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Abstract

With the rise of ad hoc networks as a new type of communication network, models are needed to get a view on the possibilities for this type of network. In the past years, various aspects of many different types of ad hoc networks have been investigated. Aspects like connectivity, path availability, capacity, mobility, throughput and other related aspects are important for the performance of such a network. Good results have already been obtained for including these aspects in models for ad hoc networks, but there still are many areas open for further research.

In this thesis we focus on the connectivity and the throughput of a specific type of ad hoc network. Modelling the network as a graph, we use extreme value analysis and graph theory to show a relation between the probability that a link between users can be established and the connectivity of the network. Taking the limitation that users can only transmit succesfully within a certain range into account, the level of connectivity is investigated, showing the possibilities of using multiple paths for the communication between users.

In ad hoc networks in general, users can be mobile and might enter or leave the network as they please. We assume that the nodes do not move around the network, but consider the property of nodes that users are entering and leaving the network. This influences the number of users that can be transmitting at the same time, which has an effect on the level of interference.

If all users would transmit data simultaneously, interference would cause transmissions to fail. Therefore a scheme is used for blocking users if their transmission would cause a collision with another transmission. Users will not start transmitting until the channel becomes free. We derive formulas for the probability that a node is transmitting and that such a transmission fails. This leads us to the expected number of transmissions simultaneously in progress.

Using multiple hops to reach a destination has an influence on the performance of the network, which is also taken into account. After discussing some possible setups for the network and their influence on the performance of the network, we present an algorithm for finding the optimal number of hops and paths that should be used for a transmission between users.

To show the influence of using multiple hops, we first determine the time a single hop is expected to take. Then, using this result, we derive the expected time it takes to transfer data over a complete path and from this we derive the throughput. Considering different possibilities for forwarding data to the next user, different results are obtained.

By comparing the throughput over a path with the throughput that could be obtained if only single hops are needed, the differences between common networks and ad hoc networks can be found. This way it is possible to discuss the advantages and limitations for the use of ad hoc networks as communication networks.

Chapter 1

Introduction

Ad hoc networks are networks that consist of mobile nodes that are connected by multihop communication routes. Unlike conventional wireless networks, there is no infrastructure or administrative support. There are no wired base stations that cover areas, so communication can only occur by forwarding information from user to user. Because of the mobility of the users, the topology of the network is subjected to changes continuously. Nodes may even (dis)appear when users choose to (dis)connect. These changes make it difficult to provide a good service for the users. Problems that are encountered are questions about connectivity, path availability, power usage, capacity, throughput and other related aspects.

There are many different kinds of ad hoc networks, each having different QoS constraints [Sant02]. One can think of a safety system to secure a building where there has to be a continuous connection between all the devices, otherwise intruders might succeed to get in without being noticed. But when the network is used for frequent communication between members of a demolition team destroying a building, temporary disconnections do not seem to be that important. Instead, costs of maintaining the network can be diminished. If the network is used to gather information about the current humidity in some remote parts of a rainforest, only once in a while a connection is needed. But in this case it might be wise to reduce the power usage, because changing the batteries might prove to be quite some work.

All the different needs an ad hoc network might have, lead to many difficult challenges to overcome [Chak01]. Effective routing, mobility management, power management, security and quality of service issues such as delay and bandwidth management are examples of subjects that still need further investigation before the wide-spread use of ad hoc networks. A combination of all these aspects inside a complete model to estimate the performance of an ad hoc network would probably be too complex, because too many factors will be of influence. Some factors might even influence each other. Therefore choices have to made what aspects are most important and what assumptions can be made regarding the factors of influence, leading to a valid model for the network. Also the impact that these factors will have on the network has to be considered. Some factors have a bigger influence on the network than others, depending on the aspect under consideration.

In this thesis the aspects that will be focused on are the connectivity of an ad hoc network, the throughput that can be achieved by the network and the expected time it will take for a flow of data to reach its destination. Since there are many different kinds of ad hoc networks and it is impossible to consider all of them, we choose a specific network to analyze. Equations for the probability of successful transmissions will be derived, together with the expected connectivity of the network. Using the results obtained for the network properties, a performance analysis of the network is made, combining both graph theoretical and stochastic aspects into one complete model. This way, a view is given on the advantages and limitations for possible future use of ad hoc networks as communication networks.

Chapter 2

Background

Many different aspects of ad hoc networks have already been investigated in the past, giving more and more insight on the advantages and disadvantages of ad hoc networks. Because of the wide diversity of ad hoc networks, many different models have been constructed to find approximations for the performance analysis of the network. In the next sections, an overview is presented of research already performed on ad hoc networks, so that the reader can get a better understanding of what an ad hoc network is, and what aspects are of importance for the future use of this kind of networks.

2.1 Ad hoc networks

When looking in a dictionary, you find that ad hoc is Latin and is translated as "to this" or "for this". The dictionary also shows that ad hoc can be translated as "improvised" or "impromptu". The expression has been applied to future networks in which new devices can be quickly added. A connection will only be made between users of an ad hoc network when it is known that they want to transmit to each other, so there is no set infrastructure. In an instant the connection is set up, but afterwards it will be gone again, explaining the choice of calling this type of network an ad hoc network.

From the perspective of this thesis, an ad hoc network is a network consisting of users only. No infrastructure is present and users can either be active or not. These users are depicted as nodes in a graph (then called an ad hoc graph or random graph). Each user has a range in which it can broadcast, meaning that all other nodes within this range can be contacted. All the users that can contact each other are connected by edges in the graph, showing the possible infrastructure. When a user wishes to contact another user that is further away, the connection will exist of multiple links. That is why the ad hoc network is also referred to as a multi-hop network.

One of the special properties that this kind of network can have, is that the nodes can move and this way edges will break and new edges will appear in time. It is also possible that vertices (dis)appear, for instance when a user turns (off) on his or her mobile phone. It therefore is rather difficult to find paths from one user to another at a considerable distance. Even if a path is found, it will probably cease to exist after some time, leading to more difficulties.

When it is clear if and how many connections can be found and how reliable they are, the next step is to investigate in what way the found connections should be used. Should one use a single connection to send all the data, running the risk it might break or should one use multiple connections, using a lot of capacity? There appears to be a trade-off between reliability and capacity. Unsolved issues with respect to quality of service and efficiency are therefore an interesting subject for future research.

Which connections exist and how they should be used depend on other aspects themselves. The more powerful the devices are that users of the network use for transmission, the bigger the range of the user will be. When more users can be reached, the connectivity increases. But there is also a down side. Users will have an influence on each other as well. They need each other to reach users further away, but they will also interfere with each other. This trade-off needs to be considered as well.

All these aspects will influence the capacity and throughput of an ad hoc network, and therefore all of these aspects have been investigated. Combination of different aspects into a model has lead to a wide variety of models to estimate the performance of ad hoc networks. Still there are many aspects that are not yet considered, showing that the field of ad hoc networks still contains many interesting possibilities for future research.

2.2 Quality of Service

When focusing on the quality of service issues as done by Chakrabarti and Mishra in [Chak01], many questions come to the surface. They state that much remains to be done, but that the attention for QoS has grown and major advances are expected in the next few years. With QoS only being of importance for a flow of packets between a source and the destination, the connection between them is of critical importance. When routing the sent packets, the routers must be aware if the resources are available for the duration of the connection. Choosing the right path to reach the required QoS, also called QoS routing, is the most important step.

Having chosen a specific path, a network is called combinatorially stable if and only if changes that destroy the connection occur sufficiently slowly to make sure that the transmission will succeed. Next to connectivity, also the availability of enough resources is a must. If a network can supply the requested QoS, regardless of topological changes, the network is called QoS-robust. A network is called QoS-preserving if it only maintains the QoS guarantees during the period a transmission ended and a new one is started. With new transmissions busy, the QoS of the network changes, influencing possible other connections.

QoS routing involves both finding a path and reserving the resources for transmission. Because of the topological changes, this process should not take too much time. The computational complexity must therefore be taken into account. Once a path has been selected and the resources are reserved, the new situation has to be reported to the other nodes. The number of network updates should nevertheless be minimized, because they use bandwidth and router capacity. Frequent changes in paths may also increase the delay jitter. If there are many QoS constraints, the problem can be proved to be NP-hard. Therefore the use of suboptimal solutions cannot be avoided.

The network state is divided into the local and global state. The local state information includes queuing delay and residual capacity for the node, together with the delay, bandwidth and cost metric of its outgoing links. The local states of all nodes together is the global state. The topology updates as mentioned before involve the global state of the network. When the nodes in the ad hoc network are highly mobile, the global state information may never be accurate.

There are different ways to determine a path from a source to a destination. When using source routing, the path is found by using the global state information available at the source. All nodes on this path are notified which nodes are their preceding and successor nodes. For distributed or hop-by-hop routing all nodes are involved in finding the path. In this case, the nodes tell the source to what node it should forward its packets. Hierarchical routing makes use of a clustered view of the complete network, using source routing per cluster. The state information is then called the aggregated global state information. Considering the QoS, flooding is not an option, unless used for control packets under certain circumstances.

All the information sent within a network does not have to have the same priority. For optimal use, the control packets should have the priority over the data packets. Even within the data packets, a hierarchy can be present. QoS routing taking in account preemption is still an open area for research. Also the development of routing policies, algorithms and protocols for handling user data with priorities are open areas.

Because of the possible inaccuracy of the state information, routing can be done by only using local state information. Nodes send probes to their neighbours with information needed to find a feasible path with the required QoS characteristics. Some adjustments to this technique are proposed to decrease the penalties for flooding the network with these probes. Routing can also be done using the imprecise global state information. Each probe then carries tickets to control the number of different paths that should be searched. Whenever a path is found, frequent checks are performed to see if it is still available and the QoS is still high enough. Refreshers are sent from the destination to the source and if they do not arrive in time, the path is declared unavailable. If a path breaks, either rerouting or repairing the path can be used to maintain the connection. Using redundant paths can increase the probability that the QoS requirements are constantly met. At the highest level, multiple paths with the same QoS are used simultaneously. At a lower level, the different paths are ranked and only used if the better paths become unavailable. For the lowest level of redundancy the resources of the alternate paths are not reserved until the optimal path breaks.

Updating the topology information should be done within a limited timespan. If changes can occur during the update, the update becomes useless. Different QoS constraints need different approaches to maintain them. Redundant routing seems to solve many problems, but this might use a lot of bandwidth that can be very valuable. More research to investigate all the benefits and disadvantages of this approach has to be done.

It is clear that there are many problems when trying to guarantee a certain level of quality of service. The inaccurate knowledge of the network state may prove to be a bottleneck that cannot be overcome if the mobility of the nodes is too high. Also the size of the network can become a problem if the global state information has to be available at every single node. State updates will then take too much time, so that new changes already have occurred before every node receives the state information.

There still are many open questions on the field of QoS. Algorithms, policies, protocols, resource reservation and routing are receiving more attention recently. Robustness of a network, priorities in traffic and cost-effective implementations on the other hand are still mostly uninvestigated.

2.3 Connectivity

Before taking a closer look if all the quality of service requirements can be met using a certain path, it has to be clear if a path from the source to the destination exists at all. In a broader point of view it might even be interesting to know if every node is connected to any other node in the network. A network is called connected if a path can be found between any two nodes of the network.

In [Sant02] the question is asked what the transmitting range r of a node should be to ensure connectivity in a network of dimension d containing n nodes. In other words, what are the conditions that must hold to ensure that a network is connected initially and remains connected as links break and nodes move? Representing the network as a graph and modeling the movement of the nodes gives a better insight in the relationships between the number of nodes, their transmitting range and the connectivity of the graph.

Consider a representation of a d-dimensional network by the pair $M_d = (N, P)$, where N is the set of nodes with $|N| = n$ and $P : N \times T \to [0, l]^d$, for some $l > 0$ as the placement function. Assume that all nodes in the network have the same transmission range r. The communication graph at a certain time t can now be drawn, viewing the positions of all the nodes and vertices between the nodes, which are present if the distance between them is not larger than r. If the number of nodes in the network is known, it may prove very useful to know the minimal required transmission range to ensure connectivity. Knowing this range can reduce the energy consumption of a network, because transmission over long distances use a lot of energy. Also the interference caused by nodes can be diminished, increasing the capacity of the network.

If the position of the nodes is not known, looking at the worst case scenario (with all the nodes situation in the corners of the network) shows that the minimal range ensuring connectivity is equal to $r = l\sqrt{d}$. It is nevertheless very unlikely that this situation will occur. For the case that $d = 1$ and the nodes are stationary, an accurate analysis can be done to find the conditions leading to disconnected graphs. For this network, connectivity is certain if the product of the number of nodes and the transmitting range is of the order of $l \log l$. This shows a substantial reduction in transmitting range from the worst-case scenario (needing a range of the order of l), but also a significant increase compared to the best-case (needing a range equal to l/n). But the assumption that the nodes are stationary does not seem applicable to ad hoc networks. The relationship between the value of the range ensuring connectivity in static graphs and the value of the range ensuring connectivity during some fraction of time in mobile graphs can be investigated.

Santi and Blough [Sant02] use simulation in a 2-dimensional network to determine the percentage of connected graphs generated, the average size of the largest connected component and the minimum size of the largest connected component, using two mobility models. The classical random waypoint model is used to model intentional movement and the drunkard model is used for random movement. The results of these simulations show that a small increase of $r_{stationary}$, the range needed in the static model, ensures connectivity for the complete time in the mobile model. Difference in mobility does not seem to have much of an impact, it is the possibility of movement that matters for connectivity. Tolerating temporary disconnections reduces the needed range significantly, so energy consumption can be brought down considerably. The average size of the largest connected component compared to n seems to grow with the network size l. Even if short disconnections are allowed, the average remains high. This indicates that only a few nodes are disconnected if the connectivity ceases to exist. These results show that one might consider using many low cost nodes and demanding a lower percentage of connectivity in stead of using less nodes with full connectivity. Comparison of the results for the different mobility models also gives some interesting new discoveries. It appears that when half or more of the nodes are not moving, the model can just as well be regarded as being totally stationary. This implies that a certain number (a large fraction) of stationary nodes will significantly increase the connectivity.

2.4 Path availability

With the connectivity of the graph in general discussed, a closer look at a specific path can now be made. If a path is found from a source to the destination (which can be done if the graph is connected), there is no certainty that this path will keep existing while transmitting the data. The mobility of the nodes causes links to break and new links will be created. Efficiency of routing is very important because of the scarcity and variability of network resources in ad hoc networks. The frequency of path failures should therefore be kept as low as possible. This can be done if the probability of the availability of a path is known. To know more about the availability of a path, mobility models to predict the future state of the network can be used. The probability that a link between nodes exists at time $t_0 + t$, given that the link existed at time t_0, is investigated in [Mcdo99] and is called link availability.

As a mobility model, random ad hoc mobility is used. The movement of each node consists of a sequence of random length intervals, mobility epochs, during which a node moves in a constant direction at a constant speed. Giving each node a mobility profile according to this model, the distribution of their movement in isolation can be derived. Combining the mobility of two nodes, the link availability between these nodes can be found. The availability of the total path is the product of the link availability when assuming that links fail independently.

When the distribution of the movement of a node is known, the relative movement between nodes can easily be found. Keeping one node at a fixed position and moving the other node in the opposite direction than the fixed node would have moved will give the same result for the distance between the nodes when both are moving separately. Since the exact location of the nodes is unimportant, this approach is valid. Because the failure of a link depends on the distance between nodes (when the assumption is made that nodes do not (dis)appear), the probability of link failure can be studied. Two situations, either with the receiving node within range or exactly on the edge, are discussed by McDonald and Znati. Combining the results, the link availability is found and used to find expressions for the path availability.

Approximating the probability that a found path from the source to the destination will fail, a ranking can be made of all the possible paths. A routing mechanism based on this ranking will improve the efficiency of the network. Choosing the most reliable link first will reduce the probability of link failures, making transmission between nodes more reliable and easy.

2.5 Multipath routing

Instead of only using the most reliable path, it is also possible to use multiple paths at the same time to increase the probability of a successful transmission. The information can be split into fragments and sent over different paths, this way preventing delay. The approach of multipath routing seems very suitable for ad hoc networks, since it diminishes the impact of unreliable links and the changing topology. A possible approach to use multipath routing is diversity coding [Tsir01]. Data load is distributed over multiple paths in order to minimize the packet drop rate, achieve load balancing and improve end-to-end delay. The assumption is made that the transmission of packets is done much faster than that variations occurs in the network topology.

Assume there are $n_{\max}$ node disjoint paths available, each path having a failure probability p_i. Each path is assumed to be an erasure channel; either all information or no information reaches the destination. All paths are assumed independent, since they have no nodes in common. When a packet has to be sent, overhead is added and the total is divided into blocks which are sent over the different paths. The original packet can be divided into N blocks and the overhead into M. This way, M-for-N diversity coding is very suitable, meaning that for a successful transmission, only N of the N+M blocks have to arrive. The probability of a successful transmission can be written as:

$$P_{succ}(n, z) = P\left\{\sum_{i=1}^{n} z_i \geq N\right\}$$

with n the number of used channels and z_i the number of blocks that actually arrive through path i. This probability has to be maximized to achieve the best results.

The choices that can be made when sending packets are the number of paths used and the distribution of the blocks over these paths. The calculation of all the possibilities to find the optimal one will take exponential time. With the frequent changes of the network, many new calculations have to be made. After every change in the network, a new optimal solution might exist. It is clear that approximations have to be used to make it possible to compute the best distribution and its probability of success. Tsirigosis and Haas propose using the normal distribution for approximating the binomial distribution and the sum of the n independent normally distributed random variables that are encountered. The newly calculated formula can be computed within acceptable time, so the proposed approach for routing can still be applied.

The results achieved using this approach show that when all paths have the same probability of success, there is a threshold value beyond which P_{succ} is increasing with the number of paths used. Under some constraints on the path failure probabilities, the probability of successful communication approaches 100% when the number of used paths tend to infinity.

2.6 Admission and power control

The number of nodes in the network is of great influence on many aspects. If there are many users, there will be much interference and the QoS may become low. But since communication is done by multi-hop routing, the connectivity improves, improving the QoS [Chia01]. This imposes the question whether a new user should be accepted into the network or if he should be blocked.

When a new user wants to connect, a choice has to be made considering whether the changes in the network are more important than the increase of interference. If no changes in the connectivity occur, only the interference has to be taken into account to decide whether the user should be admitted. If the interference will still be low enough, the user should be allowed, leaving the possibility of future improvement of the connectivity when more users want to enter the network. The trade-off between connectivity and interference makes it hard to decide what the best choice is. Provided that users are only admitted into the network if the interference doesn't grow to a value that is too high, users who enter the network later get better service, since the network is bigger. Therefore it might be considered to charge users that enter the network at a later stage more than users that enter at an earlier stage. This nevertheless is a different aspect that will not be discussed any further.

The interference appears to be an important factor to decide if users are accepted into the network. But even when a user is accepted, the interference should be controlled by the power usage. Again there is a trade-off: conserving power to keep interference at a low level or increasing power to maintain links. Because there is a deterministic mapping between the transmitting power of a user and the probability of maintaining a single-hop link, a possible choice for this transmitting power (when choosing it equal for all users) can be to set the value in such a way that all users have a specific (again equal for all nodes) number of direct neighbours. A different approach is to set a value for the maximum number of hops needed for the longest transmission, also called the diameter of the graph. A small diameter will increase the reliability of the connections and reduces the delay of transmissions. For dense networks, a lower transmitting power will be needed, since there are more potential links that can help reduce the diameter. In scarce networks, a high transmission power cannot be avoided.

Both admission and power control can be used to improve the QoS of the network. Because of the trade-off between maintaining links and interference, caused by more users or higher power, decisions are not easy to make. For the different uses of ad hoc networks, different standards should be set to keep the QoS level high enough.

2.7 Interference

It is clear that interference influences the capacity of the network, but a detailed view on this has not yet been presented. How big is for example the influence of neighbour traffic on the behavior of a node in the network? What is the impact of interference on the QoS aspects? In [Bert01], it is investigated if the impact of interference can be modeled, what the complexity of it will be and if protocols can be constructed that take this impact into account. They focus on bandwidth reservation, used to prevent congestion in the network. The bandwidth problem either consists of maximizing the number of transmitting requests or optimizing the bandwidth.

When nodes are situated far away from each other, they do not interfere. The possibility of having two simultaneous transmissions without interference because of the distance is called spatial re-use. Assume that a node is not allowed to transmit if a user within a range, say D, is already transmitting. What distance is minimally needed to ensure that no communication is jammed under this condition? To ensure that communication is not jammed, the ratio signal over noise has to be higher than a certain threshold value. Taking this into account, the following bandwidth reservation scheme can be used:

- For each node i, its interference area s_i is defined as being the set of nodes at distance at most D from this node.
- Each node i is allowed to reserve a bandwidth $c(i)$ if for all nodes j in the network, the sum of the reservations of bandwidth of the nodes of s_i does not exceed the available bandwidth of s_j: $b(s_j)$. In formula this gives:

$$\forall j \sum_{k \in s_j} c(k) \leq b(s_j)$$

If a path (longer than one hop) is established between two nodes i and j in the network, then the request $c(i)$ of node i has to be accepted on the whole path between i and j, i.e. accepted on all the nodes of the path.

The problem to find the minimal D can be translated into a multidimensional knapsack problem, which is proven to be NP-complete. The time to obtain approximate solutions, even using one of the many proposed heuristics, takes too long to be used in a protocol for the ad hoc network. It is suggested that the greedy algorithm might give the best performance. Whenever the network is sparse, good solutions are found, but in dense networks the error might increase considerably. It remains to be seen if the proposed bandwidth reservation scheme can be implemented for future ad hoc networks.

If there are variations in the network size, the network density and the traffic per node, this may have a strong influence on the interference [Hekm02]. The radio channel capacity decreases as the wanted signal carrier power to interference ratio (C/I) decreases. C/I is the ratio between the mean power of wanted signal and the mean power of the sum of interfering signals. For a performance analysis of the ad hoc network, it therefore is necessary to have a good estimate of the interference levels.

Since users move around freely in the network, accurate information regarding movement patterns and node locations is needed to obtain a good estimate of the value of C/I. A regular lattice for possible locations of mobile nodes is one of the possibilities to model this aspect. The coverage area of a node is modeled as a circle with radius r around the node. The power of the signal received by nodes within this area is higher than a threshold value γ, the receiver sensitivity. Communication between the node and all nodes within this area is direct, using a single hop. Whenever a node within the range of a node is transmitting, this node will not start transmitting itself. The distribution of the nodes over the network is assumed to be uniform. The whole area covered by all the nodes is called the service area. Normally every position within this service area is equally probable to be occupied, but using the regular lattice, the position of the nodes is restricted. This simplifies the model, making it possible to estimate the expected C/I without having accurate information about the movement patterns and exact locations of the nodes at any moment. Because of the chosen uniform distribution, all nodes that are not at the borders of the service area should have the same number of adjacent nodes, all being at the same distance from each other. As a grid, the honey-grid model is chosen because this gives the best view for the worst-case scenario. This grid divides all the nodes of the network in hexagonal rings around the source, so that each node has the maximum possible number of neighbours when using a uniform distribution on a grid. This way the amount of interference will not be underestimated.

Suppose the coverage area of a node contains two rings. Then transmission to a node on ring three can be done either by transmitting to ring one and then to ring two, finally reaching three, or directly to ring two and then to three. The rings used for hopping are called the relay rings. For the proposed model, the exact path is not relevant. Only the amount of relay traffic caused by the number of hops will have an influence on the interference. Using many hops will preserve energy, whereas fewer hops increase the reliability. An estimate of the average number of hops is calculated by:

$$E[h] \simeq 0.53N_r^{0.5} + 2(1 - \frac{N_r}{N})$$

where N is the number of nodes, N_r is the number of nodes on the relay rings and $(1 - \frac{N_r}{N})$ is the probability that either the source or destination node is not a relay node. This mean hopcount determines the expected traffic load in the entire network.

The amount of interference is directly related to the traffic produced per node. Because of the relay traffic, the total amount of traffic is strongly related to the multi-hop characteristics of the ad hoc network. New traffic that is generated is assumed to be Poisson distributed and independent from each other. The mean value of new traffic per node is denoted by λ. The total traffic per node, Λ, is then:

$$\begin{aligned} \Lambda &= \lambda + \lambda(E[h] - 1) \\ &= \lambda E[h] \end{aligned}$$

For the interference I the following expression can be found:

$$I = 6qc((a+1)\Delta)^{-\eta} \sum_{i=1}^{s} i^{-(\eta-1)}$$

where q is the probability of transmission of own signals or relay signals per node, c is a constant, a is the range of the node, Δ is the size of the side of the first hexagonal ring, η is the path loss exponent and $s = \lfloor y/(a+1) \rfloor$ is the number of interference rings, with y the total number of rings.

The expected wanted signal power, C, can be calculated from:

$$E[C] = \frac{2c\Delta^{-\eta}}{a(a+1)} \sum_{i=1}^{a} i^{-(\eta-1)}$$

Combining both formulas, including the processing gain g and substituting $q = 1 - e^{-\lambda E[h]}$ leads to:

$$E[C/I] = \frac{g \sum_{i=1}^{a} i^{-(\eta-1)}}{3a(a+1)^{-(\eta-1)}(1 - e^{-\lambda E[h]}) \sum_{i=1}^{s} i^{-(\eta-1)}}$$

It appears that the expected value of the carrier to interference ratio depends on the path-loss exponent (η) and the probability of transmission (q), which itself depends on the mean value of generated traffic per node.

Evaluating this function gives a good insight on what factors influence C/I. Looking at the network size and density, it seems that for large networks C/I tends to an asymptotic value that depends only on the path loss exponent η and the value of a, which relates to the network density. Routing overhead does not seem to have a significant influence on $E[C/I]$.

The assumption that the nodes are situated according to a specific lattice is not accurate though. However, the model is not affected by mobility of nodes, as long as the assumption of a uniform distribution is still made. Deviation from the uniform node distribution could affect the hopcount, so this influence still has to be investigated. When trying to improve the model by including the mobility, it is possible to introduce the probability of node presence at each vertex of the honey-grid.

2.8 Capacity

Interference has an influence on the capacity of the network. Instead of focusing on the interference, the focus can also immediately be placed on the capacity itself. When looking at protocols, it seems that capacity often is the limiting factor: The symptom of failure under stress is congestion losses. Achievable capacity depends on network size, traffic patterns and detailed local radio interactions [Li01]. It is expected that the total capacity of an ad hoc network will grow with the area it covers, because of spatial re-use of the spectrum. However, the throughput of a node does not only depend on the capacity, but also on the load imposed by distant users through forwarding.

Li et al. analyze static ad hoc networks, noting that in most ad hoc networks, movement of the nodes is limited to small distances. First, just one chain of nodes is regarded, as packets are sent along a path using multiple nodes. Even within a chain, interference is present. Two consecutive nodes cannot transmit at the same time, because a node cannot receive and send packets at the same time. Even node $i+2$ cannot send if node i is sending, since node $i+1$ cannot receive when node $i+2$ is sending, because of the interference it will cause. This implies that the channel utilization cannot exceed 33%. If interference reaches further than the transmitting range, the situation is even worse. Protocols often do not consider the impact of this situation. Suppose that the first node of the chain transmits as many packets as possible, taking into account the influence of interference. Because the middle nodes receive more interference (because there are nodes in front and behind them), they cannot handle the complete load. This means that packets will be dropped. While the first node was sending the packets that are lost anyway, it caused even more interference, deteriorating the situation even further.

A common protocol uses an access method to determine whether a node is allowed to start transmitting. A node that wishes to send packets, first sends a request to the destination. If the network is idle, clearance is reported back. The transmission is started and the destination will send an acknowledgement to the source when the transmission is completed. If a node overhears a request or clearance from other nodes, it knows the channel will be busy and will not start a transmission or send a clearance itself. If no response is given to a request, the sender of the request will back off and retry later. After each try that fails, the backoff time will be increased. This situation deteriorates the performance of the network in some situations. If node $i+2$ is sending data and node i is requesting permission to node $i+2$, there will be no reply, because node $i+2$ interferes with node $i+1$. While node $i+2$ keeps sending, the backoff time for node i will keep increasing. When the line is finally free, the node might still have quite some backoff time left, wasting valuable time.

If there is not a single line, but a lattice consisting of parallel chains, the interference will increase. The neighbouring chains will interfere, deteriorating the performance even further. With the same assumptions, now every third channel can broadcast at the same time without interference, leading to a utilization of 33% per chain. Within the chain, as shown earlier, the utilization was 33% per node, leading to a total of 11%. The last situation evaluated is when using a square lattice, but with traffic in both horizontal and vertical directions. In this case, the capacity of the network does not have to decrease. Using a schedule where horizontal and vertical flows are sent in different time cycles, the throughput would be half as high, but there would be twice as many flows, leaving the overall throughput at the same level. In practice this might not be possible though. If the queue of a node contains packets that have to be sent in both vertical and horizontal directions, but one for the horizontal direction is at the head of the queue, then a chance to send a packet in the vertical direction may be lost. This phenomenon is called head-of-queue blocking.

When looking at the one-hop throughput, simulation shows that it scales linearly with the area of network when taking the number of nodes proportional to the area. When using one-hop throughput to measure the capacity of the network, this accounts for path lengths. This makes it possible to compare random traffic with the traffic in the proposed lattice. It seems that the random network has less capacity than the lattice, but the differences are not large. Because of the irregular placement in the random network, some areas may not contain nodes, wasting potential spatial re-use. Furthermore, the random choice of destinations causes a tendency to send more traffic through the center of the network than along the edges, leading to more congestion.

In a larger view, the network's total capacity is of specific interest. This can be approximated by scaling: The load increases with the number of nodes and with the distance over which each node wishes to communicate. The total one-hop capacity increases with the area covered by the network. Using these relationships, the following formula can be derived:

$$\lambda < \frac{kr}{\delta}\frac{1}{L} = \frac{C/n}{L/r}$$

where λ is the capacity that is available to each node, k is a constant for the linear relationship between one-hop capacity and the area ($C = kA$, with C the one-hop capacity and A the area of the network), r is the range of radio transmission, δ is the node density, which is assumed to be uniform, L is the expected path length from source to destination and n is the number of nodes. The inequality shows that when the expected path length (L/r) increases, the bandwidth available per node decreases. This implies that the traffic pattern has a big influence on scalability.

The most commonly used traffic pattern for ad hoc networks is random traffic: each destination is chosen at random. When the node density is constant, the probability of a node communicating with a node at distance x from this node equals:

$$p(x) = x / \int_0^{\sqrt{A}} t dt$$

This gives for the expected path length:

$$L = \int_0^{\sqrt{A}} x p(x) dx = \frac{2\sqrt{A}}{3}$$

The area of the network is proportional to the number of nodes, so the capacity available to each node, λ, is $O(1/\sqrt{n})$. This is an upper bound, since the boundary situation of the network has not been taken into account.

As was derived from the presented formulas, the expected path length of the traffic pattern determines the capacity scaling. The less local the traffic pattern, the faster node capacity will degrade with network size. Considering scalable traffic patterns shows that a random traffic pattern gives the worst case scenario. The key factor for deciding whether large ad hoc networks are feasible therefore is the locality of traffic.

2.9 Mobility

In the previous sections it has become clear that mobility has a big impact on the performance of an ad hoc network. Some mobility models have been proposed to schedule the movement of the nodes. Also, making specific assumptions, the influence of mobility has been investigated without using any mobility model. All approaches have shown that mobility increases the capacity of the network (but does not have to influence the connectivity). There are multiple explanations why this is the case.

One reason is because of the exploitation of spatial diversity [Gupt01]. As mobility increases, the spatial distribution of the nodes becomes more uniform on average, thus increasing the average sparsity of the connectivity graph. With higher traffic diversity, the traffic spreads out more evenly. Nevertheless, this does not imply that the network utilization improves as well. The system often fails to take advantage of the increased available capacity. This is caused by the incapability of the protocol to keep up with the high mobility. Packets in transit will be dropped when paths break. A new path is discovered, but because of the changes it is more unlikely to get a path reply from an intermediate path. This extra time needed to find a path, causes the buffering of packets at the source, which increases idle periods of the network and possible drops caused by buffer overflow.

Optimization appears to be possible. As stated, the protocol often is not optimal, causing idle periods because of backoff time, carrier sensing delay and interframe gaps. Also, there is a lot of routing activity in the network due to apparent path breaks because congestion causes packet loss. Finally, the capacity is determined assuming that all active nodes are consuming maximal bandwidth. This is not always the case, because a bottleneck node might drop packets, while nodes further on the path do have capacity to send more packets. Increasing the packet load also increases the utilization and throughput, but the delivery fraction decreases drastically.

Another explanation is because of the exploitation of multi user diversity [Gros01]. For diversity, the idea to improve performance is by creating several independent signal paths between the source and destination. One possibility is choosing the best path only for communication. Packets on other paths have to wait then until this path becomes the best one. This will cause quite some delay though. Grossglauser and Tse also focus on applications where a long delay is tolerated. The many changes in the network can then be used to improve the performance of the network. The strategy used is for each source to send their data to all neighbours. These nodes will move around the network, and one of them will probably come close enough to the destination for a direct transmission, delivering the data as wanted. Because there are many relay nodes, the probability that this happens is high. And since only one relay node is used, the throughput will be high. The assumption is made that each node has an infinite buffer.

When looking at a network that only consists of fixed nodes, the best way to maximize the transport capacity is using short transmissions. This means only transmission to neighbours is done, which implies that many hops are necessary. The throughput in this situation is of $O(1/\sqrt{n})$, which will tend to zero for large n. When considering mobile nodes without relaying there are multiple possibilities. Transmission over long distances can be allowed, so that many source-destination couples are within reach, but then the interference will become a bottleneck. If only short transmissions are allowed, then there will only be a few source-destination couples that can transmit to each other. Neither situation will lead to a good performance. The now obvious situation to consider is mobile nodes with the use of relaying. It seems that relaying only once will be sufficient. This is because the probability for an arbitrary node to be scheduled to receive a packet from a source node is equal for all nodes and independent of the source. With no more than two transmissions needed, the total throughput is $O(n)$. The received power at the nearest neighbour is of the same order as the total interference from $O(n)$ interferers, explaining why it is possible that there are $O(n)$ concurrent nearest neighbour transmissions.

When assuming exponential path loss with exponent η, there is a clear connection with the optimal sender density θ. If θ is too small, then the exploitation of spatial re-use is suboptimal. If θ is too large, the interference power will become too dominant. For small η, interference limits the spatial channel re-use and the sender density has to be small. For large η, interference is more localized and the optimal θ and the maximum throughput are larger.

In stead of looking from the sender's viewpoint to decide where to transmit to, it also possible to look at this from the receiver's point of view. Not the closest receiver will be chosen by the sender, but the closest sender will be chosen by the receiver. Now it will not occur that two senders send to the same receiver, but it is possible that two receivers choose the same sender. This might invoke some problems. When stating that in such a situation a sender has to choose one destination, the elimination of sender-receiver pairs has to be taken into account, because the sender has to be unique. When a sender can generate signals for several receivers, multiple interference signals are created. How this will effect the performance is not yet investigated. The sender-centric viewpoint appears to be the easiest approach.

The use of multi user diversity seems to work very well. Using only point-to-point links did not work, because most of the time a source-destination pair will be situated too far apart from each other. By using other nodes as relay nodes, communication is improved considerably. There is a high throughput from the source to the relay nodes, because the probability of having a relay node close to the source is high. Also the throughput from the relay node to the destination is high, because the probability that a relay node is close to the destination is high as well. This statistical multiplexing effect is due to the large number of users in the network. This approach is different from path diversity, because there copies of the same packets are forwarded along different paths to provide redundancy against uncertain channel conditions and network connectivity. It is clear that this approach can only be used when the delay of packets is of lesser importance.

2.10 Overview

Many different aspects of ad hoc networks have an influence on the quality of the network. Many of these aspects have been investigated separately, using different models and making different assumptions. Aspects as connectivity, path availability, multipath routing, admission and power control, interference, capacity and mobility have been discussed. The aim of this chapter was to give an overview of all the research already done on the topic of ad hoc networks, but also to show the still uncovered areas.

For the future large scale use of ad hoc networks, the most important aspects will be its capacity and its throughput. How many users can be serviced by the network, taking into account the different needs they may have, and how fast can the data be transferred? All other aspects are related to these main aspects. The QoS takes into account that the needs of the users are satisfied, the connectivity will assure that all users can use the network and the path availability will make sure that users will not suddenly get disconnected. Through multipath routing it is possible to improve the reliability of the connection and the amount of data that can be transmitted and admission and power control will keep the QoS high enough. The interference in the network will cause a decrease in capacity and the mobility of nodes can be used to increase the capacity of the network.

To model the ad hoc network, graphs proved to be very useful. With a model at hand, stochastic analysis becomes possible. Graph theory and stochastics therefore are the main areas used to get a better view on the possibilities for, and the performance of ad hoc networks. A relevant problem formulation for future research should include both these areas.

Many aspects of ad hoc networks deserve more attention and many suggestions for future work have been made. One can think of the following aspects:

QoS robustness	Priority	Cost effective implementations
Pricing of network usage	State prediction	Correlation between path failures
Differences between users	Reduction of overhead	Determining of the interference area
Influences between nodes	Node distribution	Mobility in stead of using grids
Time variant models	Scalability	Improvement of capacity use
Routing performance	Mobility	

There are multiple trade-offs that have a big influence on the choices that have to be made for ad hoc networks. There are trade-offs between:

- the throughput and delay (through mobility)
- the connectivity and interference (through power usage)
- the interference and reliability (through number of paths and hops)
- the QoS and capacity (through number of users)

Even though all of these aspects are important and interesting, including all aspects in one model would be too complex and too much work for the time available, so a choice has been made which aspects will be investigated in this thesis.

Chapter 3

Motivation and problem formulation

When considering an ad hoc network as it might be used in the future for communication purposes, certain fundamental questions arise. Where do we want to use this kind of network? What do we want to use it for and on how big a scale can we use it? A single correct answer to these questions does not exist, since there are many different kinds of networks. One can for instance think of a mobile telephone network where users want to call each other. This should be a large scale network, since otherwise only a small number of people could call each other and there will be no market for such a network. But if we consider a company where the employees need to contact each other for sending data from their computer to other computers, the number of users will be limited. A different kind of network will be appropriate here. So new questions arise: What do we demand from the network? How many users should be capable of using it and what are the demands to keep the users content with the service of the network?

This leads us to the main subjects of this thesis. We will have a look at the number of users that participate in the network. It would not be appropriate to block people from the network, so all users should be included if possible. Depending on the type of network, this number of users might be unbounded. Even if we allow all users into the network, this does not have to mean that they can connect to all the other users. This aspect of the network, its connectivity, will be investigated first. The development of the possible connections when the number of users grows will be looked into. Connections between users will not always be established successfully, which influences the probability that users can connect to each other. The further users are apart, the harder it will be to get a good connection between them. The influence of the distance between users will therefore be included in the evaluation of the connectivity.

It might be desirable to have multiple paths available between users of the network. This way the probability of successfully connecting to another user will grow. Using multiple paths gives the possibility of splitting up the data over these paths. This way the risk of losing data is decreased, increasing the reliability of the network. For multiple paths, more users are needed to relay the data that is being sent over a long distance, this way increasing the interference and degrading the performance of the network. There is a trade-off that has to be investigated to find the optimal setup of the network.

Even if users are close to each other, an attempt to make a connection between them may still fail. So when all the users of the network can contact each other in theory, this does not have to mean that in practice this will also be the case. The network can be busy because other users are transmitting, or connections can fail. With the number of users growing larger, the influence they will have on each other will also grow. Therefore it is important to take these influences into account. This is the second aspect that will be investigated. Questions about the number of users that can transmit at the same time and the probability of a failure are answered, leading to a view on the performance of the network. When more users are active, more data can be transmitted, but the probability of a collision will increase. The use of multiple paths and the way these paths are used will also be of influence on the total performance of the network. The throughput of the network and of a specific path will be the central points of interest. A comparison between these two can also give more insight in the performance of the network, showing the influence of using a multihop path in stead of base stations as is done in common networks, like WLANs. This will show what the advantages of ad hoc networks are and if ad hoc networks can outperform other types of networks.

Many other aspects than can and will be of influence on the performance of the network will not be discussed in detail, but assumptions about their influences will be made and motivated throughout the thesis. Even though the influence of factors like mobility and power control can be high, only suggestions will be made for expanding this work to include these factors. The most important factors are included to give a realistic representation of an ad hoc network. This will finally lead to a model of an ad hoc network that calculates the throughput and reliability of the network. This way a good interpretation can be given of the advantages and limitations of the use of ad hoc networks as communication networks.

Chapter 4

The network model

The type of ad hoc network that will be investigated in this thesis is a large network, with many users who are all similar. The network can be depicted as a graph, where the nodes of the graph represent the users and edges show the possible links that can be established between the users. Since not all users can contact each other directly, paths are needed to make long distance connections. This means that other nodes are needed to make communication between distant nodes possible. There may be many different paths available to reach a destination, and multiple paths may be used to achieve a better throughput for the network. A possible setup of an ad hoc network is depicted in Figure 4.1.

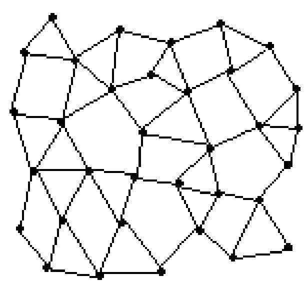


Figure 4.1: Network representation

We assume that the total network consists of immobile nodes which are distributed over a region with area A. The nodes can be situated according to a grid or they are homogeneously distributed.

Assumption 1 *Nodes are immobile and distributed over a region according to a grid or a homogeneous distribution.*

The borders of the network are not considered, leading to an identical situation for all nodes as if they are placed on a globe.

Assumption 2 *Boundary conditions are not taken into account.*

All nodes are able to transmit to nodes within a range r and the probability that a transmission within this range is successful is set at the value of p. These values are used to investigate the connectivity of the network.

Assumption 3 *Nodes can only transmit within a range r and the probability that such a transmission is successful is p.*

Of interest for the network is if all users can connect to all the other users inside the network. And if this is the case, it might be useful if there are multiple different paths leading to other users. When multiple paths are available, data might be split over these paths, increasing the reliability of the network. For the number of users tending to infinity, we will investigate the connectivity of the network and discuss the possibilities for, and consequences of, using multiple paths for the transmission of data.

With the nodes representing the users of the network, they will generate data that has to be transmitted over the network to randomly chosen other node inside the network. Every node will have new data to transmit according to a Poisson process with rate λ. The amount of data generated each time is assumed to have a distribution with mean $\frac{1}{\mu}$.

Assumption 4 *Nodes transmit a data amount with mean* $\frac{1}{\mu}$ *according to a Poisson process with rate* λ*.*

When transmitting, the network can transfer the data at a rate r_{net}. To avoid complete chaos that would happen if all nodes would start transmission at any moment they like, a specific scheme is used. Whenever a node is transmitting, all the nodes within its neighbourhood will receive a signal and will not start transmitting, otherwise collisions would occur. Whenever a user finds the channel busy, he will wait until it becomes free again. When the channel is free, the user will wait for a set period (DIFS) after which a number is drawn from a discrete uniform distribution between certain values, the contention window. This number is counted down after each time slot of length τ if there still are no other signals. Otherwise it will wait and continue counting when the channel is free again. When the countdown is completed, transmission will start. This scheme is called Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA). We consider two different ways of transmitting. Either the node will simply start transmitting its data (Basic mode), or it will send a request to send (RTS). If in the latter case the receiver replies with a clear to send (CTS), the node will start transmitting the data packets (RTS/CTS mode). When the receiver has acquired all the data, it will send an acknowledgement to the transmitter and the process is completed. The two modes are depicted in Figure 4.2.

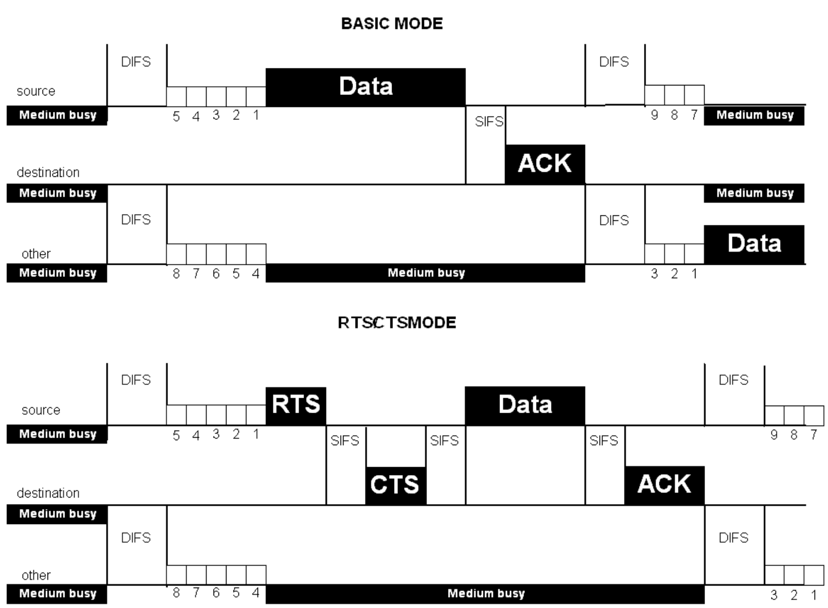


Figure 4.2: Basic and RTS/CTS mode

If for any reason the transmission fails, the process will repeat itself, starting with taking a backoff time from the contention window that has been doubled in size. To prevent backoff times from growing too large, the window will not be doubled more than r^* times. A node will not be allowed to retry transmission more than $r_{\max}$ times.

When assuming that a node always wants to transmit, a state space can be drawn showing the possible states of a node, either transmitting, or waiting to transmit. Solving this state space gives the probability that a node is transmitting and that a transmission fails. By assuming that the throughput for a single node will not change when letting go of the assumption that nodes want to transmit permanently, a processor sharing model can be used to approximate the number of flows simultaneously in progress. This approach has been introduced to evaluate the performance of a WLAN.

Assumption 5 *Whether nodes are continuously transmitting or according to a Poisson rate does not influence the throughput for a single node.*

In a WLAN, all nodes would interfere with each other, whereas in the proposed ad hoc network, only nodes within the neighbourhood of a receiving node will. By adjusting the formulas to include this difference, the properties of a node within an ad hoc network are found. Furthermore, the influence of using multiple hops is integrated in the approach, leading to an evaluation of the throughput of paths rather than the complete network, showing the difference between ad hoc networks and other networks.

For evaluating the influence of paths, we assume that a transmission will take place to a randomly chosen destination and that when multiple paths are used, these paths all have the same length, meaning that they all use the same number of hops.

Assumption 6 *Nodes transmit to a random destination, only using paths of the same length.*

The network will always choose the number of hops that will optimize the expected number of paths through which the transmission will be successful. To calculate this optimal number of hops, we use assumptions (1) and (3) that the nodes are homogeneously distributed, do not move around the network and that links exist (transmission through them will be successful) with probability p when the nodes are within each others range.

When data has to be sent over a path with multiple hops, the data has to be forwarded, which can be done according to different schemes. A distinction will be made between directly forwarding received data, or waiting until the node wants to transmit its own generated data. These different schemes will influence the throughput of a path and of the network. The total number of nodes in the network can also vary, since users may choose to join or leave the network. The process of joining and leaving can be modelled in different ways, as will be discussed.

There are more aspects that can be of influence on an ad hoc network but that are not considered. Maybe users should only be allowed to send a maximum amount of data. Or maybe users will demand a maximum time the transmission of data is allowed to take, imposing limitations on the number of paths and hops used. The influence of mobility is not included, which shows that the findings of this thesis will not apply to all kinds of ad hoc networks. Also all nodes are considered to be equal, a further simplification of the network. All the aspects mentioned together nevertheless form the model for the ad hoc network investigated in this thesis and will still be representative for ad hoc networks as they may be used in real life situations for communication purposes.

Chapter 5

Connectivity

The first aspect of influence on the performance of a network, is whether all users can reach each other. It might even be necessary that there are multiple paths through which information can flow from one user to another. When taking this into account, we are considering the connectivity of the network. We then enter the field of graph theory. Graphs prove to be very useful for modelling networks in general, including ad hoc networks. Therefore we will discuss suitable graph models for representing ad hoc networks.

Definition 1 *A graph $G(V, E)$ (or simply G) is a pair $[V(G), E(G)]$, where $V(G)$ is a finite, non empty collection, the nodes of G, and $E(G)$ is a collection of unordered pairs $\{u, v\}$ with $\{u, v\} \in V(G)$ and $u \neq v$, the edges of G. The order of a graph is the number of nodes in the graph and the size of the graph is the number of edges in the graph.*

In simple visual terms a graph is a collection of dots, the nodes, and a collection of lines between the dots, the edges. In the graph modelling the network, the nodes represent the users of the network and edges represent the connections between the users. The properties of the graph can give information about the network modelled by it.

There are many different types of graphs, all serving different purposes. A graph that often is used to model an ad hoc network is the random graph. A random graph is a graph where the number of nodes is known, but for the edges there are multiple possibilities. This implies that there are many different types of random graphs. The best known ones are the graph classes $G(N, M)$ and $G_p(N)$. $G(N, M)$ is the class of all graphs with N nodes and M edges. In some applications a random graph on N nodes and M edges is defined as a graph $G \in G(N, M)$ that is randomly chosen from $G(N, M)$ where each of its members has an equal probability of being chosen. In other applications a random graph is a graph $G \in G_p(N)$ on N nodes chosen in such a way that each potential edge is chosen with probability p, independent from the other potential edges. A refinement of the latter class is the class $G_{p_{ij}}(N)$, where each potential edge is still chosen independently, but edges between node i and j exist with probability p_{ij}. An example of a $G_{p_{ij}}(N)$ graph is the Waxman graph, using $p_{ij} = f(\overrightarrow{r_i} - \overrightarrow{r_j})$ and $f(\overrightarrow{r}) = e^{-\alpha|\overrightarrow{r}|}$. Here $\overrightarrow{r}$ stands for the vector denoting the position of node i and $|\overrightarrow{r}|$ is the length of this vector and $|\overrightarrow{r_i} - \overrightarrow{r_j}|$ gives the distance between nodes i and j. This type of graph is one of the most commonly used graphs to represent ad hoc networks. For our analysis we will use a simple version of the $G_{p_{ij}}(N)$ class, which we will denote by the $G_p^*(N)$ class. Because the influence of the distance on the probability that a link can exist between two nodes is big for long distances, we will set the probability p_{ij} to zero whenever the distance between node i and j exceeds a specific range. Within the range, the edge between nodes will exist with a fixed probability p. Having introduced different graph classes, we will in the remainder of this thesis denote a graph randomly taken from a class of graphs by the name of the class. So when talking about the graph $G_p^*(N)$, we mean a graph randomly taken from the class $G_p^*(N)$.

As noted we are interested in the connectivity of the network. A graph is called connected if any node can be reached from all other nodes using the edges of the graph. There are two kinds of connectivity, the edge connectivity $\lambda(G)$ and the node connectivity $\kappa(G)$.

Definition 2 *If G is a connected graph with ≥ 2 nodes, then the edge connectivity $\lambda(G)$ is the smallest number of edges that have to be removed from G to make it disconnected. If G is disconnected, then $\lambda(G) = 0$.*

Definition 3 *If G is a noncomplete graph, then the node connectivity $\kappa(G)$ is the smallest number of nodes that have to be removed from G (together with their incident edges) to make it disconnected. If G is a complete graph on n nodes, then $\kappa(G) = n - 1$.*

A graph is called k-connected if the node connectivity of the graph is higher than or equal to k.

Definition 4 *A subgraph $G'(V', E')$ of $G(V, E)$ is a graph with $V' \subset V$ and $E' \subset E$.*

When a graph is not connected, it is said to consist of components.

Definition 5 *A component of G is a maximal connected subgraph of G.*

Since in the ad hoc network we want to consider the influence of (dis)appearing users, the nodes of the graph, we will only consider the node connectivity and will simply denote this by the connectivity of the graph. Using a graph chosen randomly from the class $G_p^*(N)$ to model the network, we will investigate the probability that a graph is k-connected for various values of k.

5.1 Minimum degree and connectivity

Whether a graph is connected or not depends on the number of nodes and the number of edges connecting them. If every node has enough edges leading to other nodes, it will be likely that the graph will be connected. The easiest way to determine whether a graph is connected or not is by looking at the minimum degree of the graph.

Definition 6 *The degree $d(v)$ of a node $v \in V$ is the number of neighbours of v, the number of nodes it can reach using its adjacent edges. The minimum degree $\delta(G)$ of a graph is the smallest value of $d(v)$ occurring in the graph.*

First we will determine the probability that at least one of the nodes has no neighbours at all, meaning that it has no edges leading to other nodes. To do this we assume that we are dealing with an asymptotic situation that the number of nodes tends to infinity.

Assumption 7 *The number of nodes tends to infinity.*

Then we will make the link to connectivity. A node without any neighbours is called an isolated node. Before we will take into account that there is a maximum range to make a link between nodes, we will study the standard $G_p(N)$ class for its connectivity properties. In [Boll85] a theorem is stated concerning the probability of the connectedness of the graph.

Theorem 1 *If $p = \frac{\ln n + x}{n}$ then the graph $G_p(n)$ has no isolated nodes with probability $e^{-e^{-x}}$ when n tends to infinity.*

Proof. Let p with $0 \leq p \leq 1$ denote the probability that an edge is present, and let X denote the number of isolated nodes. One can easily see that the expectation of the number of isolated nodes is given by

$$EX = n(1-p)^{n-1},$$

since the probability that a node has no neighbours is $(1-p)^{n-1}$ and this probability is equal for all the n nodes. Considering the situation where n tends to infinity, we can use the following approximation, using $\rightarrow$ to denote the limit for n tending to infinity.

$$\begin{aligned}(1-p)^n &= e^{\ln(1-p)^n} \\ &= e^{n\ln(1-p)} \\ &= e^{n(-p-\frac{p^2}{2}-\frac{p^3}{3}...)} \\ &= e^{-np}e^{-np^2(\frac{1}{2}+\frac{p}{3}+...)} \\ &\rightarrow e^{-np} \text{ when } np^2 \rightarrow 0,\end{aligned}$$

We find that $EX \backsim ne^{-np}$ since $np^2 = \frac{(\ln n+x)^2}{n} \rightarrow 0$. Inserting $p = \frac{\ln(n)+x}{n}$ for some constant $x \geq 0$ we get $EX \backsim e^{-x}$. We want to find $P(X=0)$, the probability that the graph has no isolated nodes. To find this probability, we will have to look at all the possible combinations for nodes to be isolated. Let X_i be 1 if node i is isolated and 0 otherwise. To find the probability $P(X=k)$, with $X = X_1 + ... + X_n$, we need the binomial moments $S_r = \sum E(X_{l_1}...X_{l_r})$, with the sum over all sequences $1 \leq l_1 < ... < l_r \leq n$, that is all r-subsets of $\{1, ..., n\}$. It follows, with $\backsim$ showing that we are approximating, that

$$\begin{aligned}S_r &= \binom{n}{r}(1-p)^{r(n-r)+\binom{r}{2}} \\ &\backsim \frac{[n(1-p)^n]^r}{r!} \rightarrow \frac{[e^{-x}]^r}{r!}.\end{aligned}$$

See the Appendix A.1 for a more elaborate discussion of this formula. From this, using the Bonferroni inequalities as shown in Appendix A.2, we find that we are dealing with a Poisson distribution with parameter e^{-x},

$$P(X=r) = e^{-e^{-x}}\frac{[e^{-x}]^r}{r!}.$$

Finally, we find that the probability that $G_p(n)$ has no isolated nodes equals $P(X=0) = e^{-e^{-x}}$. ■

It may be easier to see this using Poisson approximation [Bill95]. We assume that we are dealing with nodes that are equal and independent of each other.

Assumption 8 *All nodes are equal and independent of each other.*

Therefore we can assume that the variables X_i, indicating whether nodes are isolated or not, are identical and independently distributed (i.i.d.).

Theorem 2 *Set variables X_i i.i.d and $X_i = 1$ with probability p_i and $X_i = 0$ with probability $1-p_i$. If $\sum_{i=1}^n p_i \rightarrow \lambda$ and $\max\limits_{1\leq i\leq n}(p_i) \rightarrow 0$, with $\rightarrow$ the limit for n going to infinity, then*

$$P(\sum_{i=1}^n X_i = k) \rightarrow e^{-\lambda}\lambda^k/k!.$$

With $p_i = (1-p)^{n-1}$ and $p = \frac{\ln n+x}{n}$, equal for all i, we find that $\max(p_i) \rightarrow 0$, since $(1-p)^{n-1} \rightarrow 0$ and

$$\begin{aligned}\sum_i p_i &= n(1-p)^{n-1} \\ &= n(1-\frac{\ln n+x}{n})^{n-1} \\ &\rightarrow n \cdot e^{-(\ln n+x)} \\ &= e^{-x}.\end{aligned}$$

The theorem then leads to

$$\begin{aligned} P(X &= k) \rightarrow e^{-\lambda}\lambda^k/k! \\ \lambda &= e^{-x}. \end{aligned}$$

The probability of a graph having no isolated nodes, or similarly $P(X = 0)$, is again proved to be equal to $e^{-e^{-x}}$.

We now have a result for the probability that nodes are isolated, but we would like to know if a graph is connected or not.

Theorem 3 *Asymptotically speaking it is the same for a random graph $G_p(n)$ with $p = \frac{\ln(n)+x}{n}$ to have no isolated nodes and being connected.*

Proof. Let A be the set of all graphs of order n that have one component of order at least 2, with all other components isolated vertices, B the set of all graphs of order n that have no isolated vertices and C the set of all connected graphs of order n. The following then holds for $n \geq 2$:

$$\begin{aligned} C &= B \cap A \\ P(B) &= P(C) + P(B \cap \overline{C}) \end{aligned}$$

Since $B \cap \overline{C} \subseteq \overline{A}$ and $P(\overline{A}) \rightarrow 0$ (this proof is omitted, see [Palm85]) it follows that $P(B \cap \overline{C}) \rightarrow 0$ and $P(B) = P(C) + o(1)$. Since we found that $P(B) \nrightarrow 0$ it follows that

$$P(B) \backsim P(C).$$

We have shown that $P(B) \backsim e^{-e^{-x}}$, so it follows that the probability of having no isolated vertices in a graph in the limit is equal to the probability of the graph being connected. ∎

This shows that there is an important relation between the connectivity of a graph and the minimum degree. If a graph needs to be connected, it may not have isolated nodes, so the minimum degree may not be zero, but it also works the other way around when considering the asymptotic situation. This only gives a result on whether a graph is connected or not, but does not touch the level of connectivity of the graph. In the next section we will show the extension to k-connectivity for any value of k.

5.2 Higher levels of connectivity

Continuing on the thought that there is a relation between the minimum degree of a random graph and its connectivity, a relation can also be found for a higher level of connectivity. Again, we will first discuss the minimum degree of the graph. By taking a convenient value for the probability p of an edge, we get to a nice expression for the expected number of nodes with degree k. From this we derive the expression for the minimum degree of the graph. Then we make the link between the connectivity and minimum degree of the graph. For determining the probability that a node has minimum degree k, we will use extreme value analysis [Embr97, Cole01]. In particular, we need the following theorems on extremal distribution theory, about the properties of the maximum M_n of n i.i.d. variables:

Theorem 4 *The exact distribution function of the maximum M_n of n i.i.d. variables $X_1, ..., X_n$ with common distribution function F can be written as*

$$P(M_n \leq x) = P(X_1 \leq x, ..., X_n \leq x) = F^n(x). \tag{5.1}$$

The distribution function F may not be known, but it is possible to investigate F^n by looking at the extreme values. This way a model can be found to approximate the distribution of M_n. Looking at the limit as $n \to \infty$, we see that $F^n(x) \to 0$ for all $x < x^+$, with x^+ the smallest value of x such that $F(x) = 1$. The distribution of M_n degenerates to a point mass on x^+. This difficulty can be avoided by using a linear renormalization of M_n,

$$M_n^* = \frac{M_n - b_n}{a_n}.$$

This way the location and scale of M_n can be stabilized as n increases, avoiding the difficulties. Now we can look at the limit distributions for M_n^*, with appropriate choices of a_n and b_n, rather than for M_n.

Theorem 5 *If there exist sequences of constants $a_n > 0$ and b_n such that*

$$P(\frac{M_n - b_n}{a_n} \leq x) \to H(x) \text{ as } n \to \infty,$$

where H is a non-degenerate distribution function, then H must be one of the standard extreme value distributions.

There are only three types of extreme value distributions:

$$\begin{array}{ll} \text{Gumbel} & \Lambda(x) = e^{-e^{-(\frac{x-b}{a})}} \\ \text{Frechet} & \Phi_\alpha(x) = \begin{cases} 0, x \leq b \\ e^{-(\frac{x-b}{a})^{-\alpha}}, x > b \end{cases} \\ \text{Weibull} & \Psi_\alpha(x) = \begin{cases} e^{-(-(\frac{x-b}{a}))^{\alpha}}, x < b \\ 1, x \geq b \end{cases} \end{array}$$

We will prove the following theorem [Erdo61] that shows the connection between the link probability and the minimum degree of the graph.

Theorem 6 *If $p(n) = \frac{(\ln(n) + k\ln(\ln(n)) + x)}{n}$ with k fixed and non-negative, then the probability that the graph $G \in G_p(n)$ has minimum degree k is given by*

$$P(\delta(G) = \ k) \to 1 - e^{-e^{-x}/k!}.$$

Proof. We are not looking at a maximum, but at the minimum of the degrees $X_i = d(v_i)$ of the nodes, so we need to make a small adjustment.

$$\begin{aligned} P(\delta(G) = \ k) &= P(\min(X_i) = k) \\ &= P(-\max(-X_i) = k) \\ &= P(\max(-X_i) = -k) \end{aligned}$$

Using 5.1 we then find for our minimum

$$P(\min(X_i) \geq k) = [P(X \geq k)]^n = [1 - P(X < k)]^n,$$

where we simply use X in stead of X_i, because the nodes are equal and independent. Looking at $P(X = k)$, the probability that a node has degree k, we find

$$\begin{aligned} P(X = k) &= \binom{n-1}{k} p^k (1-p)^{n-k-1} \\ &\backsim (np)^k \frac{e^{-np}}{k!}. \end{aligned}$$

Inserting $p = \frac{\ln n + k \ln\ln n + x}{n}$, $x \geq 0$ this gives us a convenient result:

$$\begin{aligned}
P(X = k) &\backsim (\ln(n) + k\ln(\ln(n)) + x)^k e^{-(\ln(n)+k\ln(\ln(n))+x)}/k! \\
&= \frac{e^{-x}}{n}(\ln(n))^{-k}(\ln(n) + k\ln(\ln(n)) + x)^k/k! \\
&= \frac{e^{-x}/k!}{n}(1 + \frac{k\ln(\ln(n))}{\ln(n)} + \frac{x}{\ln(n)})^k \\
&\to \frac{e^{-x}/k!}{n}, \text{ and} \\
P(X < k) &\to \sum_{j=0}^{k-1} \frac{e^{-x}/j!}{n}.
\end{aligned}$$

This finally leads to

$$\begin{aligned}
[1 - P(X < k)]^n &= [1 - \sum_{j=0}^{k-1} \frac{e^{-x}/j!}{n}]^n \\
&\to e^{-\Sigma_{j=0}^{k-1} e^{-x}/j!}.
\end{aligned}$$

For the probability that $G \in G_p(n)$ has minimum degree k we find that

$$\begin{aligned}
P(\delta(G) = \ k) &= P(\min(X_i) = k) \\
&= P(\min(X_i) \geq k) - P(\min(X_i) \geq k+1) \\
&= [1 - P(X < k)]^n - [1 - P(X < k+1)]^n \\
&\to e^{-\Sigma_{j=0}^{k-1} e^{-x}/j!} - e^{-\Sigma_{j=0}^{k-1} e^{-x}/j!} \\
&= e^{-\Sigma_{j=0}^{k-1} e^{-x}/j!}(1 - e^{-e^{-x}/k!}) \\
&\backsim 1 - e^{-e^{-x}/k!}.
\end{aligned}$$

For this approximation and a different approach we refer to Appendix A.3. Using extreme distributions an approximation for the probability that a graph $G \in G_p(n)$ has minimum degree k has been derived. We see that this formula is of the type of a Gumbel distribution, as was expected by theorem 5. ■

It still has to be proved that the minimum degree of a graph also says something about the connectivity of a graph. Using the following theorem by Erdös and Renyi [Erdo61], we can find the relationship between connectivity and minimum degree:

Theorem 7 *Let k be a fixed nonnegative integer and*

$$p(n) = (\ln(n) + k\ln(\ln(n)) + x)/n.$$

Then for $G \in G_p(n)$

$$P(\kappa(G) = k) \backsim P(\lambda(G) = k) \backsim P(\delta(G) = k).$$

As we have seen that $P(\delta(G) = k) \to 1 - e^{-\frac{e^{-x}}{k!}}$ we can conclude, using this theorem, that $P(\kappa(G) = k) \to 1 - e^{-\frac{e^{-x}}{k!}}$. This again shows the relationship between the link probability and the connectivity of the graph.

Chapter 6

Maximum range

In real networks, communication over long distances cannot take place directly since the signal will be too weak to be heard over the noise. We have been using a model that assumes that any edge can be present with the same probability (3). This does not exclude the possibility that a link between nodes far apart exists, meaning that long distance connections are assumed to possible. Nodes that geographically are far apart should have a low probability of being directly connected by an edge than nodes closer together. We will now adjust the model to make it more realistic.

We introduce a threshold distance as a boundary for direct communication. When the distance between nodes is larger than this boundary, a direct connection between these nodes is not possible. This will have a big impact on the model we want to use for representing the network. If the destination a certain source wants to transmit to is far away, a connection can only be established using multiple edges leading through other nodes, a so called path.

Definition 7 *A path between nodes a and b in a graph G is a finite sequence $v_0e_1v_1e_2...v_{k-1}e_kv_k$, consisting of nodes v_i and edges e_i of G so that $v_0 = a$, $v_k = b$ and $e_i = v_{i-1}v_i$ for $i = 1, ..., k$, with $v_i \neq v_j$ when $i \neq j$. The value of k is the length of the path. Paths are called internally disjoint when they have at most their endnodes in common.*

The graph representing the network can now only have an edge between two nodes, the users, when they are close together. An edge between two nodes represents the possibility of a direct communication between the two users. A connection over a long distance will only be possible if there are enough nodes between the source and destination, with each successive node within range of the next one. The use of an edge, representing a transmission between users, will be referred to as a hop. We will evaluate the impact of setting a distance boundary for possible transmissions by using the following model.

A total of N nodes are homogeneously distributed and all have a transmission range r. Every node within range of some node a can be reached directly from a with probability p, independent of the distance from a. Nodes out of range cannot be reached directly. We will investigate how many disjoint paths there will be between an arbitrary couple of users in the network. According to the following theorem by Whitney, this can be done by investigating what the connectivity is of the graph representing this network.

Theorem 8 *If a graph G is k-connected, then every two nodes of the graph are connected by k disjoint paths and vice versa.*

We will look at the situation by drawing the range of the nodes as circles of radius r around every node to see which nodes can be reached directly. This is used to find paths from a source s to a destination d and will give the situation as presented in Figure 6.1 when looking at a path leading straight towards the destination.

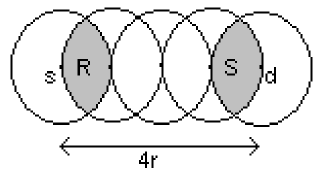


Figure 6.1: Radius of the nodes

The number of hops needed from s to d (in Fig. 6.1) will be at least four, but this will only be possible if there are nodes situated on the exact spots where the circles touch. More realistically, a hop will be made from s to a node in region R, and ideally, from there only four more hops will be needed. Having made the assumption (1) that the nodes are homogeneously distributed, the expected number of nodes in R can be calculated. It is also easy to see that when a specific path consists of h hops, then the probability that this path exists will be equal to p^h, where p still stands for the probability that a link between users exists. When there are k nodes in region R, there can be at most k disjoint paths from s to d through R. The same goes for region S, from where the destination can be reached. If there are less than k nodes in S, then there will be less than k disjoint paths to d. Continuing on this thought it is possible to find an expression for the expected number of disjoint paths in this type of network.

What we need to find, is the number of paths that can be found from a specific point to the destination, and the probability that such a path exists. The process of finding the probability that a path exists leads us to dynamic programming, because we will have to backtrack a path, starting from the destination. If we have a look at Figure 6.1, we see that from region S the probability to reach d directly will be p, since only one hop will be needed. Of course it is also possible to get there in multiple hops, but the probability of reaching d will then be p^h when using h hops, which is lower for a higher value of h and is therefore giving inferior results. To get from s to a point in S, there are multiple hops needed. To find the probability of a path of h hops from a node i to another node j being present, the following formula holds:

$$P_{ij}^h = \sum_k P_{ik} P_{kj}^{h-1}.$$

In this formula, P_{ij}^h stands for the probability that one can get from i to j in h hops and P_{ik} simply means the probability to get from i to k in one hop. We assume that the path does not contain circuits.

Assumption 9 *Paths do not contain circuits.*

Looking at Figure 6.1 again, we see that the shortest possible path will consist of four hops. This is only possible if there are nodes on the exact points where the circles touch. It is highly improbable to find nodes on those exact spots, meaning that we cannot expect to find a path of four hops. But what about five hops? It seems quite possible to reach a node in region S from s within four hops. But then of course there must be a node within region S. Since we assumed that all nodes are distributed homogeneously (1), the number of nodes in a region depends on the area of this region.

If we know the expected number of nodes within the circle, the range of a node, we also know the expected number of nodes within region S, since we can calculate the area of S. To find the area of S we use an integral after placing the circles as depicted in Figure 6.2.

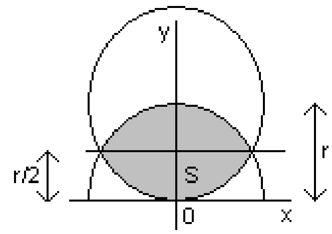


Figure 6.2: Computation of S

The area of S is equal to $\frac{2}{3}\pi r^2 - \frac{1}{2}r^2\sqrt{3}$ as shown in Appendix A.4, so the expected number of nodes in this area will be $(\frac{2}{3} - \frac{\sqrt{3}}{2\pi})m$, if the expected number of nodes within a circle with radius r is equal to m. If the number of nodes inside the circle is stochastic, then the expected number of nodes inside the circle is $\pi r^2 \sum_{m=0}^{N} P(M = m)m$. For the Poisson point process with rate λ this will then be equal to $\lambda \pi r^2$. The same goes for the area of R. For every hop to a new region there also have to be at least k nodes to leave the possibility of k-connectivity open when we only allow paths consisting of four hops. Unfortunately the presence of k nodes in a region does not assure k-connectivity, since it is likely that not every node from a region can reach another node in the following region that has to be passed going towards the destination. From every node you might find a new node to hop to, but it might be the case that two nodes have to hop to the same node. This way there will not be k internally disjoint paths and consequently no k-connectivity. More nodes are probably needed to assure k-connectivity. Can we find a formula that states the probability of k-connectivity, without having explicit knowledge of the location of the nodes? The assumption (1) that the nodes are homogeneously distributed will prove to be useful for approximating this probability. To make the problem a bit clearer, look at Figure 6.3.

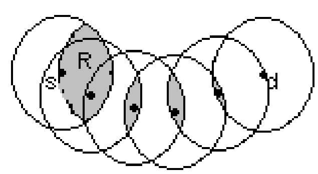


Figure 6.3: Multiple hops to a destination

We find that even though we use five hops, one more than the minimal number of hops, the probability of reaching d is extremely small. The first hop is made to a node in area R, where the area of overlap with the next circle is still quite big. But going further down the path, we see that there has to be a node in the very small region where the last three circles overlap. This probability will be very small. So even with an extra hop, the probability of reaching d within this number of hops can be small. And if the first hop would have gone to a node closer to s, but still in R, the path to d would probably have to be even longer than five hops. Can we calculate the probability that a path of h hops is possible and in how many disjoint paths of this length be found?

It appears that the number of hops that is used to communicate between two nodes has a big impact on the number of disjoint paths between these nodes. Furthermore it will be the case that when a path consists of more hops, the probability of a successful transmission becomes smaller. This is the case since every possible connection of a node to a node within its range is successful with probability p. This goes for every hop that is made, so the probability that a path of length h is successful is equal to p^h. There is a trade-off between the number of possible paths and the probability that such a path leads to a successful transmission, when considering a fixed number of hops. With possible paths we mean paths consisting of hops that can be made considering that the nodes are within each others range, but that it is not yet certain that the hop will be successful. We will investigate the number of possible paths that can be found using a fixed number of hops and the probability that a transmission through this path will be successful to get insight into the expected number of paths between two nodes.

6.1 Number of paths

To get more insight into the level of connectivity, we want to find the expected number of disjoint paths that exist between two nodes. When talking about paths that exist, we mean the paths when we take into account both that hops are only made between nodes within each others range and the probability that such a hop is successfully made. Assuming that nodes with a limited range are spread over a big area, nodes far apart can only communicate using multiple hops.

Assumption 10 *Nodes far apart can only communicate using multiple hops.*

If they are close together, a small number of hops might suffice, but paths consisting of many hops will still be possible. It is clear that there are just a few possible paths consisting of a little amount hops, but incredibly many possible paths with a large number of hops. It nevertheless is the case, as stated earlier, that a path consisting of many hops has a small probability of a successful transmission. The question thus is how many hops should be used when looking for the expected number of paths that exist. To have a look at this, we will start with investigating the number of possible paths consisting of a single hop and expand it to multiple hops, keeping track of the probability that such a path will really exist.

An important property of the network is the way the nodes are spread over the network. We use a homogeneous distribution of the nodes (33). In the model (when not assuming that nodes are situated according to a grid), we assume that nodes are distributed according to a stationary Poisson point process Φ. A description of this type of process is presented in Appendix A.5.

Assumption 11 *When nodes are not situated according to a grid, they are distributed according to a stationary Poisson point process.*

First we will have a look at the number of possible paths between a destination d and a source s when they are less that a distance r apart, so within each others range. This will lead to the situation as presented in Figure 6.4.

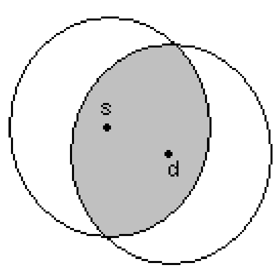


Figure 6.4: Destination within range

It is clear that the shortest way (using the least amount of hops) to reach the destination will be directly. This can be done in only one way and has a probability of success equal to p. Of course it is also possible to reach d using two hops. The first hop will then be to a node inside the grey region as shown in Figure 6.4. If it would be outside this region, it can either not reach d, or this point could not be reached from s. The expected number of possible paths using two hops therefore depends on the expected number of nodes within this grey region, which can simply be calculated. Assume that there are an expected number of m nodes within a circle of radius r around any node.

Assumption 12 *There are an expected number of m nodes within a circle of radius r.*

Since all nodes are assumed to be distributed homogeneously (1), this number is the same for every node. The expected number of nodes in the grey region with area V is then equal to $\frac{Vm}{\pi r^2}$. In a same way as done earlier, we can compute the area V. This appears to be equal to

$$
\begin{aligned}
V &= 2\int_{-\sqrt{r^2-(\frac{l}{2})^2}}^{\sqrt{r^2-(\frac{l}{2})^2}} (\sqrt{r^2-x^2} - r + \frac{l}{2})dx \\
&= \frac{(3l-4r)(\sqrt{4r^2-l^2})}{2} + 2r^2 \arcsin(\frac{\sqrt{4r^2-l^2}}{2r}).
\end{aligned}
$$

Here r is the range (and so the radius of the circle) and l is the distance between the source and destination. From this the expected number of paths using two hops can be calculated, with each path having probability p^2 of existence. Of course we should exclude the possibility of the first hop to go directly to d to get a correct calculation.

The next question then will be how many paths are expected to exist when using three or more hops. To answer this question, the result on the expected number of paths with two hops can be used. Suppose we want to find the number of possible paths consisting of $h > 2$ hops, then the probability of existence of such a path is p^h. Starting from s, a hop can be made to any node within its range. From this new node we arrived at, the connection to the destination should be made within $h-1$ hops. Considering this node as if it is the source, the process can be repeated. Eventually we reach the situation where there are two hops remaining. The number of possible paths with two hops can be calculated as described before. When, for example, we want to find the number of possible paths with three hops in the situation that s and d are within each others range, the first hop can be made to any node within the range of s. All neighbours of s are not further than a distance $2r$ apart from the destination node d, so it will still be possible to arrive at d within two hops. For the first hop we expect that there are $m-1$ possibilities (all nodes in the range except d), since according to the stationary Poisson point process there are an expected number of m neighbours for any node. We can then calculate the number of possible paths with two hops. For a correct calculation of this number we should exclude the possibility of making the second hop back to s. In the same manner we can find the number of possible paths consisting of more than three hops. This can be done for any distance between the source and the destination, so we can let go of the situation that they are within each others range and will look at the more general situation with a random distance l between the source and the destination.

A complication that has to be taken into account is that when using a fixed number of hops, we have to make sure that in some situations, hops should only be made in a specific direction, namely towards the destination. By going towards the destination we mean that the distance from the node reached after a hop to the destination should be smaller than the distance between the node and the destination before the hop. Otherwise it might be the case that the destination can no longer be reached using the remaining number of hops, because the distance between the nodes exceeds the distance that can be overcome with the remaining number of hops. This complication is shown in Figure 6.5.

The figure shows that the first hop from the source has to go to the grey region, otherwise it will no longer be possible to reach the destination within the remaining number of hops, as shown by the big circle with radius $(h-1)r$.

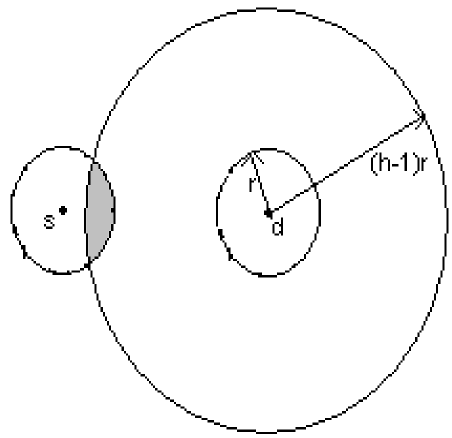


Figure 6.5: Specific region a hop has to be made to

The number of possible paths with h hops can be calculated, as long as the distance between the nodes does not exceed the distance that can be travelled within h hops. For the first hop, there will be expected to be m possibilities if there is no restriction on the region the hop should be made to (the hop is then made to any node within range of the source, assuming that d is not within range). To calculate the number of possibilities from the node we arrive after the first hop, we will use the expected distance between this node and the destination. From the starting node, the first hop can be made in any direction, say with angle $0 \leq \varphi \leq 2\pi$. The distance z travelled by the hop, the length of the hop, can be any value from the interval $(0, r]$. This leads to the integral

$$Eq = \frac{1}{\pi r^2} \int_0^{2\pi} \int_0^r z\sqrt{(z^2 + l^2 - 2zl\cos(\varphi))}dzd\varphi,$$

where Eq is the expected distance between the destination and the node arrived at after the hop and l was the distance between the destination and the node from where the hop was made. The derivation of this formula can be found in Appendix A.6. Unfortunately, we could not find an analytical solution for this integral. Therefore we use $lF(-\frac{1}{2}, -\frac{1}{2} : 2 : \frac{r^2}{l^2})$ as an approximation, where F stands for the geometric function. When plotting this function, we get Figure 6.6.

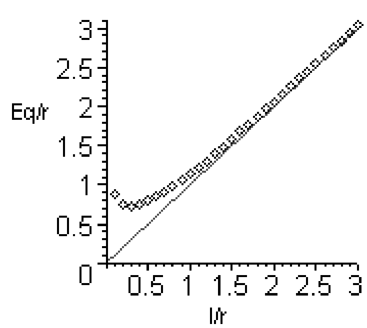


Figure 6.6: Expected new distance compared to the old distance

The figure shows the expected new distance divided by the range set out against the distance between the nodes before the hop divided by the range. The points show the results of the approximation of the integral and the straight line shows the result obtained if the distance to the destination stays the same after a hop. It appears that when the source and destination are far enough apart with respect to the range, the expected distance to the destination after one hop is expected to remain l. This means that on average, a hop does not lead towards the destination, but might even go away from it. The destination would never be reached. Therefore we will make the assumption that every hop will be made towards the destination: We assume that every hop goes to a node within that half of the circle around the sending node that is closest to the destination.

Assumption 13 *Every hop goes towards the destination.*

Only then we can be certain that the destination will eventually be reached if the number of hops is sufficient. The expectation for the distance to the destination after a hop will then be

$$Eq = \frac{2}{\pi r^2} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^r z\sqrt{(z^2 + l^2 - 2zl\cos(\varphi))}dzd\varphi.$$

The new distance can be calculated using the approximation and the calculation of the expected number of possible paths can continue, since all we need to know is the distance that still has to be overcome towards the destination and the number of hops still available. Using this approach we can calculate the expected number of paths with a fixed amount of hops, taking into account that the hop will always be made towards the destination.

Another complication that might occur is that some of the paths that are found using this approach use a hop that is also included in another path. This means that there is a dependency between these paths, they will not be disjoint. So the proposed approach does not calculate the expected number of *disjoint* paths. To find the expected number of disjoint paths, we have to use the result obtained in the previous sections: When a graph has minimum degree d, it can be expected to be d-connected under certain conditions. In the situation we are considering, we could define the degree of a node as the number of nodes it is allowed to make a hop to, while taking into account the probability that such a hop is successful. If paths consist of many hops, the region where a hop can go to will not be limited except for the half circle leading towards the destination. The expected minimum degree of the network will then be equal to $\frac{m}{2}p$, where m is the expected number of nodes inside a circle. There are $\frac{m}{2}$ possible hops inside this half circle, and recalling that a possible hop will be successful with probability p, we find that the degree, the expected number of neighbours a hop can successfully be made to, will be equal to $\frac{m}{2}p$. This brings us to the following algorithm to calculate the expected number of existing paths $ET(h)$ and the expected number of (existing) disjoint paths $ET_d(h)$, with a fixed number of h hops.

Algorithm 1 *Calculation of the expected number of paths with h hops.*

Step 1 *Set the distance l_1 between s and d, the expected number of nodes m within each circle and set $i = 1$.*

Step 2 *If $l_1 > hr$ then return $ET[h] = ET_d[h] = 0$ else go to step 3.*

Step 3 *If $h - i > 0$ go to step 4 else go to step 8.*

Step 4 *Set $i = i + 1$.*

Step 5 *Calculate the number of possible hops $ph[i-1]$ with criteria: new distance $l_i < (h - i + 1)r$.*

Step 5.1 *If $l_i < (h - i)r$ then $ph[i-1] = m/2$ and go to step 6, else go to step 5.2.*

Step 5.2 Calculate the area V of the region that is still solvable.
$V = \int_{-a}^{a}(\sqrt{[(h-i)r]^2 - x^2} - b)\ dx + \int_{-a}^{a}(\sqrt{r^2 - x^2} - c)\ dx,$
with $a = \frac{\sqrt{-r^4+2r^2l_{i-1}^2-l_{i-1}^4+2(h-i)^2r^2l_{i-1}^2+2(h-i)^2r^4-(h-i)^4r^4}}{2l_{i-1}},$
$b = \frac{((h-i)^2-1)r^2+l_{i-1}^2}{2l_{i-1}}$ *and* $c = \frac{-((h-i)^2-1)r^2+l_{i-1}^2}{2l_{i-1}}.$

Step 5.3 Calculate the expected number of nodes (= possible hops)within this region
$ph[i-1] = \frac{Vm}{\pi r^2}.$

Step 6 Calculate the expected distance l_i between the new node and the destination.

Step 6.1a If $l_{i-1} < (h-i+1)r$, the number of hops available exceeds the number of hops needed, so the first hop can be made within half a circle towards the destination.
Calculate $l_i = \frac{2}{\pi r^2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^r h\sqrt{(h^2 + l_{i-1}^2 - 2hl_{i-1}\cos(\phi))}\ dh\ d\phi.$

Step 6.1b If $l_{i-1} \geq (h-i+1)r$, there is a designated region that should be reached, calculate
$l_i = \frac{1}{V}\int_{-\beta}^{\beta}\int_{r(\phi)}^{r} h\sqrt{(h^2 + l^2 - 2hl\cos(\phi))}\ dh\ d\phi,$
where $r(\phi) = l_i cos(\phi) \pm \sqrt{l_i^2 cos^2(\phi) + (h-i)^2r^2 - l_i^2}$ *and* $\beta = \arccos(\frac{-((h-i)^2-1)r^2+l_i^2}{2l_i r}).$

Step 7 Go to step 3.

Step 8 Calculate the expected number of possible paths
$$T[h] = \prod_{i=1}^{h-1} ph[i]$$
and calculate the expected number of possible disjoint paths
$T_d[h] = \min_{i=1}^{h-1}(ph[i]).$

Step 9 Calculate the expected number of existing paths
$ET[h] = T[h]p^h$
and the expected number of disjoint paths
$ET_d[h] = T_d[h]p^h.$

Step 10 Return $ET[h]$ and $ET_d[h]$.

For every step, the solvable region, the region from which it is still possible to reach the destination within the available number of hops, is calculated. This makes sure that the algorithm will finally reach the destination or will conclude that the destination cannot be reached. So the algorithm always converges to an answer. Since the number of hops is limited, also the number of iterations is limited.

Using this algorithm, we can find the expected number of paths, either disjoint or not, that exist between any source and destination that can be used for a successful transmission between them. There is a hazard though: We would have to calculate the expected number of existing paths for any number of hops, so from 1 to $n-1$. This can mean a lot of work, especially since we want to look at the asymptotic behavior when n tends to infinity. It will nevertheless be clear that the probability that a path with $n-1$ hops can successfully be used for a transmission is very small. We suspect that there is an optimal number of hops that will lead to the highest expected number of paths. Knowing this number, we can easily derive the expected connectivity for the network: If there are k disjoint paths, the network will be k-connected. Important is to remember that we then are talking about connectivity while only allowing paths of a fixed length h.

Considering the results obtained when not taking a maximum range into consideration for the nodes, we find it quite logical to expect to find an optimal number of hops. As was seen, the minimum degree of the graph is an indicator for the connectivity. Using fewer hops, the region that is allowed to hop to may be small, giving the node fewer neighbours it can make a hop to. This means within the new definition that this node has a low degree. Adding more hops will result in more possible nodes to hop to, increasing the degree of the nodes and this way also increasing the minimum degree of the whole graph. But when we reach the situation with a large number of hops where all hops are not restricted to go to a specific region (except to the half circle closest to the destination), adding another hop will no longer increase the degree of the nodes. Therefore it will also not increase the connectivity of the graph. The connectivity might even deteriorate, since a path consisting of more consecutive hops has a smaller probability of success.

6.2 Optimal number of hops

Because it is impossible to calculate the expected number of existing paths for all possible numbers of hops, an algorithm has to be used to track the optimal number of hops within a considerably short time. A good way to start is to make a prediction of the number of hops that will probably be optimal. With the optimal number of hops, we mean the number of hops that maximizes the expected number of existing paths consisting of this number of hops. If the graph is expected to have many existing paths, the network it represents will have many options for sending data from a source to a destination. The more options there are, the better the performance of the network can be expected to be. The network might want to use multiple paths for transferring data between users, and the more paths are used, the more reliable a transmission will be. We will consider these properties of the network in more detail later.

A network might not only use paths of the same length. It is just as well possible to send data over paths of different lengths. The maximum number of paths between two nodes will probably consist of a few short paths and a large amount of long paths. When only considering the expected number of existing paths with a fixed number of hops, we disregard this. Nevertheless, it will probably be a good estimate. This can be seen as follows. When using a large number of hops, many paths can be found. If many paths exist, then almost all the neighbours of the source and destination node will be included in one of the paths, probably all in a different path. When short paths are also included, these paths will almost look like straight lines between the source and destination in the graph representation. A longer path probably will look more like a curve towards the destination. Even if the short path would include some more nodes, so consist of more hops, the long paths could still exist as a curve around it. This shows that only using long paths of the same length probably will lead to almost the same number of existing hops as when using a mix of paths with different lengths. Figure 6.7 gives an illustration.

In the upper picture the paths have different lengths, whereas in the lower picture all paths have the same length. Since we assume that the number of nodes will tend to infinity (7), there will probably be plenty of possibilities of adding an extra hop to the shorter paths, leading to almost the same result that is obtained when all paths are assumed to have the same length (6). The probability that a long path exists is smaller than that a short paths exists, so we might slightly underestimate the expected number of existing paths, but a good approximation will be found. It may be interesting to investigate the influence of assumption (6) that all paths have the same length.

It is clear that there is a minimum number of hops required to reach the destination, but it will also be clear that a large number of hops will not lead to a good result. As was seen, the probability that a path of h hops exists is equal to p^h, which will grow smaller with increasing h. Using the minimum number of hops will lead to a small number of possible paths, but with the highest probability that such a path exists. As was seen in the example introducing the problem, the number of paths will at first grow fast when adding another hop, because the region that can be hopped to will be much bigger. But adding an extra hop when the number of hops was already large will have less influence, since the region that can be hopped to is almost maximal already. We expect that the optimal number of hops will be close to, but will not exactly be, the minimum number of hops. An algorithm to find the optimal number of hops can then simply start with the minimum number of hops and keep adding a hop until we see that the expected number of existing paths decreases after adding a hop. We then have found the optimal number of hops, being the number of hops used previously.

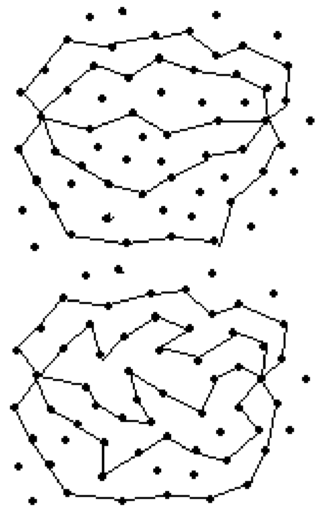


Figure 6.7: Equal number of hops

The problem of finding the optimal number of hops h_{opt} can be written as

$$\begin{aligned} & \max ET_d(h), \\ \text{such that } 0 \quad < \quad & h < n-1. \end{aligned}$$

To find the optimal number of hops, the following algorithm can be used.

Algorithm 2 *Calculation of the optimal number of hops h_{opt}.*

Step 1 *Calculate the minimum number of hops*

$$h_{\min} = \left\lceil \frac{l}{r} \right\rceil,$$

where l is the distance between the source and destination, and r is the range of a node.

Step 2 *Set $h = h_{\min}$ and use algorithm 1 to calculate the expected number of existing paths $ET_d[h]$.*

Step 3 *Calculate $ET_d[h+1]$.*

Step 4 *If $ET_d[h+1] > ET_d[h]$ then set $h = h+1$ and go to step 3 else $h_{opt} = h$.*

See Appendix A.7 for the proof that this algorithm converges to the optimal solution.

When the optimal number of hops is known for a connection between two nodes that has to be established, we can get a better view of the performance of the network that can be achieved. We will consider the performance of the complete network, as well as the performance of a single path inside this network. The number of hops used will have a big influence on the performance of the network, since every hop stands for a transmission, with new risks of delay or failure.

Chapter 7

Network performance

To determine the performance of the network as a whole, we will look at the properties of the nodes, taking into account the influence of their neighbourhood. This influence of the other nodes is the interference caused by these nodes. There are different assumptions that can be made. To get a good view on the effect that the neighbourhood has on a node itself, we can assume that all nodes are situated according to a grid.

Assumption 14 *Nodes are situated according to a grid.*

This will make it easier to determine the influence of each node on another node because nodes can be considered to be equal and the distance between the nodes is known. In the following we will only consider the Basic transmission mode for the nodes. If the RTS/CTS mode would be used, some of the possible difficulties as presented may be omitted. To show what possible problems should be taken into account in a general situation we use the Basic mode. We will acquire the probability that a node is transmitting and the probability that this transmission is successful. We will extend these results on the performance of the nodes to find the properties of the complete network, leading to a view on the performance of the network.

We will start by considering the influence nodes have on each other when all nodes are placed on a grid (14). We want the nodes to be equidistant and will assume that nodes can transmit only within a specific range (3).

Assumption 15 *Nodes on the grid are equidistant.*

The range can be depicted as a circle surrounding the node. All nodes within the range of a node are called neighbours of this node. This gives the situation as in Figure 7.1 for multiple nodes, when looking at the scenario considering the maximum number of equidistant neighbouring nodes, as discussed in Appendix A.8.

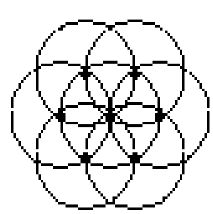


Figure 7.1: Grid construction

Continuing this grid in all directions gives a hexagonal grid, which we shall use and will simply denote by the grid. When using this grid, we can consider the number of nodes causing interference, knowing that there are multiple hops necessary to reach a specific destination. Therefore we need to consider different situations.

The first situation we will take into account is that a node is only allowed to send, when no other nodes within its direct range are sending.

Assumption 16 *Nodes within range of a transmitting node are blocked.*

This will give the situation shown by Figure 7.2.

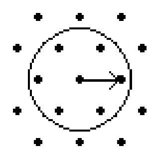


Figure 7.2: Blocked by sender

The node that is sending will cause that all nodes within the circle (the range of the node) hear the node transmitting, not only the intended receiving node. This way these nodes know that a neighbour is transmitting and that they are not allowed to transmit at the same time. We will assume that in all situations we consider, a node will know whether it is blocked or not.

Assumption 17 *A node always knows whether it is blocked or not.*

A problem we will encounter is that it is possible that a node just outside the range also wants to transmit, as depicted in Figure 7.3.

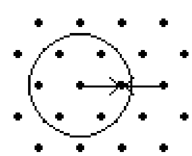


Figure 7.3: Possible collision

There is no constraint that says it is not allowed to, so it will start transmitting. This transmission will cause a collision at the node already receiving data from the other transmitting node. This causes both transmissions to fail and both nodes will backoff and try to transmit at a later stage. This problem is known as the hidden terminal problem. It can also occur that a certain node within the range wants to transmit to a node outside the range. This transmission will not be allowed even though it may not have caused a collision at a receiving node.

The question that needs to be answered is how to find the probability that a collision occurs or that a node is unnecessarily blocked. This will depend on multiple factors. To get a better view of the situation, we assume that all nodes are equal (8), meaning that they will transmit and receive at the same power. Furthermore we assume that the total network is the same at every spot, meaning that there are no special boundary conditions (2).

Taking these assumptions into account, we can look at a single node in the network and see what the influence of the nodes within its range will be. First we will look at the number of nodes that will not be allowed to send. Suppose that the range of a node contains a (hexagonal) rings. Each ring consists of $6i$ nodes, with i counting from the center node. The total number of nodes that is blocked will be

$$N_b = \sum_{i=1}^{a} 6i = 3a(a+1).$$

Of these nodes, there is a fraction that unnecessarily is not allowed to send. The amount of unnecessarily blocked nodes depends on the positions of the node that is transmitting and the receiver. It is hard to determine the expected number of unnecessarily blocked nodes and to calculate the effect of this on the throughput of the network. A different approach therefore is presented to avoid the problem of unnecessarily blocked nodes.

This approach is to let go of assumption (16) and assume that when a node transmits, all nodes within range of the receiver are blocked.

Assumption 18 *Nodes within range of a receiving node are blocked.*

To accomplish this, the node that receives the request to transmit from its neighbour will notify its neighbours. It is still possible that collisions will occur, but there are no unnecessarily blocked nodes. This can be seen in Figure 7.4.

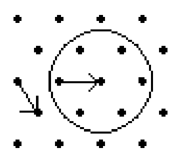


Figure 7.4: Blocked by receiver

The nodes within the circle (the range of the receiving node) are not allowed to send. If any of these nodes would start transmitting, a collision would occur at the receiving node. This shows that none of the nodes is blocked unnecessarily. But suppose that the node just outside the circle will also start receiving signals, as depicted in Figure 7.4 by the arrow outside the circle. The node that wants to send is not blocked and will start transmitting. The receiving node will receive the signals from both depicted sending nodes, which causes a collision and the loss of both signals. Using this approach, it is considerably easier to calculate the influence of other nodes, since we only have to take the probability of collisions into account.

Nevertheless, it may not be known a priori which path should be followed to transmit to the final destination using multiple hops. A specific node that is used to pass on the signal, a relay node, could be any neighbour of the transmitting node. When a node wants to transmit, it will notify all its neighbours it wants to start sending, but now all these nodes will have to notify their neighbours that they might start receiving signals. So all nodes neighbouring these nodes will also be blocked. We thus let go of assumption (18) and assume that nodes within twice the range of a sending node are blocked.

Assumption 19 *Nodes within twice the range of a sending node are blocked.*

This will lead to the situation as depicted in Figure 7.5.

The first ring is blocked because it is within range of the sender. The second ring will also be blocked since these nodes have the nodes in the first ring as neighbours. It will be clear that in this way, many nodes are unnecessarily blocked again. Take for instance the leftmost node within the second ring. If it would like to transmit to a node on its left, there would be no collision. Nevertheless this node is blocked and so it is not allowed to transmit.

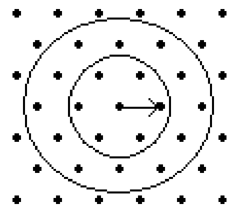


Figure 7.5: Blocking twice the range

When the range of a node contains a rings, all nodes of the surrounding $2a$ rings will be blocked, assuming that a ring is either completely included or excluded. This shows that the first node that can cause interference is at a distance of $(2a + 1)\Delta$, when the side of a hexagon is Δ. A problem we then encounter is that we should not calculate the interference for a node that is sending data, but for a node that is receiving data. For a receiving node, the signal over interference ratio C/I, the rate between the strength of the signal that should be received and the strength of all other signals that are also reaching the receiving nodes, should be higher than a threshold value for a successful transmission of the signal. The difficulty is that the circles showing the blocked nodes are not situated around this node. The number of interfering nodes at the same distance is therefore hard to calculate. The distance between the sender and receiver and the range of a node have a big influence on the interference a node will undergo. To avoid this complication, we let go of assumption (19) and we assume that the rings of blocked nodes are situated around the receiving nodes, as depicted in Figure 7.6.

Assumption 20 *Nodes within twice the range of a receiving node are blocked.*

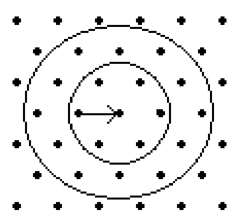


Figure 7.6: Blocking twice the range

It might be difficult for a network to notify all the nodes around a receiving node that they are blocked. If the blocked nodes are situated around the sender, these nodes can hear the transmission of the sending node and will know that they are blocked. But for a receiving node this is slightly different. When looking at a complete network, most of the traffic will be caused by relaying. To get data transferred from a source to a destination, many relay nodes may be used to accomplish this. This means that these nodes will be receivers as well as transmitters. Assuming that the rings of blocked nodes are situated around a receiving node instead of a transmitting node will have an influence, but we will assume that this can be neglected.

Assumption 21 *The influence of assuming that nodes are blocked around a receiver in stead of a sender can be neglected.*

Later on we will only consider the influence of nodes within range of a receiving node, and the amount of nodes within range is expected to be the same for any node, since all nodes are equal and independent (8). Considering the expected amount of nodes around a sending node or a receiving node will hence be of no influence. To have a better look at the situation we will consider the optimal and worst case scenarios.

7.1 Worst case scenario

The first way we assumed that a network can work (16), is that a node is only allowed to transmit when none of its neighbours is transmitting. This means that when a node starts to transmit, it will notify all its neighbours that they are not allowed to start transmitting until the transmission is finished. The blocking of nodes around the transmitter will have an influence on the interference that the node that is receiving the data will undergo. There are multiple possible situations for the nodes that cause interference, as depicted in Figure 7.7.

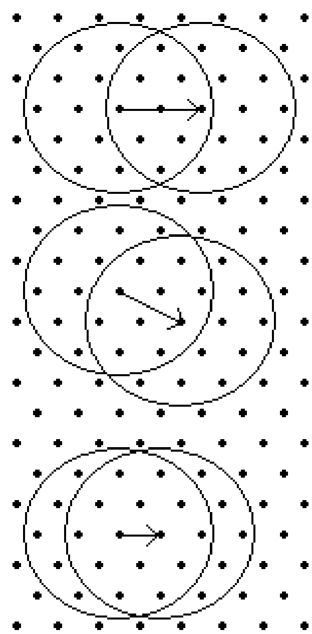


Figure 7.7: Possible situations

In this case, where the range is taken to be two, we see that the number of interfering nodes depends on where the transmission is made to. In the first situation there can be interference from three nodes within the first ring around the receiver, which would cause a collision. In the second situation there are only two nodes from the first ring that can cause interference, so in this situation there will be less interference. The third situation shows that no nodes from the first ring can cause interference. In the network, all of these situations might occur. It will be impossible to keep track of all these situations and take all of them into account separately. Therefore we assume that we are always dealing with the first situation, since in this situation the interference will be highest, giving the worst case scenario.

Assumption 22 *We only consider the worst case situation for interfering nodes.*

Depending on the range of a node, the number of blocked nodes in a ring around the receiver can be calculated. The following table gives a view on this dependency.

range/ring	1	2	3	4	5
1	3	3	0	0	0
2	3	5	5	5	0
3	3	5	7	7	7
4	3	5	7	9	9
5	3	5	7	9	11

Table 1: number of blocked nodes per ring , depending on the range

This leads to the formula for the number of blocked nodes in each ring,

$$N_b(a,r) = \begin{cases} 2r+1 \text{ if } r \leq a \\ 2a+1 \text{ if } a < r \leq 2a \\ 0 \text{ if } r > 2a \end{cases},$$

where r is the ring number and a is the range (in number of rings) of the nodes. For the number of nodes N_i in a ring that might interfere we find that

$$N_i(a,r) = \begin{cases} 4r-1 \text{ if } r \leq a \\ 6r-2a-1 \text{ if } a < r \leq 2a \\ 6r \text{ if } r > 2a \end{cases}.$$

What we would like to know is the probability that a node will cause a collision. In the descried situation, this will occur when a node within range of the receiver that is not blocked, will start transmitting. This probability depends on the number of nodes N_c within this range that are not blocked, which depends on the range of the nodes and can be written as

$$\begin{aligned} N_c(a) &= \sum_{r=0}^{a} 4r-1 \\ &= (2a+1)a. \end{aligned}$$

It follows that for a higher range, the probability of a collision will grow, since the number of nodes that can cause the collision grows. We should nevertheless also take into account that it is not possible that all these nodes cause interference at the same time. When one of the nodes starts transmitting, other nodes will be blocked again. For a higher range, the number of nodes that can simultaneously cause collisions nevertheless is equal or higher than for a lower range.

To avoid collisions, we also considered the situation that all nodes within a range of $2a$ around the transmitting node are blocked (19). This way all nodes within range of the receiver are not allowed to transmit. Avoiding collisions may seem a good approach, but then we have to take into consideration that nodes might be blocked unnecessarily. Nodes may be blocked that do not have to be blocked if their receiving node is not within range of the sender. Collisions do not have to be a problem, as long as they do not occur at the nodes that need to receive the signal correctly. This is made clear in Figure 7.8.

If the situation would be like the situation depicted in the upper part (the range is set at one) and both nodes will transmit, then both signals will be received correctly. But when the same nodes are transmitting but with the intended receiving nodes as depicted in the lower part, a collision will occur. In the first scenario one would say the node is blocked unnecessarily, but as the second situation shows, this does not have to be the case. Since it may not be the case that nodes know a priori in what direction they have to send their data, we conclude that in this case no nodes are blocked unnecessarily.

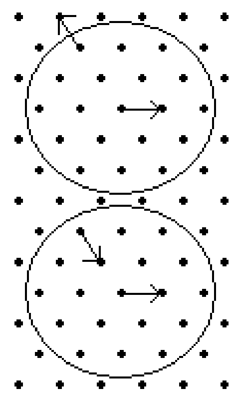


Figure 7.8: Possible collision

7.2 Best case scenario

To make a comparison between different approaches, the best case scenario is used to show how far the results of these approaches are apart from the optimal situation. To classify the situations we will consider the throughput, which is in our opinion more important than the probability of collisions or the capacity. Focussing on the throughput of the total network, we want to know the expected number of nodes that can transmit at the same time, without causing collisions.

Lets look at a strange but optimal situation when the range is one:

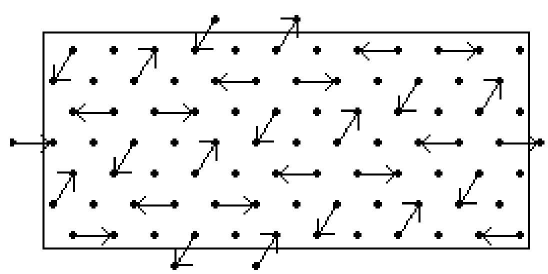


Figure 7.9: Best case scenario

We see that from the 84 nodes in the square, there are 28 nodes transmitting. This means that of every three nodes, it is possible that one node is transmitting, without causing a collision that will cause the loss of data for the node that needs to receive it. There will be many collisions, but only at places where the data of the colliding transmissions is not needed.

Nevertheless, the paths connecting a source and destination normally will not allow the setup as shown in Figure 7.9. And since multiple paths will probably be used, the flow of the data will often be in one direction. We then would get the situation presented in Figure 7.10 as an optimal situation.

In this case, from the ten nodes in the square, there are two nodes transmitting. This means that of every five nodes, it is possible that one node is transmitting. This is a better approximation for the best case scenario when considering a realistic network. Still there are collisions, but again none of them will cause an intended receiver to fail to receive the data correctly.

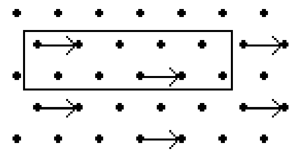


Figure 7.10: Best case in one direction

If in a network no collisions are allowed at all and the direction of the flows is expected to be in the same direction, then we would find the situation as depicted in Figure 7.11.

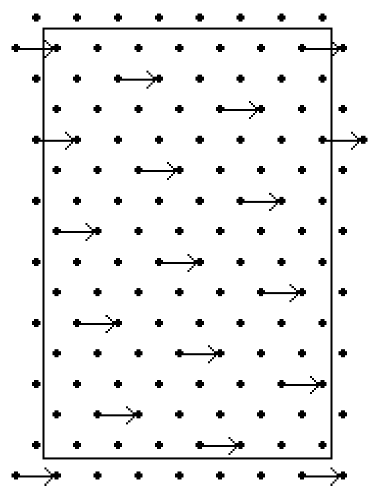


Figure 7.11: Best case with no collisions

In this case, from the 98 nodes, there are 14 nodes transmitting. This means that of every seven nodes, it is possible that one node is transmitting. There are no collisions at all, so this approach can be applied in any network. Note that in this case, the direction in which the data is sent does not effect the number of nodes that can transmit at the same time.

7.3 Comparison

A comparison can now be made between the worst and best case scenarios. How many nodes are allowed to transmit at the same time, and can they cause a collision? When we use the first approach that all nodes within range of the transmitter are blocked (16), we find (when the range is one) that of every four nodes, one node is allowed to transmit. As we will show later, it will be the case that if all these nodes are transmitting at the same time, not a single node will receive the data correctly because of collisions, regardless of the directions. This was not the case for the best case scenario when the directions where to transmit to were known and ideally placed. It is therefore clear that blocking the nodes around the receiver will not work well and hence should not be used.

If we consider the situation where we block nodes that are within a range of $2a$ around the transmitting node (19), we find that only one out of nine nodes is allowed to transmit at the same time. The best case scenario when no collisions are allowed can also be achieved, leading to one out of seven nodes transmitting at the same time. This approach can hence be used, but as noted earlier it is better to assume that the blocked nodes are situated around a receiving node.

In the situation that all nodes are blocked within the range of a receiving node (18), collisions might occur, but no nodes are blocked unnecessarily. When nodes are blocked within a range of $2a$ around the receiving node (20), then no collisions will occur, but nodes might be blocked unnecessarily.

There appear to be multiple possible approaches for the blocking of nodes. How well these approaches work, depends on the transmission of signals by the nodes. We consider two possible ways the network can work, to investigate the influence of neighbouring nodes. We either assume that all nodes that are allowed to transmit at the same time, meaning that they are not blocked, will actually do so, or we assume that a node will only transmit if it will not cause a collision. For these situations, the influence of neighbouring nodes on a specific node will be different.

7.4 Permanent transmission

If all nodes that are allowed to transmit at the same time will actually do so, the interference will be the same at any moment in time and can be considered as a constant.

Assumption 23 *Nodes will transmit if they are not blocked.*

This makes it easy to determine the signal over noise ratio. This ratio depends on the path loss exponent, describing how the influence of a signal decreases over a distance, and the range of the nodes. Using the proposed grid, the number of nodes interfering at the same time can be calculated for the worst case scenarios. When considering the situation that the nodes within a range of $2a$ around the transmitter are blocked, we found that collisions would not occur, but maybe the interference can still be high. We will now look further into this.

In the first interfering ring around the receiving node, there can be two nodes transmitting at the same time. The second ring can also contain two transmitting nodes, and the third ring can contain four. These rings will be at a distance $(ai+1)\Delta$, where a is the range and i ranges from 1 to 3. The distance between the transmitter and the receiver will be $a\Delta$. The signal over interference ratio when taking only the three closest interfering rings into account will therefore be equal to

$$C/I = \frac{\frac{1}{a^\eta}}{\left(\frac{2}{(a+1)^\eta} + \frac{2}{(2a+1)^\eta} + \frac{4}{(3a+1)^\eta}\right)}.$$

When plotting for the values of $\eta = 2, 3, 4$, we find the following results:

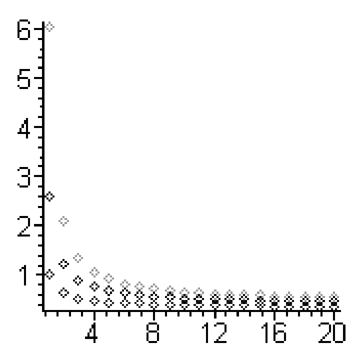


Figure 7.12: C/I plotted against a

In Figure 7.12 the lines from low to high are for $\eta = 2, 3$ and 4 respectively, with a on the x-axis and C/I on the y-axis. We see that when the range is small, the interference will be low compared to the received signal. For a high range, interference will cause the C/I ratio to drop, so that the signal will not be received correctly. This shows that even a blocking ring of $2a$ around the transmitter will not be sufficient to reduce interference far enough to avoid unsuccessful transmissions when making the assumption that all nodes will transmit if they are allowed to (23). Therefore we will have to use a different approach.

In stead of assuming that all nodes will transmit when they are allowed to and look at the interference this causes, we can assume that all nodes that want to transmit, will only do so if they are not expected to cause a collision. Whenever a node wants to transmit, it will first check if other nodes are transmitting.

Assumption 24 *Nodes will only transmit if they do not expect to cause a collision.*

Whenever this is the case, a node will wait until all transmissions have ended. After this, the node will wait for a set period (DIFS) to make sure that there really are no other transmissions. Then a backoff time is drawn from a discrete uniform distribution between certain values, the contention window, and this time will be counted down as long as there still are no other signals. When the countdown is completed, transmission will start. This way, collisions can only occur when two nodes finish counting down at the exact same moment. The probability that this will happen depends on the size of the contention window. There can be two different ways of transmitting. Either the node will simply start transmitting its data, or it will send a request to send (RTS) to see if the receiver is really ready to receive the data. If in the latter case the receiver replies with a clear to send (CTS), the node will start transmitting the data packets. For both types of transmission we will analyze the performance measures of the network.

7.5 Persistent flows

When assuming that there are persistent flows, meaning that all nodes constantly want to transmit (but will only do so if there are no other signals), the state space is as shown in Figure 7.13.

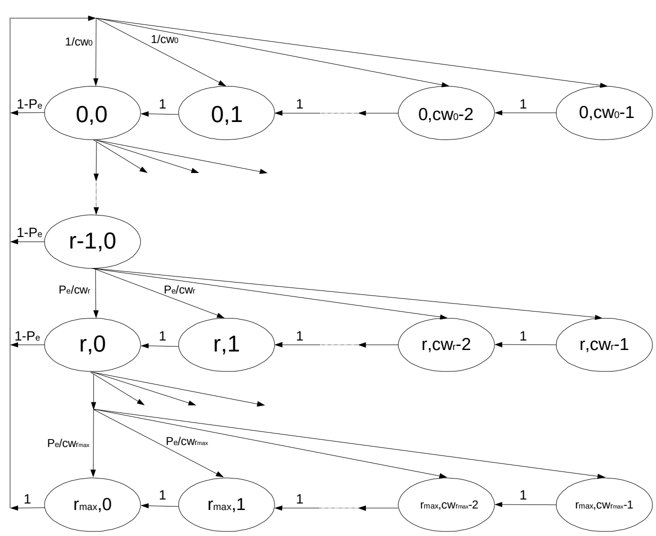


Figure 7.13: State space

Assumption 25 *Nodes always want to transmit (Persistent flows).*

There are only a finite number of states in which a node can be, irrespective of the transmission mode.The states in this statespace are presented as a couple (i, j), where i the number of times a transmission has been attempted and j is the time that it still has to wait until the node will start transmitting. A node will either be counting down or transmitting. If the node senses at some point that another node started to transmit, it will simply wait and continue the countdown when the channel is free again, so this does not affect the state space as depicted. When the backoff time ends, the transmission starts. If the transmission is successful, the process ends and repeats itself, but if the transmission fails, a new backoff time is taken. If after $r_{\max}$ attempts the transmission still fails, the system resets and will drop the data it wanted to send, after which the process repeats itself. It is also possible to consider a setup that after $r_{\max}$ attempts the process will remain in the same state until the transmission is successful, but we will not consider this.

When will a transmission fail? A node will not transmit whenever it senses that another node in its neighbourhood is receiving. So the only way a collision can occur, is when the backoff times of multiple nodes end at the same time and a receiving and transmitting node (other than the one it is receiving data from) are within each others range. Transmissions will then start at the same time and the transmitted data will not be received correctly. This leads, using the depicted state space, to the following formulas for the probability that a node is transmitting and that such a transmission fails.

$$\begin{aligned} P_t &= \sum_{r=0}^{r_{\max}} \pi(r,0) = \pi(0,0) \sum_{r=0}^{r_{\max}} P_e^r = \frac{1-P_e^{r_{\max}+1}}{1-P_e}\pi(0,0) \qquad (7.1) \\ \pi(0,0) &= \frac{2(1-P_e)}{(1-P_e^{r_{\max}+1}) + (1-P_e)\sum_{r=0}^{r_{\max}} cw_r P_e^r} \end{aligned}$$

Here $\pi(i, j)$ is the probability of being in state (i, j), P_t is the probability that a node is transmitting (either successfully or not) and P_e is the probability that a transmission fails. For this probability of failure we find that

$$P_e = \sum_{k=1}^{N} B(N, P_t, k)(1 - P_r), \qquad (7.2)$$

with

$$P_r = \frac{\binom{N-m-1}{k-1}}{\binom{N-1}{k-1}},$$

where $B(N, p, k)$ stands for the binomial distribution giving the probability of k successes out of N tries, with probability p of a success. There are N nodes in the total network, so $B(N, P_t, k)$ is the probability that there are k nodes transmitting, and thus k nodes receiving. P_r calculates the probability that none of these nodes is within range of the specific node we are assuming that wants to transmit. A transmission may still not be successful for this node when the interference reaches a level that is too high. The probability that this happens can be investigated and if this is high, we can also take that a bigger range is required to not allow nodes to start transmitting. For the remainder of this thesis we assume that blocking nodes within range of the receiver will be sufficient.

Assumption 26 *Interference is low enough for a successful transmission if none of the nodes within range of the receiving node is transmitting.*

To be able to make use of the grid model, we assume that N can be written as $3y(y + 1)$, where y is the total number of rings and that each node we consider can be viewed as the center of these rings. This way the total network only consists of complete rings.

Assumption 27 *The network consists of complete rings.*

The probability that the $k-1$ other receiving nodes are not within range, is the same as the probability that they are placed within the $N-m-1$ nodes that are not the considered node itself and the number of m nodes within its range, assuming that m is deterministic.

Assumption 28 *The number of nodes within range of a node is deterministic unless stated otherwise.*

When the nodes are situated according to the grid and a node has range a, then m will be equal to $3a(a+1)$. Since there are $\binom{N-1}{k-1}$ total possibilities to divide these other $k-1$ transmitting nodes over the network, and $\binom{N-m-1}{k-1}$ ways to place them so that these nodes are not within range, the probability of a successful transmission of a specific node when a total of k nodes are transmitting is given by P_r.

$$P_r = \frac{\binom{N-m-1}{k-1}}{\binom{N-1}{k-1}}$$

If m is stochastic we can use

$$P_r = \frac{\sum_{m=0}^{N} \binom{N-m-1}{k-1} P(M=m)}{\binom{N-1}{k-1}},$$

where $P(M=m)$ then stands for the probability that the number of nodes within range is equal to m. This can be found from the distribution of the nodes. We will work with a deterministic value of m in the remaining part of the thesis unless indicated otherwise.

Since y and a are known (and therefore also N and m) for a specific network with nodes situated according to a grid, the above equations can be used together to acquire the values for P_t and P_e. When the values for P_t and P_e are known, it is possible to have a look at the complete network.

7.6 Network properties

For a complete network, multiple properties can be determined. Because the probability that nodes are transmitting and the probability that a transmission is successful can be calculated, the expected number of transmitting nodes and the expected number of successful transmissions can also be calculated. Using these results, we can find the expected throughput of the network, when assuming that the properties for all nodes are equal (8).

The expected number of nodes transmitting at the same time is found from the binomial distribution. The probability that there are k nodes transmitting at the same time is $B(N, P_t, k)$, with mean NP_t, the expected number of nodes transmitting at the same time. Not all of these transmissions will be successful. The probability that there are s nodes transmitting successfully at the same time is given by

$$\begin{aligned} P(s \text{ nodes successfully transmitting}) &= \sum_{k=s}^{N} B(N, P_t, k) B(k, P_s, s) \\ &= B(N, P_t P_s, s). \end{aligned}$$

Here we use $P_s = 1 - P_e$. The assumption (8) that all nodes are equal is used here in the sense that the probability of a successful transmission is equal for all nodes and independent of each other. In reality this will not be the case. When for instance there are $N-3$ nodes transmitting, not all of them can be successful, because these $N-3$ nodes cannot be placed in such a way that none of them are neighbours. Furthermore it is also impossible that only one transmission is unsuccessful, since in a collision there are always two or more transmissions involved that will all fail. The impact of the assumption will nevertheless be small because these situations will only occur with a small probability, as shown in Appendix A.9, and so we stick to this model. The expected number of successful transmissions is given by

$$\sum_{s=0}^{N}\sum_{k=s}^{N} B(N,P_t,k)B(k,P_s,s)s = NP_tP_s.$$

From the viewpoint of a single node when there are N persistent flows, the data throughput will be equal to

$$R_{flow}(N) = \frac{\frac{1}{N}\sum_{s=0}^{N}\sum_{k=s}^{N} B(N,P_t,k)B(k,P_s,s)sE(P)}{B(N,P_t,0)\tau + \sum_{k=1}^{N} B(N,P_t,k)[\sum_{s=1}^{k} B(k,P_s,s)T_s + B(k,P_s,0)T_c]}.$$

The expected aggregate data throughput, the throughput of the complete network, will then be

$$R(N) = \frac{\sum_{s=0}^{N}\sum_{k=s}^{N} B(N,P_t,k)B(k,P_s,s)sE(P)}{B(N,P_t,0)\tau + \sum_{k=1}^{N} B(N,P_t,k)[\sum_{s=1}^{k} B(k,P_s,s)T_s + B(k,P_s,0)T_c]}. \quad (7.3)$$

Here we use that the throughput is equal to the event rate $\times$ expected number of successful transmissions $\times$ expected transfer volume. The event rate is equal to $\frac{1}{(\text{inter event time})}$, which is separated into three parts. The first part is when no nodes are transmitting, the second part is when at least one transmission is successful and the third part is when no transmission succeeds. When no nodes are transmitting, the system is idle for a slot duration τ. When there is at least one successful transmission, the network will be busy for a time T_s. If all transmissions fail, the network will be busy for a time T_c, which is smaller than T_s. When there are multiple transmissions, all transmissions will occur at the same time, so the system will be busy for the time that a successful transmission is taking place, even if some of the transmissions fail. Only when all transmissions fail, the system will be busy for a time T_c. The times T_s and T_c can be calculated as follows:

$$\begin{aligned}
T_s^B &= r_{net}^{-1}E(P) + \delta + SIFS + ACK + \delta + DIFS \\
T_c^B &= r_{net}^{-1}E(P) + \delta + DIFS \\
T_s^{RC} &= RTS + \delta + SIFS + CTS + \delta + SIFS + \\
&\quad r_{net}^{-1}E(P) + \delta + SIFS + ACK + \delta + DIFS \\
T_c^{RC} &= RTS + \delta + DIFS,
\end{aligned} \quad (7.4)$$

where T_*^B and T_*^{RC} stand for the Basic and RTS/CTS mode respectively, r_{net} is the channel rate, $E(P)$ the expected data volume, δ the propagation delay, SIFS the time needed to transfer from receiving to transmitting mode or the other way around, DIFS the waiting period until the next transmission starts, RTS the time needed for the request to send and CTS the time needed for the clearance to send. The length of the SIFS must be shorter than the DIFS to make sure that no other node will start transmitting while the process is still going on. Using the basic mode, a failure will be noticed when the acknowledgement does not come back, whereas for the RTS/CTS this already is the case when no CTS is received. This drastically reduces the time lost when a collision occurs since an RTS takes much shorter to send than the expected data volume, but the time needed for a successful transmission will be longer.

7.7 Non-persistent flows

The assumption (25) that all nodes will constantly want to transmit is not appropriate for an ad hoc network. Therefore we will now let go of this assumption, but will still assume that the throughput per node in the network will be $R(N)/N$ when there are N nodes in the network (5).

The network can then be considered a service center that is serving flows at various rates, as if its a Processor Sharing queueing model with state dependent service rates. When more nodes are transmitting, more data is sent and the probability of a collision increases. This has an influence on the value of $R(j)$, with j the number of nodes transmitting. The number of nodes transmitting at the same time is limited by the total number of nodes inside the network. Assuming (4) that new flow transmissions start according to a Poisson process with rate λ, we can find the equilibrium distribution of the number of flows simultaneously in progress, which is given by

$$\pi(n) = \frac{\rho^n \varphi_n}{\sum_{j=0}^{N} \rho^j \varphi_j}, \tag{7.5}$$

with

$$\varphi_n = (\prod_{j=1}^{n} \frac{R(j)}{r_{net}})^{-1},$$

where $\rho = \frac{\lambda}{\mu r_{net}}$, with $\frac{1}{\mu}$ the average data size [Cohe79]. This way we can calculate the expected number of flows present in the network and, using Little's formula, the expected flow transfer time

$$T = \sum_{n=0}^{N} \frac{n\pi(n)}{\lambda(1-\pi(N))}. \tag{7.6}$$

The conditional expected transfer time for a flow of given size x can be determined explicitly and grows linearly in x.

$$T(x) = \frac{x}{r_{net}} \sum_{n=0}^{N} \frac{n\pi(n)}{\rho(1-\pi(N))}$$

This gives the result for the situation where nodes want to transmit using a single hop according to a Poisson process with rate λ. But a transmission may not have finished after one hop, so the results obtained so far do not show anything about the throughput of the paths. Since users will be interested in the time it is expected to take for the data they transmit to arrive at the destination, this is an interesting topic to investigate. We will have a look at different situations that will influence the throughput of a path.

Chapter 8

Throughput

We have derived the formulas to calculate the throughput for a network using single hops. But with one hop the transmission may not yet be completed. When a path consists of multiple hops, the data has to be forwarded. This can be done in different ways, leading to different results for the network. How long will a transmission over a path consisting of multiple hops take and what throughput can the network achieve then? To give an answer to these questions, some aspects have to be investigated, since multiple factors are of influence.

8.1 Possibilities for paths

For an ad hoc network a transmission between a source and a destination will not be called successful until the data has reached this destination, probably needing multiple hops. The model described before only gives information about the success probability of a single hop. Now we will focus on the impact of needing multiple hops. There are several possible scenarios for sending data through an ad hoc network. We will shortly describe three of them.

1. Constant flows (CF): Whenever a node wants to transmit to a destination, a whole path between a source and destination is given free so that the second node on the path will start transmitting to the third as soon as it has received the data from the first node. This way the data will be transferred as fast as possible, but many nodes are blocked.

2. Multiple flows (MF): Whenever a node receives data that has to be forwarded to another node, it will act as if it wants to start transmitting itself. If the network is busy the node will simply wait until the network becomes free again.

3. Data saving (DS): Whenever a node receives data that has to be forwarded to another node, it will wait until it wants to transmit on its own, and then will send all the data currently waiting, together with its own data.

There are more possibilities, but these three are appropriate scenarios that can be used in communication networks. The proposed scenarios will give completely different results and may be practical in different situations. Scenario one is the only possible one if you consider a telephone network. The data has to be received instantly and cannot wait. But if for instance we are considering a network where email is delivered, both other scenarios are possible. Delay of the data that has to be transmitted is then of lesser importance, and so the requirements are lower. The difference between these scenarios is that in scenario 2 a node will want to transmit more often, whereas in scenario 3 a node will transmit more data at once. This way scenario two may lead to more collisions in comparison with scenario 3, but with a faster expected delivery time.

We will only consider a data network where delay is allowed, so scenario 1 will not be discussed. Considering scenarios 2 and 3, the model described previously can be used, making the adjustment that there are multiple hops involved. The difference will be found in either the rate at which nodes want to send or the amount of data a node wants to send. Both will depend on the expected number of hops needed for a transmission. Using scenario 2, the expected rate at which nodes want to transmit will be λh if the expected number of hops for a transmission is equal to h and the amount of data per transmission is expected to be $\frac{1}{\mu}$. This is the case since for a path with h hops, there are $h-1$ relay nodes. The probability that a specific node is a relay node for another node, will then be $\frac{h-1}{N-1}$. There are $N-1$ other nodes, so on average a node will be a relay node for another node $h-1$ times. And together with his own transmission, also at a rate of λ, the total comes to λh, not influencing the expected amount of data in one transmission, since this is equal for all nodes. For scenario 3 we find that the transmission rate λ will stay the same, but the amount of data is expected to be $\frac{h}{\mu}$, since it is expected that a node has to transmit $h-1$ data amounts as a relaying node, together with the data of its own transmission. Both approaches will result in a utilization of $\rho = \frac{\lambda h}{\mu r_{net}}$, but may lead to different characteristics for the network.

When we consider the expected time T_{total} it takes for a data packet to arrive at its destination, this time, when a packet is immediately forwarded, will be equal to

$$T_{total}^{MF} = [T_{transfer}^{MF}]h,$$

where T_{total}^{MF} stands for the total time it takes for a data packet to arrive at the destination using the MF model, $T_{transfer}^{MF}$ is the expected amount of time one hop takes using the MF model and h is the expected number of hops in the path. The expected time of one hop can be calculated using formula 7.6 with the adjustments made to the transmission rate to include the property that data will be forwarded immediately. We can simply multiply with h because the expected time will be the same for every hop and every hop is independent of the other hops, according to the assumption that all nodes are equal (8).

When we consider the situation that a node will not transmit until it generates own data again, the time for a packet to arrive at its destination will be

$$T_{total}^{DS} = [T_{transfer}^{DS}]h.$$

Here T_{total}^{DS} stands for the expected total time it takes for a data packet to arrive at the destination using the DS model, and we use $T_{transfer}^{DS}$ for the expected amount of time one hop takes using the DS model. Using formula 7.6 again, we also find this expected time. That all data will not be forwarded immediately is taken into account by leaving the transmission rate at λ and that more data packets are sent in one transmission is taken into account by changing μ and the value of $E(P)$. The expected number of packets transferred in one transmission, which we will from now on denote by N_{packet}, will be equal to h. This way the expected time it takes for a packet to reach its destination is longer than for the MF model. The difference nevertheless is that in the DS scenario more packets are transmitted than in the MF situation, so the throughput does not have to be lower.

8.2 Network and path throughput

The throughput of the network is defined as the number of packets the network is successfully transmitting within a frame of time. For the discussed models, the throughput might be different. This depends on the settings of the network, the values for the factors of influence as the transmission rate, the expected data volume and the expected number of hops. We will calculate the expected throughput of the network and of a path, using both models to take into account that a transmission may need multiple hops.

Whereas for the introductory network with transmissions over one hop the throughput of this network was considered, we will now have a look at the throughput of a path, comparing both models for forwarding data. To make a comparison between the two models, we will determine the time it takes for a fixed number of packets to arrive at the destination.

Because of relaying in the MF model, all nodes will transmit with a rate expected to be equal to λh, when there are an expected number of h hops in one path. This will influence the distribution function of the amount of nodes transmitting at the same time. The expected flow transfer time (of one hop) will then be equal to

$$T^{MF}_{transfer} = \sum_{n=0}^{N} \frac{n\pi(n)}{\widetilde{\lambda}(1-\pi(N))},$$

which is equal to the previous model, but now with $\widetilde{\lambda} = \lambda h$, and therefore $\rho = \frac{\lambda h}{\mu r_{net}}$, so the value of $\pi(n)$ will be different. Another change is caused by the dependency of $\pi(n)$ on $R(j)$. Looking at (7.3) again, we see that $R(j)$ depends on the time a transmission takes, shown in the formula by T_s and T_c, which are defined in (7.4).

Considering the DS model, the same procedure can be repeated, but this time the amount of data to be sent is expected to be equal to $\frac{h}{\mu}$, and $\widetilde{\lambda} = \lambda$. The formula for the expected flow transfer time then remains the same. The utilization is equal to that of the MF model. The time of a successful transmission of one hop is again given by (7.4), where the value of $E(P)$ will be different from the other model. This will lead to a different value for $R(j)$. Note that for both models we assume that all data is transferred in one transmission, rather than splitting it into parts and sending each part after another.

Assumption 29 *All data is transferred in one transmission.*

The throughput of a path can be calculated from the expected time a transmission takes over a path and the amount of data that is transferred through it. For both models, a path is simply the product of the hops. The time it takes to transmit packets over a path can therefore simply be found by multiplying the transfer time over a hop by the length of the path. The amount of data transferred will be taken to be known. For the MF model only one packet is transferred and for the DS model we assume that h packets are transferred.

Assumption 30 *In the DS model, a transmission contains h packets.*

From the background we know that for the grid case, the exact hop distribution is known. The assumption is made that a node will make a hop to the best possible next node, so if the range is a, a hop will always be of length a unless the hop to the destination can be smaller.

Assumption 31 *Hops are made to the best possible next node.*

This way the amount of hops needed can be kept at the lowest value. We can then see the network as if divided into normal nodes and relay nodes, always seen from the perspective of the center node. The number of relay rings is equal to $b = \lfloor \frac{y}{a} \rfloor$, with y the total number of rings. The formula for the expected hopcount is given by

$$E[h] \simeq 0.53 N_r^{0.5} + 2(1 - \frac{N_r}{N}), \tag{8.1}$$

where N_r denotes the number of nodes on the relay rings and $(1 - \frac{N_r}{N})$ is the probability that either the source or the destination node is not a relay node. The number of nodes on relay rings, can be found using $N_r = 1 + 3b(b+1)$. The source node is then included in counting the relay nodes. Using this result for the expected number of hops the transfer times can be calculated as well as the throughput.

The values of the factors determining the throughput will be different for both models, but a comparison of both models is now possible by looking at the expected throughput. The formula we will use for the throughput of a path is

$$TH_{path} = \frac{N_{packet}}{hT_{transfer}}, \tag{8.2}$$

with N_{packet} the number of packets transferred, h the number of hops in the path and $T_{transfer}$ the transfer time over a hop. Normally the throughput of a network is defined by the amount of data successfully transmitted within a set amount of time. In our approach this is slightly different since we do not include the probability that the data is lost completely if a transmission at one node fails more then $r_{\max}$ times.

Chapter 9

Non fixed number of active nodes

In a graph representation of an ad hoc network, nodes represent the users. In common graphs the number of nodes is fixed and the neighbourhood of the nodes does not change. Considering that in our case the graph is a geographical representation of the users in the network, this property means that the users stay at the same position. For an ad hoc network, neither of these assumptions have to be realistic. We nevertheless stick with the assumption that nodes do not move (1), but will consider the case where nodes are either turned on or off, so if they are present in the network or not.

Assumption 32 *Nodes in the network are either active or inactive.*

This has an influence on the number of users in the network and therefore on the properties of the network. We investigate the influence of the number of active nodes on the model described earlier when considering that the number of nodes is fixed.

There are multiple ways to model the (dis)appearing of nodes inside the network. We consider the following models:

1. The number of active nodes is given by a Poisson distribution with parameter ρ, leading to the formula
$$P(n \text{ active nodes in the network}) = e^{-\rho}\frac{\rho^n}{n!}.$$
2. The N nodes in the network are active with probability P_a, leading to the formula
$$P(n \text{ active nodes in the network}) = B(N, P_a, n).$$

In the first model, there can be an infinite number of users, whereas the other model assumes that there is a maximum of N users inside the network. For the grid model this means that we have to use the models with a limited number of nodes N, otherwise y, the number of rings, will be infinite, making further calculations impossible. For later use we will consider both possibilities and their effect on the model. This leads to the following results:

1. When the number of nodes is given by a Poisson distribution, the following adjustments have

to be made:

$$\begin{aligned}
P(k \text{ nodes transmitting}) &= \sum_{n=k}^{\infty} e^{-\rho}\frac{\rho^n}{n!}B(n,P_t,k) \\
&= \sum_{n=k}^{\infty} e^{-\rho}\rho^n \frac{P_t^k(1-P_t)^{n-k}}{k!(n-k)!} \\
&= e^{-\rho}\frac{P_t^k}{k!}\sum_{n=k}^{\infty}\rho^n\frac{(1-P_t)^{n-k}}{(n-k)!} \\
&= e^{-\rho}\frac{(\rho P_t)^k}{k!}\sum_{n=k}^{\infty}\frac{\rho^{n-k}(1-P_t)^{n-k}}{(n-k)!} \\
&= e^{-\rho P_t}\frac{(\rho P_t)^k}{k!}.
\end{aligned}$$

The probability that there are s nodes transmitting successfully becomes

$$\begin{aligned}
& P(s \text{ nodes transmitting successfully}) \\
= & \sum_{k=s}^{\infty} e^{-\rho P_t}\frac{(\rho P_t)^k}{k!}B(k,P_s,s) \\
= & e^{-\rho P_t P_s}\frac{(\rho P_t P_s)^s}{s!}.
\end{aligned}$$

2. For the binomial distribution we find:

$$\begin{aligned}
& P(k \text{ nodes transmitting}) \\
= & \sum_{n=k}^{N} B(N,P_a,n)B(n,P_t,k) \\
= & \sum_{n=k}^{N}\frac{N!}{(N-n)!(n-k)!k!}P_a^n(1-P_a)^{N-n}P_t^k(1-P_t)^{n-k} \\
= & B(N,P_aP_t,k).
\end{aligned}$$

For the number of successful transmission we find:

$$\begin{aligned}
& P(s \text{ nodes transmitting successfully}) \\
= & \sum_{k=0}^{N}\sum_{n=k}^{N} B(N,P_a,n)B(n,P_t,k)B(k,P_s,s) \\
= & \sum_{k=0}^{N}\sum_{n=k}^{N}\frac{N!}{(N-n)!(n-k)!(k-s)!s!}P_a^n(1-P_a)^{N-n}P_t^k(1-P_t)^{n-k}P_s^s(1-P_s)^{k-s} \\
= & B(N,P_aP_tP_s,s).
\end{aligned}$$

When considering the consequences of nodes turning on and off, we find that the hexagonal grid will cease to exist if some of the nodes on this grid turn on or off. Of course it is possible to assume that only the number of nodes vary, but the grid stays intact. This assumption is not realistic, so we will adjust the model by assuming that the nodes are homogeneously distributed, but not situated on a specific grid.

Assumption 33 *Nodes are homogeneously distributed.*

This will lead to a more convenient and realistic model, since if we assume that the nodes are situated according to a two-dimensional Poisson process (11) and leave out some of the nodes according to a binomial distribution, this will again lead to a homogeneous distribution. So we do not encounter the difficulty as for the hexagonal grid that the distribution of the nodes will change.

The question which model should be used depends on the type of network that has to be modeled. If the number of possible users is very large, the first model will give a good approximation and is easy to work with. As said earlier, this approach is not possible for the hexagonal grid. In that case the second model should be used. Model two is equal to model one if we would let N tend to infinity. Considering the fact that in our models we often look at the asymptotic situation that the number of nodes tends to infinity, the first model will be useful. Nevertheless we have to take into account that the number of active nodes has to remain finite for the processor sharing model for the non persistent flows to be solvable. Since the amount of nodes is allowed to be high, we can still use the proposed model.

Chapter 10

Homogeneously distributed nodes

Now that we have abandoned the assumption that the nodes are situated according to a grid (14), we have to revise the properties of the network to see which formulas still hold. Just as in the previous model, we first assume that all nodes want to transmit constantly, the persistent flows (25). Since the state space as depicted in Figure 7.13 does not depend on the position of the nodes, the expressions for the transmitting and failure probabilities P_t and P_e do not change. For P_r, the probability that a specific transmission is successful if k nodes are transmitting, we see that M, the number of neighbouring nodes is now stochastic and determined by the range of a node and the density of the nodes in the network. We still are considering the stationary Poisson point process for the placing of nodes in the network and so the distribution of M is known and $P(M = m)$ can be used in the formula. The total number of nodes N can tend to infinity, and using the Poisson or binomial distribution to model the state of the nodes (either on or off) we find the total number of active users in the network. This makes it possible again to determine the values of P_t and P_e so that the same approach can be used as described earlier.

For the hexagonal grid, we encountered the problem that all nodes should be considered to be equal, so that we were considering every node to be at the center of the rings. For the homogeneous distribution every node will be in the exact same situation when still not taking any borders of the network into account, a so called wrap around network. As mentioned earlier, the difference when nodes are considered to be either on or off has no influence on the positioning of the nodes anymore. Combining the Poisson distribution with parameter ρ for the positioning of the nodes and the binomial distribution with probability P_a of leaving out nodes leads to a Poisson distribution again with parameter ρP_a. So contrary to the destruction of the grid in the previous case, the homogeneous distribution is not disturbed.

This leads us to our final model of the network that can be investigated in a convenient way. The total number of nodes in the network, either turned on or off, can be infinite, according to the assumption made when investigating the connectivity of the network, looking at the asymptotic results (7). The total number of active nodes nevertheless still has to be finite for the model to be solvable. The nodes are either turned on with probability P_a or the number of active nodes is Poisson distributed with rate ρ. When nodes are turned on, they start transmitting with rate λ. The probability that a node is transmitting and that a transmission fails are given by P_t and P_e, which can be calculated. From these probabilities we derive the expected number of successful transmissions and using (7.5) and (7.3) with the right values depending on the model that is used, the throughput can be calculated. With the properties of a single hop transmission known, we can also find the properties of a complete path. The optimal amount of hops depends on the distance between the sender and receiver and can be calculated through algorithm 2. Using this, the expected time it takes for a certain number of packets to arrive at its destination can be calculated and the throughput of a path can be found. The throughput of the complete network can also be calculated when not considering the number of hops, but only considering the expected number of nodes to be transmitting at the same time and the time that such a transmission takes. This way a result can be found for a single-hop network, making a comparison possible between more common networks and the multihop ad hoc network. With only some small adjustments, the model that has been presented can be used for the situation that nodes are homogeneously distributed, this way giving a better representation of an ad hoc network.

Chapter 11

Multiple paths

The question that now still remains to be answered is what the link between the connectivity of the network and its throughput is. When a node wants to transmit data to a specific other node, it can use multiple paths. If the network is d-connected, a node can find d distinct paths to its destination. It would be a missed opportunity not to use these paths. By using multiple paths, the transmission can be completed in less time, and the probability of a successful reception of the data will increase. Nevertheless these paths will all cause interference. We assume that when multiple paths are used, each path will carry an equal amount of data, and no duplications are made.

Assumption 34 *Multiple paths carry the same amount of data, without duplications.*

Whenever one of the paths fails, the amount of data lost can be small enough that from the correctly received data the complete data can be reconstructed. This way retransmission does not have to take place, improving the network by saving a lot of time and making it more reliable.

Whether all paths can be used at the same time depends on the number of nodes that can transmit at the same time. In the model we are using, we do not consider a specific order in which nodes are allowed to transmit. Any node will start as soon as it has data that needs to be transmitted and the channel is free. If we consider a pair of nodes that wants to communicate through paths in between, we get the following picture.

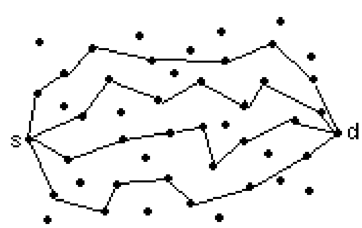


Figure 11.1: Routes

Many of the nodes are close to each other, so they will be within each others range. This means that if they transmit at the same time, a collision will occur. The model does not take into account that the nodes close to each other have a higher probability of wanting to transmit around the same time because they might be on different paths for a transmission between the same two nodes. It will nevertheless be the case that the transmitting node, the first one on each path, cannot transmit to all the second nodes of each different path at the same time, unless all data is transmitted to all neighbouring nodes, together with information which node should forward which part of the data.

We will nevertheless not consider this situation. The separate transmission to all the first nodes does not have to be a problem, since the data should not arrive at the destination at the same time, because this node can only receive data from one node at a time.

Since we are considering the situation where the number of users (nodes) are tending to infinity (7), the total number of users within each others reach will grow very large. This means that the probability that a node within the vicinity of a specific node wants to transmit at the same time is quite large anyhow, even without considering the probability that paths are close to each other. The extra influence caused by the paths lying close to each other will therefore be limited. When the density of the nodes increases, the number of paths close to each other between two nodes will also increase. Therefore there might be a dependency between nodes on different paths between the same two nodes. When nodes are closer to each other, the transmission power can nevertheless be kept at a lower level. Although the influence of dependency between paths and nodes is very interesting and should be investigated in more detail, we will stick with the assumption that all nodes can be considered to be independent (8). This does not mean that the advantage of using multiple paths will be neglected.

When calculating the optimal number of hops, we take as a criteria that the total number of paths has to be maximal. This number of paths can be seen as the connectivity of the network. As seen earlier, the minimum number of neighbours of a node is a bound on the connectivity and in the asymptotic situation is equal to the connectivity of the network. We must remember that having a node within range does not secure that there is a link to this node, since we assumed that only with probability p this link really exists (3). This means that there are more nodes that can cause interference than that there are neighbours that can actually be reached. Furthermore we only consider transmission over paths leaving from the source node that go towards the destination. The total number of paths will as a consequence be much lower than the number of nodes that can cause interference. This validates the assumption that the paths by itself will have a minor influence on the failure probability of a transmission.

Because we assume that all the nodes are acting independently (8), this implies that all paths will also act independently, so all paths can be used simultaneously.

Assumption 35 *All paths are independent, they do not influence each other.*

This is not too strange an assumption for the relaying nodes, but for the source and destination this cannot be the case. A node can only transmit to, and receive from, one node at a time. Our model will therefore slightly underestimate the time needed for a complete transmission over multiple paths.

Using multiple paths will reduce the time needed for a transmission over a path, since the amount of data that has to be transferred per path is smaller. When the amount of data to be sent is small, most of the time needed for a transmission comes from waiting for a free channel and starting up the contact if the RTS/CTS mode is in use. The time gained by using an abundance of paths might then be limited, so the number of paths does not have to be very large. It might even be the case that not all the available paths should be used, just to reduce the number of nodes that want to transmit because of relaying. It might therefore be best to use the Basic mode, since this will decrease the transmission time. The failure of a path is of a smaller influence anyway since the network can recover from some loss of data.

When more paths are used, there will be more relay nodes, increasing the probability that a collision occurs. The impact of a collision will nevertheless be smaller, since if a few paths fail, the remaining data that is successfully received might be sufficient to reconstruct the total amount of data. This depends on the coding technique used, which we will not discuss in detail.

To find the optimal number of paths to use, we should calculate the failure probability p_f of a path. We will assume that the total amount of data can be reconstructed if a fraction f of the data has been received correctly.

Assumption 36 *When a fraction f of the data arrives correctly, the total amount of data can be reconstructed.*

Of course it is important which part of the data is received correctly to make the reconstruction possible. We assume that the division of the packages over the paths is done in such a way that the failure of a fraction f of all the paths is the maximum amount of loss that can be recovered from.

Assumption 37 *When a fraction f of all the paths is successful, the total data can be reconstructed.*

Therefore the following inequality should hold:

$$p_f < 1 - f. \tag{11.1}$$

The failure probability p_f of a path depends on the number of hops of the path and the failure probability P_e of a transmission of a node. The only way the data can be lost if at one node, a transmission fails $r_{\max}$ times. This happens with probability $P_e^{r_{\max}}$. The probability that this does not occur in one of the nodes on the path is then equal to $(1 - P_e^{r_{\max}})^h$, when the path consists of h hops. The value for p_f is therefore equal to

$$p_f = 1 - (1 - P_e^{r_{\max}})^h.$$

This shows that the value of h should not grow too large, because then inequality 11.1 will no longer hold.

The reliability is optimal when the expected number of successfully used paths is maximized. This does not guarantee that the data will arrive at the destination correctly all the time. Sometimes many paths might fail, leading to a loss of data that cannot be recovered from. The sending node might not know if it still has to transmit its packets, because it does not know whether enough data arrived at the receiver to reconstruct the complete data. A way to notify the nodes that they do not have to transmit anymore will be by sending a packet back through the paths notifying the nodes that the transmission has already been completed. This will nevertheless cause a lot of extra transmissions, using a lot of the capacity of the network. We therefore assume that nodes will always keep trying to transmit the packet to the next node, regardless if this is still necessary or not.

Assumption 38 *A node will keep transmitting its data, even if this is no longer necessary.*

Of course it will still be the case that after $r_{\max}$ attempts the node will not be allowed to try again. It therefore is possible that a transmission over a path will not be completed and some of the data is lost. In extreme situations this might lead to a complete loss of the data. The probability that this occurs is expected to be low enough that it will not be necessary to build a security into the network to prevent this from happening.

Chapter 12

The complete model

The goal of this thesis has been to find a model that evaluates the performance measures of an ad hoc network. For an ad hoc network as described in the first part of this thesis we have investigated the connectivity and throughput. Many different approaches have been discussed, finally leading to a model that can find the throughput of the network of a single path, when the nodes in the network have a certain transmission range, are immobile and are homogeneously distributed. A distinction was made between nodes that are situated on a grid or not and between different modes for transmission.

In this section we present an algorithm for the different models that shows how to use the findings presented in this thesis. The algorithm is quite similar for all the different models, but still these differences between the models will be discussed, together with the advantages and limitations of each model.

Algorithm 3 *Computation of the throughput*

Step 1 *Initialization: Set area A, link probability p, network rate r_{net}, arrival rate of data λ and the expected amount of data $\frac{1}{\mu}$.*

Step 2 *Choose the distribution of the active nodes: Poisson or Binomial.*

Step 3 *Choose the path usage: MF or DS.*

Step 4 *Choose the placement of the nodes: Grid or homogeneous.*

Step 5 *Choose the transmission mode: Basic or RTS/CTS.*

Step 6 *Choose the goal: Route throughput or Network throughput.*

Step 7 *Set other parameters:*
If Poisson distribution then set rate ρ and $P(N_a = k) = e^{-\rho}\frac{\rho^k}{k!}$ else set max no of users N, active probability P_a and $P(N_a = k) = B(N, P_a, k)$.
If MF set rate $\lambda = \lambda h$ and $\mu = \mu$, else set rate $\lambda = \lambda$ and $\mu = \mu/h$.
If Grid set range a and expected number of neighbours $m = 3a(a + 1)$, else set range r and $P(M = m)$.
If Basic set $T = T^B$ else $T = T^{RC}$.
If Route throughput set $TH = TH_{path}$ else $TH = TH_{net}$.

Step 8 *Calculate the optimal number of paths and hops using algorithm 2.*

Step 9 *Calculate P_t and P_e using formulas 7.1 and 7.2.*

Step 10 *If Network throughput, then calculate the network throughput using formula 7.3.*

Step 11 If Route throughput, calculate the equilibrium distribution $\pi(n)$ of the number of flows simultaneously in progress using formula 7.5.

Step 12 If Route throughput, calculate the time needed for a single hop using formula 7.6.

Step 13 If Route throughput, calculate the throughput of a path using formula 8.2.

Step 14 Return the requested throughput value.

This algorithm will calculate the expected throughput of the network when only single hops are used, or the throughput over a path consisting of multiple hops. Since the algorithms used in step 8 both converge to an answer and the rest of the algorithm consists of using stated formulas, this algorithm also leads to a unique solution for the expected throughput. Both results, either the throughput of the network or a path, can give insight in common networks using single hops and ad hoc networks using multiple hops. A comparison between both approaches might show the differences between these networks, but then first the number of simultaneously used paths and the correlation between these paths should be investigated. Nevertheless the goal of the thesis has been achieved by combining graph theory and stochastic network theory.

12.1 Number of active nodes

As stated earlier, not all users in the network will be active. Two models were proposed, being the Poisson distribution, leaving the possibility of an infinite number of users open, or the Binomial distribution, setting a certain probability that a node is active. The question is what the difference will be between these approaches. First we will look at the expected number of active nodes for both situations.

When using the Poisson distribution, we can simply make a plot after choosing a value for ρ. The binomial distribution can also be plotted when choosing N and P_a, but for a large value of N this requires a big calculation. Since we are looking at large networks, with the number of nodes tending to infinity, the best way to approximate the binomial distribution will be by using the normal distribution with mean $\mu = NP_a$ and variation $\sigma^2 = NP_a(1 - P_a)$. Choosing $\rho = 100$, $N = 1000$ and $P_a = 0.1$, just as an example, we see the result as shown in Figure 12.1.

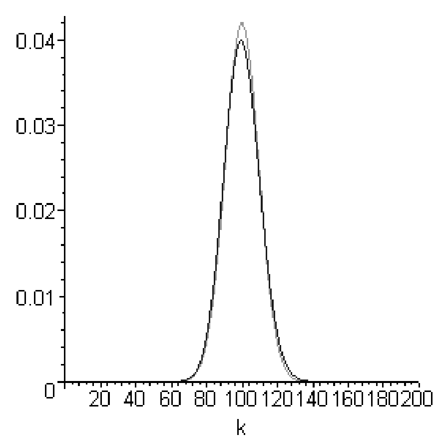


Figure 12.1: Poisson and Binomial distribution

The Poisson distribution can also be approximated by the normal distribution, with $\mu = \rho$ and $\sigma^2 = \rho$. To make a comparison possible, we choose $\rho = NP_a$. The models will lead to an almost similar result, as is known from basic probability theory. For N tending to infinity, the binomial distribution converges to the Poisson distribution. The approximation of both distributions will be the same when $NP_a(1 - P_a) = \rho$. This shows that for a very small value of P_a both models will be similar. With the assumption that N tends to infinity (7), P_a can be chosen to be very small, so the impact of the choice what kind of distribution the active nodes have will be limited.

12.2 Multiple Flows or Data Saving

The effect of using Multiple Flows (MF) in stead of Data Saving (DS) has already been discussed in the section concerning the path possibilities. The difference is found in the parameters for the transmission rate, being either λh or λ, and the amount of data having an average of either $\frac{1}{\mu}$ or $\frac{h}{\mu}$ for MF and DS respectively. The effect this will have on the throughput of the network can are through a difference in the amount of data and the time needed to transmit this amount. When considering a network with the values simply λ and $\frac{1}{\mu}$, changing to the MF model will only influence the value of ρ, which will be multiplied by h. If we would change to the DS model, the same change would occur for ρ, but now also the values of E(P) and T_s (and maybe T_c) will influence R(j) and so also $\pi(n)$ and the throughput. When using MF in stead of DS, more nodes will want to transmit at the same time, leading to a higher probability of failure, and in that way causing a lower throughput per node compared to the DS model.

12.3 Grid or uniformly distributed

The influence of the nodes being placed on a grid or not only depends on the hopcount that can be expected. The total number of nodes in the neighbourhood of a node can be chosen to be expected to be equal by setting the range a and r in such a way that

$$3a(a+1) = \pi r^2 \sum_{m=0}^{N} P(M=m)m$$

When using the grid, we found that the formula (8.1) was

$$E[h] \simeq 0.53 N_r^{0.5} + 2(1 - \frac{N_r}{N})$$

For the values $y = 100$ and $a = 4$ for the total number of rings and the number of rings within the range of a node, we find N_r to be 1950 and N to be 30.000, leading to an $E[h] \simeq 25$. This is also the maximum hopcount, since $\frac{y}{a} = 25$. When using the approach for uniformly distributed nodes, we should know the distance between the source and destination. If we consider the total area of the network to be a circle of radius r^*, then the expected distance between two arbitrary nodes is $\frac{128}{45\pi} r^*$. A bottom value for the needed hopcount was $\frac{l}{r}$, so this gives $\frac{128 r^*}{45 \pi r}$. With a range a of 4, a node will have 60 neighbours, so $\frac{r}{r^*}$ must be equal to $\sqrt{\frac{60}{30.000}}$. This brings us to a lower bound of approximately 20 hops. This shows that there is a difference between the expected hopcount for both approaches.

12.4 Basic or RTS/CTS mode

The use of the Basic or RTS/CTS mode has an obvious influence. When the number of nodes and therefore the number of transmissions increase, there will be more collisions. To reduce the impact of these collisions, RTS/CTS will be the better of the two approaches. This is the case since then after not receiving a CTS the node will backoff, losing only the time needed to send the RTS. When in

Basic mode the transmission fails, the time of sending the data was lost, which will be much longer than the time it takes to transmit an RTS. When few collisions occur or the influence of collisions is negligible, the Basic mode will take less time for the whole transmission, leaving out the time needed for sending the RTS and CTS. Depending on the type of network that will be considered and the precautions taken to prevent collisions a choice can be made which mode will lead to the best performance of the network.

12.5 Network or path throughput

Looking at the path throughput is different from looking at the network throughput. When using the MF mode and assuming that the paths between the same nodes have no significant influence on each other (35), the throughput per node is the same when considering a path or the whole network. In this case it is therefore possible to simply divide the network into paths that can operate at the exact same time. If the DS mode is used, the throughput in a path is influenced by the time data has to wait until the node wants to transmit itself. This will decrease the throughput of a node in a path. But for a single node in the network, this waiting time is not considered. In this situation the throughput of a path will give a better view of the performance of the network, than considering the network as a whole, consisting of single nodes. It may be interesting to investigate whether comparing both results can say something about the number of paths that can or will be in use at the same time in this situation.

Chapter 13

Conclusion

As a model for future communication network, ad hoc networks might prove to be very useful. Therefore a lot of research is currently done to find models that can give insight in the advantages and limitations of ad hoc networks. In this thesis an overview has been presented considering many interesting aspects of this type of networks. From these aspects, the most critical properties, being the connectivity and throughput, have been investigated in more detail.

Since there are many different types of ad hoc networks, a specific network model has been presented. In this model many characteristic properties of ad hoc networks have been included, like the maximum transmission range of users and the possibility of users joining and leaving the network. Nevertheless there also are properties that have not been included, like the mobility of the users. The presented model gives a representation of a communication network as it might be used in real life.

Considering the connectivity of the network, it was shown that there is a threshold value for the probability that a link exists to assure the connectivity of the network. When the number of users in the network grows very large, the number of neighbours of a node shows the grade of connectivity. But this only is the case, when links can be made between any two nodes. When there is a maximum range in which nodes can transmit, a new approach has been presented. The more hops you allow to reach a final destination, the more possible paths there are, so the connectivity improves. Using the result that the number of neighbours is an indicator for the connectivity, together with an algorithm to find the number of possible paths when using a certain amount of hops over a specific distance led to an algorithm to find the optimal number of hops and the expected connectivity of the network.

The network performance depends on many factors, but mostly on the environment of a node and the probability that these nodes interfere with each other. By assuming that all nodes are situated on a grid (14), a first evaluation of the influence of the neighbours of a node was made, showing that permanent transmission will almost always lead to interference that is too high to assure a successful transmission. Therefore the assumption is made that nodes can sense if the network is busy and will only transmit if this is not the case (17,24). Because nodes should not start transmitting at the same time, a procedure is followed before the actual transmission of the data. By drawing the state space, formulas were found to determine the probability that a node is transmitting and the probability of a transmission failure, when all nodes would like to transmit at all times. Making the link with the situation that data is generated according to a Poisson process finally led to a model to find the throughput of the network when single hops are considered. The time of a single hop could be calculated and through this, the time needed for a total transmission from a source to a destination multiple hops away. This led to the final result that was required, being the throughput of a path inside the network. This naturally depends on the number of hops in the path and the total number of possible paths, so the knowledge of the connectivity of the network was very useful.

This way both aspects of the network were combined, leading to a final model of the network to determine its performance measures. In that way, this thesis brings a contribution to the research for the possibility of future use of ad hoc networks as communication networks. Still, there are many aspects that should be investigated. Possible changes and additions to the work presented here are including mobility in the model, considering small networks in stead of networks that tend to grow to infinity, have a more detailed look at the influence of interference in a path itself instead of between paths and nodes and different ways of transmitting, meaning that nodes might not know the direction in which they should send, or that the nodes might not sense whether the network is busy or not. This shows that even though a lot of research has already been done on the topic of ad hoc networks, it still remains an interesting area of research with many unanswered questions.

Appendices

A.1 The formula for S_r

To find the formula for the binomial moments $S_r = \sum E(X_{l_1}...X_{l_r})$, with the sum over all sequences $1 \leq l_1 < ... < l_r \leq n$, that is all r-subsets of $\{1, ..., n\}$, we start from $r = 0$ and will increase r. Set $X = X_1 + .. + X_n$, with $X_i(G)$ equal to 1 or 0 if a node is isolated or not. By definition we know that $S_0 = 1$ and further

$$S_1 = EX = n(1-p)^{n-1}$$

and

$$\begin{aligned} S_2 &= \frac{E(X^2) - E(X)}{2} \\ &= \binom{n}{2}(1-p)^{2(n-2)+1}, \end{aligned}$$

since

$$E(X^2) = \sum_{i=1}^{n} E(X_i^2) + \sum_{i \neq j} E(X_i X_j).$$

With $X_i^2 = X_i$, either 0 or 1, and all the elements of the second sum equal, this can be written as

$$\begin{aligned} E(X^2) &= E(X) + n(n-1)E(X_1 X_2) \\ &= E(X) + n(n-1)(1-p)^{2(n-2)+1}, \end{aligned}$$

since from both nodes no edge goes to another node with probability $(1-p)^{n-2}$ and there is no edge between both nodes with probability $(1-p)$, together giving the stated formula.
Now it can be seen that continuing this way we find

$$S_r = \binom{n}{r}(1-p)^{r(n-r)+\binom{r}{2}}.$$

A.2 The Bonferroni inequalities

The Bonferroni inequalities show that for each k and m,

$$P(X = k) \leq \sum_{j=0}^{2m} (-1)^j \binom{k+j}{j} S_{k+j}$$

and

$$P(X = k) \geq \sum_{j=0}^{2m-1} (-1)^j \binom{k+j}{j} S_{k+j}$$

Suppose that for each r (as is the case in our situation)

$$\lim_{n\to\infty} S_r = \frac{\mu^r}{r!},$$

with $0 < \mu < \infty$, then the Bonferroni inequalities show that for fixed k and m,

$$\begin{aligned}
\lim_{n\to\infty} P(X = k) &\leq \sum_{j=0}^{2m} (-1)^j \binom{k+j}{j} \frac{\mu^{k+j}}{(k+j)!} \\
\lim_{n\to\infty} P(X = k) &\leq \frac{\mu^k}{k!} \sum_{j=0}^{2m} \frac{(-\mu)^j}{j!},
\end{aligned}$$

for any m. Thus we find that

$$\lim_{n\to\infty} P(X = k) \leq e^{-\mu} \frac{\mu^k}{k!}$$

and similarly for the reverse inequality. Then we can conclude

$$P(X = k) \to e^{-\mu} \frac{\mu^k}{k!},$$

showing that X is distributed in the limit according to Poisson's law with mean μ.

A.3 Approximation for the probability that a graph has minimum degree d

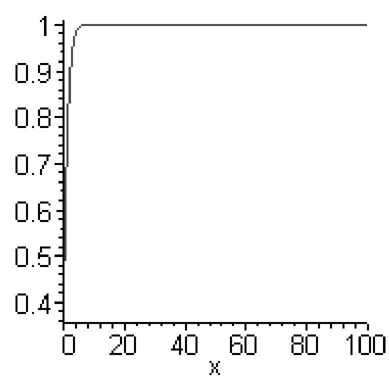
As was shown, the probability that a random graph has minimum degree d can be approximated by

$$e^{-\Sigma_{j=0}^{d-1} e^{-x}/j!} (1 - e^{-e^{-x}/d!}).$$

The first part can be rewritten as

$$\begin{aligned}
e^{-\Sigma_{j=0}^{d-1} e^{-x}/j!} &= (e^{-e^{-x}})^{\Sigma_{j=0}^{d-1} \frac{1}{j!}} \\
&= (e^{-e^{-x}})^{1+1+\frac{1}{2}+\ldots+\frac{1}{d+1!}}.
\end{aligned}$$

Since $1/e \leq e^{-e^{-x}} \leq 1$ this part will be of no influence if $e^{-e^{-x}}$ is close to one. Plotting the function shows the following



Already for $x = 3$ it will equal 0.95, and 0.999 is already reached for $x = 7$. Whenever x is small, there will be a small influence of this factor, but this can be neglected and so the approximation

$$e^{-\Sigma_{j=0}^{d-1} e^{-x}/j!} (1 - e^{-e^{-x}/d!}) \sim (1 - e^{-e^{-x}/d!})$$

can be made.

A different approach is by looking at the expected number of nodes with degree d

$$\begin{aligned} E(X_d) &= n\binom{n-1}{d}p^d(1-p)^{n-1-d} \\ &\backsim n(np)^d(\frac{e^{-np}}{d!}). \end{aligned}$$

Taking $p = \frac{a ln(n)}{n}$ and setting the right side equal to $\frac{e^{-x}}{d!}$, we find that

$$\begin{aligned} n(np)^d(\frac{e^{-np}}{d!}) &= n(a\ln(n))^d\frac{e^{-a\ln(n)}}{d!} \\ &= n^{1-a}a^d(\ln(n))^d/d! \\ &= e^{-x}/d!. \end{aligned}$$

Now we can determine a:

$$\begin{aligned} n^{1-a}a^d(\ln(n))^d &= e^{-x} \\ (1-a)\ln(n) + d\ln(a) + d\ln(\ln(n)) &= -x \\ \ln(n) + d\ln(a) + d\ln(\ln(n)) + x &= a\ln(n) \\ 1 + d\ln(a) + \frac{d\ln(\ln(n))}{\ln(n)} + \frac{x}{\ln(n)} &= a \end{aligned}$$

This shows that if we assume that $a \to 1$, we find $p = \frac{\ln(n)+d\ln(\ln(n)+x}{n}$. The distribution of the number of nodes with degree d will then be Poisson

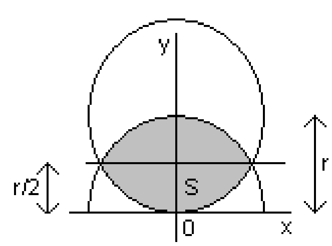
$$\begin{aligned} P(X_d = k) &= e^{-\mu}\frac{\mu^k}{k!} \\ \mu &= e^{-x}/d!. \end{aligned}$$

The probability that a graph has minimum degree d will then be equal to

$$\begin{aligned} P(X_d > 0) &= 1 - P(X_d = 0) \\ &= 1 - e^{-e^{-x}/d!}. \end{aligned}$$

A.4 Calculation of area S

For more information about the number of neighbours of a node, we need to know how many nodes we can expect in the region surrounding it. This depends on the area of such a region. For a first impression we considered the situation as depicted in 6.1. For this situation we wanted to know the number of nodes in region S, which could be depicted as follows when placed on a grid.



To find the area of S, we need more information on the depicted circles. The outer line of the half circle with radius r and origin 0 can be written as

$$\begin{aligned} y &= \sqrt{r^2 - x^2}, \\ -r &\leq x \leq r. \end{aligned}$$

The half circle cuts the other circle at the points $(-\frac{r\sqrt{3}}{2}, \frac{r}{2})$ and $(\frac{r\sqrt{3}}{2}, \frac{r}{2})$. To calculate the area of S above the line $y = \frac{r}{2}$ as drawn in the picture, we can shift the picture down until the line $y = \frac{r}{2}$ becomes $y = 0$. The formula for the part of the half circle that still remains above the x-axis can be written as

$$\begin{aligned} y &= \sqrt{r^2 - x^2} - \frac{r}{2}, \\ -\frac{r\sqrt{3}}{2} &\leq x \leq \frac{r\sqrt{3}}{2}. \end{aligned}$$

For the area of S still above the x-axis we then use the integral

$$\begin{aligned} & \int_{-\frac{r\sqrt{3}}{2}}^{\frac{r\sqrt{3}}{2}} (\sqrt{r^2 - x^2} - \frac{r}{2})\, dx \\ = & [\frac{x\sqrt{r^2 - x^2}}{2} + \frac{r^2}{2} \arctan(\frac{x}{\sqrt{r^2 - x^2}}) - \frac{xr}{2}]_{-\frac{r\sqrt{3}}{2}}^{\frac{r\sqrt{3}}{2}} \\ = & [\frac{r^2\sqrt{3}}{8} + \frac{r^2}{2} \arctan(\sqrt{3}) - \frac{r^2\sqrt{3}}{4}] - [-\frac{r^2\sqrt{3}}{8} + \frac{r^2}{2} \arctan(-\sqrt{3}) + \frac{r^2\sqrt{3}}{4}] \\ = & [\frac{\pi r^2}{6} - \frac{r^2\sqrt{3}}{8}] - [-\frac{\pi r^2}{6} + \frac{r^2\sqrt{3}}{8}] \\ = & \frac{1}{3}\pi r^2 - \frac{1}{4} r^2 \sqrt{3}. \end{aligned}$$

This is exactly half of the area of S, so the total area of S will be

$$\frac{2}{3}\pi r^2 - \frac{1}{2} r^2 \sqrt{3}.$$

If there are m nodes to be expected within the range of a node, a total of $(\frac{2}{3} - \frac{\sqrt{3}}{2\pi})m$ nodes are expected to be present in S.

A.5 The stationary Poisson point process

A stationary Poisson point process Φ is a random counting measure that can be applied to Borel sets. In our case we can simply use regions as the Borel sets. For a region B, $\Phi(B)$ denotes the number of points of Φ in this region B. The stationary Poisson point process has two fundamental properties.

1. *Poisson distribution of point counts.* The number of points in a bounded region has a Poisson distribution of mean $\lambda v_d(B)$ for some constant λ.

2. *Independent scattering.* The number of points in q disjoint regions form q independent random variables, for arbitrary q.

With λ the intensity of the process and $v_d(B)$ the Lebesgue measure of B, in our case the area of region B. From the stated properties, the whole distribution of the stationary Poisson process can be determined, once the intensity λ is known, leading to some basic properties of the process.

- Finite dimensional distribution. If B$_1, ..., B_k$ are disjoint sets, then $\Phi(B_1), ..., \Phi(B_k)$ are are independent Poisson random variables with means $\lambda v_d(B_1), ..., \lambda v_d(B_k)$. This leads to

$$\begin{aligned} P(\Phi(B_1) &= n_1, ..., \Phi(B_k) = n_k) \\ &= \frac{\lambda^{n_1+...+n_k}(v_d(B_1))^{n_1}...(v_d(B_k))^{n_k}}{n_1!...n_k!}. \end{aligned}$$

- Stationarity and isotropy. The point process is stationary since $\Phi_x = \{x_n + x\}$ has the same distribution for all x. It is also isotropic since the same is true for all rotated processes $r\Phi = \{rx_n\}$. A point process having both these properties is said to be motion invariant.

An important property of the stationary Poisson point process concerns the Palm distribution. A Palm distribution probability is a conditional probability of a point process event, given that a point is observed at a specific location. This means that we can derive properties of the distribution of the nodes, assuming that we already know a specific position of a node. For the Palm distribution probability we use the following notation

$$P(\Phi \text{ has property } Y \,\|x) = P(\Phi \text{ has property } Y \,|x \in \Phi),$$

where $P(.\,\|x)$ means that Φ contains a point at position x. In short form we write

$$P(\Phi \in Y \,\|x) = P(\Phi_{-x} \in Y \,\|o).$$

Here Φ_{-x} denotes the shifted point process $\{x_1 - x, ..., x_k - x\}$. Now we can use Slivnyak's theorem:

$$P(\Phi \in Y \,\|o) = P(\Phi \cup \{o\} \in Y).$$

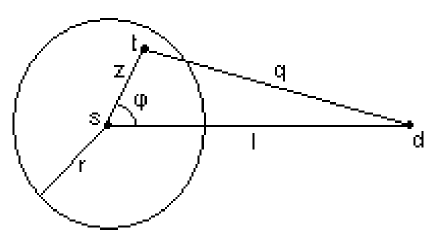
The proof of this theorem can be found in [Stoy95]. Using this theorem and applying it to our situation, we find

$$P(\Phi(B) = n \,\|o) = P(\Phi(B) = n).$$

This means that if the expected number of node found in circle of radius r is equal to m, the number of neighbours of a node at the center of this circle (assuming that we know it is situated there) will also be equal to m.

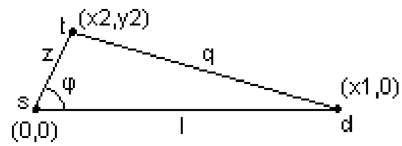
A.6 Expected distance to the destination after a hop

To find the number of possible paths when using a fixed number of hops, we use the expected distance to the destination from a node reached after a hop from a node we know the distance to the destination from. This distance depends on the direction the hop was made to and the length of the hop. Because a hop can only be made to a node within the range, the length of the hop will not exceed r. The hop can not go the node itself, so we also know that the length of the hop will be higher than 0. Since we did not put any constraints on the direction the hop will be made to, the angle ϕ of the hop can have any value between 0 and 2π. Here we take the angle to be 0 if the hop is made straight towards the destination. The following figure gives an illustration.



The figure shows the situation when a hop of length z with an angle φ has been made from s to t.

Placing the triangle on a grid with s on the origin $(0, 0)$, d on $(x_1, 0)$ and t on (x_2, y_2) as in the following figure, we can find the expression for q.



Using this figure, it will be easy to see that q is given by

$$q = \sqrt{(x_1 - x_2)^2 + y_2^2}.$$

Because the hop has to stay within the range, the values of x_2 and y_2 are bounded by

$$0 \leq x_2^2 + y_2^2 \leq r,$$

which is equal to

$$-\sqrt{r^2 - x_2^2} \leq y_2 \leq \sqrt{r^2 - x_2^2}.$$

Together with

$$-r \leq x_2 \leq r,$$

we find that the expectation of q is given by

$$Eq = \frac{1}{\pi r^2} \int_{-r}^{r} \int_{-\sqrt{r^2 - x_2^2}}^{\sqrt{r^2 - x_2^2}} (\sqrt{(x_1 - x_2)^2 + y_2^2})\ dy_2\ dx_2,$$

since we make the hop to any spot within the circle of area πr^2, all having the same probability. Now we switch to polar coordinates with

$$\begin{aligned} x_1 &= l, \\ x_2 &= z\cos(\varphi), \\ y_2 &= z\sin(\varphi), \end{aligned}$$

and find that the shown integral is equal to

$$Eq = \frac{1}{\pi r^2} \int_{0}^{2\pi} \int_{0}^{r} (z\sqrt{(z^2 + l^2 - 2zl\cos(\varphi))})\ dz\ d\varphi.$$

A.7 Proof of algorithm 2

Algorithm 2 always has a fixed starting point, being the minimum number of hops. This value will always exist and so the algorithm can start. By raising the amount of hops that can be used, new values of the expected number of disjoint paths are calculated until the value drops after raising the amount of hops. This implies that we have found a maximum for $ET_d[h]$, but this could be a local maximum. It remains to be proved that this will also be a global maximum. If at one point the value of $ET_d[h]$ drops for the higher value $h+1$, then this shows that the value of $\frac{T_d[h+1]}{T_d[h]}p$ (with the variables as defined in algorithm 1) is lower than 1. What needs to be shown is that this implies that also the following inequality holds:

$$\frac{T_d[h+i]}{T_d[h]}p^i < 1.$$

This can be shown as follows. We know that $T_d[h] \leq T_d[h+1] \leq T_d[h+2]$ since adding a hop cannot decrease the minimum number of nodes to hop to. When adding a hop, the area that is still feasible will grow, but the grow of this area will decrease per extra hop. So we also know that $T_d[h+2] - T_d[h+1] < T_d[h+1] - T_d[h]$. We already found that $\frac{T_d[h+1]}{T_d[h]}p < 1$. Then we also know that $\frac{T_d[h+1]}{T_d[h]}px < 1$ if $x \leq 1$. Take $x = \frac{T_d[h+2]T_d[h]}{T_d[h+1]^2}$, then we find $\frac{T_d[h+2]}{T_d[h+1]}p < 1$, as long as $x \leq 1$. Take $T_d[h+2] = T_d[h+1] + \varepsilon_2$ and $T_d[h+1] = T_d[h] + \varepsilon_1$, then we know that $\varepsilon_2 \leq \varepsilon_1$ and $\varepsilon_1, \varepsilon_2 \geq 0$, and so

$$\begin{aligned} x &= \frac{(T_d[h+1] + \varepsilon_2)(T_d[h+1] - \varepsilon_1)}{T_d[h+1]^2} \\ &= \frac{T_d[h+1]^2 + (\varepsilon_2 - \varepsilon_1)T_d[h+1] - \varepsilon_2\varepsilon_1}{T_d[h+1]^2} \\ &\leq \frac{T_d[h+1]^2}{T_d[h+1]^2} = 1 \end{aligned}$$

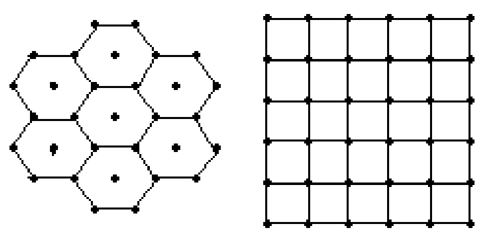
So indeed we find that $x \leq 1$ and it follows that since $\frac{T_d[h+2]}{T_d[h+1]}p < 1$ and $\frac{T_d[h+1]}{T_d[h]}p < 1$ also $\frac{T_d[h+2]}{T_d[h]}p^2 < 1$. We now can easily use induction to prove that for any i we find that $\frac{T_d[h+i]}{T_d[h]}p^i < 1$ and the maximum we found has to be global.

What remains to be proved is that at one point we will find that $ET_d[h+1] < ET_d[h]$. This is the case when $\frac{T_d[h+1]}{T_d[h]}p < 1$. Suppose that at one point the number of hops that can be used is very large. The hops will then only be restricted to the half circle leading towards the destination. Adding another hop will not increase this area, and so the value of $T_d[h+1]$ will be equal to $T_d[h]$ for this high value of h. We then see that $\frac{T_d[h+1]}{T_d[h]}p < 1$ and so this situation will always occur, though this might be for a high value of h.

This shows that algorithm 2 always converges to an answer that indeed is the global optimal solution.

A.8 Grid explanation

When using a regular grid to position the nodes, we want the nodes to be equidistant and that all nodes have the same number of neighbours. These are the two requirements that also hold for a uniform distribution. Only two grids that fulfill these requirements are the rectangular and hexagonal grid as shown in the following figure.



Of both possible situations, the hexagonal grid shows that a node will have more neighbours that in the rectangular grid. Therefore we will use the hexagonal grid for considering the worst case situation.

A.9 Influence of the possibility of a single transmission failure

As stated in section 7.6 the influence of including the possibility that only one transmission fails is limited. In the network as described, a single failure is impossible, since a failure only happens when two or more transmissions collide. When this happens, all colliding transmissions fail and so a single transmission failure cannot occur. The probability that one transmission fails if there are K nodes transmitting, which is possible in the model, is given by

$$B(K, P_e, 1) = KP_e(1 - P_e)^{K-1}.$$

Since we know that $0 < P_e < 1$, we see that for the limit with K growing large

$$\lim_{K\to\infty} KP_e(1 - P_e)^{K-1} = 0,$$

and so in this case the probability that this situation occurs is small. If we look at the situation that K is small, then hardly any nodes are transmitting. Recalling formula 7.2 for the failure probability

$$P_e = \sum_{k=0}^{N} B(N, P_t, k)\left(1 - \frac{\binom{N-m-1}{k-1}}{\binom{N-1}{k-1}}\right),$$

we see that for a small value of K nodes transmitting, the last part, then equal to $1 - \frac{\binom{N-m-1}{K-1}}{\binom{N-1}{K-1}}$ will be almost 0, since N tends to infinity. So also then the probability of the situation occurring is small. The impact of the assumption will hence be small, since both situations have a low probability of occurring.

Complete algorithm

Algorithm 4 *Computation of the throughput*

Step 1 *Initialization: Set area* A*, link probability* p *,network rate* r_{net}*, arrival rate of data* λ *and the expected amount of data* $\frac{1}{\mu}$*.*

Step 2 *Choose the distribution of the active nodes: Poisson or Binomial.*

Step 3 *Choose the path usage: MF or DS.*

Step 4 *Choose the placement of the nodes: Grid or Uniform.*

Step 5 *Choose the transmission mode: Basic or RTS/CTS.*

Step 6 *Choose the goal: Path throughput or Network throughput.*

Step 7 *Set other parameters:*
If Poisson distribution then set rate ρ *and* $P(N_a = k) = e^{-\rho}\frac{\rho^k}{k!}$ *else set max no of users* N*, active probability* P_a *and* $P(N_a = k) = B(N, P_a, k)$*.*
If MF set rate $\lambda = \lambda h$ *and* $\mu = \mu$*, else set rate* $\lambda = \lambda$ *and* $\mu = \mu/h$*.*
If Grid set range a *and the expected number of neighbours* $m = 3a(a+1)$*, else set range* r *and* $P(M = m)$*.*
If Basic set $T = T^B$ *else* $T = T^{RC}$*.*
If Path throughput set $TH = TH_{path}$ *else* $TH = TH_{net}$*.*

Step 8 *Calculate the optimal number of paths and hops using algorithm 2.*

- *Calculation of expected number of paths with* h *hops*

Step 1 *Set the distance* l_1 *between* s *and* d*, the expected number of nodes* m *within each circle and set* $i = 1$*.*

Step 2 *If* $l_1 > hr$ *then return* $ET[h] = ET_d[h] = 0$ *else go to step 3.*

Step 3 *If* $h - i > 0$ *go to step 4 else go to step 8.*

Step 4 *Set* $i = i + 1$*.*

Step 5 *Calculate the number of possible hops* $ph[i-1]$ *with criteria: new distance* $l_i < (h-i+1)r$*.*

Step 5.1 *If* $l_i < (h-i)r$ *then* $ph[i-1] = m/2$ *and go to step 6, else go to step 5.2.*

Step 5.2 *Calculate the area* V *of the region that is still solvable.*
$V = \int_{-a}^{a}(\sqrt{[(h-i)r]^2 - x^2} - b)\ dx + \int_{-a}^{a}(\sqrt{r^2 - x^2} - c)\ dx$,
with $a = \frac{\sqrt{-r^4+2r^2l_{i-1}^2-l_{i-1}^4+2(h-i)^2r^2l_{i-1}^2+2(h-i)^2r^4-(h-i)^4r^4}}{2l_{i-1}}$,
$b = \frac{((h-i)^2-1)r^2+l_{i-1}^2}{2l_{i-1}}$ *and* $c = \frac{-((h-i)^2-1)r^2+l_{i-1}^2}{2l_{i-1}}$.

Step 5.3 *Calculate the expected number of nodes (= possible hops)within this region*
$ph[i-1] = \frac{Vm}{\pi r^2}$.

Step 6 Calculate the expected distance l_i between the new node and the destination.

Step 6.1a If $l_{i-1} < (h-i+1)r$, the number of hops available exceeds the number of hops needed, so the first hop can be made within half a circle towards the destination.
Calculate $l_i = \frac{2}{\pi r^2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^r h\sqrt{(h^2+l_{i-1}^2-2hl_{i-1}\cos(\phi))}\ dh\ d\phi$.

Step 6.1b If $l_{i-1} \geq (h-i+1)r$, there is a designated region that should be reached, calculate $l_i = \frac{1}{V}\int_{-\beta}^{\beta}\int_{r(\phi)}^{r} h\sqrt{(h^2+l^2-2hl\cos(\phi))}\ dh\ d\phi$,
where $r(\phi) = l_i cos(\phi) \pm \sqrt{l_i^2 cos^2(\phi)+(h-i)^2r^2-l_i^2}$ *and* $\beta = \arccos(\frac{-((h-i)^2-1)r^2+l_i^2}{2l_i r})$.

Step 7 Go to step 3.

Step 8 Calculate the expected number of possible paths
$T[h] = \prod_{i=1}^{h-1} ph[i]$
and calculate the expected number of possible disjoint paths
$T_d[h] = \min_{i=1}^{h-1}(ph[i])$.

Step 9 Calculate the expected number of existing paths
$ET[h] = T[h]p^h$
and the expected number of disjoint paths
$ET_d[h] = T_d[h]p^h$.

- *Calculation of optimal number of hops h_{opt}*

Step 1 Calculate the minimum number of hops

$$h_{\min} = \left\lceil \frac{l}{r} \right\rceil,$$

where l is the distance between the source and destination, and r is the range of a node.

Step 2 Set $h = h_{\min}$ and use algorithm 1 to calculate the expected number of existing paths $ET_d[h]$.

Step 3 Calculate $ET_d[h+1]$.

Step 4 If $ET_d[h+1] > ET_d[h]$ then set $h = h+1$ and go to step 3 else $h_{opt} = h$.

Step 9 Calculate P_t and P_e using formulas 7.1 and 7.2.

- $P_t = \sum_{r=0}^{r_{\max}} \pi(r,0) = \pi(0,0)\sum_{r=0}^{r_{\max}} P_e^r = \frac{1-P_e^{r_{\max}}}{1-P_e}\pi(0,0)$,
  $\pi(0,0) = \frac{2(1-P_e)}{(1-P_e^{r_{\max}+1})+(1-P_e)\Sigma_{r=0}^{r_{\max}} cw_r P_e^r}$

- $P_e = \sum_{k=1}^{N} B(N,P_t,k)(1-P_r)$,
  $P_r = \frac{\Sigma_{m=0}^{N}\binom{N-m-1}{k-1}P(M=m)}{\binom{N-1}{k-1}}$

Step 10 If Network throughput, then calculate the network throughput using formula 7.3.

- $R(N) = \frac{\Sigma_{s=0}^{N}\Sigma_{k=s}^{N} B(N,P_t,k)B(k,P_s,s)sE(P)}{B(N,P_t,0)\tau+\Sigma_{k=1}^{N} B(N,P_t,k)[\Sigma_{s=1}^{k} B(k,P_s,s)T_s+B(k,P_s,0)T_c]}$

Step 11 If Route throughput, calculate the equilibrium distribution $\pi(n)$ of the number of flows simultaneously in progress using formula 7.5.

- $\pi(n) = \frac{\rho^n\varphi_n}{\sum_{j=0}^{N_a}\rho^j\varphi_j}$, $\varphi_n = (\prod_{j=1}^{n}\frac{R(j)}{r})^{-1}$

Step 12 If Route throughput, calculate the time needed for a single hop using formula 7.6.

- $T = \sum_{n=0}^{N_a} \frac{n\pi(n)}{\lambda(1-\pi(N_a))}$

Step 13 If Route throughput, calculate the throughput of a path using formula 8.2.

- $TH_{path} = \frac{N_{packet}}{hT_{transfer}}$

Step 14 Return the requested throughput value.

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