k, |z| \leq 1$, and sum both sides of the equations over k. We now obtain

$$\begin{array}{rl} \left(\lambda_i + j\mu_i\right)\sum_{k=0}^{\infty} P_i\left(j,k\right)z^k & = \lambda_i \sum_{k=0}^{\infty} P_i\left(j-1,k\right)z^k \\ & \quad + \left(j+1\right)\mu_i \sum_{k=0}^{\infty} P_i\left(j+1,k\right)z^k \\ & \quad + \mu_i(1-z)\sum_{k=0}^{\infty} kP_i\left(j,k\right)z^{k-1}, \\ P_i\left(-1,k\right)=0, & \quad j=0,1,..,c_i-1, \end{array} \tag{3.9}$$

$$\begin{array}{l} \left(\left(p_{1,i}\left(1-z\right)+p_{3,i}\right)\lambda_i + c_i\mu_i\right)\sum_{k=0}^{\infty} P_i\left(c_i,k\right)z^k \\ \quad = \lambda_i \sum_{k=0}^{\infty} P_i\left(c_i-1,k\right)z^k \\ \quad\quad + \left(j+1\right)\mu_i \sum_{k=0}^{\infty} P_i\left(c_i+1,k\right)z^k \\ \quad\quad + \mu_i\left(1-z\right)\sum_{k=0}^{\infty} kP_i\left(c_i,k\right)z^{k-1}, \end{array} \tag{3.10}$$

$$\begin{array}{l} \left(\left(p_{1,i}\left(1-z\right)+p_{3,i}\right)\lambda_i + j\mu_i\right)\sum_{k=0}^{\infty} P_i\left(j,k\right)z^k \\ \quad = p_{3,i}\lambda_i \sum_{k=0}^{\infty} P_i\left(j-1,k\right)z^k \\ \quad\quad + \left(j+1\right)\mu_i \sum_{k=0}^{\infty} P_i\left(j+1,k\right)z^k \\ \quad\quad + \mu_i\left(1-z\right)\sum_{k=0}^{\infty} kP_i\left(j,k\right)z^{k-1}, \\ \quad j=c_i+1,c_i+2,... \end{array} \tag{3.11}$$

Next, let $G_j^{(i)}\left(z\right)$ be the marginal generating function

$$G_j^{(i)}\left(z\right) = \sum_{k=0}^{\infty} P_i\left(j,k\right)z^k.$$

with $|z| \leq 1$. The expectation and variance of the overflow can be calculated by using first and second order derivatives of $G_j^{(i)}\left(z\right)$ with respect to z:

$$\sum_{j=0}^{\infty} \frac{\partial}{\partial z} G_j^{(i)}\left(z\right)\bigg|_{z=1} = \sum_{j=0}^{\infty}\sum_{k=0}^{\infty} kP_i\left(j,k\right),$$

$$\sum_{j=0}^{\infty} \frac{\partial^2}{\partial z^2} G_j^{(i)}\left(z\right)\bigg|_{z=1} = \sum_{j=0}^{\infty}\sum_{k=0}^{\infty} k(k-1)P_i\left(j,k\right).$$

The expectation and variance are now:

$$\boxed{E_i = \sum_{j=0}^{\infty} \frac{\partial}{\partial z} G_j^{(i)}\left(z\right)\bigg|_{z=1},} \tag{3.12}$$

$$\boxed{V_i = \sum_{j=0}^{\infty} \frac{\partial^2}{\partial z^2} G_j^{(i)}\left(z\right)\bigg|_{z=1} + E_i - \left(E_i\right)^2.} \tag{3.13}$$

Therefore we next differentiate both sides of equations (3.9), (3.10) and (3.11) with respect to z and take $z=1$. This results in the following equations:

$$\begin{array}{l} \left(\lambda_i + \left(j+1\right)\mu_i\right)E_j^{(i)} = \lambda_i E_{j-1}^{(i)} + \left(j+1\right)\mu_i E_{j+1}^{(i)} \\ E_{-1}^{(i)} = 0, \quad j=0,1,..,c_i-1, \end{array} \tag{3.14}$$

$$\begin{array}{rl} \left(p_{3,i}\lambda_i + \left(c_i+1\right)\mu_i\right)E_{c_i}^{(i)} & = \lambda_i E_{c_i-1}^{(i)} + \left(c_i+1\right)\mu_i E_{c_i+1}^{(i)} \\ & \quad + p_{1,i}\lambda_i P_i\left(c_i\right), \end{array} \tag{3.15}$$

$$\begin{aligned}(p_{3,i}\lambda + (j+1)\,\mu_i)\,E_j^{(i)} &= p_{3,i}\lambda_i E_{j-1}^{(i)} \\ &\quad + (j+1)\,\mu_i E_{j+1}^{(i)} + p_{1,i}\lambda_i P_i\,(j)\,, \\ j = c_i+1, c_i+2, \ldots\end{aligned} \tag{3.16}$$

with $P_i\,(j)$ the probability that there are j patients at ICU i, and $E_j^{(i)} = \frac{\partial}{\partial z} G_j^{(i)}\,(z)\Big|_{z=1}$, the expected number of patients in the overflow with j patients in ICU i. To obtain the expected number of patients in the overflow we sum these equations over j. It now follows that

$$\boxed{E_i = p_{1,i}\rho_i \sum_{j=c_i}^{\infty} P_i\,(j)\,,} \tag{3.17}$$

with ρ_i the load λ_i/μ_i.

To find the variance of the number of patients in the overflow, we take the second order derivatives of equations (3.9), (3.10) and (3.11) with respect to z and take $z = 1$. This results in the following equations:

$$\begin{aligned}&(\lambda_i + (j+2)\,\mu_i)\,E_j^{\prime(i)} = \lambda_i E_{j-1}^{\prime(i)} + (j+1)\,\mu_i E_{j+1}^{\prime(i)} \\ &E_{-1}^{\prime(i)} = 0, \quad j = 0,1,2,..,c_i-1,\end{aligned} \tag{3.18}$$

$$\begin{aligned}(p_{3,i}\lambda_i + (c_i+2)\,\mu_i)\,E_{c_i}^{\prime(i)} &= \lambda_i E_{c_i-1}^{\prime(i)} + (c_i+1)\,\mu_i E_{j+1}^{\prime(i)} \\ &\quad +2p_{1,i}\lambda_i E_{c_i}^{(i)},\end{aligned} \tag{3.19}$$

$$\begin{aligned}(p_{3,i}\lambda_i + (j+2)\,\mu_i)\,E_j^{\prime(i)} &= p_{3,i}\lambda_i E_{j-1}^{\prime(i)} + (j+1)\,\mu_i E_{j+1}^{\prime(i)} \\ &\quad +2p_{1,i}\lambda_i E_j^{(i)}, \\ j = c_i+1, c_i+2, \ldots,\end{aligned} \tag{3.20}$$

with $E_j^{\prime(i)} = \frac{\partial^2}{\partial z^2} G_j^{(i)}\,(z)\Big|_{z=1}$. When we sum these equations over j we find the expectation of $(K\,(K-1))$. This, together with E_i, gives the variance of the number of patients in the overflow:

$$\boxed{V_i = p_{1,i}\rho_i \sum_{j=c_i}^{\infty} E_j^{(i)} + E_i - (E_i)^2\,.} \tag{3.21}$$

It now remains to find $\sum_{j=c_i}^{\infty} P_i\,(j)$ and $\sum_{j=c_i}^{\infty} E_j^{(i)}$. The probabilities $P_i\,(j)$ that there are j patients at ICU i can be found by using the fact that $G_j^{(i)}(1) = P_i\,(j)\,.$ We take $z = 1$ in equations (3.9), (3.10) and (3.11) to obtain

$$\begin{aligned}&P_i\,(j+1) = \left(\tfrac{\lambda_i}{(j+1)\mu_i} + \tfrac{j}{j+1}\right) P_i\,(j) - \tfrac{\lambda_i}{(j+1)\mu_i} P_i\,(j-1)\,, \\ &P_i\,(-1) = 0, \quad j = 0,1,..,c_i-1.\end{aligned} \tag{3.22}$$

$$P_i\,(c_i+1) = \left(\tfrac{p_{3,i}\lambda_i}{(c_i+1)\mu_i} + \tfrac{c_i}{c_i+1}\right) P_i\,(c_i) - \tfrac{\lambda_i}{(c_i+1)\mu_i} P_i\,(c_i-1)\,, \tag{3.23}$$

$$\begin{aligned}&P_i\,(j+1) = \left(\tfrac{p_{3,i}\lambda_i}{(j+1)\mu_i} + \tfrac{j}{j+1}\right) P_i\,(j) - \tfrac{p_{3,i}\lambda_i}{(j+1)\mu_i} P_i\,(j-1)\,, \\ &j = c_i+1, c_i+2, \ldots\end{aligned} \tag{3.24}$$

From this it follows that

$$P_i\,(j) = \begin{cases} \frac{1}{j!}\,(\rho_i)^j\,P_i\,(0)\,, & j = 0,1,..,c_i, \\ \frac{1}{j!}\,(p_{3,i})^{j-c_i}\,(\rho_i)^j\,P_i\,(0) & j = c_i+1, c_i+2, \ldots, \end{cases} \tag{3.25}$$

with $\rho_i = \lambda_i/\mu_i$. From equations (3.25) together with the fact that $\sum_{j=0}^{\infty} P_i(j) = 1$ we obtain $P_i(0)$:

$$\begin{aligned} P_i(0) &= \left[\sum_{j=0}^{c_i} \frac{(\rho_i)^j}{j!} + \sum_{j=c_i+1}^{\infty} \frac{(\rho_i)^j}{j!} (p_{3,i})^{j-c_i}\right]^{-1}, \\ &= \left[\sum_{j=0}^{c_i} \frac{(\rho_i)^j}{j!} (1-(p_{3,i})^{j-c_i}) + (p_{3,i})^{-c_i} e^{p_{3,i}\rho_i}\right]^{-1}. \end{aligned} \tag{3.26}$$

From formulas (3.25) together with formula (3.26) we obtain

$$\boxed{\begin{aligned} \sum_{j=c_i}^{\infty} P_i(j) = 1 - \sum_{j=0}^{c_i-1} & \frac{(\rho_i)^j}{j!} \\ & * \left[\sum_{j=0}^{c_i} \frac{(\rho_i)^j}{j!}(1-(p_{3,i})^{j-c_i}) \right. \\ & \left. + (p_{3,i})^{-c} e^{p_{3,i}\rho_i}\right]^{-1}. \end{aligned}} \tag{3.27}$$

Next we want to find $\sum_{j=c_i}^{\infty} E_j^{(i)}$. We know that

$$\sum_{j=c_i}^{\infty} E_j^{(i)} = 1 - \sum_{j=0}^{c_i-1} E_j^{(i)}.$$

Therefore we need to find formula for $E_j^{(i)}, j = 0..c_i - 1$. By iterating equations (3.14) from $j = 0$ to $c_i - 1$ we obtain

$$E_j^{(i)} = \sum_{l=0}^{j} \frac{1}{l!} (\rho_i)^l E_0^{(i)}, \quad j = 0, 1, .., c_i. \tag{3.28}$$

To find $E_0^{(i)}$ we need to find $E_j^{(i)}, j > c_i$, so that we can use the fact that $E_i = \sum_{j=0}^{\infty} E_j^{(i)}$. By iterating equations (3.15) and (3.16) from $j = c_i$ to ∞ we obtain

$$\begin{aligned} E_j^{(i)} &= \left(\left[1 - (p_{1,i} + p_{2,i}) \sum_{l=c_i+1}^{j} \frac{(p_{3,i})^{l-(c_i+1)} (\rho_i)^{l-c_i} c_i!}{l!} \right] \right. \\ & \left. * \sum_{l=0}^{c_i} \frac{1}{l!} (\rho_i)^l \right) E_0^{(i)} + \sum_{l=c_i+1}^{j} \frac{(p_{3,i})^{l-(c_i+1)} (\rho_i)^l}{l!} E_0^{(i)} \\ & - p_{1,i} P_i(0) \sum_{l=c_i+1}^{j} (l - c_i) \frac{(p_{3,i})^{l-(c_i+1)} (\rho_i)^l}{l!}, \end{aligned} \tag{3.29}$$

with $P_{0,i}$ given by formula (3.26). Next $E_0^{(i)}$ can be calculated:

$$\begin{aligned} E_0^{(i)} &= p_{1,i} P_i(0) * \left(\sum_{j=c_i+1}^{\infty} \frac{(\rho_i)^{c_i+1}}{c_i!} + \sum_{j=0}^{c_i} \frac{(p_{3,i})^{j-c_i} (\rho_i)^{j+1}}{j!} \right. \\ & - (p_{3,i})^{-c_i} \rho_i e^{p_{3,i}\rho_i} + (p_{3,i}\rho_i - c_i) \\ & \left. * \sum_{j=c_i+1}^{\infty} \sum_{l=c_i+1}^{j} \frac{(p_{3,i})^{l-(c_i+1)} (\rho_i)^l}{l!} \right) \left[\sum_{l=0}^{c_i} \frac{c_i+1-l}{l!} (\rho_i)^l \right. \\ & + \sum_{j=c_i+1}^{\infty} \sum_{l=0}^{c_i} \frac{1}{l!} (\rho_i)^l + \left(1 - (p_{1,i} + p_{2.i}) \frac{c_i!}{(\rho_i)^{c_i}} \sum_{l=0}^{c_i} \frac{1}{l!} (\rho_i)^l \right) \\ & \left. * \sum_{j=c_i+1}^{\infty} \sum_{l=c_i+1}^{j} \frac{(p_{3,i})^{l-(c_i+1)} (\rho_i)^l}{l!} \right]^{-1} . \end{aligned} \tag{3.30}$$

The sum $\sum_{j=c_i}^{\infty} E_j^{(i)}$ is now given by

$$\boxed{\sum_{j=c_i}^{\infty} E_j^{(i)} = 1 - E_0^{(i)} \sum_{l=0}^{c_i-1} \frac{c_i-l}{l!} (\rho_i)^l .} \tag{3.31}$$

Formulas (3.17) and (3.21) together with formulas (3.27) and (3.31) give the expectation and variance of the overflow.

3.4 The ERM for ICUs

In this section we give a method to calculate the number of regional beds required, given the expectation and variance of the overflow of the ICUs in the region. The expectation and variance of the overflow of each ICU in the region can be calculated using the formulas from Section 3.3. Let E_i be the expectation of the overflow of ICU i and V_i the variance of the overflow of ICU i. The total expectation and variance of the overflow of IC patients in the region is

$$\boxed{E = \sum_{i=1}^{n} E_i,} \tag{3.32}$$

$$\boxed{V = \sum_{i=1}^{n} V_i.} \tag{3.33}$$

Using the ERM we have considered two approaches. In the first approach we model the equivalent ICU as an ICU with only one patient stream, the stream of regional emergency patients. This equivalent ICU is depicted in Figure 3.6. The second approach is less straightforward but probably more realistic. In this approach we model the equivalent ICU as an ICU with all three patient streams. This equivalent ICU is depicted in Figure 3.7. In Section 3.4.1 we discuss the first approach. In Section 3.4.2 we discuss the second approach which is not completely worked out. Finally, in Section 3.4.3, an algorithm is given to determine the number of regional beds required. For the algorithm the results of either the simplified approach or the more complex approach can be used.

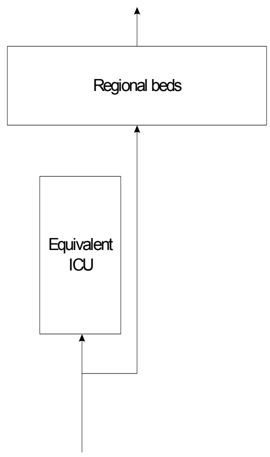

Figure 3.6: Equivalent Random IC with regional emergency patients.

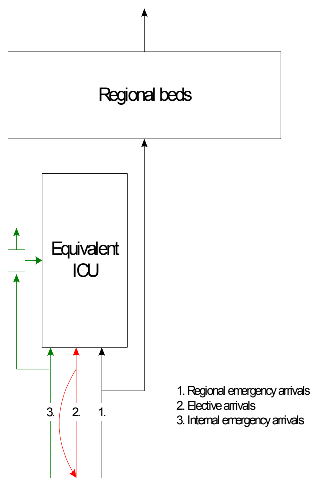

Figure 3.7: Equivalent Random IC with three patient types

3.4.1 Simplified Equivalent Random ICU

In this section we give formula for the expected number of patients that overflow the simplified equivalent ICU and the regional beds. The equivalent ICU is modelled like the classical equivalent station, with only regional emergency patients in the system. In case all operational IC beds are occupied but a regional bed is available, (regional emergency) patients that arrive are placed in a regional bed. If all regional beds are occupied too, the patient is sent to the (virtual) overflow. The patients in the overflow are patients for whom a bed in another hospital has to be sought. For a given number of regional beds r, we will give formula for the probability that a patient has to be sent outside the region (to the virtual overflow). Formulas from the classical ERM are used.

First we need to find a random load ρ and an equivalent random capacity c such that (see [8]):

$$\begin{aligned} E &= \rho B\left(c,\rho\right), \\ V &= E\left(1 - E + \frac{\rho}{c+1+E-\rho}\right), \end{aligned}$$

with $B\left(c,\rho\right)$ the blocking probability of an ICU with load ρ and capacity c. The load ρ and capacity c represent the load and capacity that one ICU with only regional emergency patients would have if the expectation and variance of the overflow were equal to E and V. To estimate ρ and c we use equations given by Rapp [24]:

$$\rho = V + 3\tfrac{V}{E}\left(\tfrac{V}{E} - 1\right), \tag{3.34}$$

$$\boxed{c = \frac{\rho\left(E+\frac{V}{E}\right)}{E_r+\frac{V}{E}-1} - E - 1.} \tag{3.35}$$

Cooper [8] states that these estimates of ρ and c are generally on the high side of the exact values. Rounding c down to an integer $\lfloor c \rfloor$ and then finding ρ by

$$\boxed{\rho = \frac{(\lfloor c \rfloor + E + 1)\left(E+\frac{V}{E}-1\right)}{E+\frac{V}{E}},} \tag{3.36}$$

gives a better approximation.

Let r be the capacity of the overflow (the number of regional IC beds). The expected number of patients that overflow the first ICU as well as the regional ICU is then:

$$\boxed{\begin{aligned} \overline{E} &= \rho B\left(c+r,\rho\right), \\ &= \rho \frac{\rho^{c+r}/(c+r)!}{\sum_{k=0}^{c+r}\rho^k/k!} \end{aligned}} \tag{3.37}$$

3.4.2 Complex Equivalent Random ICU

In this section we give a first step to find formula for the expectation of the number of patients that overflow the more complex equivalent random ICU and the regional beds. This complex equivalent ICU has, like the original ICUs, three patient types (regional emergency patients, elective patients and internal emergency patients). In case all operational IC beds are occupied, regional emergency patients first flow to the regional beds. If all regional beds are occupied too, the regional patient is sent to the (virtual) overflow. The patients in the overflow are regional emergency patients for who a bed in another hospital has to be sought. In case all operational beds are occupied at the ICU, arriving elective patients are sent home and internal emergency patients are placed in an over bed. We give formula for the probability that a patient has to be sent outside the region (to the virtual overflow) for a given number of regional beds r.

The probabilities p_1, p_2 and p_3 that a patient arriving at an ICU in the region is a regional emergency patient, an elective patient or an internal emergency can be calculated as follows

$$p_t = \frac{\sum_{i=1}^{n} p_{t,i}}{n}, \quad t = 1, 2, 3,$$

with n the number of ICUs in the region. We now need to find a random load ρ and an equivalent random capacity c such that

$$E = p_1 \frac{\lambda}{\mu} \sum_{j=c}^{\infty} P(j), \tag{3.38}$$

$$V = p_1 \rho \sum_{j=c}^{\infty} E_j + E - E^2, \tag{3.39}$$

with $\sum_{j=c}^{\infty} P(j)$ given by formula (3.27) and $\sum_{j=c}^{\infty} E_j$ given by formula (3.31). These formulas are the same as those in Section 3.3 because the equivalent random ICU is similar to an original ICU. A solution to equations (3.38) and (3.39) still has to be found.

We use the same method as in Section 3.3 to calculate the expected number of patients in the overflow. The model of the equivalent ICU with regional beds and virtual overflow with infinite capacity is depicted in Figure 3.8. Let $P(j, l, k)$ be the probability $P(J = j, L = l, K = k)$ of having $J = j$ patients in the operational IC beds, $L = l$ patients in the regional beds and $K = k$ patients in the overflow, with $j = 0, 1, .., c + 1, ...; l = 0, 1, .., r$ and $k = 0, 1, ...$ The balance equations for these probabilities are

$$\boxed{\begin{aligned} &(\lambda + j\mu + l\mu + k\mu) P(j, l, k) \\ &\qquad = \lambda P(j-1, l, k) + (j+1)\mu P(j+1, l, k) \\ &\qquad\quad + (l+1)\mu P(j, l+1, k) + (k+1)\mu P(j, l, k+1) \\ &P(-1, l, k) = 0, \quad j = 0, 1, .., c-1, \quad l = 0, 1, .., r-1, \quad k = 0, 1, ... \end{aligned}} \tag{3.40}$$

$$\boxed{\begin{aligned} &((p_1 + p_3)\lambda + j\mu + l\mu + k\mu) P(j, l, k) \\ &\qquad = \lambda P(j-1, l, k) + p_1 \lambda P(j, l-1, k) \\ &\qquad\quad + (j+1)\mu P(j+1, l, k) + (l+1)\mu P(j, l+1, k) \\ &\qquad\quad + (k+1)\mu P(j, l, k+1) \\ &j = c, \quad l = 0, 1, .., r-1, \quad k = 0, 1, ... \end{aligned}} \tag{3.41}$$

$$\boxed{\begin{aligned} &((p_1 + p_3)\lambda + j\mu + l\mu + k\mu) P(j, k, l) \\ &\qquad = p_3 \lambda P(j-1, l, k) + p_1 \lambda P(j, l-1, k) \\ &\qquad\quad + (j+1)\mu P(j+1, l, k) + (l+1)\mu P(j, l+1, k) \\ &\qquad\quad + (k+1)\mu P(j, l, k+1) \\ &j = c+1, c+2, ... \quad l = 0, 1, .., r-1, \quad k = 0, 1, ... \end{aligned}} \tag{3.42}$$

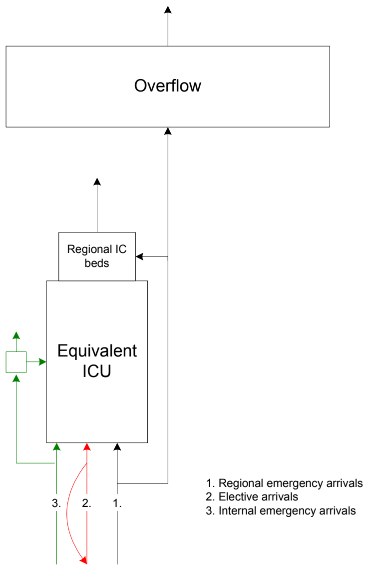

Figure 3.8: Equivalent ICU with regional beds and overflow

$$\boxed{\begin{aligned}&(\lambda + j\mu + l\mu + k\mu)\, P\,(j,l,k)\\ &\qquad = \lambda P\,(j-1,l,k) + (j+1)\,\mu P\,(j+1,l,k)\\ &\qquad\quad + (k+1)\,\mu P\,(j,l,k+1)\\ &j = 0,1,2,..,c-1, \quad l = r, \quad k = 0,1,...\end{aligned}} \tag{3.43}$$

$$\boxed{\begin{aligned}&((p_1+p_3)\,\lambda + j\mu + l\mu + k\mu)\, P\,(j,l,k)\\ &\qquad = \lambda P\,(j-1,l,k) + p_1\lambda P\,(j,l-1,k)\\ &\qquad\quad + p_1\lambda P\,(j,l,k-1) + (j+1)\,\mu P\,(j+1,l,k)\\ &\qquad\quad + (k+1)\,\mu P\,(j,l,k+1)\,.\\ &j = c, \quad l = r, \quad k = 0,1,...\end{aligned}} \tag{3.44}$$

$$\boxed{\begin{aligned}&((p_1+p_3)\,\lambda + j\mu + l\mu + k\mu)\, P\,(j,k,l)\\ &\qquad = p_3\lambda P\,(j-1,l,k) + p_1\lambda P\,(j,l-1,k)\\ &\qquad\quad + p_1\lambda P\,(k,l,k-1) + (j+1)\,\mu P\,(j+1,l,k)\\ &\qquad\quad + (k+1)\,\mu P\,(j,l,k+1)\\ &j = c+1, c+2,... \quad l = r, \quad k = 0,1,...\end{aligned}} \tag{3.45}$$

We now multiply both sides of these equations by z^k and sum them over j and l:

$$\begin{aligned}&(\lambda + j\mu + l\mu)\sum_{k=0}^{\infty} P\,(j,l,k)\, z^k\\ &\qquad = \lambda \sum_{k=0}^{\infty} P\,(j-1,l,k)\, z^k\\ &\qquad\quad + (j+1)\,\mu \sum_{k=0}^{\infty} P\,(j+1,l,k)\, z^k\\ &\qquad\quad + (l+1)\,\mu \sum_{k=0}^{\infty} P\,(j,l+1,k)\, z^k\\ &\qquad\quad + (1-z)\,\mu \sum_{k=0}^{\infty} kP\,(j,l,k)\, z^{k-1}\\ &P(-1,l,k) = 0, \quad j = 0,1,..,c-1, \quad l = 0,1,..,r-1, \quad k = 0,1,...\end{aligned} \tag{3.46}$$

$$\begin{aligned}&\left(\left(p_1+p_3\right)\lambda+j\mu+l\mu\right)\textstyle\sum_{k=0}^{\infty}P\left(j,l,k\right)z^k\\&\qquad=\lambda\textstyle\sum_{k=0}^{\infty}P\left(j-1,l,k\right)z^k\\&\qquad\quad+p_1\lambda\textstyle\sum_{k=0}^{\infty}P\left(j,l-1,k\right)z^k\\&\qquad\quad+\left(j+1\right)\mu\textstyle\sum_{k=0}^{\infty}P\left(j+1,l,k\right)z^k\\&\qquad\quad+\left(l+1\right)\mu\textstyle\sum_{k=0}^{\infty}P\left(j,l+1,k\right)z^k\\&\qquad\quad+\left(1-z\right)\mu\textstyle\sum_{k=0}^{\infty}kP\left(j,l,k\right)z^{k-1}\\&j=c,\quad l=0,1,..,r-1,\quad k=0,1,...\end{aligned}\tag{3.47}$$

$$\begin{aligned}&\left(\left(p_1+p_3\right)\lambda+j\mu+l\mu\right)\textstyle\sum_{k=0}^{\infty}P\left(j,l,k\right)z^k\\&\qquad=p_3\lambda\textstyle\sum_{k=0}^{\infty}P\left(j-1,l,k\right)z^k\\&\qquad\quad+p_1\lambda\textstyle\sum_{k=0}^{\infty}P\left(j,l-1,k\right)z^k\\&\qquad\quad+\left(j+1\right)\mu\textstyle\sum_{k=0}^{\infty}P\left(j+1,l,k\right)z^k\\&\qquad\quad+\left(l+1\right)\mu\textstyle\sum_{k=0}^{\infty}P\left(j,l+1,k\right)z^k\\&\qquad\quad+\left(1-z\right)\mu\textstyle\sum_{k=0}^{\infty}kP\left(j,l,k\right)z^{k-1}\\&j=c+1,c+2,...,\quad l=0,1,..,r-1,\quad k=0,1,...\end{aligned}\tag{3.48}$$

$$\begin{aligned}&\left(\lambda+j\mu+l\mu\right)\textstyle\sum_{k=0}^{\infty}P\left(j,l,k\right)z^k\\&\qquad=\lambda\textstyle\sum_{k=0}^{\infty}P\left(j-1,l,k\right)z^k\\&\qquad\quad+\left(j+1\right)\mu\textstyle\sum_{k=0}^{\infty}P\left(j+1,l,k\right)z^k\\&\qquad\quad+\left(1-z\right)\mu\textstyle\sum_{k=0}^{\infty}kP\left(j,l,k\right)z^{k-1}\\&j=0,1,..,c-1,\quad l=r,\quad k=0,1,...\end{aligned}\tag{3.49}$$

$$\begin{aligned}&\left(\left(p_1\left(1-z\right)+p_3\right)\lambda+j\mu+l\mu\right)\textstyle\sum_{k=0}^{\infty}P\left(j,l,k\right)z^k\\&\qquad=\lambda\textstyle\sum_{k=0}^{\infty}P\left(j-1,l,k\right)z^k\\&\qquad\quad+p_1\lambda\textstyle\sum_{k=0}^{\infty}P\left(j,l-1,k\right)z^k\\&\qquad\quad+\left(j+1\right)\mu\textstyle\sum_{k=0}^{\infty}P\left(j+1,l,k\right)z^k\\&\qquad\quad+\left(1-z\right)\mu\textstyle\sum_{k=0}^{\infty}kP\left(j,l,k\right)z^{k-1}\\&j=c,\quad l=r,\quad k=0,1,...\end{aligned}\tag{3.50}$$

$$\begin{aligned}&\left(\left(p_1\left(1-z\right)+p_3\right)\lambda+j\mu+l\mu\right)\textstyle\sum_{k=0}^{\infty}P\left(j,l,k\right)z^k\\&\qquad=p_3\lambda\textstyle\sum_{k=0}^{\infty}P\left(j-1,l,k\right)z^k\\&\qquad\quad+p_1\lambda\textstyle\sum_{k=0}^{\infty}P\left(j,l-1,k\right)z^k\\&\qquad\quad+\left(j+1\right)\mu\textstyle\sum_{k=0}^{\infty}P\left(j+1,l,k\right)z^k\\&\qquad\quad+\left(1-z\right)\mu\textstyle\sum_{k=0}^{\infty}kP\left(j,l,k\right)z^{k-1}\\&j=c+1,c+2,..,\quad l=r,\quad k=0,1,...\end{aligned}\tag{3.51}$$

Next, let $G_{j,l}\left(z\right)$ be the marginal generating function

$$G_{j,l}\left(z\right)=\sum_{k=0}^{\infty}P\left(j,l,k\right)z^k.$$

with $|z|\leq 1$. The expectation and variance of the overflow can be calculated by using first and second order derivatives of $G_j^{(i)}\left(z\right)$ with respect to z:

$$\sum_{j=0}^{\infty}\sum_{l}^{\infty}\frac{\partial}{\partial z}G_{j,l}\left(z\right)\bigg|_{z=1}=\sum_{j=0}^{\infty}\sum_{l=0}^{\infty}\sum_{k=0}^{\infty}kP\left(j,l,k\right).$$

The expectation can therefore be calculated through

$$\overline{E}=\sum_{j=0}^{\infty}\sum_{l}^{\infty}\frac{\partial}{\partial z}G_{j,l}\left(z\right)\bigg|_{z=1}.\tag{3.52}$$

Therefore we next differentiate both sides of equations (3.46), (3.47), (3.48), (3.49), (3.50) and (3.51) with respect to z and take $z = 1$. This results in the following equations:

$$\begin{aligned}&(\lambda + j\mu + l\mu + \mu) E_{j,l} = \lambda E_{j-1,l} + (j+1)\mu E_{j+1,l} + (l+1)\mu E_{j,l+1} \\ &E_{-1,l} = 0, \quad j = 0,1,..,c-1, \quad l = 0,1,..,r-1, \quad k = 0,1,...\end{aligned} \tag{3.53}$$

$$\begin{aligned}&((p_1 + p_3)\lambda + j\mu + l\mu + \mu) E_{j,l} \\ &\qquad = \lambda E_{j-1,l} + p_1\lambda E_{j,l-1} + (j+1)\mu E_{j+1,l} \\ &\qquad\quad + (l+1)\mu E_{j,l+1} \\ &j = c, \quad l = 0,1,..,r-1, \quad k = 0,1,...\end{aligned} \tag{3.54}$$

$$\begin{aligned}&((p_1 + p_3)\lambda + j\mu + l\mu + \mu) E_{j,l} \\ &\qquad = p_3\lambda E_{j-1,l} + p_1\lambda E_{j,l-1} + (j+1)\mu E_{j+1,l} \\ &\qquad\quad + (l+1)\mu E_{j,l+1} \\ &j \geq c+1, \quad l = 0,1,..,r-1, \quad k = 0,1,...\end{aligned} \tag{3.55}$$

$$\begin{aligned}&(\lambda + j\mu + l\mu + \mu) E_{j,l} = \lambda E_{j-1,l} + (j+1)\mu E_{j+1,l} \\ &j \leq c-1, \quad l = r, \quad k = 0,1,...\end{aligned} \tag{3.56}$$

$$\begin{aligned}&(p_3\lambda + j\mu + l\mu + \mu) E_{j,l} \\ &\qquad = \lambda E_{j-1,l} + p_1\lambda E_{j,l-1} + (j+1)\mu E_{j+1,l} + p_1\lambda P(j,l) \\ &j = c, \quad l = r, \quad k = 0,1,...,\end{aligned} \tag{3.57}$$

$$\begin{aligned}&(p_3\lambda + j\mu + l\mu + \mu) E_{j,l} \\ &\qquad = p_3\lambda E_{j-1,l} + p_1\lambda E_{j,l-1} + (j+1)\mu E_{j+1,l} + p_1\lambda P(j,l) \\ &j \geq c+1, \quad l = r, \quad k = 0,1,...\end{aligned} \tag{3.58}$$

with $E_{j,l}$ the expected number of patients in the overflow with j patients in the equivalent ICU and l patients in the over beds, and $P(j,l)$ the probability of having j patients in the ICU and l patients in the regional beds.

To obtain the expected number of patients in the overflow we first sum equations (3.53), (3.54) and (3.55), and equations (3.56), (3.57) and (3.58) over $j \geq 0$. This results in:

$$\begin{aligned}&p_1\lambda \textstyle\sum_{j=c}^{\infty} E_{j,l} + l\mu \sum_{j=0}^{\infty} E_{j,l} + \mu \sum_{j=0}^{\infty} E_{j,l} = \\ &\qquad (l+1)\mu \textstyle\sum_{j=0}^{\infty} E_{j,l+1} + p_1\lambda \sum_{j=c}^{\infty} E_{j,l-1}, \\ &\qquad\quad l = 0,1,..,r-1\end{aligned} \tag{3.59}$$

$$r\mu \sum_{j=0}^{\infty} E_{j,r} + \mu \sum_{j=0}^{\infty} E_{j,r} = p_1\lambda \sum_{j=c}^{\infty} E_{j,r-1} + p_1\lambda \sum_{j=c}^{\infty} P(j,l) \tag{3.60}$$

Summing equations (3.59) and (3.60) over $l = 0,1,..,r$ results in:

$$\boxed{\overline{E} = p_1\rho \textstyle\sum_{j=c}^{\infty} P(j,r)} \tag{3.61}$$

It now remains to find a formula for $\sum_{j=c}^{\infty} P(j,r)$. To find the sum, we first need to find $P(j,r), j = 0,1,..$ To this end we use formulas (3.46) to (3.51). Taking $z = 1$ in these equations results in the following equations:

$$\begin{aligned}&(\lambda + j\mu + l\mu) P(j,l) \\ &\qquad = \lambda P(j-1,l) + (j+1)\mu P(j+1,l) \\ &\qquad\quad + (l+1)\mu P(j,l+1) \\ &P(-1,l) = 0, \quad j = 0,1,..,c-1, \quad l = 0,1,..,r-1, \quad k = 0,1,...\end{aligned} \tag{3.62}$$

$$\begin{aligned}((p_1+p_3)\,\lambda+j\mu+l\mu)\,P\,(j,l)&\\ =\lambda P\,(j-1,l)+p_1\lambda P\,(j,l-1)+(j+1)\,\mu P\,(j+1,l)&\\ +\,(l+1)\,\mu P\,(j,l+1)&\\ j=c,\quad l=0,1,..,r-1,\quad k=0,1,...&\end{aligned} \tag{3.63}$$

$$\begin{aligned}((p_1+p_3)\,\lambda+j\mu+l\mu)\,P\,(j,l)=&\\ p_3\lambda P\,(j-1,l)+p_1\lambda P\,(j,l-1)+(j+1)\,\mu P\,(j+1,l)&\\ +\,(l+1)\,\mu P\,(j,l+1)&\\ j=c+1,c+2,...,\quad l=0,1,..,r-1,\quad k=0,1,...&\end{aligned} \tag{3.64}$$

$$\begin{aligned}(\lambda+j\mu+l\mu)\,P\,(j,l)=\lambda P\,(j-1,l)+(j+1)\,\mu P\,(j+1,l)\\ j=0,1,..,c-1,\quad l=r,\quad k=0,1,...\end{aligned} \tag{3.65}$$

$$\begin{aligned}(p_3\lambda+j\mu+l\mu)\,P\,(j,l)=\lambda P\,(j-1,l)+p_1\lambda P\,(j,l-1)\\ +\,(j+1)\,\mu P\,(j+1,l)\\ j=c,\quad l=r,\quad k=0,1,...\end{aligned} \tag{3.66}$$

$$\begin{aligned}(p_3\lambda+j\mu+l\mu)\,P\,(j,l)=p_3\lambda P\,(j-1,l)+p_1\lambda P\,(j,l-1)\\ +\,(j+1)\,\mu P\,(j+1,l)\\ j=c+1,c+2,...\quad l=r,\quad k=0,1,...\end{aligned} \tag{3.67}$$

Finding formula for $P\,(j,l)$ is not straightforward. However, making the realistic assumption that the number of patients in the ICU is restricted by the number of constructional beds in the ICU, $P\,(j,l)$ can easily be found through iterating the equations (3.62) to (3.67) for j from 0 to c and for l from 0 to r.

We have given a first start to this more complex method. To work the method out, solutions to equations (3.38) and (3.39) need to be found and λ and μ need be determined.

3.4.3 Algorithm to calculate the number of regional beds required

In Section 3.4.1 and 3.4.2 we have discussed two different approaches to determine the expected number of patients who are blocked at the equivalent ICU and at the regional beds, given a certain number of regional beds. The approach in Section 3.4.1 is worked out, whereas for the method discussed in Section 3.4.2 a complete solution is not yet given. In this section we therefore use the results of Section 3.4.1. We give an algorithm for the approximation of the number of regional beds needed such that the probability that a regional patient needs to be transferred to an ICU outside the region is satisfying. Let α be the maximum proportion of regional emergency patients that is allowed to overflow all ICU beds in the region and ρ and c calculated by formulas (3.34), (3.35) and (3.36). The number of regional beds r required can then be found through the following algorithm:

1. Initialise $r = 0$
2. While $\epsilon > \alpha$ do
   - $r := r + 1$;
   - Calculate $\overline{E}$ through (3.37);
   - $\epsilon = \overline{E} / \sum_{i=1}^{n} p_{1,i}\rho_i$;

   end;

(3.68)

The total number of regional beds r will need to be distributed over the ICUs in the region. It is reasonable not to evenly distribute these beds over the ICUs in the region, but for example distribute them in proportion to the number of IC beds an ICU has and taking into account the mean number of regional patients an ICU already takes care of. For the proper functioning of these regional IC beds it is of importance that only the regional emergency patients use these beds and not elective patients or emergency patients from within the hospital. Therefore there should be some kind of regional control on these beds. This can for example be realised by obliging ICUs to register data on the use of these regional beds and putting a fine on refusing regional patients when regional patients do not occupy the regional beds.

3.5 Discussion of the method

In this chapter we have developed a method with which the required number of regional beds can be approximated. For the development of this method we made some simplifications that do not correspond to reality. In this Section we will discuss the implications of these simplifications.

We have not considered the patients that can be predischarged in case of emergency because their discharge is already at hand. This will most likely result in a larger proportion of refused patients in the model, compared to reality. Therefore we advise to consider this patient flow by interviewing intensivist. When this patient stream is better defined the ERM for ICUs can be modified. For this the balance equations should be adapted.

We assumed the LOS to be exponentially distributed although this is not realistic. More realistic is a LogNormal distribution. However, we do not know for certain if this bad assumption has implications. It is possible that the method is insensitive to the distribution and only requires a mean LOS. Therefore we advise examining whether the method is insensitive to the distribution of the LOS. If this is not the case we recommend modelling the LOS as a phase type distribution and modifying the model to this end. The work of Machiahara ([20] and [21]) and Shortle [29], who modified the ERM for hyper-exponential service times, can serve as an example

We have assumed all three patient types to have the same mean LOS. This assumption is not a realistic assumption. The mean LOS of the elective patients differs from the mean LOS of the emergency patients. We believe the method will provide more realistic results if it is adjusted to this difference. As discussed in Section 3.2.1, Fredericks [13] has developed a modified ERM with different mean service times. Presumably Fredericks ERM can be used to modify the ERM for ICUs so that different patient types can have different mean LOS.

The assumption that elective patients arrive according to a Poisson process is not realistic. The arrival of elective patients is in reality less variable than if modelled by a Poisson process. Therefore we think that, if computed with the ERM for ICUs, the number of refused elective patients is larger than it is in reality. We recommend finding a distribution that better models the arrivals of elective patients and examining whether the ERM for ICUs can be adjusted to this end. Adjusting the ERM so that the arrivals are not Poisson is not straightforward.

The most difficult thing about modelling patient streams is that in reality each patient is treated as a separate case and much is done to admit an arriving patient requiring intensive care to the ICU. This can not be modelled, as it is not possible to treat each arriving patient in a model as a separate case. Therefore it has to be taken into account that the number of refused patients determined with the method, will most likely always be higher than in reality.

We did not consider the fact that different ICUs might offer different services. Therefore, in reality, a patient requiring intensive care cannot just be placed in any ICU. When distributing the regional beds among the ICUs in the region this should be taken into account. This might be difficult as it has to be taken into account how many regional patients need the special care that is only provided at certain ICU. Considering these difficulties is beyond the scope of this research.

Chapter 4

Simulation model

In the previous chapter we discussed a method to approximate the number of required regional beds. In this chapter we will discuss the simulation model we developed to validate the analytic method. Further the simulation model can be used to demonstrate the patients flows through a region with regional beds. The simulation was developed in eM-Plant version 7.0.2. EM-Plant is software for object-oriented, graphical modelling for simulating and visualising systems and business processes. In Appendix F a short introduction into the basic elements of eM-Plant is given, for the reader unfamiliar with eM-Plant.

The simulation model developed is generic in the sense that it can be used for any region with any number of ICUs. Like in the analytic model, when we talk about an ICU, we mean one or more Intensive Care Units that act as one Unit (e.g. the IC department of cluster 17 of the Erasmus MC is modelled as one ICU). The number of ICUs in the region, the number of beds per ICU, the arrival times and Length Of Stay (LOS) can all be adjusted according to the wishes of the user.

4.1 Description of the simulation model

The main frame of the simulation model represents the region. The region contains several ICUs and a unit with several regional beds. The three types of patients arrive at an ICU according to a poisson process (i.e. exponential interarrival times), each with its own rate. Elective patients do not arrive on weekenddays. If a bed is available the patient is treated at this ICU. The length of stay of the patient is modelled through a LogNormal distribution, each having a different mean LOS. In case no beds are available and an internal emergency patients arrives, an over bed is created for this patients. When no bed is available at the arrival of an elective patient, the patient is deleted from the system. When a regional emergency patient arrives and no bed is available the patient is sent to a regional bed (in the frame of the region). Figure 4.1 shows the basic patient streams in the simulation model with one ICU. In the rest of this section we discuss the simulation model in more detail.

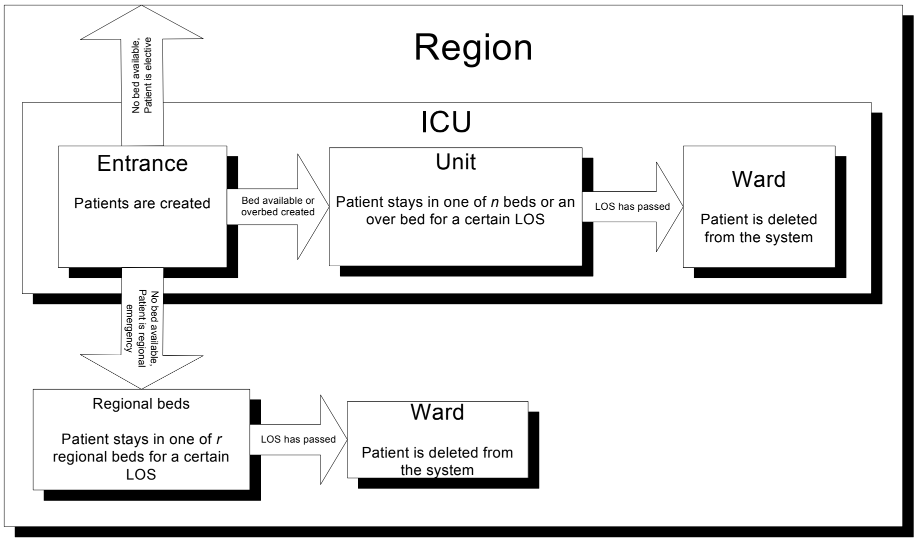

Figure 4.1: The basic patient streams in the simulation model

4.1.1 The simulation model of the region

The region is the top level of the simulation model. In this level the simulation has to be started, stopped and reset. Figure 4.2 shows the Frame window of the region. Before the simulation is run, the number of ICUs and the number of regional beds used for the experiment have to be chosen. This can be done by adjusting the variables 'NumberofICS' and 'NumberofRegionalbeds' and running the method 'CreateRegion'. By running this Method, new units, regional beds, a CardFile and a TableFile are created. In the CardFile 'ListofRegionalbeds' the location of all regional beds is registered. In the TableFile 'TableofICs' the location of the ICUs is recorded together with an initial value for the number of beds per ICU and the arrival rates per ICU. These initial values can be adjusted according to the wishes of the user. After having adjusted these values in the TableFile, the ICUs can be completed according to the chosen values by running the Method 'CreateICs'. This Method calls in each ICU the Methods 'SetArrivalRates' and 'CreateBeds' that take care of adjusting the arrival rates and creating the correct number of beds.

In the Object 'Regionalbeds' in the Frame window of the Region, the regional beds can be found. A regional bed is similar to a normal bed, which is explained later on.

The Methods 'Start' 'Stop' and 'ResetInit' are used to start the simulation, stop the simulation and reset the simulation. The EventController can be used for more extensive regulation of the simulation, e.g. for adjusting the rate of the simulation or by going through the simulation step by step.

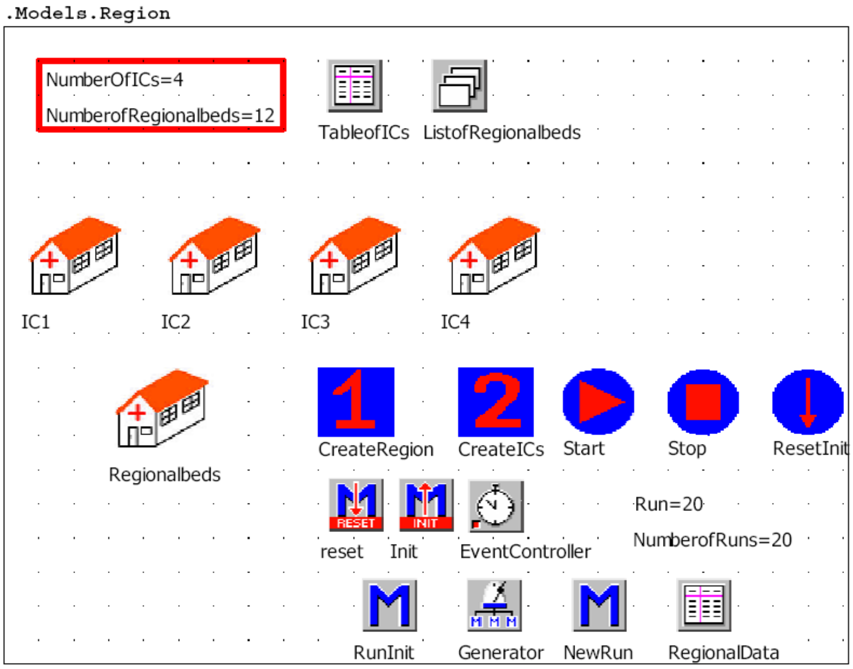

Figure 4.2: The top Frame window of the simulation model

4.1.2 The simulation model of the ICU

The Frame window of the ICU is depicted in figure 4.3. The model of the ICU consists of three basis elements: the entrance, where patients arrive, the unit where the patients stay in an IC bed and the ward, where patients leave the system. In the next three subsections we discuss these three main elements of the model of the ICU.

Entrance

In the model we distinguish three patient types: elective patients, internal emergency patients and regional emergency patients. Like in our analytic model, we assume all three patients arrive according to a Poisson process. The three patient types can have different mean interarrival times. In Section 4.1.1 we discussed how to adjust the arrival rates.

The internal emergency patients and the regional emergency patients are created by the Methods "CreatingIntEmerg" and "CreatingExtEmerg" that are activated by Generators with exponential interval times. The patients are represented by different entities. The emergency patients arrive in the Parallelproc "Arrivals" where the Method "MovingArrivals" is activated. The program code in this Method regulates what to do with the patient. The Method "BeginDay" is activated each weekday at 9:00 a.m. and creates elective patients. The number of elective patients to be created that morning is pulled from a Poisson

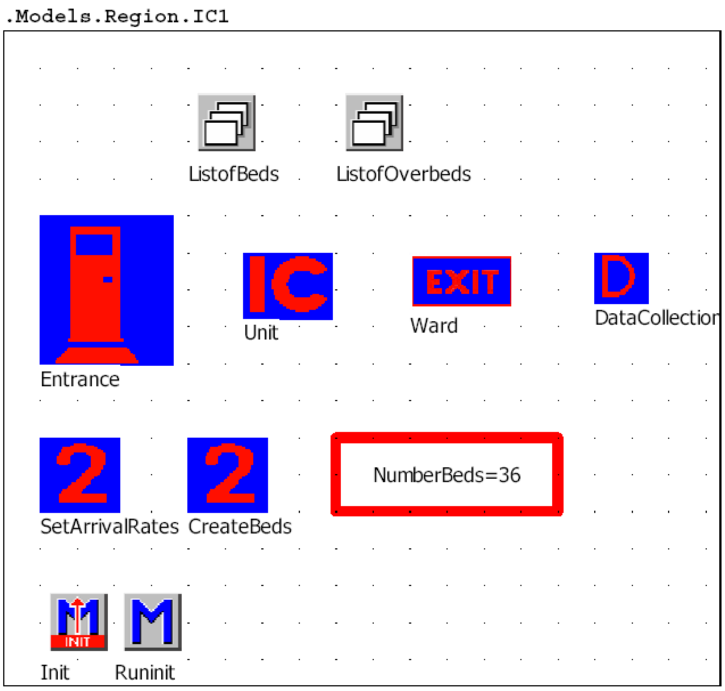

Figure 4.3: The Frame window of an IC

distribution.

When a patient arrives, it is immediately verified whether one of the beds in the ICU is available (for emergency patients this happens through the Method "MovingArrivals", for elective patients through the Method "BeginDay" that also creates the elective patients). If a bed is available, the bed is given a 'processing time' and the patient is sent to the available bed. In the following subsection, we discuss the processing time of the bed (this is the LOS of the patient). If no bed is available and the patient is an internal emergency patient, the Method "CreateOverbed" is executed. When this Method is executed, an over bed is created, the processing time is set and the patient is placed in this over bed. If no bed is available and the arriving patient is a regional emergency patient, the Method "TransExtEmerg" is called upon. This Method verifies whether a regional bed is available. If this is the fact, the processing time is set and the patient is placed in the available regional bed. If no regional bed is available the patient is deleted from the system. If not all elective patients planned for an operation that day can be placed in a bed, the remaining patients are deleted from the system. This is done through the Method "BeginDay".

Unit

The unit consists of a chosen number of beds. In Section 4.1.1 we discussed how to create the chosen number of beds for each ICU. While running the simulation, over beds can be added to the ICU. A bed (normal-, regional- or over bed) is modelled as a SingleProc. The ProcessingTime of the SingleProc is the LOS of the patient. For the LOS we distinguish three patient types: elective, internal emergency and regional Emergency. Although it is plausible that more distinction can be made, like difference in the treating specialism, we have chosen not to get in too much detail. We do not think that making more distinction in patients concerning the LOS improves the simulation enough (the extra work of adding details has to be considered against the improvement of the simulation).

We assume that the LOS has a Lognormal distribution. Marazzi et. al. [22] established the adequacy of the Lognormal, the Weibull and the Gamma distribution for describing the distribution of the LOS. Section 2.1.3 discussed that the Lognormal distribution fits well to the data concerning the LOS of the Erasmus MC. Therefore we have chosen to use a Lognormal distribution for the LOS in the simulation model. The exponential distribution can be used to study the different outcomes when using a Lognormal distribution or an exponential distribution. We have used a different mean LOS for each patient type but have not made a distinction for each ICU. We have used the means as calculated in Section 2.1.3.

When the patient has finished his stay at the ICU, the Method "LeaveIC" is executed, which moves the patient to the ward. In case of a normal bed, this Method checks if an over bed is in use. If this is the fact, then the Processing Time of the bed is adjusted to the remaining LOS of the patient in the over bed, the patient in the over bed is placed in the normal and the over bed is deleted.

Ward

The ward is modelled as a Drain. The Method "PatientLeavesIC", that is run when a patient enters the Drain, verifies whether the patient comes from an over bed. If this is the case, the over bed is deleted. After execution of the Method "PatientLeavesIC", the patient is removed from the system.

4.2 Data collection

During a simulation run several data is collected. For each ICU data is collected and for the total region data is collected.

The average bed occupation
For each ICU and for the regional beds, the average bed occupation is registered. This is calculated every time a patient leaves the ICU or a regional bed. The average bed occupation is calculated as follows:

$$\begin{aligned} Ld &: = Ld + simtime - Et, \\ Bo &: = \frac{Ld}{simtime * n}, \end{aligned}$$

with Ld the current total number of lying days of al patients that have left the ICU during the simulation run, $simtime$ the current simulation time (the time the simulation run has taken up to now), Et the simulation time at which the patient entered the bed, Bo the average bed occupation and n the total number of operational beds at the ICU (when the bed occupation is calculated for an ICU) or the total number of regional beds (when the bed occupation is calculated for the regional beds).

The proportion of cancelled operations
For each ICU the proportion of cancelled elective operations on patients that require intensive care after the operation, is calculated. This is done every time an elective patient has been admitted to the ICU or an elective operation has been cancelled due to a shortage of IC beds.

The proportion of refused regional emergency patients
For each ICU and for the region in total, the proportion of regional emergency patients that is refused due to a shortage of IC beds is calculated. This is done each time a regional emergency patient is admitted or refused.

The average number of over beds
For each ICU, the average number of over beds is calculated, each time a patient leaves an over bed:

$$\begin{aligned} Od &: = Od + simtime - Et, \\ Ao &: = \frac{Od}{simtime}, \end{aligned}$$

with Od the current amount of time over beds have existed, $simtime$ the current simulation time, Et the simulation time at which the patient entered the bed and Ao the average number of over beds.

4.3 Validation of the simulation model

Before using the simulation model for predictions, we want to validate the simulation model. That is, we want to know if the simulation is a good representation of the reality. Therefore we have run the simulation model with one ICU consisting of 36 IC beds and no regional beds. This is the situation of the IC department of cluster 17 of the Erasmus MC. We have used the arrival rates as given in Section 2.1.3. We made 20 runs, each consisting of 30 years. The first 10 runs were used for calculating the warm-up period, the last 10 runs were used to get the results. For calculating the warm-up period we used Welch's graphical method as discussed in [18]. The confidence intervals are calculated using the Replication / Deletion Method as discussed in [18]:

$$\begin{aligned} \overline{X} \pm t_{n-1,1-\alpha/2}\sqrt{\frac{S^2(n)}{n}}, \\ S^2(n) = \frac{\sum_{i=1}^{n}[X_i - \overline{X}]^2}{n-1}, \end{aligned} \tag{4.1}$$

with $\overline{X}$ the mean of the observations, excluding the warm-up period, n the number of runs, $t_{n-1,1/\alpha/2}$ the upper $1-\alpha/2$ critical point for the t distribution with $n-1$ degrees of freedom and X_i the mean of run i excluding the warm-up period. First we compare the outcomes with the data of the IC department

	Simulation results	Actual data
Mean bed occupation	0.89 ± 0.001	± 0.87
Proportion of cancelled operations	0.26 ± 0.005	0.07 ± 0.01
Proportion of refused regional emergencies	0.18 ± 0.004	0.09 ± 0.01

Table 4.1: Comparing the simulation results with the reality

of cluster 17 of the Erasmus MC. At the end of this section, we discuss these outcomes.

Bed occupation
Using formula (4.1) the 95% confidence interval for the mean bed occupation is 0.89 ± 0.001. Because the actual number of operational beds can vary from day to day (depending among others on the available staff) we can only give an estimate of the actual bed occupation, using the average number of beds. This results in a mean bed occupation of 0.87, which is close to the confidence interval of the mean bed occupation in the simulation model.

Proportion of cancelled operations
The Replication / Deletion method gives a 95% confidence interval of 0.26 ± 0.005 for the proportion of cancelled operations. Using data from the year 2003 of the IC department of cluster 17 of the Erasmus MC we find a confidence interval of 0.07 ± 0.01 for the proportion of cancelled operations. That means the simulation gives a high proportion of cancelled operations.

Proportion of refused regional emergency patients
During the simulation we have collected data on the proportion of refused regional emergency patients. These are the patients that at present are transferred to an ICU in another hospital and later on in our model will be transferred to a regional bed. When a large number of observations is taken, the proportion of refused regional emergency patients approaches the probability a regional emergency patient is refused. The 95% confidence interval for the proportion of refused regional emergency patients is 0.18 ± 0.004. Using data from the year 2003 of the ICUs of cluster 17 of the Erasmus MC we find a confidence interval of 0.09 ± 0.01 for the proportion of refused regional emergency patients. That means the proportion of refused regional emergency patients in the simulation model is high compared to the actual proportion of refused regional emergency patients.

In Table 4.1 an overview is given of the results presented in this section. It can be seen that the proportion of cancelled operations and the proportion of refused regional emergency patients are on the high side. Several explanations can be found for that. First of all, as already mentioned in Section 2.1.3, the mean LOS is most likely to be too large. The only solution to this is to collect correct data on the lying days. Another explanation for the high proportion of cancelled operations and refused regional emergency patients in the simulation is that in reality it may happen that IC patients are predischarged to release the bed for an arriving patient whereas in the simulation model it is possible that one minute before a patient is released, another patient is refused. More research on the predischarging of patients has to bed done before this can be added to the simulation model. Adapting the simulation model to this end is not difficult. The high proportion of cancelled operations can also be explained by the assumption on the arrival process. We have assumed that elective patients arrive according to a Poisson process although we believe that this is not correct. The variability of elective arrivals is assumed to be smaller than the variability of the Poisson process. With a higher variability it is more likely that patients are being rejected. The solution to this is to consider other distributions for the arrival of elective patients.

Chapter 5

Results

In this chapter we discuss the results concerning the analytic model and the simulation model as treated in Chapters 3 and 4. To this end, the data of the IC department of cluster 17 of the Erasmus MC, as discussed in Section 2.1.3, is used. In addition, we have used estimated data, concerning arrivals, from three other hospitals in the region Rijnmond[1]. Information on these hospitals can be found in the Appendix C. As we only have information on the arrivals at the other hospitals but not on the LOS, we will assume the mean LOS to be the same for each ICU. Besides, it has to be noted that the data from these hospitals are estimated values. Moreover, we only have information from a few hospitals in the region Rijnmond. Therefore the results in this chapter are merely used to evaluate the analytic model and to show the benefit of using regional emergency beds.

For the implementation of the analytical method we have used Maple v. 8.00 (software to analyse and solve mathematical problems). We have only implemented the simplified method with one patient type in the Equivalent ICU, as discussed in Section 3.4.1. The rationale behind this is that we do not yet have practical formula to compute the capacity and the load of the equivalent ICU with three patient types as discussed in Section 3.4.2. We, however, do recommend implementing this adjusted ERM so that the differences between these two methods can be studied. The Maple code for the implementation of the adjusted ERM with one patient stream at the equivalent ICU can be found in Appendix D. In the implementation of the adjusted ERM, we have made one difference compared to the theoretical method as discussed in this chapter. For the implementation we have assumed the possible number of over beds to be limited. The reason for this is that the calculations are difficult for an unlimited possible number of over beds. Moreover, in practice the total number of beds (operational beds plus over beds) is actually limited by the number of constructional beds available. For the correctness of the assumptions we have verified how much the outcome depends on the number of possible over beds. For several ICUs we have calculated the expectation and variance of the number of patients in the overflow for different maximum number of over beds. In Appendix E, graphs containing the expectation and the variance as a function of

[1] The data was obtained after requesting 11 hospitals in the region for information on the arrivals at their IC.

the maximum number of beds, for different ICUs, are given. It can be seen that both the expectation and the variance converge fast. Therefore the difference in outcomes do not differ much when using an infinite maximum number of over beds or a finite maximum number of over beds. For correctness we, however, advise to take a finite maximum number of over beds equal to the total number of beds available in the ICU. The ERM takes approximately one minute to approximate the blocking probabilities for 0 to 50 regional beds.

To obtain results from the simulation model we have made 20 runs for each test. Results of the first 10 runs are used to determine the warm-up period. This is necessary because the system starts empty, which is not the case in reality. Results of the second 10 runs are used to calculate the 95% confidence intervals for the results of the simulation model. To this end we have used the Replication / Deletion Method as shortly discussed in Section 4.3. Formula (4.1) is then used to calculate the confidence interval. Each run simulates 30 or 60 years, depending on the amount of observations required for the warm-up period and the confidence interval. A run of 60 years, including four hospitals takes approximately half an hour. A run of 30 years with one hospital takes approximately 10 minutes.

In Section 5.1 we first compare the outcomes of the simulation model using an exponential distributed LOS with the outcomes of the simulation model using a lognormal distributed LOS. To this end we use the data of the Erasmus MC. Next, in Section 5.2, we give the results of the simulation model for the three other hospitals, without regional beds. We use these results to discuss the assumption that the mean LOS is the same for each ICU. Then, in Section 5.3, we discuss the results for the situation of a region with several hospitals without cooperation. In this case each ICU reserves beds only for the regional patients arriving at their ICU. We use the results to compare the analytic model with the simulation model. Finally, in Section 5.4, we demonstrate the advantages of jointly reserving regional IC beds. To this end we use the data of the four hospitals in the region.

5.1 Comparing exponential LOS with lognormal LOS

In Section 2.1.3 we have seen that the lognormal distribution best fits the data of the Length Of Stay (LOS) at the ICU. In the analytic model, however, we have assumed the LOS to be exponentially distributed. To test what the influence of the exponential assumption is, we have run the simulation model with the data of the Erasmus MC for exponential distributed LOS and for lognormal distributed LOS. We now compare the outcomes of the mean bed occupation, the proportion of cancelled operations, the proportion of refused emergency patients and the average number of over beds. Table 5.1 gives the confidence intervals, when using exponential distributed LOS, when using Lognormal distributed LOS, and for the difference between these two situations. From the confidence intervals of the differences it can be seen that, although there is a slight difference for the mean bed occupation, there is no significant difference for the other outcomes. We are particularly interested in the proportion of refused regional emergency patients, for which it does not make a difference if the exponential distribution or the Lognormal distribution is used for the LOS. Therefore we may use the exponential distribution for the LOS, even though it does not correspond to reality.

	Exponential confidence interval	Lognormal confidence interval	Difference confidence interval
Mean bed occupation	0.90 ± 0.002	0.89 ± 0.001	0.003 ± 0.002
Proportion of cancelled operations	0.26 ± 0.006	0.26 ± 0.005	0.003 ± 0.009
Proportion of refused regional patients	0.18 ± 0.005	0.18 ± 0.004	-0.002 ± 0.007
Average number of over beds	0.08 ± 0.004	0.08 ± 0.004	-0.002 ± 0.008

Table 5.1: Comparing Exponential LOS with LogNormal LOS with data of the Erasmus MC

5.2 Evaluating the assumption of one mean LOS

To generate results, we have made the assumption that the mean LOS is the same in every hospital. To verify if this assumption is realistic we have run the simulation model, without regional beds, for the three hospitals in the region of which we have obtained estimated data concerning the arrivals of patients. For the mean LOS we have, for each ICU, taken the mean LOS of the IC department the Erasmus MC. The outcomes of the simulation model are given in Table 5.2. We do not have enough data to statistically draw conclusions, but given the extreme outcomes for the Sint Franciscus Gasthuis and the Albert Schweizer Hospital, we presume these outcomes are not realistic. Adding to this the fact that the Erasmus MC, as a university hospital, receives patients with complicated disorders, we can expect that the mean LOS at the ICU is not the same for different hospitals. It is therefore wise to verify this conclusion and if necessary, to collect data concerning the LOS of each ICU and use this in the analytic model and the simulation model to determine the number of regional beds required in the region.

5.3 Results for the region without cooperation

In this section we discuss the results for the situation when the ICUs in the region are not cooperating. In this situation we examine the case if the ICUs each would reserve there own beds (reserved beds) for regional emergency patients. We have obtained results from the analytic model and from the simulation model for several hospitals. For the analytic model we have obtained the blocking probability for regional emergency patients and for the simulation model we

[2] The number of cancelled operations was too small to calculate a 95% confidence interval.

	Sint Franciscus Gasthuis	Dirksland	Albert Schweizer
Mean bedoccupation	1.23 ± 0.02	0.18 ± 0.01	1.23 ± 0.02
Proportion of cancelled operations	0.74 ± 0.01	$\pm 0^2$	0.72 ± 0.01
Proportion of refused regional patients	0.71 ± 0.02	0.001 ± 0.001	0.69 ± 0.02

Table 5.2: Outcomes of the simulation without regional beds

have obtained the proportion of refused regional emergency patients. These two should approximately be the same as we have taken a large number of observations for the proportion of refused regional emergency patients. In Table 5.3 the results for several numbers of reserved beds are given for the Erasmus MC. It might seem laborious to use the ERM for the blocking probability of one ICU. However, we already have formulas for ERM and the approximation should be as good as when we using the ERM to calculate blocking probabilities for several ICUs. When we compare the blocking probabilities obtained by the ERM, with the proportion of refused regional emergency patients obtained by the simulation, it can be seen that the differences are small. In Section 4.3 we established a difference between the outcomes of the simulation and the real data. Assuming that these differences are a result of an error in the mean and standard deviation of the LOS and as a results of the assumption that the elective patients arrive according to a poisson process, this error should also influence the outcomes of the ERM. As the difference in results from the simulation and from the ERM are small we can conclude that the ERM, when using a correct mean LOS, leads to a good approximation of the blocking probability of the regional emergency patients. In Appendix H the outcomes of the ERM and of the simulation of the other three hospitals without cooperation are given. These results too, show the small difference in outcomes between the ERM and the simulation. This appendix also gives graphs containing the blocking probabilities of the ERM as a function of the number of reserved beds for the four hospitals.

5.4 Results for the region with cooperation

We now discuss the results for the region when there is cooperation. We have obtained outcomes from the ERM and from the simulation model using the data from the four hospitals. We have done this for a several number of regional beds that are jointly reserved for regional emergency patients arriving at any hospital. By means of the ERM we have obtained the total blocking probability for all regional emergency patients. Using the simulation model, we have obtained the total proportion of refused regional emergency patients, as well as the proportion of refused regional emergency patients for each hospital separately. Table 5.4 and 5.5 show these results.

We will now illustrate the advantage of cooperation within the region by means of an example, using the simulation results given in this and the previous sec-

Number of reserved beds	ERM blocking probability	Proportion refused regionals in simulation
0	0.207	0.182 ± 0.004
1	0.168	0.148 ± 0.003
2	0.133	0.119 ± 0.003
3	0.102	0.093 ± 0.004
4	0.077	0.070 ± 0.004
5	0.056	0.053 ± 0.003
6	0.039	0.039 ± 0.003
7	0.026	0.026 ± 0.001
8	0.017	0.019 ± 0.003
9	0.011	0.012 ± 0.002
10	0.006	0.009 ± 0.002
11	0.004	0.005 ± 0.001
12	0.002	0.003 ± 0.001
13	0.001	0.002 ± 0.001
14	0.001	0.001 ± 0.000
15	0.000	0.001 ± 0.000
16	0.000	0.000 ± 0.000

Table 5.3: Blocking probability regional emergency patients at the Erasmus MC without cooperation

Number of regional beds	ERM blocking probability	Proportion refused regionals in simulation
0	0.255	0.232 ± 0.006
1	0.215	0.195 ± 0.003
2	0.177	0.162 ± 0.006
3	0.142	0.134 ± 0.005
4	0.112	0.107 ± 0.004
5	0.085	0.083 ± 0.004
6	0.063	0.065 ± 0.002
7	0.045	0.049 ± 0.001
8	0.030	0.036 ± 0.001
9	0.020	0.026 ± 0.002
10	0.013	0.018 ± 0.001
11	0.008	0.011 ± 0.001
12	0.004	0.007 ± 0.000
13	0.002	0.005 ± 0.000
14	0.001	0.003 ± 0.000
15	0.001	0.002 ± 0.000
16	0.000	

Table 5.4: Blocking probability regional emergency patients in the region with cooperation

Nr of regional beds	Erasmus MC	Albert Schweizer	Dirksland	Sint Franciscus
0	0.182 ± 0.004	0.698 ± 0.016	0.001 ± 0.001	0.705 ± 0.016
1	0.161 ± 0.004	0.543 ± 0.012	0.001 ± 0.001	0.540 ± 0.009
2	0.134 ± 0.003	0.401 ± 0.012	0.001 ± 0.001	0.417 ± 0.007
3	0.118 ± 0.004	0.302 ± 0.018		0.318 ± 0.008
4	0.096 ± 0.002	0.222 ± 0.015		0.224 ± 0.007
5	0.074 ± 0.002	0.154 ± 0.011		0.168 ± 0.007
6	0.060 ± 0.002	0.115 ± 0.008		0.121 ± 0.005
7	0.046 ± 0.001	0.085 ± 0.008		0.085 ± 0.005
8	0.034 ± 0.002	0.059 ± 0.005		0.061 ± 0.005
9	0.025 ± 0.002	0.039 ± 0.005		0.038 ± 0.003
10	0.018 ± 0.002	0.024 ± 0.004		0.026 ± 0.003
11	0.011 ± 0.001	0.015 ± 0.004		0.017 ± 0.002
12	0.007 ± 0.001	0.010 ± 0.001		0.012 ± 0.002
13	0.005 ± 0.004	0.006 ± 0.003		0.006 ± 0.002
14	0.003 ± 0.000	0.004 ± 0.001		0.004 ± 0.001
15	0.002 ± 0.000			

Table 5.5: Blocking probability regional emergency patients for each hospital with cooperation

tion. The reason we do not use the ERM in this example is that currently the ERM only gives the blocking probabilities for the whole region and not per ICU. Suppose the management of the ICUs in the region decides to accept a probability of 0.01 that a regional patient is refused and needs to be transferred to an ICU outside the region. From Table 5.4 it can be seen that 11 regional beds are required to obtain a probability of 0.011. If we now look in Table 5.5 in the row with 11 beds, we see that this results in a blocking probability of approximately 0.01 for regional patients arriving at the Erasmus MC, approximately 0.02 for regional patients arriving at the Albert Schweizer Hospital and the Sint Franciscus Gasthuis and approximately 0 for regional patients arriving at the Dirksland Hospital. We now use Tables 5.3, H.1, H.2 and H.3 to determine how many regional beds were required to obtain these same results without cooperation. For the Erasmus MC 9 regional beds would be required, for the Albert Schweizer hospital 3 beds would be required and for the Sint Franciscus Gasthuis 4 beds would be required, resulting in 16 beds in total. Accordingly, cooperation between the hospitals, in this case, results in a saving of 5 beds (31%). The reservation of regional beds does not influence the blocking of elective patients or the use of over beds.

5.5 Discussion

In Section 1.3 we have shortly discussed the differences between simulation models and analytic models. Because of these differences, the combination of using an analytic model with a simulation model gives a good understanding of the overflow of IC patients. Moreover, we think that a combination of these two can be a good tool for the management of the ICUs in the region to make decisions concerning the reservation of regional beds. When the necessary data has been obtained, the ERM can easily be used to approximate the number regional beds required. If preferred, this Method can be adjusted such that the blocking probabilities for each ICU separately can be determined. The simulation model can then be used to demonstrate the flow of patients through the hospitals. The analytic model has the advantage of giving a concrete answer in a short amount of time. The simulation model demonstrates the behaviour of the system but requires more time to obtained results.

Chapter 6

Conclusions and recommendations

In this chapter we give the conclusions and recommendations that result from the research.

6.1 Conclusions

The joint reservation of regional IC beds for the overflow of regional emergency patients is a good solution to the problem that regional emergency patients needing intensive care can not always be treated within the region. The cooperation between the ICUs in the region, by means of reserving regional beds, leads to a saving of beds compared to the solution where each ICU tries to solve this problem single-handedly. Given a satisfying probability of having to refuse a regional emergency patient, the adjusted Equivalent Random Method can be used to give an approximation of the total number of regional beds needed. Subsequently, the management of the ICUs in the region will need to determine how to distribute this number of beds among the ICUs, taking into account the size of the ICUs and the type and level of care provided at each ICU. The current available data is not correct and sufficient, as a result of which we can not give a concrete answer to the question how many regional beds are needed in the region Rijnmond.

For computations with the analytic method and the simulation model we have made the assumption that the mean LOS is the same for each ICU. This assumption appears to be incorrect. For a better approximation of the number of beds required, data concerning the mean LOS of each ICU need to be collected. The simulation model will need minor adjustments to use a different mean LOS for each ICU.

For the analytic method we have made the questionable assumption that the LOS is exponentially distributed. The model of the ICU, however, appears to be insensitive to the choice between the exponential and the (more presumable) lognormal distribution for the LOS. Therefore we may use the exponential distribution even though this does not correspond to reality.

For the analytic method we have made the (unrealistic) assumption of an unlimited possible number of over beds. For the implementation of this method we needed to limit the possible number of over beds. Apart from the fact that this is more realistic, the expectation and variance of the number of regional patients overflowing the ICU as a function of the maximum possible number of over beds turn out to converge fast. Consequently the assumption of an unlimited possible number if over beds does not influence the outcomes of the computations, as long as the number of constructional beds is reasonably large.

6.2 Recommendations

General
Although we conclude that the reservations of IC beds for the overflow of regional emergency patients is a good solution to the problem of regional emergency patients overflowing the region, we recommend to do further research before actually introducing the regional beds.

- We have not taken the costs of having more operational beds into account. Therefore we advise to first determine how many extra beds are needed in the region for the reservation of regional beds and consequently consider the costs of these extra beds.

- We did not consider the fact that different hospitals offer different services. When distributing the regional beds over the different ICUs in the region, it is possible that this severely complicates the introduction of regional beds. We recommend examining the implications of the different ICUs offering different care.

- We have only considered one solution to the problem of the regional overflow. Although we believe it is a good solution we do not know if it is the best solution. We recommend investigating other options of solving the overflow problem and comparing the different options. An example of another option is the reservation of regional beds not only for the overflow of regional emergency patients, but for all regional emergency patients. In such a case there would be separate beds for the internal emergency and elective patients and separate beds for the regional emergency patients. An advantage of this compared to the reservation just for the overflow of regional emergency patients is that it is more orderly and therefore more likely to succeed in implementing. A disadvantage is that by separating beds for different patients, the use of the bed capacity is less flexible.

The adjusted ERM
The adjusted Equivalent Random Method is an appropriate tool to assist the management of a region on the number of regional beds needed. However, some adjustments are advised to improve the method.

- The assumption that the LOS is exponentially distributed is supported by our results, but not proven. Therefore we recommend examining whether the results are insensitive to the type of distribution chosen. If this is the fact, only the mean LOS is of importance.

- The assumption that all patient types have the same mean LOS is incorrect. We advise examining the implications of this assumption on the outcomes of the ERM. If it turns out that the assumption of having different mean LOS for the different types of patients has a bad implication, we advise to adjust the analytic method. We expect that the work of Fredericks [13] can be used for this purpose.

- We advise to work out the more complex version of the analytic method, in which the equivalent ICU receives three patient streams. This version is more realistic and therefore might give better approximations of the number of regional beds needed. Comparing the outcomes of the two versions with each other and with results of the simulation model will give an answer.

The simulation model

In the simulation model we have not gone in too much detail, as we do not think that more detail will make the simulation model better. We do however recommend considering one thing.

- We recommend considering the predischarging of patients. For this purpose the predischarging of patients as it currently happens should be studied before the simulation model can be adjusted. Adjusting the simulation will not be difficult.

Data

Correct and sufficient data is important in order to advise a region on the number of regional emergency beds needed. The following recommendations are given concerning the data.

- The Length of Stay (LOS) registered at cluster 17 of the Erasmus MC does not equal the actual LOS. We recommend examining if the actual LOS can be determined from the existing data. If this is not possible we recommend starting the registering of the actual LOS. The correct LOS is needed as input for the method to calculate the number of regional beds required.

- We recommend collecting the required data from all the ICUs in the region. This data is necessary to give an approximation of the number of regional beds needed. Required is the average number of regional patients arriving per day, the average number of elective patients arriving per day, the average number of internal emergency patients arriving per day, the number of operational beds, the number of constructional beds and the mean LOS. To use a different mean LOS for each ICU, the simulation model needs minor adjustment.

- We have made the assumption that elective patients arrive according to a Poisson process. This assumption is questionable and therefore we advise to examine if there exists a distribution that better fits the arrivals of elective patients than the Poisson distribution. If it is necessary and possible, we advise to adjust the model to this.

- In the introduction we mentioned that the number of trauma patients offered to the IC department of cluster 17 of the Erasmus MC is most likely to further increase. We recommend investigating this increase and the implications it might have on the average number of patients arriving and on the mean LOS.

- Currently the IC department of cluster 17 of the Erasmus MC is being reorganised. We recommend investigating whether this change has implications on the average number of patients arriving and on the mean LOS.

Introduction of the regional beds

This document reports a mathematical study and therefore does not discuss the management side of introducing regional beds. However, we do recognise this is an import and complicated process. Therefore we highly recommend to carefully consider the following:

- Carefully consider what probability of refusing a regional patient is satisfying. The costs of having more beds should be weighed against the costs of transporting patients. With costs we do not only aim at money, but also at e.g. low bed occupations (thus more staff than work) and possible deteriorating of a patients health because of transportation.

- We recommend to strictly watch over the beds reserved for regional patients. If these beds are used for elective patients or internal emergency patients, the probability of refusing a regional emergency patient will be higher than agreed on by the management of the ICUs in the region. The costs of having the extra beds are then consequently paid for other patients than intended.

Other research

During the research documented in this report, we have run into matters that can be interesting to investigate, but were not of direct relevance for this research. For more research, however, we give some recommendations.

- In the region Rijnmond, information on the available IC capacity is not shared among hospitals or with the ambulance service. We recommend investigating possibilities to better share information in order to minimise the transportation of patients. To this end we refer to the national database for the registration of available IC capacity [14]. We recommend investigating the options of exploiting this service.

- Currently, elective patients, although planned, arrive unexpectedly at the ICU. We have the impression that by planning the operations, after which an IC bed is needed, in cooperation with the intensivists, will result in a better utilisation of the IC beds. We recommend investigating the options to improve the planning, and subsequently the arrival of the elective IC patients.

- Only assumptions have been made on the available IC bed capacity in the region Rijnmond. We advise to explore the capacity and the use of it. If some ICUs turn out to have a very low bed occupation it might be possible to use some of these beds for the regional emergency patients.

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Appendix A

Definitions, abreviations and symbols

A.1 List with abbreviations

Erasmus MC	Erasmus Medical Centre
ERM	Equivalent Random Method
IC	Intensive Care
ICU	Intensive Care Unit
LOS	Length Of Stay

A.2 List with definitions

Bed occupation	The proportion of beds that is occupied.
Capacity of the ICU	The number of equiped beds and the quantity of staff.
Constructional bed	Bed that is not staffed and can therefore officially not be occupied.
Critical Care	See Intensive Care
Elective Operation	Operation that is planned.
Elective patient	Patient that is planned. An elective IC patient comes from the Operation Theatre and the arrival can be postponed if necessary.
Emergency patient	Patient that requires immediate treatment.
High care	Guarding, nursing and treatment of patients with a threatening or existing disorder at one of the vital organ functions.
Inpatient	Patient that needs a hospital bed for at least one night

Intensive care	Guarding, nursing and treatment of patients with a threatening or existing disorder at more than one of the vital organ functions.
Intensive Care bed	Bed that is equipped to provide intensive care.
Intensive Care department	Department that consists of one or more Intensive Care Units.
Intensive Care Unit	Hospital ward staffed and equipped to provide intensive care.
Intensive Care Patient	Patient requiring Intensive Care treatment.
Intensivist	Physician who is specialised in the care of critically ill patients.
Internal Emergency Patient	Emergency patient admitted to the hospital before the emergency.
Length of Stay	The number of days the patient has spent at the unit.
Medium Care	Guarding, nursing and treatment of critically ill patients of whom the vital organs are functioning on themselves.
Operational bed	Staffed bed that is occupied or can be occupied by a patient.
Over bed	Constructional bed that is not staffed. The bed is used when all staffed beds are occupied and an emergency patient arrives from within the hospital.
Overflow	(Fictitious) Intensive Care Unit that takes care of Intensive Care Patients refused at an Intensive Care Unit due to the unavailability of operational beds.
Post Operation Care Unit	Hospital ward staffed and equipped to provide monitoring of the vital organ functions of patients who need intensive monitoring for at most 24 hours after an operation
Recovery Room	Hospital ward where the vital organ functions of the patient are strictly guarded after surgery.
Region	A region in which the ICUs cooperate.
Regional bed	Intensive Care bed reserved for the treatment of regional emergency patients.
Regional emergency patient	Emergency patient in the region not admitted to a hospital before the emergency.
Special Care	Care that is provided to patients who at arrival at the IC department require medium care or high care.

Step down patient	Patient who steps down from intensive care to medium care or high care.
Step up patient	Patient who needs Medium Care or High Care at arrival on the IC.
Trauma Centre	Integral care facility for patients from severe accidents.
Unit	Recognizable part of a medical department.

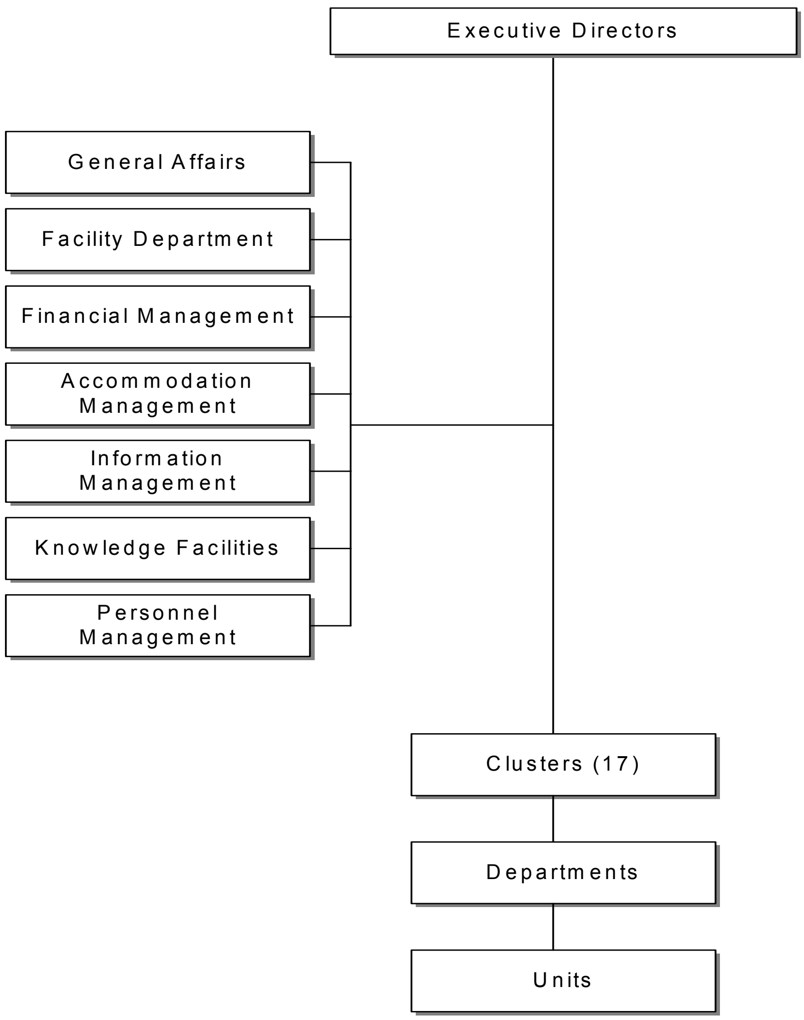
A.3 List with symbols

λ	Mean arrival rate at equivalent ICU.
λ_i	Mean arrival rate at ICU i.
μ^{-1}	Mean LOS of equivalent ICU.
μ_i^{-1}	Mean LOS of ICU i.
ρ	Load of equivalent ICU: λ/μ.
ρ_i	Load of ICU i: λ_i/μ_i.
$B(c,\rho)$	Erlang loss blocking probability with capacity c and load ρ.
c	Capacity of equivalent ICU.
c_i	Capacity if ICU i.
E	Expected number of patients in the overflow of the equivalent ICU.
$\overline{E}$	Expected number of patients that overflow the region (that are refused in the ICUs and the regional beds).
E_i	Expected number of patients in the overflow of ICU i.
$E_j^{(i)}$	Expected number of patients in the overflow of ICU i,with j patients in ICU i (first derivative of $G_j^{(i)}(z)$ with respect to z).
$E_j^{\prime(i)}$	Second derivative of $G_j^{(i)}(z)$ with respect to z.
$E_{j,l}$	Expected number of patients in the overflow with j patients in the equivalent ICU and l patients in the regional beds.
$G_j^{(i)}(z)$	Marginal generating function of the probabiltiy $P_i(j,k)$.
$G_{j,l}(z)$	Marginal generating function of the probability $P(j,l,k)$.
j	Number of patients in the ICU, $j = 0,1,2,....$
k	Number of patients in the overflow, $k = 0,1,2,....$
l	Number of patients in the regional beds, $l = 0,1,2,...,r$.
n	Number of ICUs.
p_t	Probability patient arriving at equivalent ICU is of type t, $t = 1,2,3$.
$p_{t,i}$	Probability patient arriving at ICU i is of type t, $t = 1,2,3$.
$P_i(j)$	Probability of having j patients in ICU i.
$P_i(j,k)$	Probability of having j patients in ICU i and k patients in the overflow.
$P(j,l,k)$	Probability of having j patients in the equivalent ICU, l patients in the regional beds and k patients in the overflow.
r	Number of regional beds.
V	Variance of the number of patients in the overflow of the equivalent ICU.
V_i	Variance of the number of patients in the overflow of ICU i.

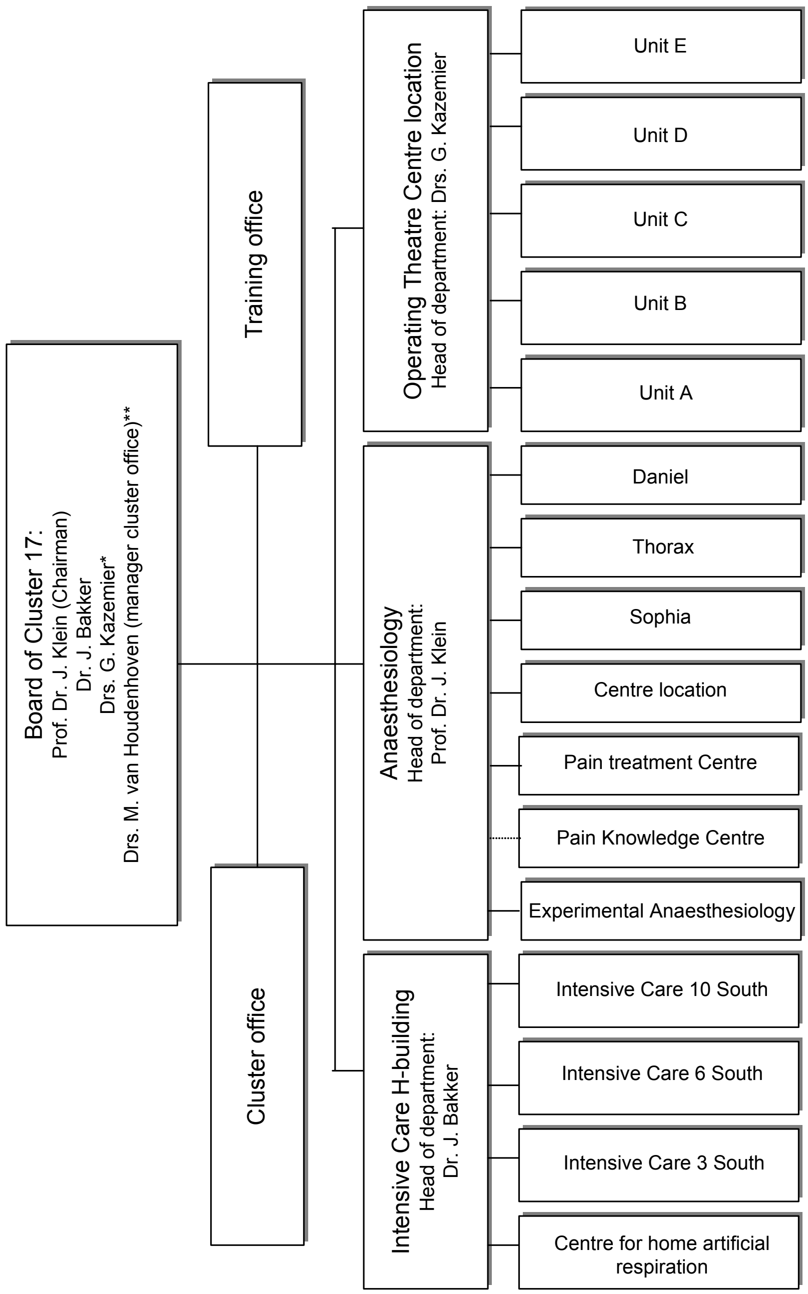
Appendix B

Organisational charts

B.1 Organisational chart of the Erasmus MC



B.2 Organisational chart of cluster 17



*Drs. G. Kazemier is, on behalf of the board, responsible for the cluster office and the training office.
**Drs. M. van Houdenhoven performs the task of manager of the cluster office and the training office and the function of care manager within the cluster.

Appendix C

Data from several hospitals in the region

We have obtained estimated values for data concerning the arrivals at the IC department from three hospitals in the region: Albert Schweizer hospital, Dirksland hospital and Sint Franciscus Gasthuis. Albert Schweizer hospital [1] is a general hospital with four locations. Two locations are situated in Dordrecht, one is situated in Zwijndrecht and one is situated in Sliedrecht. The Albert Schweizer hospital has 1045 beds of which 13 IC beds, 180 medical specialists and 3755 employees. Dirksland hospital [9] is a protestant-christian hospital situated in Dirksland. Dirksland hospital has 140 beds of which 5 IC beds, 35 medical specialists and 420 employees. Sint Franciscus Gasthuis [28] is a general hospital situated in Rotterdam. They have 613 beds of which 11 IC beds, 96 medical specialists and 2000 employees. Table C.1 shows the estimated data as given by these hospitals

Hospital	Number of IC beds	Mean number of regional emergency arrivals per day	Mean number of elective arrivals per day	Mean number of internal emergency arrivals per day
Albert Schweizer	13	0.09	0.95	1.66
Dirksland	5	0.07	0.06	0.02
Sint Franciscus	11	0.18	0.96	1.37

Table C.1: Data of several hospitals in the region Rijnmond

Appendix D

Maple code for the adjusted ERM

Expected number of patients in the overflow

```
> restart:
```

The input

```
> n := 4:
  lambda[1,1] := 1/.46:
  lambda[2,1] := 1/.58:
  lambda[3,1] := 1/.62:
  lambda[1,2] := 0.18:
  lambda[2,2] := 0.96:
  lambda[3,2] := 1.37:
  lambda[1,3] := 0.07:
  lambda[2,3] := 0.06:
  lambda[3,3] := 0.02:
  lambda[1,4] := 0.09:
  lambda[2,4] := 0.95:
  lambda[3,4] := 1.66:

  mu[1] := 1/6.93:
  c[1] := 36:
  cmax[1] := 52:
  mu[2] := 1/6.93:
  c[2] := 11:
  cmax[2] := 25:
  mu[3] := 1/6.93:
  c[3] := 5:
  cmax[3] := 25:
  mu[4] := 1/6.93:
  c[4] := 13:
  cmax[4] := 25:

  for i from 1 to n do
     lambda[i] := lambda[1,i] + lambda[2,i] + lambda[3,i]:
     p[1,i] := lambda[1,i] / lambda[i];
     p[2,i] := lambda[2,i] / lambda[i];
     p[3,i] := lambda[3,i] / lambda[i];
     rho[i] := lambda[i] /mu;
```

```
end:
```

The expected number of patients in the overflow

```
>  for i from 1 to n do
      t:=cmax[i]:
      P[0,i] := 1 / (sum('1 / j!  * rho[i]^j', 'j' = 0..c[i])
                + sum('p[3,i]^(j - c[i]) * rho[i]^j / j!',
                'j' = c[i] + 1..t))
      EK[i] := p[1,i] * rho[i] * (1 - sum('1 / j!  *rho[i]^j','
               j'=0..c[i]-1) * P[0,i]):
   end:

   for i from 1 to n do
      EK[i] := EK[i]:
   end:
```

The variance of the number of patients in the overflow

```
>  for i from 1 to n do

      t:=cmax[i]:
      E[0,i]:=0:
      unassign('E[0,i]'):
      E[-1,i] := 0:

      for j from 1 to c[i] do
         E[j,i] := E[j-1,i] + rho[i]^j / j!  * E[0,i];
      end:

      E1[i] := E[0,i]-(p[1,i] + p[2,i]) * c[i]!
               * rho[i]^(-c[i]) *E[c[i],i]:

      for j from c[i]+1 to t do
         E[j,i] := E[j-1,i] + p[3,i]^(j-(c[i]+1)) * rho[i]^(j)
                   / j!  *(E1[i]- p[1,i] * P[0,i] * (j - c[i]) ):
      end:

      E[0,i] := solve(sum('E[j,i]','j' = 0..t) = EK[i],E[0,i]):

      Var[i] := p[1,i] * rho[i] * sum('E[j,i]','j' = c[i]..t)
                + EK[i] - (EK[i])^2:

    end:

    for i from 1 to n do
      Var[i]:=Var[i];
    end:
```

The Equivalent Random Method 1

```
>  EK[ERM] := sum('EK[i]','i'=1..n):

    Var[ERM] := sum('Var[i]','i'=1..n):

    rho[ERM] := Var[ERM] + 3 * Var[ERM] / EK[ERM]
                * (Var[ERM] / EK[ERM] - 1):

    c[ERM] := rho[ERM] * (EK[ERM] + Var[ERM] / EK[ERM])
              / (EK[ERM] + Var[ERM] / EK[ERM] - 1) - EK[ERM] - 1:
```

```
    rho[ERM] := (floor(c[ERM]) + EK[ERM] + 1) * (EK[ERM] + Var[ERM]
                / EK[ERM] - 1) / (EK[ERM] + Var[ERM] / EK[ERM]):

    for r from 0 to 30 do
      EK[regional,r] := rho[ERM] * rho[ERM] ^(floor(c[ERM]) + r)
                        /(floor(c[ERM]) + r)!/sum(rho[ERM]^k/k!,
                        k = 0 ..floor(c[ERM])+r);
    end:

    for r from 0 to 30 do
      P[blocking,r] := EK[regional,r]/sum('p[1,i]*rho[i]',
                       'i' = 1..n);
end:

    E:={}:
    Pr:={}:
    for r from 0 to 30 do
      E := E union {[r,EK[regional,r]]}:
      Pr := Pr union {[r,P[blocking,r]]}:
    end:
    plot(E,style=point,labels=['r','expectation'],color=black):
    plot(Pr,style=point,labels=['r','Blockingprobability'],color=black):
```

Appendix E

Graphs of the expectation and variance of the overflow

The Graphs in this section represent the expectation or the variance of the number of patients in the overflow of several hospitals (without regional beds) as function of the possible number of over beds.

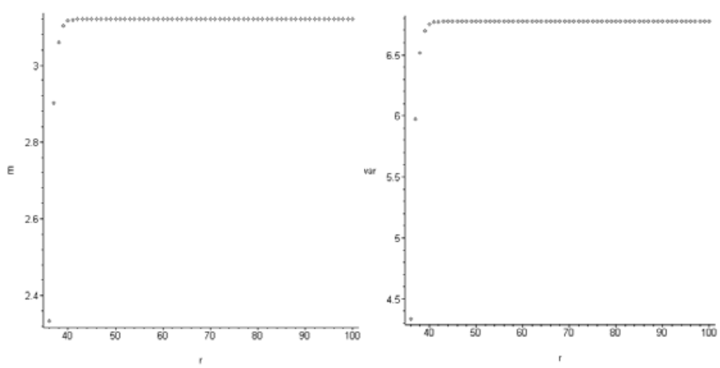


Figure E.1: Expectation and Variance of the number of patients in the overflow of the Erasmus MC as a function of the possible number of over beds

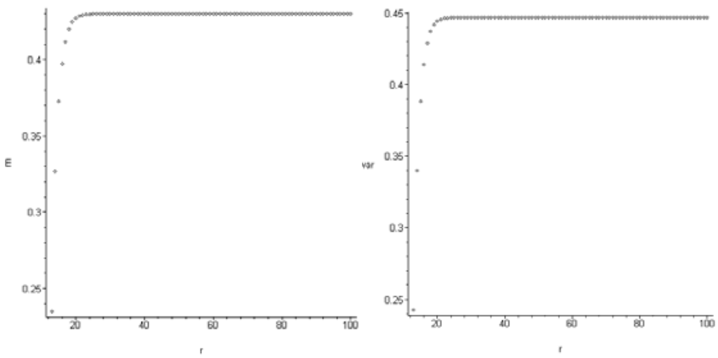


Figure E.2: Expectation and Variance of the number of patients in the overflow of the Albert Schweizer Hospital as a function of the possible number of over beds

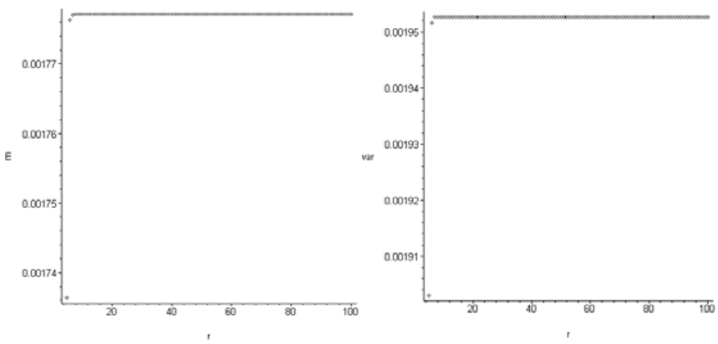


Figure E.3: Expectation and Variance of the number of patients in the overflow of the Dirksland Hospital as a function of the possible number of over beds

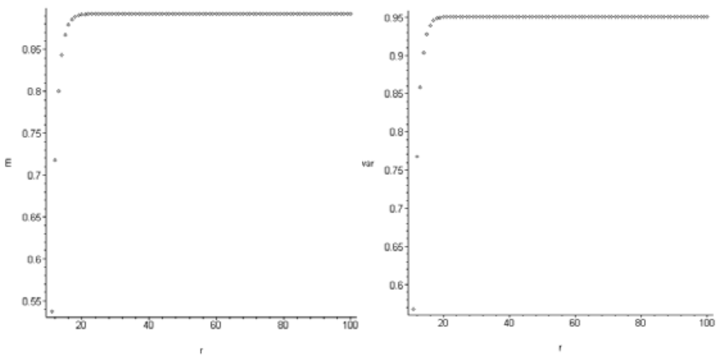


Figure E.4: Expectation and Variance of the number of patients in the overflow of the Sint Franciscus Gasthuis as a function of the possible number of over beds

Appendix F

Basic elements of eM-Plant

EM-Plant is software for object oriented graphical modelling for simulating and visualising systems and business processes. In this Chapter we shall give a short description of the basic eM-Plant elements used in the simulation model discussed in Chapter 4. Table F.1 gives an overview of these elements. For more information see the eM-Plant tutorial [32].

Each new model in eM-Plant starts with a *Frame*. A Frame serves for grouping objects and building hierarchically structured features. For example, the Region and the ICU in the simulation model are both built in a frame. The Region is in this case in top of the hierarchy and the ICU is one frame below the region.

An *entity* is a moving object. In the simulation model the patients are represented by entities. Each entity (patient) can be given special *attributes* (characteristics), e.g. the creation time of the entity (the patients time of arrival).

A *Single Processing Unit* (SingleProc) receives and processes moving objects, one at a time. In the simulation model the SingleProc is used to model a bed, that 'processes' patients. A *Parallel Processing Unit* (ParallelProc) has more than one station to process moving objects. A *Drain* is a single processing station that destroys moving objects after processing them. In the simulation model, a Drain is used to delete patients from the system.

A *Method* can be used to program user-defined models, extending the functionality of the basic material flow objects. In the simulation model, for example, the program code in a Method regulates what to do with arriving patients. A *Generator* is used to activate Methods at specified times. In the simulation generators are, for example, used to create patients with exponential intervals.

The object *Variable* is a global variable that, during a simulation run, can be accessed by other Objects and Methods during. In the simulation model Variables are, for example, used for the number of ICUs and the number of beds. *CardFiles* and *TableFiles* are lists in which, during a simulation run, the individual cells can be accessed by other Objects and Methods. A CardFile has one column and a TableFile has several columns. In the simulation model CardFiles and TableFiles are, for example, used to access the bed objects.

The *EventController* coordinates and synchronizes the different events taken place during a simulation run. It can be used to start, stop and reset the simulation.

Element	**Description**
CardFile	The CardFile is a list with one column providing random access to the contents of the individual cells using their row number.
Drain	The drain consists of a single processing station and destroys the Moving Unit after processing it.
Entity	The Entity is a moving material flow object without loading capacity.
EventController	The EventController coordinates and synchronizes the different events taking place during a simulation run.
Generator	The Generator activates Method objects at specified times.
Method	The Method can be used to program user-defined models, extending the functionality of the basic material flow objects.
ParellelProc	The ParellelProc has several stations for processing Moving Units in parallel at the same time.
SingleProc	The SingleProc has a single station for processing a Moving Unit.
TableFile	The TableFile is a list with several columns, from which the individual cells can be accessed by using their index.
Variable	The variable is a global variable that other objects and methods can access during a simulation run.

Table F.1: Basic elements of eM-Plant

Appendix G

Methods of simulation model

G.1 Method Create Region

This method creates the number of ICUs as chosen, and the number of regionalbeds as chosen.

```
is
    i, x, y : integer;
    IC, rb : object;
    ICstring : string;
do

    The existing ICUs are deleted.
    y := TableofICs.yDim;
    For i := 1 to y loop
        IC := TableofICs[1,i];
        ICstring := (to_str(TableofICs[1,i]));
        if existsObject(ICstring) then
            IC.deleteobject;
        end;
    next;

    The existing ward with regional beds is deleted.
    if existsObject("Regionalbeds") then
        Regionalbeds.deleteobject;
    end;

    The existing table containing the ICUs is deleted.
    if existsObject("TableofICs") then
        TableofICs.deleteobject;
    end;

    The existing list containing the regional beds is deleted.
```

```
if existsObject("ListofRegionalbeds") then
    ListofRegionalbeds.deleteObject;
end;
```

A new table for the ICUs and a new list for the regional beds are created.

```
InformationFlow.TableofICs.createobject(current, 200, 35);
InformationFlow.ListofRegionalbeds.createObject(current,260,35);
```

The ICUs are created.

```
i := 0; – The loopnumber
x := 30; – The x-coordinate
y := 130; – The y-coordinate

while i <= NumberOfICs loop

    IC := .Models.IC.createObject(current,x,y);

    TableofICs.writeRow(1,i + 1,IC);
    TableofICs.writeRow(2,i + 1,36);
    TableofICs.writeRow(3,i + 1,2.4);
    TableofICs.writeRow(4,i + 1,53568);
    TableofICs.writeRow(5,i + 1,39744);

    x := 30 + (i - 10 * floor(i / 10)) * 80;
    y := 130 + floor(i / 10) * 80;
    i := i + 1;

end;
```

The 'unnumbered' ICU is deleted.

```
Models.Region.IC.deleteobject;
TableofICs.cutrow(1);
```

A ward for the regional beds is created:

```
Objects.Regionalbeds.createobject(current,80,220);

i := 0; – Loopnumber
x := 30; – x-coordinate
y := 50; – y-coordinate
while i <= NumberofRegionalbeds loop

    rb := .Objects.regionalbed.createObject(Regionalbeds,x,y);

    ListofRegionalbeds.insert(i + 1, rb);

    x := 30 + (i - 10 * floor(i / 10)) * 80;
    y := 50 + floor(i / 10) * 80;
    i := i + 1;

end;
```

```
    The 'unnumbered' bed is deleted:
    Regionalbeds.regionalbed.deleteObject;
    ListofRegionalbeds.cutRow(1);

end;
```

G.2 Method createICs

This method creates the number of beds and the arrival rates as chosen per ICU in the table containing the ICUs.

```
is
    y, i : integer;
    IC : object;
do

    i := 1; – The loopnumber
    y := TableofICs.yDim; – The number of units

    while i <= y loop

        The beds are created:
        IC := TableofICs[1,i];
        IC.Numberbeds := TableofICs[2,i];
        IC.CreateBeds;

        The arrival rates are set:
        IC.Entrance.ElAR := TableofICs[3,i];
        IC.Entrance.IEAR := TableofICs[4,i];
        IC.Entrance.EEAR := TableofICs[5,i];
        IC.SetArrivalRates;

        The parameter is updated:
        i := i + 1;

    end;

end;
```

G.3 Method CreateBeds

This method creates the unit with the number of beds as chosen with the variables numberbeds and numberofRegionalbeds.

```
is
    x, y, i : integer;
    a : object;
    do
```

The existing unit is deleted:

```
if existsObject("Unit") then
    Unit.deleteObject;
end;
```

The existing list of beds is deleted:

```
if existsObject("ListofBeds") then
    ListofBeds.deleteObject;
end;
```

The existing list of over beds is deleted:

```
if existsObject("ListofOverbeds") then
    ListofOverbeds.deleteObject;
end;
```

New lists are created:

```
InformationFlow.ListofBeds.createObject(current,120,50);
InformationFlow.ListofOverbeds.createObject(current,200,50);
```

A new unit is created:

```
Objects.Unit.createObject(current,150,140);

i := 0;
x := 30;
y := 50;

while i <= NumberBeds loop

    a := .Objects.bed.createObject(Unit,x,y);

    ListofBeds.append(a);

    x := 30 + (i - 9 * floor(i / 9)) * 80;
    y := 50 + floor(i / 9) * 80;
    i := i + 1;

end;
```

The 'unnumbered' bed is deleted:

```
    Unit.bed.deleteObject;
    ListofBeds.cutRow(1);

end;
```

G.4 Method BeginDay

This method creates the elective patients on weekdays at 9 a.m.

```
is
    ne, r, i, j, nco, nae, n : integer;
    patient, obj, COKD, dcic : object;
do

The method that changes the day to the next day is activated.
    newday;

if it is a weekday an certain number of elective patients is created. If a bed
is available they are give a bed, otherwise the patient is deleted.
    if dayname = "monday" or
    dayname = "tuesday" or
    dayname = "wednesday" or
    dayname = "thirsday" or
    dayname = "friday" then
        n := location.NumberBeds;
        dcic := location.DataCollection;
        r := location.location.run;

        The number of elective patients to be created is Poisson distributed.
        ne := z_poisson(r,ElAR);

        for i := 1 to ne loop

            patient := .MUs.ElectivePatient.create(ElectivesWaiting);

            if dcic.NumberofOccupiedbeds < n then

                j := 1; – loopnumber
                while not ElectivesWaiting.empty and j <= n loop

                    The beds are checked for their availability
                    obj := location.ListofBeds.read(j).bed;
                    if not obj.full then

                        If the bed is available, the processing time is set
                        obj.procTime.setParam("lognorm", 3*(r-1) +
                            patient.stream, patient.meanLOS, patient.sdLOS);

                        The number of admitted patients is increased
                        dcic.Numberadmittedpatients :=
                            dcic.Numberadmittedpatients + 1;

                        The number of occupied beds is increased by one.
                        dcic.NumberofOccupiedBeds :=
                            dcic.NumberofOccupiedbeds + 1;
```

```
                        Data for this patient type is collected through the
                        method DataCollection.
                        patient.DataCollection.execute;

                        The entering time of the patient is set.
                        patient.EnteringTime := EventController.SimTime;

                        The patient is placed on the bed
                        patient.move(obj);
                    end;

                    j := j + 1;
                end;
            end;

            if not ElectivesWaiting.empty then

                If nog bed is available the patient is deleted.
                patient.delete;

                The number of cancelled operations is increased by one.
                dcic.NumbercancelledOperations :=
                    dcic.NumbercancelledOperations + 1;

                The proportion of cancelled operations is calculated
                nco := dcic.NumbercancelledOperations;
                nae := dcic.NumberAdmittedElectives;
                if nco + nae > 0 then
                    dcic.ProbabilityOperationCanceled :=
                        nco /(nco + nae);
                end;
                COKD := dcic.CanceledOKData;
                COKD.writerow(r,nco + nae,dcic.ProbabilityOperationCanceled);

            end;
        next;
    end;
end;
```

G.5 Method MovingArrivals

This method deals with arriving emergency patients.

```
is

    n, i , r :integer;
    obj :object;
    locpat : string;
    patmoved : boolean;
do
```

```
    patmoved := false;
    n := location.NumberBeds;

    An available bed is sought
    if location.DataCollection.NumberofOccupiedbeds < n then
        i := 1;
        while not patmoved and i <= n loop

            The next operational bed in the list is found:
            obj := location.ListofBeds.read(i).bed;

            if not obj.full then
                The processing time of the available bed is set:
                r := location.location.Run;
                obj.procTime.setParam("lognorm", 3*(r-1)
                    + @.stream, @.meanLOS,@.sdLOS);

                The number of patients admitted to this ICU is increased:
                location.DataCollection.Numberadmittedpatients :=
                    location.DataCollection.Numberadmittedpatients + 1;

                The number of occupied beds is increased by one:
                location.DataCollection.NumberofOccupiedBeds :=
                    location.DataCollection.NumberofOccupiedbeds + 1;

                Data for this patient type is collected:
                @.DataCollection.execute;

                The entering time of the patient is noted:
                @.EnteringTime := EventController.SimTime;

                The patient is moved to the available bed:
                @.move(obj);
                patmoved := true;

            end;

            i := i + 1;

        end;

    end;

    If all operational beds are occupied, the method handling the overflow
    of this patient type is executed.
    if not patmoved then
        @.Method.Execute;
    end;

end;
```

G.6 Method CreateOverbed

This Method deals with the internal emergency patients when no operational bed is available.

```
is
    ob, b : object;
    naie, r, y, i : integer;
do

    An over bed is created and added to the list with over beds.
    ob := .Objects.Overbed.createObject(location.Unit,750,30);
    location.ListofOverbeds.append(ob);

    The processing time is set to equal the length of stay of the patient
    r := location.location.Run;
    ob.bed.procTime.setParam("lognorm", 3*(r-1) + @.stream,@.meanLOS,
        @.sdLOS);

    The entering time of the patient is set.
    @.EnteringTime := EventController.SimTime;

    The number of admitted internal emergency patients is increased by one.
    location.Datacollection.NumberAdmittedIntEmerg :=
        location.DataCollection.NumberAdmittedIntEmerg +1;

    It is noted in which over bed the patient is placed.
    @.overbed := ob;

    The patient is placed in the over bed.
    @.move(ob.bed);

    The number of admitted patients is increased by one.
    location.DataCollection.NumberAdmittedPatients :=
        location.DataCollection.NumberAdmittedPatients + 1;

    The number of over beds created is increased by one.
    location.DataCollection.NumberOverbedsCreated :=
        location.DataCollection.NumberOverbedsCreated + 1;

    The number of existing over beds is increased by one.
    location.DataCollection.NumberofOverbeds :=
        location.DataCollection.NumberofOverbeds + 1;

end;
```

G.7 Method TranspExtEmerg

This Method deals with the regional emergency patients when no operational bed is available at the ICU.

```
is
    n, i , arp, trp, ntp, naee, r : integer;
    obj, dcr, dcic, RD, PPTD : object;
    locpat : string;
    patmoved : boolean;
do

    patmoved := false;
    If a regional bed is available, the patient is placed in this bed.
    n := location.location.NumberofRegionalBeds;
    r := location.location.Run;
    dcr := location.location.Regionalbeds.DataCollection;
    dcic := location.DataCollection;
    locpat := to_str(@);

    i := 1; – Loopnumber
    while not patmoved and i <= n loop

        obj := location.location.ListofRegionalBeds.read(i);
        if not obj.bed.full then

            The processing time is set according to the length of stay.
            obj.bed.procTime.setParam("lognorm",3*(r-1) +
            @.stream,@.meanLOS, @.sdLOS);

            The entering time of the patient is noted.
            @.EnteringTime := EventController.Simtime;

            The number of admitted patients is increased by one.
            dcic.NumberadmittedPatients :=
                dcic.NumberadmittedPatients + 1;

            The number of admitted regional patients at the ICU is increased.
            dcic.NumberadmittedExtEmerg :=
            dcic.NumberAdmittedExtEmerg + 1;

            The number of admitted regional patients in the region is
            increased by one.
            dcr.AdmittedRegionalPatients :=
                dcr.AdmittedRegionalPatients + 1;

            The proportion of transferred regional patients is calculated for
            the region.
            trp := dcr.TransportedRegionalPatients;
            arp := dcr.AdmittedRegionalPatients;
```

```
            dcr.ProbabilityPatientTransported := trp / (trp + arp);

            The proportion of transferred regional patients is calculated for
            the ICU.
            ntp := dcic.NumbertransferedPatients;
            naee := dcic.NumberadmittedExtEmerg;
            if ntp + naee > 0 then
                dcic.ProbabilityPatienttransfered := ntp /(ntp + naee);
            end;

            The patient is placed in the over bed.
            @.move(obj.bed);
            patmoved := true;

        end;
        i:=i+1;
    end;

    If the regional beds are all occupied, an IC bed in another hospital has to be
    found (the patient is deleted from the system).
    if not patmoved then

        @.delete;

        The number of transferred patients is increased by one for the ICU.
        dcic.NumberTransportedPatients :=
            dcic.NumberTransportedPatients + 1;

        The number of transferred patients is increased by one for the region.
        dcr.TransportedRegionalPatients :=
            dcr.TransportedRegionalPatients + 1;

        The proportion of transferred regional patients is calculated
        for the region.
        arp := dcr.AdmittedRegionalPatients;
        trp := dcr.TransportedRegionalPatients;
        dcr.ProbabilityPatientTransported := trp / (trp + arp);

        The proportion of transferred regional patient is calculated for the ICU.
        ntp := dcic.NumberTransportedPatients;
        naee := dcic.NumberAdmittedExtEmerg;
        if ntp + naee > 0 then
            dcic.ProbabilityPatientTransported := ntp /(ntp + naee);
        end;
    end;

    Data concerning the proportion of transferred regional patients is written
    to a file.
    RD := location.location.RegionalData;
    RD.writerow(r,arp + trp,dcr.ProbabilityPatientTransported);
```

```
    PPTD := dcic.ProbPatientTransData;
    PPTD.writerow(r,ntp + naee,dcic.ProbabilityPatientTransported);

end;
```

Appendix H

Tables and graphs containing results

H.1 Tables with blocking probabilities

Number of reserved beds	ERM blocking probability	Proportion refused regionals in simulation
0	0.732	0.698 ± 0.016
1	0.230	0.251 ± 0.008
2	0.049	0.070 ± 0.004
3	0.007.	0.013 ± 0.002
4	0.001	0.001 ± 0.000
5	0.001	0.000 ± 0.000
6	0.000	

Table H.1: Blocking probability regional emergency patients at the Albert Schweizer Hospital, without cooperation

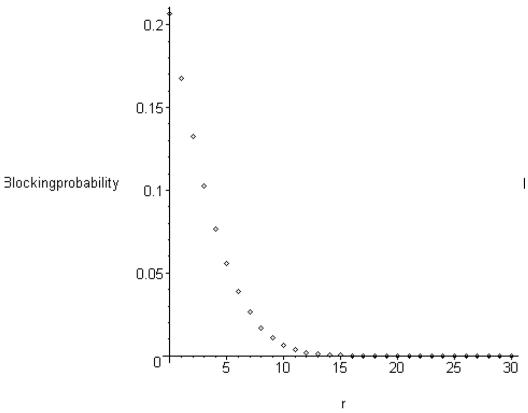
Number of reserved beds	ERM blocking probability	Proportion refused regionals in simulation
0	0.016	0.001 ± 0.001
1	0.001	0.000 ± 0.000
2	0.000	

Table H.2: Blocking probability regional emergency patients at the Dirksland Hospital, without cooperation

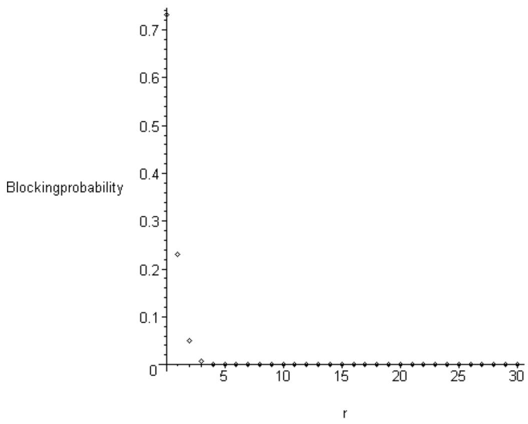
Number of reserved beds	ERM blocking probability	Proportion refused regionals in simulation
0	0.742	0.705 ± 0.016
1	0.357	0.367 ± 0.014
2	0.135	0.158 ± 0.008
3	0.039	0.058 ± 0.005
4	0.009	0.018 ± 0.004
5	0.002	0.004 ± 0.002
6	0.000	0.001 ± 0.000

Table H.3: Blocking probability regional emergency patients at the Sint Franciscus Gasthuis, without cooperation

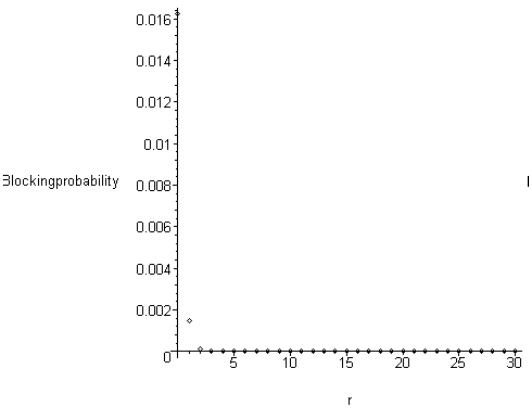
H.2 Graphs of blocking probabilies



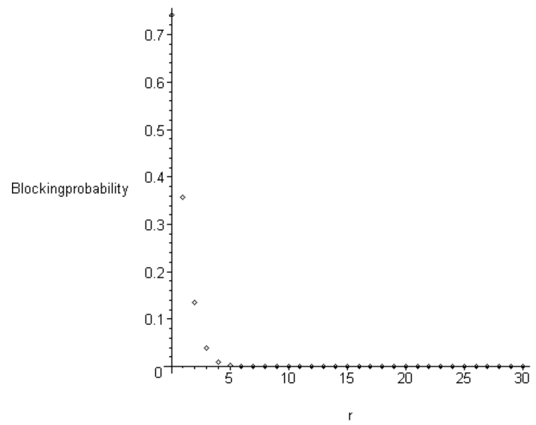
ERM blocking probability Erasmus MC



ERM blocking probability Albert Schweizer hospital



ERM blocking probability Dirksland hospital



ERM Blocking probability Sint Franciscus Gasthuis