A Mathematician's view on normative liability studies

Master Thesis Applied Mathematics

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The answer to life, the universe and everything:

```
for (i=0; ; i++)
   Thread.new(execute i);
```

Abstract

When accidents occur, the law prescribes how damage should be divided between the parties involved. This law therefore influences the decision making of those parties. The government and judge should aim to maximize social welfare by choosing laws which encourage beneficial behavior. Because of the importance of the subject many scholars from the field of law and economics estimate the effects of various laws. The law resulting in the highest social welfare is then recommended.

There are two problems governing this field. First is the ambiguity concerning the assumptions on which the analysis is based. For every claim by a scholar there can be found someone disagreeing with this claim, but based on different assumptions. It indicates there is consensus about the analysis, but not about the proper starting point of the analysis. This is a problem which seems to slow down the progress in this field.

This research therefore provides a systematic way to denote these assumptions. The ease of use and its generality is demonstrated by classifying some of the major studies. This classification shows that there are a couple of simplifications present in all studies. Typically people are assumed to be perfectly rational and risk-neutral and the judge is often assumed to be omnipotent and working for free. To estimate the impact of these simplifications a simulation model is developed. This model mimics the behavior of reality and can be used to predict the effect of laws and study the effect of assumptions. Combining the classification and the simulation model reveals the second problem in this field: risk-averse and irrational behavior are hardly studied but the impacts are very significant.

This research therefore also contains an analysis on the effect and implications of risk-averse and irrational behavior. Preliminary findings show that analysis of this behavior is possible and in some cases even easier to analyze than rational behavior. Furthermore it is shown that under this extensive model the only criterion for the judge to consider is: 'how easy would it have been for the party to avoid the accident'. This differs from contemporary conclusions which recommend to consider 'due care'. It will also be concluded that these irrational games can be simplified to games where both parties have only two actions. This is an important finding as this greatly simplifies analysis and provides a guideline for judiciary decisions.

Acknowledgements

I rarely enjoyed this much freedom in pursuing what I like. I'd like to thank my supervisors for allowing me this freedom and, just in case, apologize for using it. This freedom allowed me to strike a fine balance between (i) work I like, (ii) work I think my supervisors like and (iii) the rest of life. This often turned out to mean: doing fun things when possible and working when not doing fun things. I really appreciate the subtle guidance of Judith and Werner, thanks.

In my research I ventured into the unknown and slightly foggy lands of Law. Mathematics proved to be a fire in wet surrounding, hard to start but then easily yielding many results. I'd like to thank Michiel Heldeweg and Steven Shavell for showing and explaining me the foreign things I encountered.

Finally I'd like to thank my friends for the great time I had the past couple months. In particular I'd like to thank Stan, Koen, Jaap en Stefan for their guidance (and sometimes a bit more than that) in my work. I hope to be able to return the favor.

Contents

Chapter 1

Introduction

When accidents occur the law prescribes how the damage should be distributed among those involved. In simple cases this results in the one who caused the accident being liable and thus paying for the effects. In many cases however there is a more complex structure leading to the accident and it is no longer feasible to determine a single cause. Research on the topic of law and economics aims to provide a systematic way to analyze the law and compare laws based on their effect.

Before 1960 economic analysis of the law focused primarily on social costs. Pigou ([15],1912) defines social costs to be external effects not affecting the decision maker. Private costs are those which influence the decision maker. Pigou shows that if social costs can be internalized in the private cost by, for example, taxes or subsidies this will make selfish actors choose socially optimal actions. Coase ([5],1960) shows that in circumstances where bargaining is free, social costs are automatically included and thus there is no need for the government to intervene. The focus then shifts to the effects of liability rules. Calabresi ([3],1965) indicates that private cost should also be considered in liability cases by noting that it is not socially desirable for everyone to do the utmost to prevent accidents. Posner ([16],1972) investigates the effects of a negligence liability rule: the person not to exercise 'due care' is liable. The basic theory as still used today is set when Brown ([2],1973) formalizes the analysis and is able to prove the optimality of some liability rules under certain assumptions. An overview of the many considerations when dealing with liability rules can be found in Shavell ([18],2007).

The goal of these studies is to maximize the social welfare. The social welfare can be thought of as 'total happiness' and is often approximated by total wealth. Liability rules affect the social welfare in two ways (Figure 1.1). First the liability rule affects the behavior of the people and second it determines how the wealth is distributed and therefore has a direct impact as well. So the goal of a liability rule is two-fold: (1) encourage socially desirable behavior and (2) allocate damage such that it falls where it hurts least. It will be found that both factors are important but (1) is dominant.

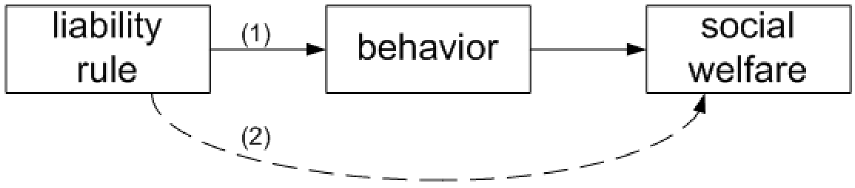

Figure 1.1: the environment of liability rules

For every combination of actions by the players the liability rule states the division of the accident cost between these players. Usually games are considered to involve two players, the injurer and the victim. The victim is assumed to incur the damage. There are three basic liability rules: 'strict liability', 'no liability' and 'negligence'. Under the strict liability rule the injurer pays all. This has the attractive feature that it provides the strongest incentives for the injurer whom is generally considered to be best able to avoid the accident. Under the no liability rule the victim pays all, which avoids people going to court and which saves the cost of the trial. The negligence rule states the victim is liable unless the injurer did not exercise due care (i.e. did not take appropriate action). When due care is set at the social optimal level this, in theory, persuades both injurer and victim to take optimal levels of care, unlike the two other rules.

In order to state that a certain rule is better it is necessary to be able to predict the resulting social welfare. This requires assumptions about the natural world and on the behavior and desires of people. On these assumptions studies disagree. Up until this report studies agreed only on assuming risk-neutral and perfectly rational behavior. Studies already disagreed on whether or not to include probabilistic size of the accident, court costs and on which liability rules to consider. We believe it is not the analysis which can be improved, but the more fundamental question: 'what to analyze?'

This research therefore aims to provide a structured and formal basis for the analysis of liability rules. In Chapter 2 the notation is introduced to communicate a specific set of assumptions, restrictions and optimality criteria. Chapter 3 classifies part of the literature according to this notation and determines which topics have been studied so far. Chapter 4 contains four background articles which contain important information to supplement our conclusion. In Chapter 5 simulation model which predicts the effect of liability rules is treated. The simulation model is used to determine the accuracy of the assumptions. The combined results of these chapters lead to our conclusion Chapter 6 on which topics are important to study in.

Chapter 2

Liability notation

This chapter contains the notation as it will be used to classify literature in Chapter 3 and to the foundation for the simulation model in Chapter 5. Section 2.1 explains the easiest game called the fundamental liability game for which extensions will be summarized in Section 2.2. Sections 2.3-2.6 contain the explanation of the notation on restrictions, assumptions, optimality criteria and liability rules respectively.

2.1 The fundamental liability game

In this section we propose a simple liability game which captures the essence of liability problems. This will then provide the basis for modifications and extensions in the following sections. The notation is similar to the one proposed in Brown ([2], 1973). The most significant change is that we consider players to choose actions from a finite set of actions instead of choosing the value of a continuous variable. This is done in order to provide more flexibility for modifications later on.

The fundamental liability game considers two players, the injurer and the victim. Before the potential accident the injurer chooses an action ($i \in I$) and the victim chooses an action ($v \in V$). The choice of action affects the probability and size of an accident. Furthermore the action incurs an immediate avoidance cost, $a_i^{(i)}$ and $a_v^{(v)}$. These avoidance costs are made regardless of whether the accident occurs. For example consider the possible action of buying and wearing a helmet when biking, this reduces the size of the accident but results in an immediate cost equal to the price of the helmet. An accident occurs with probability P_{iv}. We will consider the simplification where the size of an accident is independent of the actions as the more complex games can be reduced to the game with fixed size. The size of the accident is denoted by s.

A liability rule $L \in [0,1]^{I \times V}$ defines for all combination of actions which proportion of the expected accident cost should be allocated to the injurer. This means the 'no liability' rule will be $L_Z = \mathbf{0}$ and the 'strict liability' rule is $L_S = \mathbf{1}$. $\mathbf{0}$ and $\mathbf{1}$ denote the matrix of all zeros and all ones respectively.

The behavior of the players is modeled as follows. First the players compute their expected utility for all combination of actions,

$$U_{iv}^{(i)} = w^{(i)} - L_{iv} P_{iv} s - a_i^{(i)} \qquad U_{iv}^{(v)} = w^{(v)} - (1 - L_{iv}) P_{iv} s - a_v^{(v)}.$$

Here $U_{iv}^{(i)}$ is the expected utility for the injurer, $w^{(i)}$ is the initial wealth of the injurer. Secondly they compute the expected utility for their actions based on the strategy of the other player

$$u_i^{(i)} = \sum_{v \in V} p_v^{(v)} U_{iv}^{(i)} \qquad u_v^{(v)} = \sum_{i \in I} p_i^{(i)} U_{iv}^{(v)}.$$

Here $u_i^{(i)}$ is the expected utility for the injurer of action i, $p_v^{(v)}$ denotes the probability the victim chooses action v. Finally the players choose actions with a higher expected utility with a higher probability,

$$p_i^{(i)} = g(u^{(i)}) \qquad p_v^{(v)} = g(u^{(v)}).$$

The definition and specification of the function g is treated in Section 2.5 on optimality conditions. Clearly the strategies of the players are interdependent so these steps are iterated to find the behavior of the players.

The quality of a liability rule is measured by the expected social welfare. The expected social welfare $w^{(s)}$ is the sum of the individual welfare of the players after the accident

$$\bar{w}^{(s)} = \bar{w}^{(i)} + \bar{w}^{(v)} = \sum_{i \in I} \sum_{v \in V} p_i^{(i)} (U_{iv}^{(i)} + U_{iv}^{(v)}) p_v^{(v)} = \sum_{i \in I} \sum_{v \in V} p_i^{(i)} U_{iv}^{(s)} p_v^{(v)}.$$

Here $U_{iv}^{(s)}$ denotes the total social utility given actions and $\bar{w}$ denotes the expected wealth after the accident for the injurer, victim and society. In consequent chapters $w^{(i)}$ and $w^{(v)}$ will denote the welfare of the players prior to the accident and $w^{(s)}$ will denote the social welfare after the accident. The set of all liability rules which result in the highest social welfare is called the efficient rule L_E. The efficient rule depends on the specific instance of the game.

$$L_E = \{L \in [0,1]^{I \times V} : w^{(s)}(L) = \max_{L'} w^{(s)}(L')\}.$$

Thus if one was to try all possible liability rules, the efficient rule would have the highest social welfare. The efficient rule will play an important role in determining the loss of essential features for the various simplifications. Throughout the rest of the report the optimal actions for society (e.g. minimal expected accident cost plus avoidance costs) are denoted by i^* and v^*.

2.2 Extensions

This section provides a notation for liability games which allows for quick communication of restrictions, assumptions and results. Table 2.1 summarizes the notation and indicates on which page more detailed information can be found. The notation is divided in four parts denoting: (2.3) absence of restrictions which make results more generally applicable, (2.4) more realistic assumptions which make results more accurate, (2.5) optimality conditions and (2.6) liability rules. Notation will be presented as:

Restrictions/Assumptions/Optimality conditions($\implies$ statement about liability rules).

The difference between a restriction and an assumption is that a restriction is sometimes exactly true and an assumption is always approximately true. So the statement: *if there is a lunar eclipse then the moon is red* is a restriction whereas the statement: *the moon is a perfect sphere* is an assumption. The notation has been designed in such a way that the absence of symbols imply the easier or more common form of the game. So by default very little notation is required, and as the model becomes more general, realistic or uncommon, more symbols are included.

This notation aims to provide a model for proving theories in liability games as well as classify studies on liability. In order to be able to classify a wide range of studies we introduce some additional notation. We use a plus-sign (i.e. +) to denote a more general/realistic version of the restriction/assumption, a minus-sign (-) to denote a less general or less realistic version and we use a prime-sign (') to denote a different assumption, neither clearly better nor worse. We also allow for subnotation which provides further distinction within a certain restriction or assumption, this will be of the form Ur (denoting we take into account risk aversion). If a study uses all the default assumptions we omit the assumption part (e.g. U/NE instead of U/ /NE)

There are multiple ways to use this notation. Most common uses will be to classify a study or to state a theorem or conclusion. A theorem can be stated as U/NE $\implies$ S $\in L_E$ (in a unilateral game the strict liability rule is efficient) or R/C/NE $\implies$ $w^{(s)}$(N)> $w^{(s)}$(S) (in a regular game including court costs, the negligence rule is superior to the strict liability rule).

Restrictions	U	Unilateral Game	p. 9
	R	Regular Game	p. 9
	C	Coordination Game	p. 9
	M	Multiplayer Game	p. 9
	L	Level of Activity	p. 10
	B	Prior Bargaining possible	p. 10
Assumptions	C	Court costs > 0	p. 10
	D	Damage $\neq \sum$(liability)	p. 11
	U	Utility = f(resource)	p. 11
	E	Evidence uncertain	p. 12
Optimality Criteria	NE	Nash Equilibrium	p. 13
	F	Fuzzy Choices	p. 13
Liability Rules	Z	No (zero) liability	p. 15
	S	Strict liability	p. 15
	N	Negligence rule	p. 16
	R	Relative negligence	p. 16
	C	Comparative negligence	p. 16
	E	Set of efficient (optimal) liability rules	p. 17

Table 2.1: a summary of the notation

2.3 Restrictions

Restrictions state under which circumstances the results of a study are valid. A restriction therefore does not alter the accuracy of the conclusions, only how often it applies. This section will treat the restrictions. First the various types of games will be treated (U, R, C) followed by restrictions on the level of activity (L) and finally the possibility of bargaining (B).

2.3.1 (U) Unilateral Game

In a unilateral game only the injurer is assumed to be able to influence the probability and size of an accident. This imposes the additional constraint $|V| = 1$ (victim has only 1 action) on the fundamental liability game. An example of a unilateral game is when an aviator makes an error in judgment because of a lack of sleep; the victim has no influence on this.

2.3.2 (R) Regular Games

Regular games are the most commonly analyzed type of liability games. In regular games the actions chosen by players increase or decrease the expected accident cost regardless of the choice of action by the other player. The size of this increase or decrease can and will differ depending on the action of the other player. So wearing a helmet on a bicycle is an action typical for a regular game as this increases safety regardless of other choices (although exceptions can be found). The choice of driving on a certain side of the road cannot possibly occur in a regular game as the safety of driving on the 'wrong' side of the road depends on the choices by the other players if no laws are in effect. The advantage of applying theory to regular games is the fact that it is easy to state which actions are more desirable than others from the perspective of society.

Mathematically a game is a regular game if,

$$U^{(s)}_{i'v} > U^{(s)}_{iv} \implies U^{(s)}_{i'v'} > U^{(s)}_{iv'} \quad \text{and} \tag{2.1}$$

$$U^{(s)}_{iv'} > U^{(s)}_{iv} \implies U^{(s)}_{i'v'} > U^{(s)}_{iv} \quad \forall i, i' \in I \text{ and } v, v' \in V. \tag{2.2}$$

This definition implies that every unilateral game is also a regular game ($U \subset R$).

2.3.3 (C) Coordination Games

In a coordination game the optimal action for a player may depend on the actions of the other player. Driving on the 'wrong' side of the road is a simple example, but there are also more subtle liability games belonging to C (not to R). Consider a bicyclist who can buy a helmet and a driver who can choose to go and drive or stay at home. In case the driver decides to drive, the optimal action for the cyclist is to buy and wear the helmet. On the other hand if the driver stays at home it is be optimal not to buy and wear a helmet.

This is the most general type of game and it is immediately clear that $R \subset C$. It is often argued that most games are regular. While this is to a large extend true, there certainly are many games which are not regular, so analysis should be extended to coordination games whenever possible.

2.3.4 (M) Multiplayer Game

The fundamental liability game is a two person game. In practice however accidents will also occur where there are more than two players involved. This can be denoted as M_n, where n is the number of people. The notation without a subscript indicates any number of players.

2.3.5 (L) Level of Activity

Including the level of activity in the analysis adds another dimension which is 'invisible' to the judge. In the fundamental liability game the liability rule affects the level of care. Under L the liability rule also affects how often people engage in a certain activity. The level of activity therefore determines how often a player encounters a potentially hazardous situation. This is 'invisible' to the judge because the judge cannot award higher liability to players who often engage in a certain activity.

For example if drivers become liable more often, the use of cars will decrease. When a car hits a pedestrian the liability will not depend on how often the driver used the car in the past year. Studies including the level of activity are very different in analysis and results and we therefore believe this to be very relevant.

2.3.6 (B) Prior Bargaining possible

This indicates that the players can come to an agreement on their actions prior to the potential accident and can transfer wealth for free to compensate others. If prior bargaining is indeed possible for free this usually reduces the complexity of the problem because of the Coase theorem.

The Coase theorem [5] states that if prior bargaining is possible, the liability rule does not affect the social welfare and therefore the liability rule does not matter. There are a few limitations for the Coase theorem to hold. As mentioned the Coase theorem only holds when there are no transaction costs for bargaining. Also, since the liability rule does influence the distribution of wealth, this distribution should not matter either (hence assumption U should be absent). Finally we argue that one can only pay someone for doing something. Paying everyone for not starting a factory is not plausible in the long run. This is a logical consequence of the implicit assumption of the Coase theorem that distribution of resources should not matter. It indeed might not, but people are still limited by their wealth and people might not always be able to pay the 'optimal' amount.

2.4 Assumptions

Assumptions are premises which are taken for granted. Assumptions facilitate the analysis but reduce the accuracy of the conclusions. The fundamental liability game contains many assumptions. The upcoming extensions will denote an absence of one of these assumptions. More symbols typically imply a more realistic and accurate research.

2.4.1 (C) Court Costs > 0

In the fundamental liability game going to court is assumed to impose no additional cost on society. In reality going to court is expensive and therefore does affect social welfare. Under assumption C, it is assumed that additional fixed cost c is incurred unless the victim decides not to press charges. The victim is assumed to bring the case to court if the expected benefit of going to court is positive for him/her.

Under the assumption C the judge can divide the court costs between the parties. In order to denote the case that the court costs are paid for by the society we use Cs. C is the case in most countries, but since Cs is easier to solve this is also included.

Settlement of cases influences these costs, but not in a way that significantly affects the analysis. If a case is settled this avoids the fixed costs c. This does not significantly change the distribution of money between injurer and victim, since if it would then one party would benefit from not settling. Settling can therefore be modeled simply by reducing the fixed cost c and only has no impact on the analysis.

2.4.2 (D) Damage $\neq \sum$(liability)

In the fundamental liability game the court is assumed to divide the damage between the parties. Under the assumption D, the judge is not limited to dividing the damage but can also impose an additional fine or offer a subsidy. This means that the judge can also take money from, and give money to the tax payer. This gives the judge the flexibility to provide optimal incentives and optimal distribution of welfare at the same time. The judge will impose an additional fine in case both parties exhibit reckless behavior to discourage this behavior. On the other hand, if the parties exercised due care and the accident is 'caused' by an unlucky turn of events it might be optimal to reduce the total liability.

This assumption will be denoted with the symbol D. If the liability is only allowed to exceed the amount of damage (e.g. only money is going to the tax payer) we denote this with De (Liability Exceeds Damage). This assumption might not be the most realistic but because it makes analysis very simple and liability rules much more efficient we choose to include this assumption.

2.4.3 (U) Utility = f(resource)

The fundamental liability game assumes the distribution of resources to be irrelevant. In practice we notice two things: (i) people tend to be risk-averse and (ii) punishing wealthier people has a lower impact on their happiness. While risk and distribution of welfare seem two separate notions, both appear in the analysis once one acknowledges the difference between utility and resources. We define resources to be tangible goods (e.g. money). For utility we consider happiness or more formally 'everything that matters'. We see that in practice, these indeed might and will differ. The resources can be changed and the utility is what needs to be optimized. Furthermore players are assumed to try to optimize their utility.

It is generally accepted that under ordinary circumstances utility is a concave function of resources, meaning that as we become richer the impact on happiness of one additional unit of resource becomes less. Common utility functions are $u = ln(r)$ and $u = r^x$ with $0 < x < 1$, here 'ln' denotes the (natural) logarithm, u is the utility and r is the wealth. Realistic utility functions are assumed to be strictly concave twice differentiable increasing functions: $f''(r) < 0$ and $f'(r) > 0$ on the interval $(0, \infty)$. The fundamental liability game can be considered to use a utility function $u = r$.

We will assume every player has the same utility function. Not because we believe this is true, but because it is hard to make a difference between people on this. The fairness of punishing the wealthy more severely is discussed in more detail in section 4.3.

We will denote this as U, if a study claims wealth/equity should not be considered this will be denoted as Ur (risk only). If a study considers only equity considerations this will be denoted as Ue (equity only). A way to achieve this is by assuming all accidents happen with certainty. The possibility of insurance is an argument which is often put forth to back up their claim of the superiority of studying Ue instead of U. This is because insurance nullifies the risk for those involved. However for many types of accidents insurance is not feasible. We therefore choose to consider both equity and risk by default.

2.4.4 (E) Uncertain evidence

In the fundamental liability game the judge is assumed to be omnipotent. Every action taken, payoff, possible action and probability is assumed to be known. In reality it often occurs that evidence and the facts are not entirely known. We denote this wide class with the symbol E. There are many variables which are uncertain in practice and we therefore limit the notation to the three most distinctive and common uncertainties: uncertainty of action chosen, due care and damage.

(Ea) Action uncertain

A common type of uncertainty is when the action the injurer took is uncertain for the court. In this case the injurer usually argues that he took a safe action and the victim tries to prove the injurer took a negligent action.

We assume the evidence, after proper statistical analysis, yields a probability that the injurer took the negligent action. The judge then decides based on this information on which action to base the liability. As is the common approach in the analysis of liability rules, we will again assume that in this decision the judge will optimize expected social welfare. Since evidence in practice is never entirely conclusive there will always be a probability of a mistake, however small.

(Ed) Due care uncertain

Under negligence rules the player who fails to meet the due care standard is negligent and therefore is liable. Under the assumption that players are perfectly rational it is optimal to set 'due care' as the optimal actions for society. Due care can then be computed by considering the avoidance costs and the probability and size of an accident. In reality this will not be as explicitly computed by the judges. It is therefore likely that the judges make slight errors in estimating the level of due care. Under liability rules based on the Nash equilibrium this may have a high impact as this can make the difference between the injurer paying everything or nothing.

We suggest uncertain due care is modeled using the same approach used for the fuzzy choices of player in Section 2.5.2.

(Es) Size of accident uncertain

In the fundamental liability game the monetary value of an accident is assumed to be known exactly. In reality the monetary value is often not known exactly. Either because victims try to make the damage seem high to increase their claim, or because there are no easy valuation principles of non-tangible resources.

Under Es the monetary value of the damage of the accident s is assumed to be uncertain. The judge is assumed to have a report stating the expected value of the damage and the accuracy (standard deviation) of the estimation. We furthermore assume this estimator is unbiased because we believe people either believe it to be unbiased or this would be corrected. We recommend approximating this uncertainty by an unbiased normal distribution, a method which can also be used for uncertain avoidance costs or uncertain probability of an accident.

2.5 Optimality conditions

In the previous sections the restrictions and assumptions have been discussed. This section shows the notation in order to communicate what is defined as optimal. Both optimality criteria can be said to maximize total social welfare but they differ in the way in which they expect people to behave. The Nash equilibrium assumes perfect rational behavior and is therefore also the easiest and most common choice. The fuzzy condition assumes irrational behavior and is therefore arguably the most realistic.

2.5.1 (NE) Nash Equilibrium

The simplest form of optimality is the Nash equilibrium. A liability rule is NE optimal if it can be shown that neither player has an incentive to play anything other than the socially optimal action if he knows the other player chooses the socially optimal action. A Nash equilibrium is what possibly occurs if all players behave perfectly rational and they know others do so as well.

A common way to show a socially optimal Nash equilibrium emerges is by showing that the 'goal function' of every selfish player is results in optimization of the the social welfare function. If all players optimize the social welfare it follows that an optimal Nash equilibrium emerges.

A unique Nash equilibrium is a Nash equilibrium with the additional constraint that no other Nash equilibria exist. This ensures that if all players play perfectly rational and know others do so as well, they will choose the actions such that the unique Nash equilibrium emerges. This is harder to prove because besides proving something is a Nash equilibrium, it is also necessary to show all other combinations of actions are no Nash equilibria.

2.5.2 (F) Fuzzy Choices / irrational behavior

The fuzzy choices optimality condition assumes players choose more likable actions with a higher probability instead of choosing the best action with probability 1. This ensures that two actions with similar utility (i.g. likability) will be equally likely chosen and as the difference increases the more likable action becomes more probable. This is much like what we observe in real life and this model therefore seems the most suitable for analyzing liability, even more so because accidents often occur because of irrational behavior.

The approach proposed here is the same as used by McKelvey ([14],1996). We define the increasing likelihood function $l(u)$ which denotes the likability and thus the likelihood of a certain action. We then set the probability of an action being chosen proportional to the likelihood of an action so for the injurer

$$p_i^{(i)} = \frac{l(u_i)}{\sum_{x \in I} l(u_x)}. \tag{2.3}$$

Here $p_i^{(i)}$ is the probability of action i being chosen, l is the likelihood function u_x is the expected utility of action x and I is the action set of the injurer.

Because likelihood functions are hard to obtain empirically and will differ from one situation to the next we do not restrict the notation to a specific function. We will often consider the likelihood function proposed by McKelvey as this has some attractive features. McKelvey proposes $l(u) = e^{\lambda u}$ where λ is the rationality of the player (higher is more rational). This means that taking $\lambda \to \infty$ will result in the NE optimality condition. He also shows that fuzzy games with this likelihood function have a fixed point (i.e. solution).

We propose two extensions to the model of McKelvey for increased realism. First is an extension which allows non-disjoint actions to be modeled, correcting for the bias of the default model to overestimate the likelihood of similar actions. Secondly an extension is proposed to take into account habits or biases in the behavior of people.

(Fs) Similarity between actions

A drawback of the model by McKelvey is that adding an action very similar to an already included action (someone wiggling his/her toes during their action) will result in a change in the equilibrium. To correct for this we define for every action a variable called uniqueness and set the probability of an action proportional to the likelihood times the uniqueness of an action. This allows similar actions to be adjusted downwards to reduce the effect of the way of modeling and unique actions to become more likely. The concept of using uniqueness is intuitively appealing as the uniqueness can be directly taken from an extensive form game. The effect of using uniqueness is illustrated in Example 2.1.

Example 2.1

Alice, being slightly tipsy after a night of partying, has to choose between driving home or taking a taxi. Driving home might result in a fine of 200 British pounds with a probability of 10%, and an accident costing 4000 pounds with 0.5%. Taking a taxi home costs 30 pounds. Making it home is assumed to yield Alice a utility of 70. We assume Alice that night uses the likelihood function $l(u) = u^2$.

This problem can now be modeled in three ways. Table 2.2a shows the basic type of modeling by comparing the action drive and the action taxi. Second alternative in Table 2.2b is to include the taxi+ alternative where Alice takes the taxi but in a slightly different way which has very little impact on the utility, for example taking one taxi company over another or wiggling her toes during the trip. Table 2.2c shows how the unrealistic change in expected probabilities of behavior is avoided using uniqueness. If the taxi and the taxi+ alternative become more unlike each other still a proper uniqueness can be found.

action	drive	taxi
u	30	40
$l(u)$	900	1600
$p_i^{(i)}$	36%	64%

(a) Basic modeling

action	drive	taxi	taxi+
u	30	40	40
$l(u)$	900	1600	1600
$p_i^{(i)}$	22%	39%	39%

(b) F, without uniqueness including taxi+ alternative

action	drive	taxi	taxi+
uniqueness	1	0.5	0.5
u	30	40	40
$l(u)$	900	1600	1600
$l_c(u)$	900	800	800
$p_i^{(i)}$	36%	32%	32%

(c) Fs modeling including uniqueness

Table 2.2: Three ways of determining Alice her behavior

□

(Fh) Habits

In reality we observe people sometimes irrationally favor one action over another. Bob might drive a certain route to his work every day because he does not know the other route is much faster and prettier as well. Eve might sit in front of the television all night, while she would rather have done something else. Fh will be used to denote the hardest and most realistic assumption on the behavior of people: actions can have different likelihood functions. So one action might have $l^{(1)}(u) = ln(u)$ and another might have $l^{(2)}(u) = ln(u) + 1$ to model a bias towards action 2.

2.6 Liability rules

The fundamental liability game allows for all liability matrices to occur. Typically many of these are inefficient or too random for practical use. This section explains some basic rules which have been studied and/or used in practice.

2.6.1 (Z) No liability

Under no liability or zero liability, nobody is held liable for what they do or cause. This means damage is borne by those on whom it falls and therefore there will be no lawsuits. This is represented as $L = \mathbf{0}$.

2.6.2 (S) Strict liability

Strict liability in simple games states that those who cause the damage are held liable. In more complex games it somewhat counterintuitively states that the damage should be paid for by those on whom it does not fall (i.e. the injurer). Strict liability is represented as: $L = \mathbf{1}$.

We use Sn to denote strict liability with contributory negligence. This means the injurer is liable unless the victim did not exercise due care. Sdn denotes strict liability with dual contributory negligence. This means the injurer is liable unless he/she exercised due care and the victim did not. Due care is treated in more detail in the section on negligence (2.6.3).

2.6.3 (N) Negligence rule

The negligence rule states the victim pays for the damage unless the injurer has neglected his duty to take due care. The first formal formulation of due care is due to Judge Learned Hand. He states that due care has been taken if more care would result in reduced social welfare because the penalty of avoidance exceeds the benefits of reduced accident costs. In the fundamental liability game and extensions this means the level of due care is set to be the optimal action. So $L_{iv} = 0$ for $i \geq i^*$ and $L_{iv} = 1$ for $i < i^*$.

We use Nn to denote the negligence rule with contributory negligence. Which means the victim is liable for the damage unless the victim exercised due care and the injurer did not.

2.6.4 (R) Relative Negligence

Relative negligence states that the party who could have most easily reduced the size of or altogether avoided the accident is liable. So the party which has the highest reduction in expected accident cost per extra unit of money spent as avoidance cost is held negligent. In the discrete case we find for the reduction in expected accident cost divided by avoidance cost for injurer and victim:

$$n_i = -\frac{P_{(i+1)v}s - P_{iv}s}{a^{(i)}_{(i+1)} - a^{(i)}_i} \tag{2.4}$$

$$n_v = -\frac{P_{i(v+1)}s - P_{iv}s}{a^{(v)}_{(v+1)} - a^{(v)}_v}. \tag{2.5}$$

The party which could have most easily reduced the expected accident cost is now liable

$$L_{iv} = \begin{cases} 1 \text{ if } n_i > n_v \\ 0 \text{ if } n_i \leq n_v. \end{cases} \tag{2.6}$$

When this is analyzed with a continuous action set the definitions above turn into derivatives. We see that relative negligence is only properly defined for regular games.

2.6.5 (C) Comparative Negligence

Comparative negligence is similar to relative negligence. It also considers the reduced expected accident cost per unit of extra avoidance cost invested. Comparative negligence divides the cost according to the degree of negligence instead of making the one whom is most negligent fully liable. Under C the liability rule is defined as follows,

$$L_{iv} = \frac{f(n_i)}{f(n_i) + f(n_v)}. \tag{2.7}$$

Here $f(n)$ is a strictly increasing function of the amount of negligence, n_i and n_v the marginal negligence as defined in equations 2.4 and 2.5. The most common form studied is where $f(n) = n$.

As can be seen, relative negligence is a limiting case of comparative negligence (take $f(n) = n^k$ with $k \to \infty$) . So one can argue that relative negligence does not need to be included in the notation. We choose to keep these separate in the notation because they are often studied separately.

2.6.6 (E) Efficient liability rule

The efficient rule is defined as the rule which maximizes social welfare. So contrary to the more common rules there is no easy way to determine the efficient rule. To find the efficient rule one would for every rule estimate the social welfare and pick the best rule. So we see that the efficient rule is very much tailored to the situation whereas the more common rules apply a somewhat rigid scheme to many different games. For a game G the efficient rule is defined as

$$L_E = \{L \in [0,1]^{I \times V} : w^{(s)}(L) = \max_{L'} w^{(s)}(L')\}.$$

The efficient rule is therefore flexible, accurate and efficient. The biggest drawback is that it is complicated and might therefore not be understood by the people or even the court. We believe the efficient rule is therefore best suited for analysis of situations to infer what defines the quality of a liability rule. Extensive simulation on the efficient rules is done in Chapter 5.

Chapter 3

Classification of Literature

This chapter classifies some of the major studies on liability. The goal is three-fold: (i) to show how the notation can be applied, (ii) to provide a short overview as a reference for the interest of the reader and (iii) to determine how much the different aspects are studied to support our conclusion on further study. The results are given in Table 3.1 and 3.2. Table 3.1 contains the more mathematical/economical oriented studies; Table 3.2 contains the less formal background articles.

Year	Classification	Author(s)	Description
1973	R/Ed'/NE	Brown [2]	First formal modeling of continuous regular games.
1980	RL/NE $\implies w^{(s)}(\text{Nn}) > w^{(s)}(\text{S}), w^{(s)}(\text{N}) > w^{(s)}(\text{Z})$	Shavell [17]	Level of activity.
1985	R/Es$^+$/NE	Haddock, Curran [9]	Comparative negligence and dichotomous liability rules.
1988	C$^-$/E'/NE	Feldman, Frost [7]	Comparison of many different negligence rules.
2001	UEd'/NE $\implies w^{(s)}(\text{N}') > w^{(s)}(\text{N})$	van Wijck, Winters [22]	The restated negligence rule.
2003	C/E'/NE $\not\Rightarrow$ (C$^- \in E, w^{(s)}(\text{C}) > w^{(s)}(\text{S})$)	Bar-Gill [1]	Comparative Negligence under uncertain evidence.
2003	C/Es$^+$/NE $\implies$ Z, S, Sn $\notin$ E	Singh [19]	Dichotomous rules under court errors.
2004	R/Ed'/NE $\not\Rightarrow$ (N' $\subset$ E) RLb/NE	Singh [20]	Van Weijck, Winters [22] under regular games.
2007	R/NE	Singh [21]	Derivation of necessary /sufficient conditions for the efficient rule.
2009	R/NE	Feldman, Singh [8]	Comparative vigilance.

Table 3.1: Mathematical/economical approaches

Year	Classification	Author(s)	Description
1960	CB/NE	Coase [5]	Prior bargaining results in efficient choices.
1965		Calabresi [3]	Proper modeling, avoidance costs.
1968	CB/NE	Calabresi [4]	Solution which arises from bargaining approximates the efficient liability rule.
1972	-N	Posner [16]	Empirical considerations for negligence.
1974		Ehrlich, Posner [6]	Desired detail level of the law, standards vs rules.
1974		Leff [13]	An ethical and philosophical critique.
1998	/U/F$^+$	Jolls, Substein, Thaler [10]	Realistic behavior of people.
2000	/F$'$	Korobkin, Ulen [12]	Realistic behavior of people in law
2007	RLE'/U/-, (*/*/NE)$^-$	Shavell [18]	Extensive overview of semi-mathematical modeling

Table 3.2: background

We see that all studies focus on the Nash equilibrium as the optimality criteria. Furthermore no studies were found which applied risk-averse or distributional welfare to liability. We see many studies focus on uncertain evidence, mostly on the uncertain size of the accident. The uncertain size is often used as an argument in favor of comparative negligence. Another well studied topic is the level of activity (L).

It also becomes clear why the notation as proposed in this document is useful. As mentioned in the introduction, the studies highly vary in assumptions and restrictions which result in (too) many different conclusions. Without consensus on the assumptions, practical and accepted conclusions will stay out of reach.

Chapter 4

Background articles

Before proceeding to the analysis of the assumptions this chapter will treat four unrelated subjects. These should be read as words of caution, suggestions for direction of analysis and more in-depth results.

Section 4.1 will caution about proper modeling, Section 4.2 provides background analysis on the conventionality of actions. Section 4.3 argues against the common notion that the amount of liability should be independent of the wealth of the parties involved. Finally Section 4.4 will suggest an approach for the analysis of Fuzzy behavior.

4.1 On proper modeling

As one attains more knowledge this increasingly starts to define ones world. As one becomes more familiar with a model it becomes harder to find new flaws and remember the old ones. Thus sometimes naivety can achieve more than experience. To minimize this effect it is good to continue to realize how much we do not know and be heedful of the difference between theory and practice. In this section we attempt to understand the limitations of our notation for models and note how the results should be used. First we take a quick look at some of the many things which have not been included nor discussed as part of the notation. This will be followed by an example to introduce the second topic which is the proper use of the results.

One of the aspects not incorporated in the notation is the intrinsic value of the law. Currently citizens are supposed to like a law if an economist determines it is 'efficient'. In practice we should not attribute this rational behavior to citizens and should consider the possibility that the 'best' law might be the one which is liked best. Another aspect which has not included is the indirect effect liability rules might have. Punishing someone poor might result in him/her turning to theft to make a living. This aspect will often be insignificant, but in specific cases will be very important. A final aspect which the author believes is likely to make our house of cards come tumbling down is the tendency of humans to return to an equilibrium level of happiness regardless of resources (also known as hedonic adaptation). Because of this the distribution of resources might have very little impact on happiness and any optimizations should be looked for on a more spiritual or fundamental level.

Example 4.1

In [5, p. 8] Coase uses the following example:

[...] A confectioner (in Wigmore Street) used two mortars and pestles in connection with his business (one had been in operation in the same position for more than 60 years and the other for more than 26 years). A doctor then came to occupy neighbouring premises (in Wimpole Street). The confectioner's machinery caused the doctor no harm until, eight years after he had first occupied the premises, he built a consulting room at the end of his garden right against the confectioner's kitchen. It was then found that the noise and vibration caused by the confectioner's machinery made it difficult for the doctor to use his new consulting room.

In the subsequent text it became clear that the judges and Coase modeled this problem as follows. The payoff for society $U^{(s)}$=

$$\begin{array}{ccc} & \textit{use} & \textit{not use} \\ \textit{not move} & -2100 & -2000 \\ \textit{move} & -600 & -3600 \end{array}.$$

For the sake of simplicity we added the following values: constructing the consulting room: €2000, moving the confectioners machinery: €1600, using the consulting room with machines moved €3000, using consulting room with machinery in place €-100.

From this we can conclude that without intervention (not move, not use) is a Nash equilibrium. This is undesirable as the payoffs are low. If the confectioner was held liable, (move,use) becomes a Nash equilibrium which increases the payoffs from €-2000 to €-600. This is why they argue in favor of strict liability (confectioner liable).

This however neglects to take into account that the optimal action is in fact if the doctor did not build the consulting room in the first place. So the proper way to model this is to also include the action: ' not build':

$$\begin{array}{cccc} & \textit{use} & \textit{not use} & \textit{not build} \\ \textit{not move} & -2100 & -2000 & 0 \\ \textit{move} & -600 & -3600 & -1600 \end{array}.$$

So we see that applying 'no liability' in this case results in the confectioner not building the consulting room whereas applying 'strict liability' results in the construction of the consulting room. We see that the proper way of modeling results in the opposite conclusion. From this example we learn that the person to choose last is more often liable. So if the consulting room would have been there and the confectioner installed their machinery it would be optimal to make the confectioner liable. □

Results can always be found on different levels of abstraction. You can know the answer to life, the universe and everything yet know nothing at all. Similarly results following from these types of liability models are several steps removed from practice. When this is overlooked this results in misuse as in Example 4.1. This example is no exception and we therefore believe there is room for studies on the intermediate level between the current research, and practice. We believe that applying rather than refining current results will yield the following. Higher liability should be assigned to:

- the rich (follows from R/U/NE)
- the one who could most easily have avoided the accident (R/F)
- the unpredictable (R/NE)

- the one exercising the less common action (R/NE)
- the smart (R/Fh)
- the individual (rather than the group) (M/NE)
- the one who chooses last (R/NE)
- the repeating offender (R/Ea/NE)
- the action with other negative (probabilistic) side-effects.

These are merely a couple of expected results. The next section will give a short example of the less common action mentioned above. All in all we recommend modesty concerning the correctness and completeness of this research, liability research and research in general.

4.2 On conventionality of action

This section explains just one aspect of liability rules which is easily overlooked, but in theory could be properly modeled. For this we consider the often modeled accident between a car and a pedestrian.

Think a moment on the way you would model this based on the information so far. Now consider this law in two different countries, one with a single pedestrian and many cars and the other with a single car and many pedestrians. Clearly the efficient law very much depends on the ratio cars/pedestrians. For a country with only a single car it does not make sense to require all pedestrians to watch for cars before crossing. So we conclude the more common action ought to take more care and therefore should be punished more severely.

Including the conventionality of action is vital and yet so easily overlooked.

4.3 On the distribution of welfare

Under assumption Ue it was argued that the utility of wealthier people suffers less from the same loss of resources. This resulted in the conclusion that in the efficient rule wealthier people should be punished more severely. One might wonder about the validity of this conclusion and argue that punishing the wealthy more severely leads to reduced incentive to become wealthy which in turn reduces overall welfare.

Strictly speaking this is correct, but we believe that the effect of wealth dependent punishment should be considered in the larger framework of ways to redistribute wealth. There should be little question about the optimality of taxing the wealthiest slightly disproportional and giving it to the poorest. So the question which remains is whether to redistribute when accidents occur, or in another (non-probabilistic) way.

We see that the question of the effect of wealth dependent punishment is actually irrelevant. The correct and relevant question is whether this effect is better or worse compared to imposing the same penalty on being wealthy another way. It should by now be clear that because of the reduced risk-aversion of the wealthy the penalty on wealth is optimally done by redistributing wealth when accidents occur. This way the risk of bad luck in accidents is borne by those least affected by it, while keeping overall burden on the wealthy constant. This effect is demonstrated in Example 4.2.

Example 4.2

Consider two players i and v with a wealth of \$100 000 and \$60 000 respectively. There is a possible accident of size \$50 000 occurring with probability 10% which we will assume is mostly due to bad luck instead of negligence from one of the parties. For this system we will compare the proportional with the non-proportional liability system. Under proportional liability higher liability is assigned to the wealthier, under non-proportional liability both have equal liability. Moving from the current non-proportional liability to proportional liability wealthier may be 'compensated' for this higher liability by a slightly lower tax, keeping their wealth equal for both systems. Using the natural logarithm as utility function (see Section 2.4.3 for more information) the effects are demonstrated in Table 4.1.

	proportional		**non-proportional**	
	i	v	i	v
Initial wealth	\$100000	\$60000	\$100000	\$60000
Accident ($p = 0.1, s = \$50k$)	-\$35000	-\$15000	-\$25000	-\$25000
Tax	-\$8500	-\$6500	-\$10000	-\$5000
E[resource loss]	-\$12000	-\$8000	-\$12500	-\$7500
E[utility] (logarithmic)	11.37588	10.85453	11.37502	10.85447

Table 4.1: a comparison between a non-proportional and proportional liability rule

We see that the proportional liability rule yields a higher utility for both participants whilst keeping the tax revenue constant. Thus forcing wealth-independence reduces the utility of both players and is therefore not Pareto-efficient. One might be tempted to put aside this conclusion because the effects seem very small. However in Chapter 5 it will be found that this is the assumption with the second-highest impact so at this point it is sufficient to know such an effect exists.

□

This conclusion in favor of proportional liability rules differs from what is usually assumed in studies. The remainder of this section will consider three arguments which can and have been used to argue for non-proportional liability. First is an argument by Kaplow and Shavell [11]. They argue that including fairness as a separate notion is incorrect by showing that taking into account only (social) welfare is correct. While there should be little doubt that taking into account only welfare is indeed correct on an abstract level, they incorrectly imply this means separate notions of fairness should not be included in many fields of practical research.

We believe the flaw in Kaplow and Shavell's reasoning is that they confuse resources and welfare. Resources are the tangibles that the government can observe and change and welfare is the utility or 'happiness' which the government wants to optimize. In this sense we agree with Kaplow and Shavell that it is clear that fairness should not be added to welfare because welfare by definition includes everything that matters. This does not mean that separate notions of fairness should not be added to the total of resources. Loosely we can think of the relation as:

$$\text{Total Resources} + \text{Fairness} = \text{Welfare}.$$

We can see that combining the goal of maximizing total resources with concerns of fair distribution of resources leads to optimal welfare. While both arguments are correct we believe that the addition of fairness to the total number of resources is the only approach with practical relevance.

Secondly we treat the common argument that the judge cannot accurately base his/her decision on the wealth of players. Similarly one might wonder about the acceptance of strongly proportional liability rules in practice. Both arguments are valid concerns and we concede that laws requiring the judge to use proportional liability rules might not be yield optimal social welfare in practice. This does not render our knowledge of the optimality of proportional liability rules useless however. Rather it should be used when considering which law to implement. Laws should favor the party which is likely to be poorest. In a sense this favors the no liability rule as the size of an accident is bound to have some correlation with the wealth of the victim. One might take this one step further and limit the liability for the damage of an expensive object to the value of an ordinary object with the same properties. Therefore if damage occurs to a very expensive car the injurer, if found liable, would be liable up to the amount of an ordinary car.

A final argument used to argue against proportional liability rules states that private insurance nullifies risk. Furthermore any inefficiencies of private insurance also apply to proportional liability rules. Although more arguments can be found, our main argument against private insurance is that in reality private insurance does not function properly. We see many uninsured accidents happening and this is unlikely to change. Using a proportional liability scheme rather than private insurance offers more flexibility and control and can therefore be used to more effect.

4.4 Fuzzy behavior and the Efficient rule (C/U/F)

This section describes a method to analyze irrational behavior. In current analyses people are assumed to behave perfectly rational in order to simplify analysis. Our approach is based on the opposite: assuming near-perfect irrational behavior. Near-perfect irrational behavior is defined as behavior where a random action is chosen with probability $\to 1$.This will also simplify analysis, but at a lower loss of accuracy: we show that behavior in practice is more similar to near-perfect irrational behavior than to perfect rational behavior. Our first conclusion is therefore that combining results from perfect rational behavior and near-perfect irrational behavior is a significant improvement from studying only the Nash Equilibrium.

The use of assuming near-perfect irrational behavior will be demonstrated by showing that solutions for games where both players have two actions can be generalized to games with any number of actions. This conclusion greatly simplifies analysis as these 2x2 games are much easier to solve than games with more or even an infinite number of actions. A third and final conclusion which becomes clear during the analysis is that the amount of liability for a player only depends on how easy a player can avoid an accident. Remarkable is that we do not find any evidence supporting the use of discontinuous criteria such as 'due care'.

The start of the analysis is the necessary condition for a global optimum: $\frac{\delta w^{(s)}}{\delta L_{iv}} = 0 \quad \forall i, v$. If we can compute this derivative and set it equal to zero we find a candidate for the efficient rule. We will find this derivative can be computed to some extent for near-perfect irrational behavior.

4.4.1 Notation repeated

This section briefly describes the notation as used in the coming proofs. The notation is consistent with the notation as presented in Chapter 2 and this section may therefore be skipped.

The injurer and victim choose an action $i \in I$ and $v \in V$ respectively. Given these actions there is a probability P_{ij} that an accident of the fixed size s occurs. Actions i and v incur avoidance costs $a_i^{(i)}$ and $a_v^{(v)}$ regardless of whether or not the accident occurs. The liability rule L_{iv} denotes the part s which will be paid for by the injurer.

Risk-aversion and distribution of wealth is modeled as follows. $w^{(i)}$ and $w^{(v)}$ denote the initial wealth of the injurer and victim (i.e. pre-accident, pre-avoidance cost). $f(r)$ denotes the function which defines the relation between resources and utility. f is a strictly increasing convex function. Players are assumed to maximize expected utility. $U_{iv}^{(i)}$ and $U_{iv}^{(v)}$ denote the expected utility for the injurer and victim given their actions. u_i and u_v denote the expected utility for the action i and v, given the strategy of the other player.

Irrational or fuzzy behavior is modeled by assuming an action with a higher expected utility is more likely to be chosen. $l(u)$ is the likelihood function which is strictly increasing. The probability of an action being chosen is proportional to the likelihood of that action, e.g. for the injurer $p_{\bar{i}}^{(i)} = \frac{l(U_{\bar{i}})}{\sum_{i \in I} l(U_i)}$. Under near-perfect irrational behavior we will use $l(u) = e^{\lambda u}$ with $\lambda \to 0$.

4.4.2 Near-perfect irrational behavior

In this section we demonstrate a way to analyze irrational behavior by approximating the effect of a small change in the liability rule. The small change in liability rule $L_{\bar{i}v}^{(new)} = L_{\bar{i}v}^{(original)} + h$ results in a change in the expected utility for the players $U_{iv}^{(i)}$ and $U_{iv}^{(v)}$, which using the original probabilities that actions will be chosen ($p_i^{(i)(original)}$ and $p_v^{(v)(original)}$) will in turn change $u_i^{(i)}$ and $u_v^{(v)}$ which in turn will change the likelihood $l_i^{(i)}$ and $l_v^{(v)}$ of the actions being chosen. So when both players assume the strategy p of the opponent will not change this method will yield a new strategy for both players called $p^{(i)(1)}$ and $p^{(v)(1)}$. This change in strategy caused by the change in liability rule is called the primary reaction.

Because of the primary reaction, the strategy of the players change from $p^{(i)(original)}$ to $p^{(i)(1)}$. This changes $u_i^{(i)}$ which in turn will give rise to another change in the strategy. This is called the secondary reaction. The secondary reaction uses the strategy after the primary reaction $p^{(i)(1)}$ and $p^{(v)(1)}$ to compute the new utility $u_i^{(i)(2)}$ and $u_v^{(v)(2)}$, likelihood and finally the new strategy $p^{(i)(2)}$ and $p^{(v)(2)}$. Because of the secondary reaction the expected utility for their actions of the players change. This gives rise to the tertiary etc.

We will see that following reactions will become smaller every time and the sequence converges to the new solution of the game for the changed liability rule. We find that we can now approximate the total change by considering only the primary reaction. For h small enough we find:

Theorem:

$$|p^{(new)} - p^{(original)}| \equiv |p^{(\infty)} - p^{(0)}| < (1 + \epsilon)|p^{(1)} - p^{(0)}|.$$

ϵ is some error due to rational behavior (i.e. everything but the primary reaction), we will show that $\epsilon = \frac{1}{1 - \lambda s(\max_{iv}(P_{iv}) - \min_{iv}(P_{iv}))} - 1$ (valid if this gives an $\epsilon > 0$). The important conclusion is that the primary reaction (i.e. $p^{(1)} - p^{(0)}$) becomes more important as people behave more irrational ($\lambda \to 0$). So under near-perfect irrational behavior we need to consider only the primary reaction.

As an indication of the accuracy of assuming near-perfect irrational behavior we compute ϵ for a couple of realistic games. These tests indicate that the rational part (ϵ) is dominated by the irrational part. For example, an estimation of ϵ based on the realistic game of Section 5.2 yields $\epsilon = 0.65$. This is an upper bound and we expect the real epsilon to be much lower. (This estimate is found using the realistic game of Section 5.2, excluding utility, using Equation (4.10) and the solution of this game $p^{(i)} = [0.13, 0.48, 0.38]$ $p^{(v)} = [0.22, 0.29, 0.49]$.)

The general outline of the proof is as follows:

$$p^{(\infty)} - p^{(0)} = \sum_{n=0}^{\infty}(p^{(n+1)} - p^{(n)})$$

which by showing that for h small enough it holds that

$$|p^{(n+1)} - p^{(n)}| < q \cdot |p^{(n)} - p^{(n-1)}| \quad \exists q$$

yields

$$\begin{aligned} |p^{(\infty)} - p^{(0)}| &< \sum_{n=0}^{\infty} q^n \cdot |p^{(1)} - p^{(0)}| \\ &= (1+\epsilon) \cdot |p^{(1)} - p^{(0)}| \end{aligned}$$

with

$$\epsilon = \frac{1}{1-q} - 1 \quad \text{for } q < 1$$

Proof We will study the change in actions played when the liability rule changes slightly (i.e. $\frac{\delta p}{\delta L_{\bar{i}\bar{v}}}$). For this we define the sequence to formalize the primary, secondary,... reactions $p_i^{(i)(0)}, p_i^{(i)(1)}, ..., p_i^{(i)(n)}$ and $p_v^{(v)(0)}, p_v^{(v)(1)}, ..., p_v^{(v)(n)}$ where $p_i^{(i)(n)}$ is the probability that the injurer plays action i at iteration n. $p_i^{(i)(0)}$ and $p_v^{(v)(0)}$ is the solution of the game before the change. Then the rest of the sequence is defined as follows $p_i^{(i)(n+1)} = R(p_v^{(v)(n)})$, where R denotes the strategy given the strategy of the opponent. This means that $p^{(1)} - p^{(0)}$ is a reaction on the change of the liability rule and subsequent reactions are reactions on the reaction of the opponent.

Without proof we will state that a small change in the liability rule ($L_{\bar{i}\bar{v}}$) can equivalently be modeled as a small change in the expected utility for the players (see Section 4.4.4). So our starting point is

$$U_{\bar{i}v}^{(i)(1)} = U_{\bar{i}v}^{(i)(0)} + h, \quad U_{\bar{i}v}^{(v)(1)} = U_{\bar{i}v}^{(v)(0)} - ch, \quad h \to 0. \tag{4.1}$$

Here $U_{iv}^{(i)}$ is the expected utility for the injurer if the injurer chooses action i and the victim chooses action v. Similarly $U_{iv}^{(v)}$ is the expected utility for the victim. We now proceed to find the reactions of the players on this disturbance. The primary disturbance (4.1) results in a change in the utility of action $\bar{i}$ of

$$u_{\bar{i}}^{(1)} = \sum_{v \in V} p_v^{(v)(0)} U_{\bar{i}v}^{(i)(1)} = u_{\bar{i}}^{(0)} + h p_{\bar{v}}^{(v)(0)}. \tag{4.2}$$

Using the Taylor approximation of the likelihood function l around $u_{\bar{i}}^{(0)}$ yields

$$l_{\bar{i}}^{(1)} = l(u_{\bar{i}}^{(1)}) = l(u_{\bar{i}}^{(0)}) + l'(u_{\bar{i}}^{(0)}) \cdot (u_i^{(1)} - u_i^{(0)}) + o(u_i^{(1)} - u_i^{(0)}) \tag{4.3}$$

which combined with (4.2) yields

$$l_{\bar{i}}^{(1)} = l(u_{\bar{i}}^{(0)}) + l'(u_{\bar{i}}^{(0)}) h p_{\bar{v}}^{(v)(0)} + o(h). \tag{4.4}$$

The change in the probability that action $\bar{i}$ is chosen can now be expressed as

$$p_{\bar{i}}^{(i)(1)} - p_{\bar{i}}^{(i)(0)} = \frac{l_{\bar{i}}^{(1)}}{\sum_{i\in I} l_i^{(1)}} - \frac{l_{\bar{i}}^{(0)}}{\sum_{i\in I} l_i^{(0)}}$$

which using (4.4) can be rewritten as

$$\begin{aligned}
&= \frac{l_{\bar{i}}^{(0)} + l_{\bar{i}}'^{(0)} h p_{\bar{v}}^{(0)} + o(h)}{\left(\sum_{i\in I} l_i^{(0)}\right) + l_{\bar{i}}'^{(0)} h p_{\bar{v}}^{(0)} + o(h)} - \frac{l_{\bar{i}}^{(0)}}{\sum_{i\in I} l_i^{(0)}}. \\
&= \frac{\left(\sum_{i\in I} l_i^{(0)}\right)\left(l_{\bar{i}}^{(0)} + l_{\bar{i}}'^{(0)} h p_{\bar{v}}^{(0)} + o(h)\right) - l_{\bar{i}}^{(0)}\left(\left(\sum_{i\in I} l_i^{(0)}\right) + l_{\bar{i}}'^{(0)} h p_{\bar{v}}^{(0)} + o(h)\right)}{\left(\left(\sum_{i\in I} l_i^{(0)}\right) + l_{\bar{i}}'^{(0)} h p_{\bar{v}}^{(0)} + o(h)\right)\left(\sum_{i\in I} l_i^{(0)}\right)} \\
&= \frac{\left(\left(\sum_{i\in I} l_i^{(0)}\right) - l_{\bar{i}}^{(0)}\right)\left(l_{\bar{i}}'^{(0)} p_{\bar{v}}^{(v)(0)}\right)}{\left(\sum_{i\in I} l_i^{(0)}\right)^2} \cdot h \,(\text{for } h \text{ small enough}) \\
&= \frac{(1 - p_{\bar{i}}^{(i)(0)}) \cdot l_{\bar{i}}'^{(0)} p_{\bar{v}}^{(v)(0)}}{\sum_{i\in I} l_i^{(0)}} \cdot h.
\end{aligned} \tag{4.5}$$

This completes the derivation of the first iteration of the injurer. Similarly we find for the first iteration of the victim,

$$p_{\bar{v}}^{(v)(1)} - p_{\bar{v}}^{(v)(0)} = \frac{(1 - p_{\bar{v}}^{(v)(0)}) \cdot l_{\bar{v}}'^{(0)} p_{\bar{i}}^{(i)(0)}}{\sum_{v\in V} l_v^{(0)}} \cdot -ch. \tag{4.6}$$

This first step is called the primary reaction of the players. Increasing or decreasing a payoff will result in increased/decreased probability of the corresponding action being played. There is a secondary effect where players optimize their strategy to match the changed strategy of the opponent. In irrational games this reaction also occurs but is generally small compared to the primary reaction. To see this and show convergence we continue the algorithm:

$$u_i^{(n+1)} = \sum_{v\in V} p_v^{(v)(n)} U_{iv}^{(i)(1)} = u_i^{(n)} + \sum_{v\in V} (p_v^{(v)(n)} - p_v^{(v)(n-1)}) U_{iv}^{(i)(1)} \quad \forall n \geq 1.$$

This is of the same form as (4.2) so repeating steps (4.2)-(4.5) yields

$$p_i^{(i)(n+1)} - p_i^{(i)(n)} = \frac{(1 - p_i^{(i)(n)}) \cdot l_i'^{(n)}}{\sum_{i\in I} l_i^{(n)}} \cdot \left(\sum_{v\in V} (p_v^{(v)(n)} - p_v^{(v)(n-1)}) U_{iv}^{(i)(1)}\right) \quad \forall n \geq 1 \tag{4.7}$$

Here $U_{iv}^{(i)(n)}$ was replaced by $U_{iv}^{(i)(1)}$ as the expected utility given actions does not change after the initial disturbance. There is one final modification which makes (4.7) easier to work with. We see that if up until n there is convergence (i.e. $p^{(k)} - p^{(k-1)} < p^{(k-1)} - p^{(k-2)}$ for $1 \leq k \leq n$) we find $l^{(n)} = l^{(n-1)} + o(1)$. So then without further proof we state that now it follows from induction that,

$$p_i^{(i)(n+1)} - p_i^{(i)(n)} = \frac{(1 - p_i^{(i)(0)}) \cdot l_i'^{(0)}}{\sum_{i \in I} l_i^{(0)}} \cdot \left(\sum_{v \in V} (p_v^{(v)(n)} - p_v^{(v)(n-1)}) U_{iv}^{(i)(1)} \right) \quad \forall n \geq 1$$
$$\text{if } p_i^{(i)(n)} - p_i^{(i)(0)} \to 0 \tag{4.8}$$

So we see (4.8) can be used both to check if the series converge and if it indeed converges it gives the correct solution of the new game.

At this point it is easy to state a sufficient condition for the convergence of the algorithm to a solution of the game (e.g. spectral radius of the corresponding matrix < 1) which can then be used to accurately determine the impact of the primary and secondary reaction. Instead we choose a less accurate but more intuitive method which overestimates the impact of the secondary reaction. For this we will use the likelihood function $l(u) = e^{\lambda u}$, this gives

$$\begin{aligned} p_i^{(i)(n+1)} - p_i^{(i)(n)} &= \frac{(1 - p_i^{(i)(0)}) \cdot l_i'^{(0)}}{\sum_{i \in I} l_i^{(0)}} \cdot \left(\sum_{v \in V} (p_v^{(v)(n)} - p_v^{(v)(n-1)}) U_{iv}^{(i)(1)} \right) \\ &= \frac{(1 - p_i^{(i)(0)}) \cdot \lambda l_i^{(0)}}{\sum_{i \in I} l_i^{(0)}} \cdot \left(\sum_{v \in V} (p_v^{(v)(n)} - p_v^{(v)(n-1)}) U_{iv}^{(i)(1)} \right) \\ &= \lambda (1 - p_i^{(i)(0)}) p_i^{(i)(0)} \cdot \left(\sum_{v \in V} (p_v^{(v)(n)} - p_v^{(v)(n-1)}) U_{iv}^{(i)(1)} \right) \end{aligned} \tag{4.9}$$

so then is also holds

$$\begin{aligned} \sum_{i \in I} |p_i^{(i)(n+1)} - p_i^{(i)(n)}| &= \sum_{i \in I} \left| \lambda (1 - p_i^{(i)(0)}) p_i^{(i)(0)} \cdot \left(\sum_{v \in V} (p_v^{(v)(n)} - p_v^{(v)(n-1)}) U_{iv}^{(i)(1)} \right) \right| \\ &= \lambda \sum_{i \in I} (1 - p_i^{(i)(0)}) p_i^{(i)(0)} \cdot \left| \sum_{v \in V} (p_v^{(v)(n)} - p_v^{(v)(n-1)}) U_{iv}^{(i)(1)} \right| \\ &= \lambda \sum_{i \in I} (1 - p_i^{(i)(0)}) p_i^{(i)(0)} \cdot \left(\sum_{v \in V^{(n)+}} (p_v^{(v)(n)} - p_v^{(v)(n-1)}) U_{iv}^{(i)(1)} \right. \\ &\quad \left. - \sum_{v \in V^{(n)-}} (p_v^{(v)(n)} - p_v^{(v)(n-1)}) U_{iv}^{(i)(1)} \right) \end{aligned}$$

with $V^{(n)+} = \{v \in V : p_v^{(v)(n)} - p_v^{(v)(n-1)} \geq 0\}$ and $V^{(n)-} = \{v \in V : p_v^{(v)(n)} - p_v^{(v)(n-1)} < 0\}$

$$
\begin{aligned}
&\leq \lambda \sum_{i \in I} (1 - p_i^{(i)(0)}) p_i^{(i)(0)} \cdot \left(\max_v (U_{iv}^{(i)(1)}) - \min_v (U_{iv}^{(i)(1)}) \right) \\
&\cdot \sum_{v \in V} \left| p_v^{(v)(n)} - p_v^{(v)(n-1)} \right| \qquad (4.10) \\
&< \lambda \sum_{i \in I} p_i^{(i)(0)} \cdot \left(\max_{i,v} (U_{iv}^{(i)(1)}) - \min_{i,v} (U_{iv}^{(i)(1)}) \right) \\
&\cdot \sum_{v \in V} \left| p_v^{(v)(n)} - p_v^{(v)(n-1)} \right|
\end{aligned}
$$

which because $\sum_{i \in I} p_i^{(i)(0)} = 1$ becomes

$$
= \lambda \cdot \left(\max_{i,v} (U_{iv}^{(i)(1)}) - \min_{i,v} (U_{iv}^{(i)(1)}) \right) \sum_{v \in V} \left| p_v^{(v)(n)} - p_v^{(v)(n-1)} \right|. \qquad (4.11)
$$

Now everything comes together. Taking $c = 1$ for the sake of brevity, we first find for the victim

$$
\sum_{v \in V} |p_v^{(v)(n+1)} - p_v^{(v)(n)}| < \lambda \cdot \left(\max_{i,v} (U_{iv}^{(v)(1)}) - \min_{i,v} (U_{iv}^{(v)(1)}) \right) \sum_{i \in I} \left| p_i^{(i)(n)} - p_i^{(i)(n-1)} \right|. \qquad (4.12)
$$

Furthermore since $\sum_{i \in I} (p_i^{(i)(n+1)} - p_i^{(i)(n)}) = 0$ we find

$$
|p_i^{(i)(n+1)} - p_i^{(i)(n)}| \leq 0.5 \cdot \sum_{i \in I} |p_i^{(i)(n+1)} - p_i^{(i)(n)}|. \qquad (4.13)
$$

Finally, from the definition it follows that

$$
\sum_{i \in I} \left(p_i^{(i)(1)} - p_i^{(i)(0)} \right) = 2 \cdot \left(p_{\bar{i}}^{(i)(1)} - \cdot p_{\bar{i}}^{(i)(0)} \right). \qquad (4.14)
$$

Now combining (4.11)-(4.14) yields our conclusion

$$
|p_i^{(i)(n+1)} - p_i^{(i)(n)}| < (\lambda q)^n \cdot |p_i^{(i)(1)} - p_i^{(i)(0)}| \qquad \text{for } n \geq 1 \qquad (4.15)
$$

or

$$
|p_i^{(i)(n)} - p_i^{(i)(1)}| < \left(\frac{1}{1 - \lambda q} - 1 \right) |p_i^{(i)(1)} - p_i^{(i)(0)}| \qquad \text{as } n \to \infty \qquad (4.16)
$$

with $q = \max \left(\max_{i,v} (U_{iv}^{(i)(1)}) - \min_{i,v} (U_{iv}^{(i)(1)}), \max_{i,v} (U_{iv}^{(v)(1)}) - \min_{i,v} (U_{iv}^{(v)(1)}) \right)$ which is valid for h small enough and if $\lambda q < 0$ from this definition. In games where no pair of actions exist such that one action has lower avoidance cost and yields lower expected accident cost as well we find the rough approximation: $q \leq s \left(\max_{iv} (P_{iv}) - \min_{iv} (P_{iv}) \right)$.

4.4.3 Simplification

In the previous section it has been found that under near-perfect irrational behavior only primary reactions need to be considered. Consider an accident with any number of actions for both injurer and victim, furthermore assume they chose action $\bar{i}$ and $\bar{v}$ and an accident occurred. How should the judge rule?

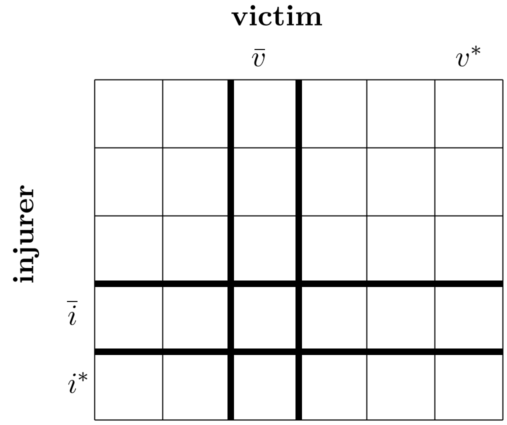

Figure 4.1: A schematic presentation of the choices of the players

The judge will attempt to set $L_{\bar{i}\bar{v}}$ such that $\frac{\delta w^{(s)}}{\delta L_{\bar{i}\bar{v}}} = 0$. Due to near-perfect irrational behavior and results from the previous subsection we know that changing $L_{\bar{i}\bar{v}}$ will not affect the ratio p_x/p_y $(x, y \neq \bar{i})$. The game can now be rewritten to a game where both players have only two actions $\bar{i}$ and i_{agg}. $l_{i_{agg}}$ as follows

$$U^{(i)(agg)}_{i_{agg}v_{agg}} = \frac{\sum_{i\in I_{agg}}\sum_{v\in V_{agg}} p_i^{(i)} p_v^{(v)} U_{iv}^{(i)(orig)}}{\sum_{i\in I_{agg}}\sum_{v\in V_{agg}} p_i^{(i)} p_v^{(v)}}$$

$$U^{(i)(agg)}_{i_{agg}\bar{v}} = \frac{\sum_{i\in I_{agg}} p_i^{(i)} p_{\bar{v}}^{(i)} U_{i\bar{v}}^{(i)(orig)}}{\sum_{i\in I_{agg}} p_i^{(i)} p_{\bar{v}}^{(v)}}$$

$$U^{(i)(agg)}_{\bar{i}v_{agg}} = \frac{\sum_{v\in V_{agg}} p_{\bar{i}}^{(i)} p_v^{(v)} U_{iv}^{(i)(orig)}}{\sum_{v\in V_{agg}} p_{\bar{i}}^{(i)} p_v^{(v)}}$$

$$U^{(i)(agg)}_{\bar{i}\bar{v}} = U^{(i)(orig)}_{\bar{i}\bar{v}}$$

where $I_{agg} = \{i \in I : i \neq \bar{i}\} = I \backslash \{i\}$ and agg stands for aggregate.

Under the near-perfect irrational behavior assumption any game can therefore be simplified to a 2x2 Fh game. It is important to realize that this aggregation changes the likelihood function for the aggregated action, hence Fh instead of F. This does not affect our conclusion at the end of this section.

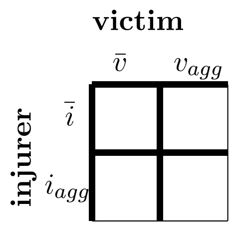

Figure 4.2: The simplified game

4.4.4 Including utility

This section shows how U can be included in the proofs. We choose to include U while excluding L, restriction C, E and Fh because it will be shown that U is important (Chapter 5) and is easily incorporated. We do expect the results can be applied to the omitted restrictions and assumptions as well after some modifications.

From the definition it directly follows that

$$U_{iv}^{(i)(0)} = (1 - P_{iv})f(w^{(i)} - a^{(i)}) + P_{iv}f(w^{(i)} - L_{iv}^{(0)}s - a^{(i)}) \tag{4.17}$$
$$U_{iv}^{(v)(0)} = (1 - P_{iv})f(w^{(v)} - a^{(v)}) + P_{iv}f(w^{(v)} - (1 - L_{iv}^{(0)})s - a^{(v)}). \tag{4.18}$$

Decreasing L_{iv} by an amount of $\bar{h} \to 0$ the new payoff can be derived using a Taylor approximations of f

$$L_{iv}^{(1)} \equiv L_{iv}^{(0)} - \bar{h} \tag{4.19}$$
$$\begin{aligned} U_{iv}^{(i)(1)} &= (1 - P_{iv})f(w^{(i)} - a^{(i)}) + P_{iv}f(w^{(i)} - L_{iv}^{(0)}s - a^{(i)}) \\ &\quad + P_{iv}f'(w^{(i)} - L_{iv}^{(0)}s - a^{(i)})\bar{h} + o(\bar{h}) \\ &= U_{iv}^{(i)(0)} + P_{iv}f'(w^{(i)} - L_{iv}^{(0)}s - a^{(i)})\bar{h} + o(\bar{h}) \end{aligned} \tag{4.20}$$
$$U_{iv}^{(v)(1)} = U_{iv}^{(v)(0)} - P_{iv}f'(w^{(v)} - (1 - L_{iv}^{(0)})s - a^{(v)})\bar{h} + o(\bar{h}). \tag{4.21}$$

Since $h \to 0$ this can equivalently be modeled as

$$U_{iv}^{(i)(1)} = U_{iv}^{(i)(0)} + h \tag{4.22}$$
$$U_{iv}^{(v)(1)} = U_{iv}^{(v)(0)} - ch \tag{4.23}$$

with $c = \frac{f'(w^{(v)} - (1 - L_{iv}^{(0)})s - a^{(v)})}{f'(w^{(i)} - L_{iv}^{(0)}s - a^{(i)})}$ and $h = P_{iv}f'(w^{(i)} - L_{iv}^{(0)}s - a^{(i)})\bar{h} + o(\bar{h}) \to 0$ as $\bar{h} \to 0$. If the avoidance cost and accident cost are low compared to the wealth this can accurately be simplified to the intuitively more appealing $c = \frac{f'(w^{(v)})}{f'(w^{(i)})}$.

4.4.5 Conclusion

Noticing the lack of accuracy of using the Nash equilibrium condition this section investigated fuzzy behavior. Games can than be expressed as a combination of a rational component and an completely irrational component. Estimation of these components in Subsection 4.4.2 yields that the irrational component dominates the rational component. This leads us to assume near-perfect irrational behavior to simplify the study of fuzzy behavior. It has been shown that under near-perfect irrational behavior players only react to an initial disturbance of the game and do not react on each other. This is then used to show that all games can be simplified to games where both players have only two actions.

On an abstract level these games are easily solved. The judge needs to set the liability such that the derivative of the social welfare to the liability of the current action is zero. This depends on the expected social welfare for the aggregated action of the player. It is interesting to see that the efficient rule is based on an aggregate of what the player also could have done and no evidence is found for anything as discontinuous as 'due care'.

Chapter 5

Simulation Model (R/CsUEs/F)

In this chapter a newly developed simulation model is described and used to evaluate the impact of assumptions. The simulation model can simulate all games satisfying R/Cs U Es/F and approximate the efficient rule. It is therefore a useful tool for analysis and for gathering information about the quality of liability rules. It will be used to estimate the impact of the various assumptions which will be combined with the classification of literature to conclude which assumptions are most important for further research.

Section 5.1 describes the inner workings of the model followed by the application of the model in Section 5.2. The following sections will in turn investigate the effect of equity considerations (Ue), risk-aversion (Ur), Fuzzy behavior (F), Court costs (Cs) and uncertain size of the accident (Es).

A note of warning about using the simulation model. The results and model are only valid as an indication and should not be considered a proof. Even though the results presented here are consistent with findings from many other simulations, the high complexity suggests exceptions may be found. We took great care in checking the results, but since no other liability simulation models nor studies on these type of simulations were found there were no results to check our findings. Instead the most important results were manually checked (pencil, paper and patience) to ensure their validity. All in all we are confident the results are sound, but we stress the importance of not overestimating the generality of these results.

5.1 Model description

This section contains a short summary of the structure and assumptions of the model. A short version of the most important code is included in Appendix A. The model works exactly as described in Chapter 2 therefore we limit ourselves to discussing only new assumptions and implementation here.

The main module of the simulation model computes the social welfare given all restrictions, assumptions and liability rule. This module has three main steps (1) for every action i and v compute the utility for the injurer, victim and society ($U^{(i)}, U^{(v)}, U^{(s)}$), (2) compute the probability that action i and v are played ($p_i^{(i)}, p_v^{(v)}$) and (3) compute the social welfare ($w^{(s)}$). The resulting social welfare can then be used to compare the quality of liability rules. The fourth important part is a module to compute the efficient rule by finding the liability rule which maximizes the social welfare (L_E) (4). These modules are again divided in multiple parts which take into account various aspects of the problem. A final module (5) takes care of fairness, this computes the proper tax reduction to match the increased liability for the wealthier as described in Section 4.3.

(1) Computation of the utility for the injurer, victim and society given the actions of the players. This is a straightforward computation since for every combination of actions the probability, size, division of liability and utility function is given: $U_{iv}^{(i)} = P_{iv} f(w^{(i)} - sL_{iv} - a_i^{(i)}) + (1 - P_{iv}) f(w^{(i)} - a_i^{(i)})$. There are two complications: the uncertain evidence and the court costs. The uncertain evidence is modeled by introducing a normal random variable S which is the size of the liability. For fast computation the normal distribution is approximated by a vector of 100 selected values of the normal distribution at 0.5%, 1.5%... 99.5%. If court costs are present ($L_{iv} > 0$ and $c > 0$) c is assumed to be divided among an infinite number of tax payers of wealth 80 000 (so $U_{iv}^{(s)} = U_{iv}^{(i)} + U_{iv}^{(v)} - P_{iv} f'(80000)c$).

(2) Compute the solution of the game (i.e. find $p_i^{(i)}$ and $p_v^{(v)}$) using utilities of the injurer and victim from step (1). This is done iteratively by setting the strategy of each player to match the latest strategy of the other player. By default the utility function $l(u) = e^{\lambda u}$ as proposed by McKelvey [14] is used. The most attractive feature is that this is the only 'starting utility'-independent function in the sense that the probability of choosing an alternative with $u_1 = a + c$ over an alternative with $u_2 = a$ with the same probability independent of a. λ is estimated by finding the λ such that that the probability that an alternative which results in 20 100 resources (e.g. euro) is chosen with a probability $\frac{2}{3}$ over an alternative with 20 000 resources, so $\lambda = \frac{ln(2)}{f(20100) - f(20000)}$.

(3) Compute total social welfare using the results of the first steps: $w^{(s)} = p^{(i)T} U^{(s)} p^{(v)}$.

(4) Computation of the efficient rule is done by choosing L_E to maximize the social welfare. This is the hardest part of the simulation model because the efficient rule is very irregular and unpredictable. The three main problems encountered when optimizing the liability rule are: (i) many local maxima may exist, (ii) in coordination games liability can increase when a player takes more care therefore these games are excluded (iii) court costs make the social welfare function discontinuous. In an attempt to find the global maximum three consecutive steps are taken

(4a) try all combinations of dichotomous liability rules where for every combination of actions only one party is liable
(4b) optimize in a random order all individual cells using the golden ratio method
(4c) Set the lowest L_{iv} to zero. If this increases social welfare keep this and return to (b).

This method seems to work well, although there is no guarantee it indeed finds the optimal liability rule. A final optional addition to speed up the computation is that every step can be set to consider only liability rules where liability decreases as care increases.

(5) In Section 4.3 we found that punishing the wealthier more severely can increase the welfare of every player if this is compensated by a slightly lower tax for the rich. This module computes the size of the tax reduction such that the wealthier party does not suffer from wealth-dependent liability rules.

5.2 Application of the model

The simulation model is used to determine the accuracy of the assumptions. In order to measure the accuracy we define a realistic game and remove one assumption (i.e. make it less realistic) and compute the efficient rule for this simplified game. This efficient rule is then applied to the realistic game and compared to the efficient rule for that realistic game. The resulting difference is the welfare loss caused by the simplification. This can be represented in a formula as follows

$$M = f'(80000)^{-1} \cdot (w^{(s)}(L_E(R)|R) - w^{(s)}(L_E(R - \text{assumption})|R)).$$

Here M is the monetary equivalent of the welfare loss caused by the simplification and is used as the measure for the quality of an assumption. $w^{(s)}(L|G)$ is the social welfare of liability rule L for the game G, $L_E(G)$ is the efficient rule for game G and R is the realistic game. Finally we reverse the resource to utility calculation to make the results more tangible, hence the multiplication by $f'(80000)^{-1}$. Thus a higher value of$M$ indicates a higher importance. The major strength of this method is that it allows the effect of different assumptions to be compared.

For the sake of brevity we will restrict ourselves to presenting a single realistic game as an example. More simulations were done and found consistent with the results presented. The realistic game R as used has the following characteristics. All monetary amounts that are mentioned are in 2010 euros. The size of the possible accident is 15000 euro

$$s = 15000.$$

The probability of an accident depends on the actions of the injurer and victim. The injurer chooses vertically, victim chooses horizontally.

$$P = \begin{bmatrix} 3\% & 2.4\% & 1.8\% \\ 1.8\% & 1.3\% & 1\% \\ 1\% & 0.5\% & 0.1\% \end{bmatrix}.$$

We see that the injurer can more easily reduce the accident, but this will be done at a higher cost

$$a^{(i)} = \begin{bmatrix} 0 \\ 90 \\ 200 \end{bmatrix}, \quad a^{(v)} = [0, 30, 70].$$

Still for the injurer can slightly cheaper avoid the accident. We also see the game is regular. The injurer is assumed to be much wealthier than the victim

$$w^{(i)} = 80000, \quad w^{(v)} = 20000.$$

This concludes the specification of the restrictions and we now proceed to the variables which define the assumptions.

$$c = 80$$

We assume the court costs for an accident of the size 15000 are 8000 euros and that only 1% of the accidents is actually brought to court. So we assume 99% of the cases are solved by the injurer and victim themselves.

$$es = 0.1$$

We assume that the coefficient of variation for the uncertainty in the determination of the size of the accident by the court is 0.1. This means that the standard deviation is 0.1 times the size of the accident. So if $s = 15000$ then $\sigma = 1500$ meaning that in 65% of the cases the estimated size falls between 13500 and 16500 and in 95% of the cases the estimated size falls between 12000 and 18000.

$$f(r) = ln(r)$$

We choose to use the logarithmic utility function. The logarithmic utility function states that every doubling of someones welfare resources increases the utility by a constant number. Analysis shows that experiments with $f(r) = \sqrt{r}$ yields similar results.

Now that the realistic game is defined we provide the results of the simulation as background information for the reader.

$$L_E(R) = \mathbf{1}$$

$$p^{(i)} = \begin{bmatrix} 0.288 \\ 0.356 \\ 0.356 \end{bmatrix}, \quad p^{(v)} = [0.4728, 0.3846, 0.1427].$$

For equal wealth:

$$L_E(R + w^{(v)} = 80000) = \begin{bmatrix} 0.6278 & 0.8682 & 0.9364 \\ 0.2275 & 0.4672 & 0.5354 \\ 0 & 0.3794 & 0.4476 \end{bmatrix}$$

$$u^{(i)} = 11.286943$$

using a tax scheme keeping the wealth of the wealthier constant

$$L_E(R + \text{tax}) = \begin{bmatrix} 0 & 0.8648 & 1 \\ 0 & 0.7625 & 0.8780 \\ 0 & 0.7298 & 0.8426 \end{bmatrix}$$

$$u^{(i)} = 11.286945$$

5.3 Impact of wealth (Ue)

In Chapter 3 it became clear that liability studies incorporating differences in wealth are rare at best. Many studies rationalize this absence using an argument which is shown to be incorrect in Section 4.3. So it is interesting to find the welfare loss caused by disregarding differences in wealth. In order to provide the most realistic view on the welfare loss we show the results where the tax scheme is used to ensure no player suffers from this proportional liability scheme. Thus the wealthier will pay more as liability but pay a slightly lower tax, keeping their wealth at the same level as without proportional liability. Wealth turns out to have the second highest impact. The results are shown in Figure 5.1.

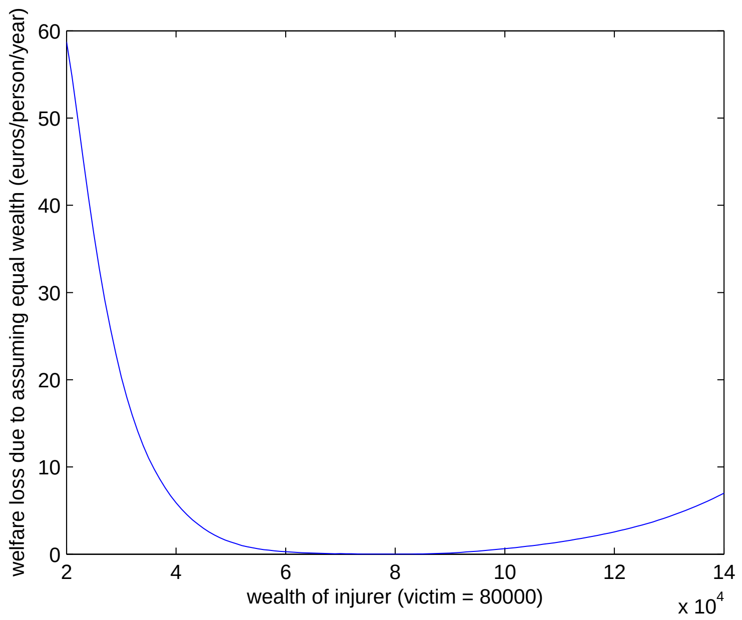

Figure 5.1: impact of wealth

The welfare loss due to assuming equal wealth increases as the difference in wealth increases, and is zero when the wealth is equal. So the impact depends on the situation. We estimate the potential effect to be M_{Ue} = **€30**.

5.4 Impact of risk aversion (Ur)

People tend to avoid risk so players rather have a high probability of an accident and lower size than vice versa. Due to the setup of the resource to utility function we are forced to test this as an incremental effect to welfare. Under equal wealth for both players ($w^{(i)} = w^{(v)} = 80000$) the difference between using a resource to utility and stating that resources equal utility is as follows

$$L_E(R + f = r) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$L_E(R + f = ln(r)) = \begin{bmatrix} 0.6278 & 0.8682 & 0.9364 \\ 0.2275 & 0.4672 & 0.5354 \\ 0 & 0.3794 & 0.4476 \end{bmatrix}$$

$$M_{Ur} = 3.72.$$

This effect is small and becomes even smaller as the difference between wealth increases.We therefore estimate $M_{Ur} = $ **€3** .

5.5 Impact of fuzzy behavior (F)

Similar to (U) for fuzzy behavior no prior research has been found. We argued that the current assumption of the Nash equilibrium is not entirely accurate. Further study in section 4.4 showed that it is possible to analyze irrational behavior by assuming near-perfect irrational behavior. The relevance of studying fuzzy behavior becomes clear as we look at the results of the simulation in Figure 5.2. Only the impact of wealth comes close in impact, but since results for fuzzy behavior are easier to apply and studies will be easier we believe this to be the most important factor to study.

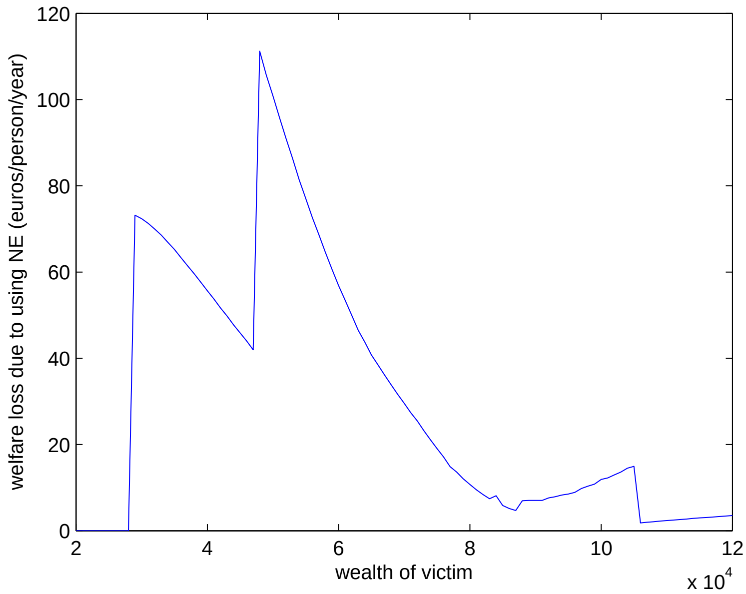

Figure 5.2: impact of fuzzy behavior

We see that the welfare loss of the NE assumption is very variable. This makes sense as the only goal of the NE-liability rule is to make the best action a Nash Equilibrium. This becomes clear by looking at the sharp increase of welfare loss around wealth equals 30 000. Between 20 000 and 30 000 both NE and F suggest the strict liability rule (L = **1**). Then at $w^{(v)} = 30000$ we see that using the NE assumption suggest the following liability rule

$$L_E(R + NE, w^{(v)} = 30000) = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

while $L_E(F)$ remains at strict liability. This is caused by the fact that under NE it follows that $p_1^{(v)} = 0$ while this is actually about 38%. So using the Nash equilibrium oversimplifies the reality resulting in a significant welfare loss. The only reason that the liability is specified for $p_1^{(v)}$ the is because the simulation model approximates rational behavior, so the impact will in practice be even higher. All in all, we estimate $M_F = $ **€60**.

5.6 Impact of the Court costs (Cs)

A common argument in favor of no liability is that it avoids court costs as victims have nothing to gain by going to court. This simulation shows that this effect is relatively small. It should be noted that this holds in cases where the government pays the court costs. We believe that if the players pay the court costs this can greatly increase welfare.

Simulations show that under the default realistic game there is no welfare loss at all by ignoring court costs as both methods suggest the strict liability rule. When both parties have equal wealth ($w^{(i)} = w^{(v)} = 80000$) we find:

$$L_E(R+C) = \begin{bmatrix} 0.6295 & 0.8654 & 0.9323 \\ 0.2309 & 0.4671 & 0.5345 \\ 0.1442 & 0.3799 & 0.4473 \end{bmatrix}$$

$$L_E(R) = \begin{bmatrix} 0.6294 & 0.8704 & 0.9384 \\ 0.2260 & 0.4672 & 0.5358 \\ 0 & 0.3789 & 0.4474 \end{bmatrix}$$

$$M_{Cs} = 0.0144.$$

Even setting the court costs to $c = 500$ and setting the wealth of the victim to maximize M ($w^{(v)} = 100000$) we find $M_{Cs} = 2.8$. All in all we estimate $M_{Cs} =$ **€0.01**.

5.7 Impact of uncertain evidence (Es)

Various studies consider the implications of uncertain costs of the accident. Since we do not believe biased uncertain evidence can exist for a longer time we investigate unbiased uncertain evidence. From the simulation we conclude that this is completely irrelevant to study.

Similar to the Court costs, at low levels of wealth there is no difference at all. Considering equal wealth ($w^{(i)} = w^{(v)} = 80000$) we find:

$$L(es = 0) = \begin{bmatrix} 0.6295 & 0.8654 & 0.9323 \\ 0.2309 & 0.4671 & 0.5345 \\ 0.1442 & 0.3799 & 0.4473 \end{bmatrix}$$

$$L(es = 0.1) = \begin{bmatrix} 0.6294 & 0.8704 & 0.9384 \\ 0.2260 & 0.4672 & 0.5358 \\ 0 & 0.3789 & 0.4474 \end{bmatrix}$$

$$M_{Es} = 0.0000895.$$

Even setting the uncertainty to the unrealistic value of 1 (e.g. the court is only 65% certain the damage is between 0 and 30 000 euro) yields a welfare loss of only 0.8393. Based on this we conclude $M_{Es} =$ **€0**.

5.8 Impact of dichotomous liability rules

A couple of studies consider only dichotomous liability rules, liability rules where one party is fully liable. While it is slightly dubious if this indeed simplifies the study, it is nevertheless interesting to know the accuracy of dichotomous liability rules. Based on the simulation we conclude that seems justified to study dichotomous liability rules. Results are shown in Figure 5.3.

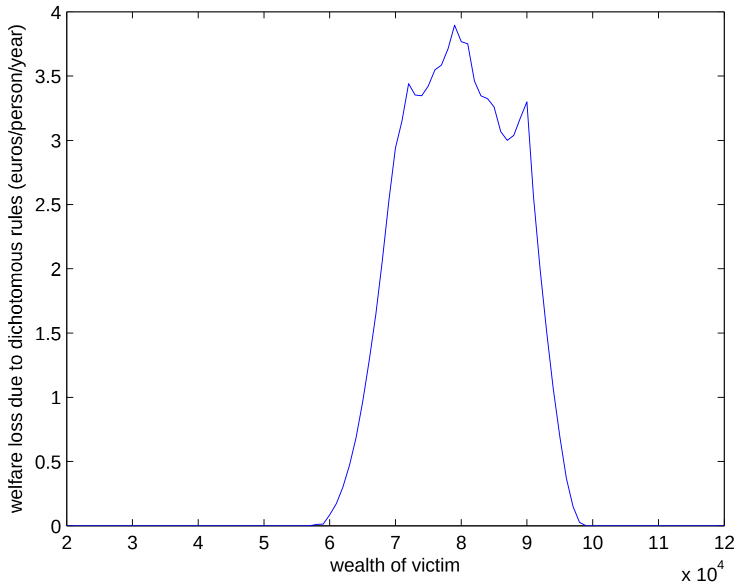

Figure 5.3: impact of dichotomous liability rules

We see that the restriction to dichotomous liability rules has the highest loss of welfare when the efficient liability rule would divide the damage. Many accidents in practice are more decisive than our realistic game so these results are an upper bound. Therefore we estimate M_{Dich} = **€2.50**. So we conclude that limiting study to dichotomous liability rules is acceptable if this results in better incorporation of Ue or F.

Chapter 6

Conclusions

This report presents the results of an extensive study on liability rules. This integral approach revealed the limited coherence and sense of direction in literature. Our goal was therefore twofold: (i) to make sure all potential relevant aspects are known and structured and (ii) to provide insight on the importance of studying these aspects. We found that, despite their importance, irrational behavior and difference in wealth were little studied. We also found that court costs, uncertain size of the accident and games with more than two actions are studied, but need not be. These findings are substantiated in the remainder of the conclusion. We believe studies on liability to be relevant and to have much potential for further study.

Relevance

For the restrictions, assumptions and optimality conditions found in the second chapter we determine the relevance for further study. The overall relevance of a topic is based on three factors:

impact The impact denotes the extend of the contribution of the topic to the liability problem. For a restriction a high score indicates it is often applicable. For an assumption a high score indicates studying the assumption is important to obtain accurate results. A high impact therefore makes the topic more important for study.

studied indicates the amount of research already conducted on this topic. If little research is available on a topic (lower score) this makes the topic more interesting to study.

complexity The final factor is the expected difficulty for further study. This can be high because the topic seems hard to study or because results will not easily be accepted in practice. An easier topic (i.e. lower score) is more likely to yield results and therefore more important to study.

The impact of many assumptions has been found using the newly developed simulation model (Chapter 5). The number of studies already available has been taken from the classification of literature (Chapter 3). Finally the complexity and the impact of restrictions is based on an educated guess. These factors are combined into an overall score which denotes the estimation of the relevance of further study on the topic (overall = impact $\cdot$ (1-studied) $\cdot$ (1-complexity) $\cdot$ 10). The exact numbers are slightly subjective but since the differences are significant this will alter the conclusion. Furthermore we believe that results can be traced to their corresponding sections and the results are unbiased. The results are shown in Table 6.1.

	impact	studied	complexity	Overall
R	1	3/4	1/2	1.3
C	1/2	1/4	3/4	0.9
M	1/4	0	3/4	0.6
L	1/2	2/3	3/4	0.4
B	1/4	1	0	0
C	1/10	1/3	1/2	0.3
D	1	9/10	0	1.0
Ur	1/4	1/2	1/2	0.6
Ue	3/4	1/10	1	1.4
Ea	?	?	1/2	-
Ed	?	1/4	?	-
Es	0	1/2	1/2	0
F	1	0	3/4	2.5

Table 6.1: Three criteria and the overall relevance of the topics

It is clear that we believe there is a mismatch between what is studied, and what turns out to have a high impact. The two newly introduced topics of distributional welfare and fuzzy behavior have the highest impact. The court costs which are commonly used as an argument in favor of 'no liability' turns out to have a very low impact. We recommend further study on fuzzy behavior (F) and distributional welfare (Ue) and not start new studies on uncertain or variable size of the accident.

Simplifications for further study

The report also features two 'tricks' to simplify future research. Most important is the conclusion that games where both players have only two possible actions can be used to approximate games with any number of actions. This greatly reduces the complexity of the analysis.

This simplification is based on the theory of irrational games. These irrational games appear when combining a modest level of irrational behavior with low difference in payoff between actions. In irrational games actions of the players are largely independent of the action of the other player. Irrational games are easier to analyze as the actions of both players in equilibrium can be assumed to react only on an initial disturbance of the game, not on the reaction of the other player on this disturbance. It has been found that many games in practice are irrational games.

Finally, analysis can also be simplified by considering only dichotomous liability rules, rules where only one party is fully liable. Simulations show that considering only dichotomous liability rules results in a small welfare loss. It is unclear whether dichotomous liability rules indeed facilitate analysis, but we believe this simplification is justified if this allows fuzzy behavior or distributional welfare to be better studied.

Deterrence versus low impact

In the introduction the liability rule was described to be a result of balancing two factors. On one hand punishing severely leads to better deterrence of undesirable actions, on the other hand punishing too severely reduces welfare for the punished parties. Based on the analysis we now state that deterrence seems to be dominant. The main argument in favor of this is that liability rules not incorporating risk-aversion result in similar social welfare in simulations. It should be noted that this conclusion does not extend to welfare differences (Ue) but only to individuals.

Proper modeling

Finally a note is made on the proper use of the analysis. In Section 4.1 it is shown that that contemporary liability research precariously leans on assumptions about happiness. While one might be tempted to dismiss this as being off-topic we believe that a house is never stronger than its foundation.

We also noted that results on liability rules are often misused or misinterpreted. We believe there is an intermediate level of study between results of contemporary analysis and practice. All in all we recommend modesty and caution if one cares about making an improvement.

Further research

Beside the study of the restrictions, assumptions and optimality conditions we encountered various interesting hypotheses and topics for further study. These were often not pursued because they did not fit the general flow of this report.

- Hypothesis: In a regular, near-perfect irrational game (R/F) a player taking more care results in a lower or equal share of liability under an efficient rule (counterexamples under C and NE have been found).
- A conclusion from the analysis of fuzzy behavior is that games can be simplified to a 2x2 game. This implies that the judge need only consider how easy it would have been for the parties to choose a better/worse action. Intuitively we put more emphasis on what might have been done *better* rather than on what could have been done *worse*. This is not directly clear. This will not be a hard topic but would be interesting to investigate this mismatch.
- Now that it is clear that some assumptions need not be included in further analysis, we believe the way is clear for more fundamental research. A graph of the liability rules is shown in Figure 6.1. Every line denotes the liability in one of the nine combinations of actions by the players. The important thing to notice is that it seems to be well possible to approximate this behavior by straight lines. Further study might reveal the slope and position of these lines

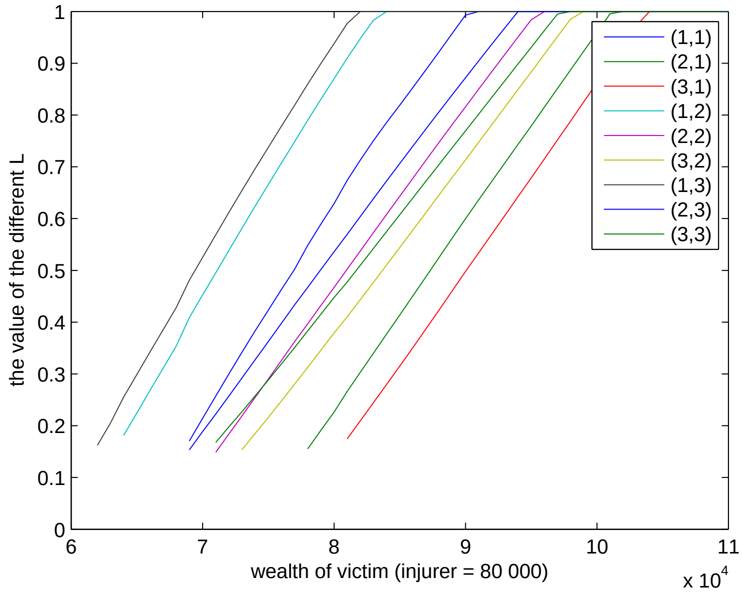

Figure 6.1: the liability rule as a function of wealth

- This research started as an investigation of individual enforcers rather then a government (e.g. a mother correcting her child). As it became clear that much more research was going on on the topic of liability, this question got pushed to the back. Still we believe that from a game theoretic perspective it would be interesting to investigate the role of punishment and blame in a non-governmental setting.
- This research focus on liability rules. While the theories developed are most suited for analysis of liability we do believe that many ideas can be extended and applied to other topics in law.
- This research started out as a very general research aiming to provide a very fundamental and encompassing study of liability. But gradually, as excesses were removed it became more of an argument in favor of studying irrational behavior and wealth-dependent liability. We believe it would be very worthwhile to write an article which is sure to encompass all possible issues around liability rules.
- A topic which is not really related but which was encountered when reading [17] is the nonlinearity of big law issues. It is common practice in economics to linearly approximate curves when many parties are considered (e.g. price elasticity). We doubt that this is also good practice when 'big' laws are considered, i.e. laws which have a significant impact. It would certainly be interesting to study if this linear approximation is valid when used for liability analysis.

The section on proper modeling (4.1) contains some more hypotheses which may be interesting for further study.

Appendix A

Liability simulation model - Source code

A.1 EUtility

```
function [Ui, Uv, Us] = EUtility(s,P,q,r,L,W,c,es,fu)
% returns the utility for the injurer, the victim and the society
% utility is defined as: that what the player wants optimized.

%s, perceived size of an accident (fixed)
%P, probability of an accident (given actions)
%q, col vector avoidance cost injurer (g. action)(must be sorted ascending)
%r, row vector avoidance cost victim (g. action) (must be sorted ascending)
%L, Liability rule (part for injurer) (given actions)
%W, wealth of the players W(1)=injurer, W(2)=victim
%c, court costs corrected for settlement (c if L(iv)<1) 0 otherwise)
%es, coefficient of variation for uncertainty in evidence (sigma/s=es)
%fu, function utility (handle). utility as a function of wealth
%not included here: uncertain due care (should be included in L)

%Ui, expected utility for injurer (given actions)
%Uv, expected utility for victim (given actions)
%Us, expected utility for society (given action) = Ui+Uv + sometimes c

% fuzzy choices is hardcoded in Fuzzy.m
% risk aversion is approximated (overestimated?)
% if the liability rule has more than 5% chance to assign a liability
%   larger than the players wealth, this routine might crash depending on
%   the utility function (so: (i) choose cv low enough, or (ii) choose a
%   proper utility function, or (iii) take care in L)
%   the average wealth (to compute the impact of court costs) is assumed to
%   be 80.000 (source:
%   http://www.cpb.nl/nl/pub/cepmev/mev/2009/kaders/kader09.pdf)
% Average wealth is hardcoded at Us (bottom of this function)


%------START--------
```

```
%Utility part (risk aversion & equity)
if (es ~= 0)
    S = norminv(0.005:0.01:0.995,s,s*es); %approximate normal distribution
else
    S = s;
end;

Ui = zeros(size(P)); Uv = zeros(size(P)); %initialize Ui and Uv
for i=1:size(P,1)
    for v=1:size(P,2)
        Ui(i,v) = P(i,v) * fu(W(1) - q(i) - L(i,v)* s) ...
            + (1- P(i,v)) * fu(W(1) - q(i));
        Uv(i,v) = P(i,v) *sum(fu(W(2) - r(v) + L(i,v)* s - S))/size(S,2)...
            + (1- P(i,v)) * fu(W(2) - r(v));
    end
end

% society is injurer + victim -court costs on society
% court cost on society assumes everyone in society pays. So we take the
% derivative because every person will get a very small amount.
Us = Ui+Uv -(fu(80001)-fu(80000)) * c * ne(zeros(size(L)),L) .* P;
```

A.2 Fuzzy

```
function [p1,p2] = Fuzzy(G1,G2,fu) %choices p1 given G player1, G player2
%returns the fuzzy equilibrium
%pn(a) is the probability that player n plays action a (1 is the injurer)
%Gn is the utility for player n (given actions)

%Rationality. Assuming someone with 66% probability chooses for 20100
    %wealth (money!) instead of 20000 and we solve for lambda.
lambda = 0.693147180559945./(fu(20100)-fu(20000)); %ln(2)/(u20100-u20000);
G1 = lambda* G1; G2 = lambda* G2; %everything is always done times lambda
    %so we might as well do it here

%numerical issues might arise if exp() becomes too high. To make sure this
    %is not the case we shift everything such that the highest utility does
    %not exceed 100 (up until exp(700) works fine)(does not alter solution)
trans = max(max(max(max(G1,G2))) - 100, 0); %find the amount to shift
G1 = G1 - trans; G2 = G2 - trans; %shift

fl = @(u) exp(u); %the logit utility function

p1 = ones(size(G1,1),1);     %initialize p1: all actions are chosen with the
p1 = p1./sum(p1);            %same probability
p2 = ones(1,size(G1,2));
p2 = p2./sum(p2);
```

```
p1old = p1 * 1000;          % p1 from the previous run to track improvement
p2old = p2 * 1000;          % times 1000 to pass the first while test

while(sum((p1-p1old).^2) + sum((p2-p2old).^2) > 0.00000001) %accuracy
%this function currently suffers from a restriction, it does not always
%converge (but if it does, it is correct)
    p1old = p1;
    p2old = p2;

    %let player 2 play his/her (best) strategy against p1
    u2 = p1'*G2;                            %expected utility p2
    l2 = fl(u2);                            %likelihood of actions
    p2 = 0.5.*(l2./sum(l2))+0.5.*p2old;  %probability

    u1 = G1*p2';
    l1 = fl(u1);
    p1 = 0.5.*(l1./sum(l1))+0.5.*p1old;
end
```

A.3 SocialWelfare

```
function [tsw] = SocialWelfare(s,P,q,r,L,W,c,es,fu)
% returns the social welfare of a certain liability rule under certain
% circumstances.
% All symbols are explained in fuzzy.m

%enable the next two lines to include the compensatory tax scheme to keep
%the wealth of the wealther constant
%tsw = SocialWelfareRedis(s,P,q,r,L,W,c,es,fu);
%return

% Calculate the expected utility for the different actions
[Ui,Uv,Us] = EUtility(s,P,q,r,L,W,c,es,fu);

% See how often each action is played
[pi,pv] = Fuzzy(Ui,Uv,fu);

% compute social welfare
tsw = pi' * Us * pv'
```

A.4 LEfficient

```
function [L] = LEfficient(s,P,q,r,W,c,es,fu, increasing)
%optimize L
%if increasing is true then only liability rules are considered where if
%care decreases liability increases this greatly improves performance.

%1) Step 1 of the calculation is to check all dichotomous liability rules
%(only 1's and zeros)
```

```
L = zeros(size(P));
if (increasing == false) %check all liability rules
    Lbest = zeros(size(L));
    Lbest_val = SocialWelfare(s,P,q,r,Lbest,W,c,es,fu);
    for i = 1:(2^(numel(L))-1)
       L(1:numel(L)) = dec2bin(i,numel(L))-'0';
       L_val = SocialWelfare(s,P,q,r,L,W,c,es,fu);
       if L_val > Lbest_val
           Lbest = L;
           Lbest_val = L_val;
       end
    end
else                    %check only increasing liability rules
    Lbest = ones(size(P)); %take strict liability as a start
    Lbest_val = SocialWelfare(s,P,q,r,Lbest,W,c,es,fu);
    i = 0;
    v = size(L,2);
    L = zeros(size(P)); %start with all zeros
    while ~isequal(L,ones(size(P)))
        %at this point all 'likely' liability rules L will pass once
        %(except for strict liability)
        L_val= SocialWelfare(s,P,q,r,L,W,c,es,fu);
        if L_val > Lbest_val
            Lbest = L;
            Lbest_val = L_val;
        end

        %construct all increasing dichotomous liability rules:
        if i == size(L,1)   %in case the bottom is reached
            v = v-1;        % move one to the left
            while i~=1 && L(i-1,v) == 0 %if not valid
                L(i:size(L,1):numel(P)) = 0; % clear that row
                i = i-1;    % move one up
            end
        else
            i = i+1;
            while v~=size(L,2) && L(i,v+1)==0 %if there is a 0 to the right
                v = v+1;    %go there
            end
        end
        L(i,v) = 1;
    end
end

%2) now optimize one cell at a time
L = OptimizeLiabilityRule(s,P,q,r,Lbest,W,c,es,fu, true, increasing);
%L = Lbest;
```

```
function [L] = OptimizeLiabilityRule(s,P,q,r,L,W,c,es,fu, changezeros,...
    increasing)
% this function optimizes one cell at a time in a random order
% changezeros = false implies the zeros cannot be changed

Lold = L*10 + 1000; %to pass the first while loop
while sum(sum((L-Lold).^2)) > 0.0000000001
    Lold = L;
    Order = randperm (numel(L)); %optimize all cells once in a random order
    for count = 1 : numel(Order);
        i = mod(Order(count)-1,size(L,1))+1;
        v = (Order(count)-i)/size(L,1) + 1;
        if (changezeros || (L(i,v)~=0))
            L = OptimizeCell(s,P,q,r,L,W,c,es,fu, i, v, increasing);
        end;
    end
end


function [L] = OptimizeCell(s,P,q,r,Lorig,W,c,es,fu, i, v, increasing)
%optimizes a certain cell using golden ratio optimization
%if increasing = true the adjacent cells are taken as boundary

%Lorig: original L (before optimization)
%Lopt: optimized L
%Lzero: when the cell is set to zero

Lopt = Lorig;
Lzero = Lorig;

%find the boundaries
lmin = 0; lmax = 1;
if increasing == true
    if (i>1) lmax = Lopt(i-1,v); end;
    if (v < size(P,2)) lmax = min(lmax, Lopt(i,v+1)); end;
    if (v>1) lmin = Lopt(i,v-1); end;
    if (i < size(P,1)) lmin = max(lmin, Lopt(i+1,v)); end;
end

%golden ratio optimization
high = 0.618033989 * (lmax-lmin) + lmin;
Lopt(i,v) = high;
highvalue = SocialWelfare(s,P,q,r,Lopt,W,c,es,fu);
low  = 0.381966011 * (lmax-lmin) + lmin;
Lopt(i,v) = low;
lowvalue = SocialWelfare(s,P,q,r,Lopt,W,c,es,fu);

while((lmax-lmin) > 0.00001)
    if (highvalue > lowvalue)
```

```
        lmin = low;
        low = high; %the high of the previous round is now low
        lowvalue = highvalue;
        high = 0.618033989 * (lmax-lmin) + lmin;
        Lopt(i,v) = high;
        highvalue = SocialWelfare(s,P,q,r,Lopt,W,c,es,fu);
    else
        lmax = high;
        high = low;
        highvalue = lowvalue;
        low  = 0.381966011 * (lmax-lmin) + lmin;
        Lopt(i,v) = low;
        lowvalue = SocialWelfare(s,P,q,r,Lopt,W,c,es,fu);
    end;
end;
Lopt(i,v) = (high + low)/2;

%Compare original, optimized and zero
LoptVal = SocialWelfare(s,P,q,r,Lopt,W,c,es,fu);
LorigVal = SocialWelfare(s,P,q,r,Lorig,W,c,es,fu);
Lzero(i,v) = 0;
LzeroVal = SocialWelfare(s,P,q,r,Lzero,W,c,es,fu);

if LoptVal >= LorigVal && LoptVal >= LzeroVal
    L = Lopt;
elseif LorigVal >= LzeroVal
    L = Lorig;
else
    L = Lzero;
end
```

A.5 SocialWelfareRedis

```
function [tsw] = SocialWelfareRedis(s,P,q,r,L,W,c,es,fu)
%exactly the same as socialwelfare except that this module also takes into
%account the compensation for the rich due to bearing a higher risk

%manually fill in the welfare of the injurer and victim under equal wealth
%no compensatory tax for the wealthier in the next two lines
Eui_equal = 11.286943637883974
Euv_equal = 11.288148924333155
Eus_equal = Eui_equal + Euv_equal;

[Ui,Uv,Us] = EUtility(s,P,q,r,L,W,c,es,fu);
[pi,pv] = Fuzzy(Ui,Uv,fu);
Eui_inequal = pi' * Ui * pv'
Euv_inequal = pi' * Uv * pv'

if W(1) > W(2) %injurer is wealthier
```

```
    URedis = Eui_equal - Eui_inequal; % the amount of utility to redistrib.
    RRedis = URedis * 1/(fu(W(1)+1)-fu(W(1)));
else %victim is wealthier
    URedis = -(Euv_equal - Euv_inequal); % the amount of utility to redistrib.
    RRedis = URedis * 1/(fu(W(2)+1)-fu(W(2)));
end

WRedis = [W(1) + RRedis, W(2) - RRedis]
[Ui,Uv,Us] = EUtility(s,P,q,r,L,WRedis,c,es,fu);
tsw = pi' * Us * pv';
Eui_redis = pi' * Ui * pv';
Euv_redis = pi' * Uv * pv';
```

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