Technische Universiteit Eindhoven University of Technology

EURANDOM

Delays at signalised intersections with exhaustive traffic control

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Model description

In this paper we provide an analysis for signalised traffic intersections with general renewal arrivals and vehicle-actuated, *exhaustive* traffic control. Nonconflicting traffic flows are divided into M groups that receive a green light simultaneously (e.g., flows 1, 2, 6, and 7 in Figure 1). The control policy is exhaustive, meaning that a green phase ends after the departure of the last vehicle in any flow that faces a green light.

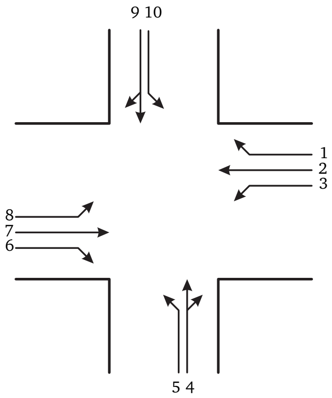

Figure 1: A traffic intersection consisting of 10 flows.

To model the intersection, we use a *polling system* with multiple queues being served simultaneously. First, we analyse this system under Heavy-Traffic (HT) conditions and Light-Traffic (LT) conditions. Subsequently, we develop a closed-form approximation for the mean delay of an arbitrary vehicle using an interpolation between the LT and HT limits.

Stability

We order the flows within the groups according to their traffic intensities. Denote by $\{g, 1\}$ the flow with the highest traffic intensity $\rho_{g,1}$ in group $g = 1, \ldots, G$, and so on. We refer to this flow as the *dominant* flow of group g. We show that only the dominant flows play a role in the stability condition:

$$\sum_{g=1}^{M} \rho_{g,1} < 1.$$

The total amount of traffic arriving per time unit is denoted by $\rho = \sum_{g,j} \rho_{g,j}$.

Heavy traffic

We introduce the *relative* traffic intensities $\hat{\rho}_{g,j} = \rho_{g,j}/\rho$, and use them to define $L = \sum_g \hat{\rho}_{g,1}$, the total relative load of the dominant flows. The distribution of the scaled delay of a vehicle in flow $\{g, j\}$ under HT conditions ($L\rho \uparrow 1$) is

$$(1 - L\rho)W_{g,j} \xrightarrow{d} \begin{cases} 0 & \text{w.p. } \frac{\hat{\rho}_{g,1}-\hat{\rho}_{g,j}}{L-\hat{\rho}_{g,j}}, \\ U \times \Gamma_I, & \text{w.p. } \frac{1-\hat{\rho}_{g,1}/L}{1-\hat{\rho}_{g,j}/L}, \end{cases}$$

where U is a uniformly distributed random variable and Γ_I has a Gamma distribution. This is shown using the Heavy Traffic Averaging Principle (from the polling literature) combined with HT fluid limits for the intersection (see Figure 2).

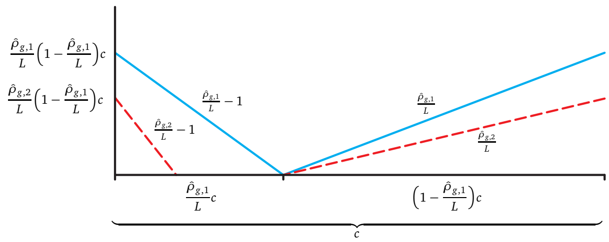

Figure 2: Heavy-traffic fluid limits for the scaled delays of two flows in group g.

Approximation

We have developed a closed-form approximation

$$\mathbb{E}[W_{g,j}] \approx \frac{A + B\rho + C\rho^2}{1 - L\rho}, \tag{1}$$

where A, B, and C are chosen such that that (1) is asymptotically correct in the LT and HT limits. Figure 3 shows an accurate approximation of the mean delay in a real-life example.

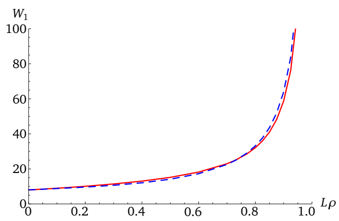

Figure 3: Approximated (blue) and simulated (red) mean delays.