CTIT

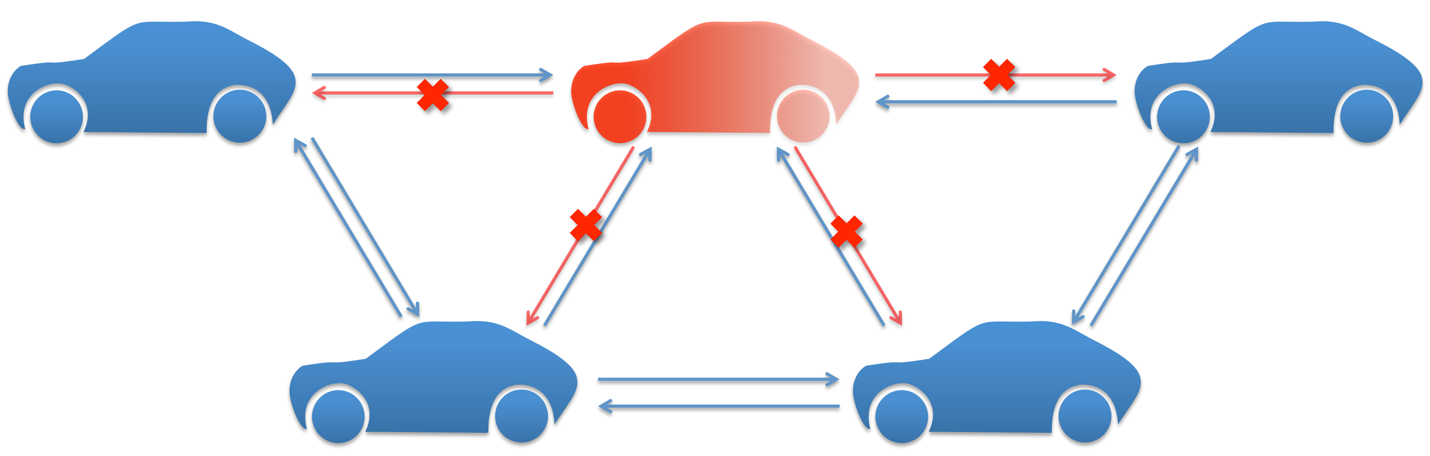
Average consensus over unreliable networks: an improved compensation method.

Introduction

One challenge for platooning vehicles is to achieve a common goal, e.g. agree on a common speed to pursue. This is the so-called (average) consensus problem, which can be compromised by communication loss. To prevent failures in the whole vehicles network, some kind of compensation must be introduced.

Assumptions

- All-to-all wireless communication
- Broadcast communication failures
- Probability of failure equal for every vehicle



The dynamics

The system dynamics is described by

$$x_i(t+1) = \sum_{j=1}^{n} P_{ij} x_j(t)$$

Which will not converge in presence of communication losses, requiring some compensation.

The compensation

Biased compensation method

The matrix is changed according to

$$P(t)_B = \overline{P}L(t) + diag(1 - \overline{P}L(t)1)$$

The system will now converge, but the final value of the nodes will not be the exact average of the initial conditions.

Improved compensation method

The matrix is changed according to

$$G(t) = L(t)\overline{P}L(t)I - L(t)$$

$$P_I(t) = G(t) + \frac{(1 - G(t)1)(1^T - 1^T G(t))}{(1^T - 1^T G(t))1}$$

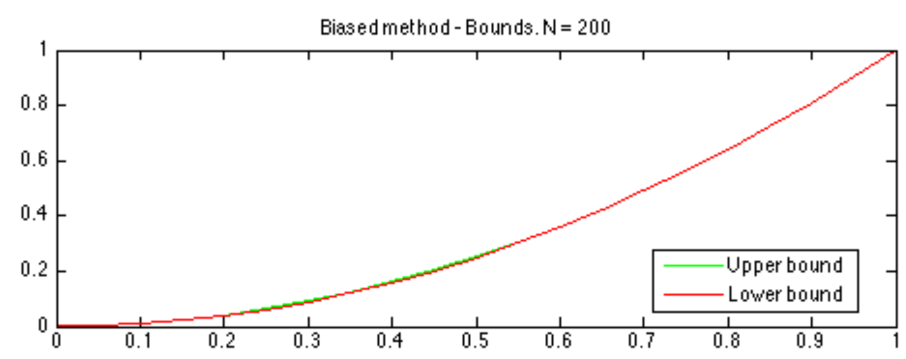
The system will again converge, moreover the final value will be the exact average of the initial values

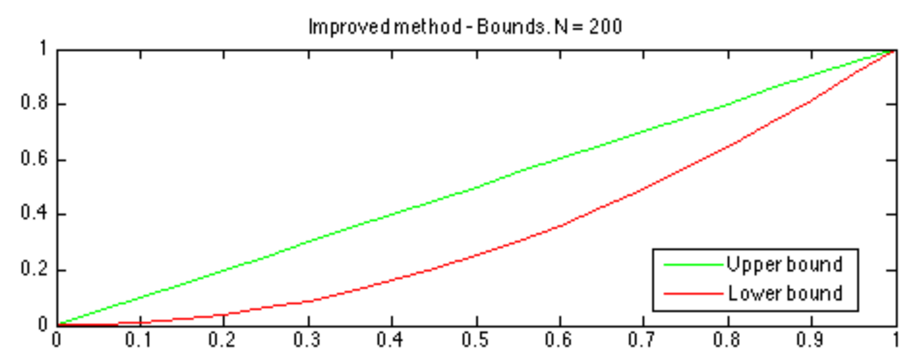
Results

As a result, the speed of convergence is evaluated.

$$R = \sup_{x(0)} \limsup_{t\to\infty} E\left[\left\| x(t) - 1x_A(t) \right\|^2\right]^{\frac{1}{t}}$$

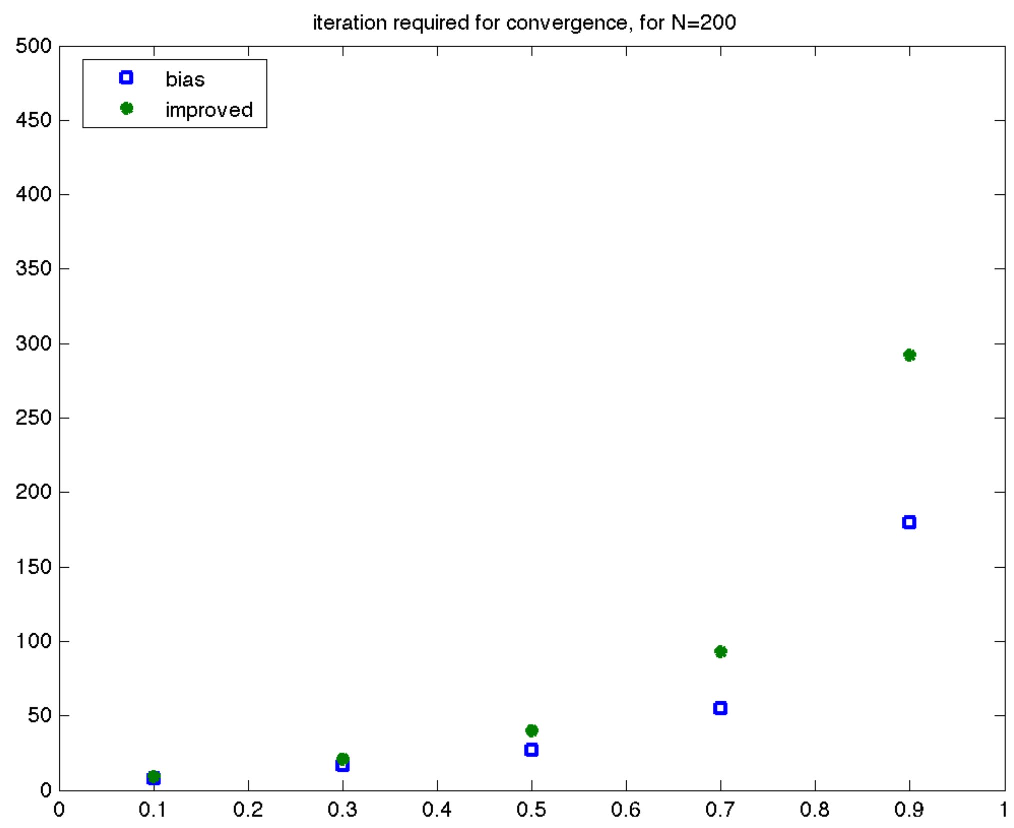
Rate	Lower bound	Upper bound
Biased compensation	p^2	$\frac{2p + p^2(n-2)}{n}$
Improved compensation	$\left(\frac{pn - p^n}{n-1}\right)^2$	$\frac{pn - p^n}{n-1}$





For big network, the convergence rate of the *biased* algorithm is p^2, while the rate of the *improved* one is upper bounded by p. We expect the *improved algorithm* to be slower.

The exact average comes at the price of more iterations of the algorithm, which increase when the probability of failure is high.
The graph is the *full graph*: the number of iteration required for the convergence is the same for every network size.



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One challenge for platooning vehicles is to achieve a common goal, e.g. agree on a common speed to pursue. This is the so-called (average) consensus problem, which can be compromised by communication loss. To prevent failures in the whole vehicles network, some kind of compensation must be introduced.

Assumptions

- All-to-all wireless communication
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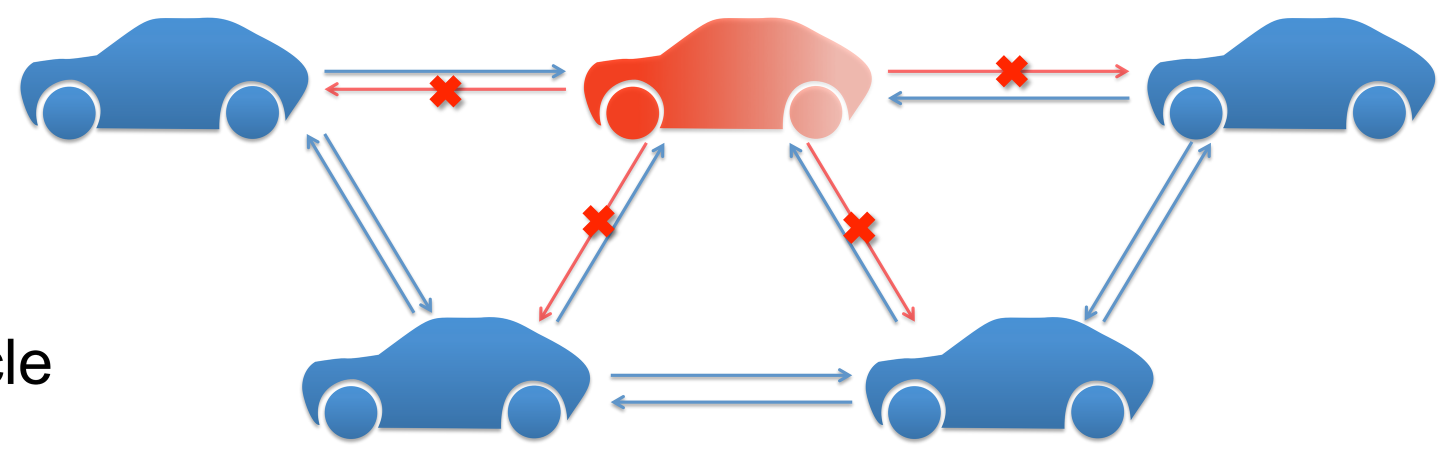


The consensus dynamics

The system dynamics is described by

$$x_i(t+1) = \sum_{j+1}^{n} \overline{P}_{ij} x_j(t)$$

which does not converge in presence of communication losses, requiring some compensation.

Definitions

- $\overline{P}$: Original matrix, without losses, describing the network topology.
- $L(t)$: diagonal matrix, where $L_{ii} = 0$ when the i-th car fails the communication.
- $x(t)$: state vector.

The compensation

Biased compensation method

The matrix is changed according to

$$P_B(t) = \overline{P}L(t) + diag(\mathbb{1} - \overline{P}L(t)\mathbb{1})$$

the system converges, but the final value of the nodes is not the average of the initial conditions.

Improved compensation method

The matrix is changed according to

$$G(t) = L(t)\overline{P}L(t) + I - L(t)$$

$$P_I(t) = G(t) + \frac{(\mathbb{1} - G(t)\mathbb{1})\left(\mathbb{1}^T - \mathbb{1}^T G(t)\right)}{\left(\mathbb{1}^T - \mathbb{1}^T G(t)\right)\mathbb{1}}$$

the system converges to the exact average of the initial values.

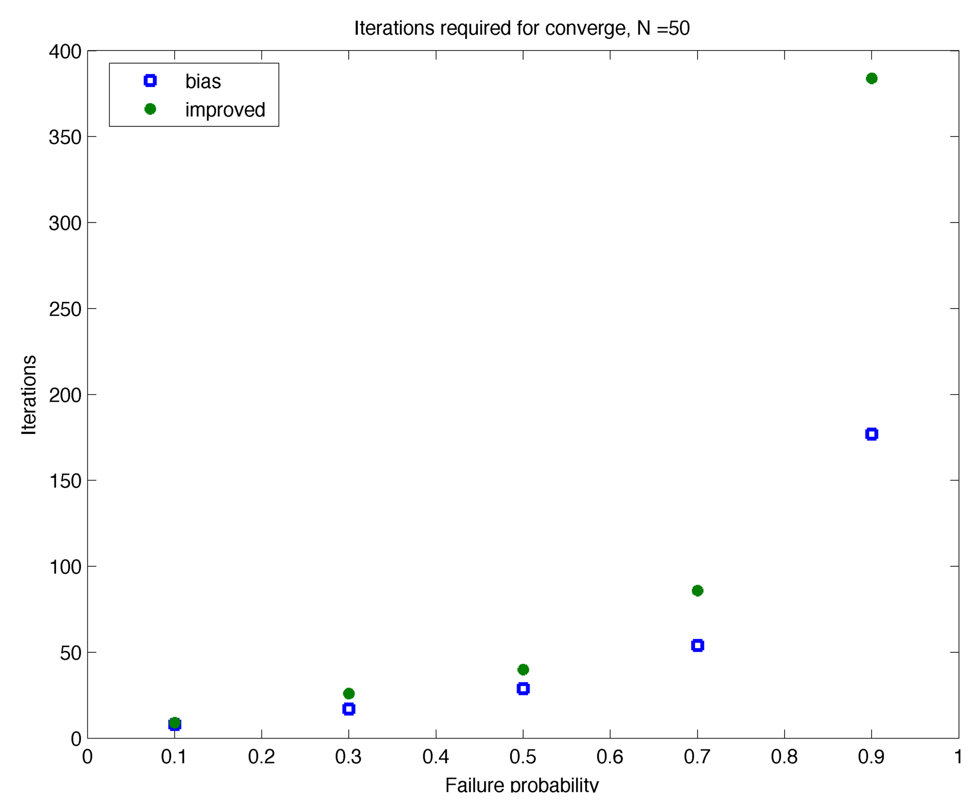
Results

The converge rate and the iterations required for the convergence are evaluated.

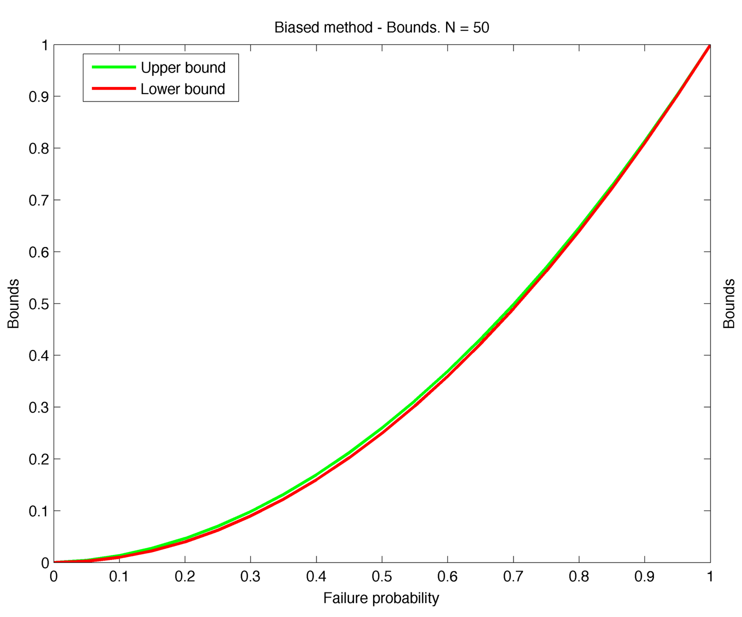
$$R = \sup_{x(0)} \limsup_{t\to\infty} E\left[\left\|x(t) - 1x_A(t)\right\|^2\right]^{\frac{1}{t}}$$

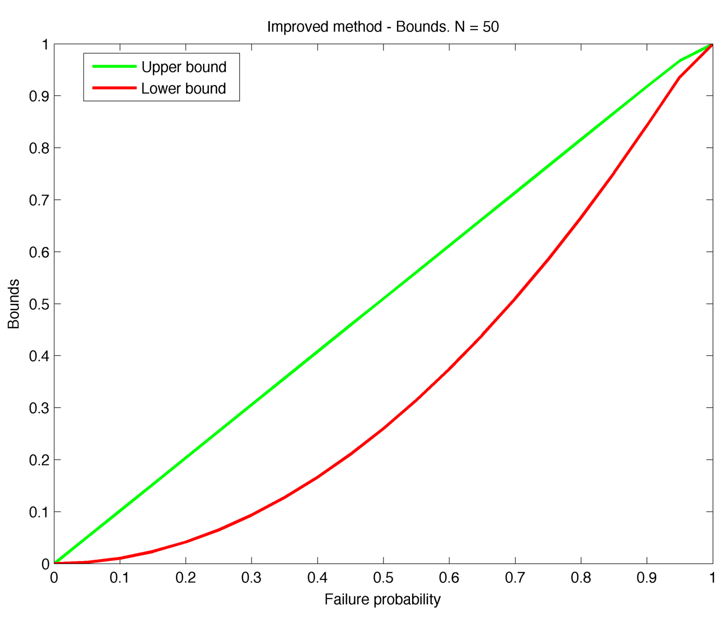
Rate	Lower bound	Upper bound
Biased compensation	p^2	$\frac{2p+p^2(n-2)}{n}$
Improved compensation	$\left(\frac{pn-p^n}{n-1}\right)^2$	$\frac{pn-p^n}{n-1}$

For large networks, the rate of the *biased* algorithm is p^2, while the rate of the *improved* one is upper bounded by p. We expect the *improved algorithm* to be **slower**.

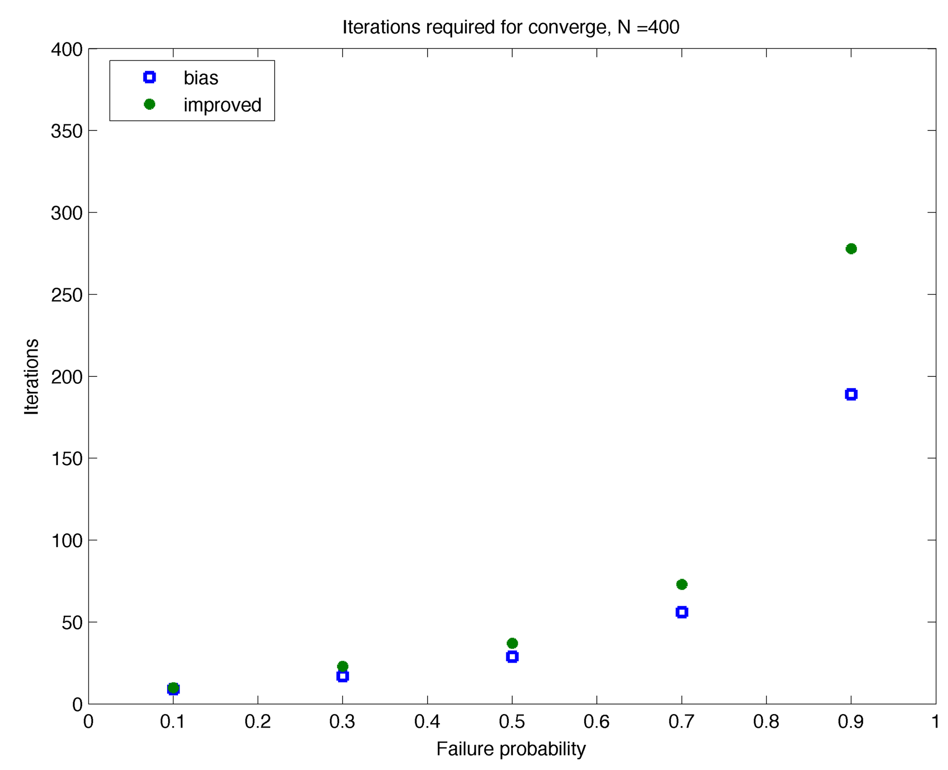


The exact average comes at the price of more iterations required by the algorithm, which increase when the failure probability is higher.





The number of iteration required for the convergence is almost the same for every network, because the communication is all-to-all.



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F. Acciani, G. Heijenk, P. Frasca *"Average consenus over unreliable networks: an improved compensation method"* 34th Benelux Meeting on Systems and Control, 2015

F. Fagnani, S. Zampieri *"Average consensus with packet drop communications"*, SIAM Journal of Control Optimization, vol 48, No. 1, pp 102-133