A decomposition result for group representations on L^p-spaces

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Representation theory

Aim: to decompose a group representation into irreducible components.

Results have been found for groups acting unitarily on Hilbert spaces.

Less is known about actions on **Banach spaces**.

Banach lattices

Banach spaces endowed with an **ordering** having nice properties.

Representations on such spaces should be decomposed into **order irreducible** components.

In particular, **L^p-spaces** of p-integrable scalar valued functions are Banach lattices.

Group actions

If G is a group acting measurably on a measure space (X, μ) with μ a **G-invariant measure**, then we have an associated representation

$$\pi : G \rightarrow B\left(L^p(X,\mu)\right)$$

of G as a group of isometric lattice isomorphisms on $L^p(X, \mu)$ for each $1 \leq p \leq \infty$.

Main question

Is it possible to decompose the representation π of G into order irreducible representations?

Why is this interesting?

Because it could help to better understand such group actions.

Also, it might give a model for **decomposition results** for representations on other Banach lattices.

Decomposition results for group representations have applications in various areas of mathematics.

Useful property

The representation π on $L^p(X, \mu)$ is order irreducible if and only if μ is **ergodic**.

Measure disintegration

For a locally compact Polish transformation group (G,X) we can decompose an **invariant probability measure** μ:

$$\mu(Y) = \int_X \lambda_x(Y)\, d\mu(x)$$

for measurable subsets Y of X, with λ_x ranging over the ergodic measures on X.

Banach bundles

A **bundle** B over a Hausdorff space X having Banach spaces as **fibers**.

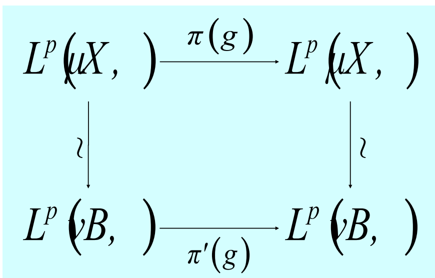
We can consider Banach spaces $L^p(B, \nu)$ of **p-integrable sections of B.**

This provides us with a way to form a \`**p-integral of Banach spaces**' of sorts.

Results

If (G,X) is a locally compact Polish transformation group, μ a finite invariant measure and $1 \leq p < \infty$, then $L^p(X, \mu)$ is **isometrically lattice isomorphic** to a space of p-integrable sections of a Banach bundle over the ergodic measures on X.

We have a **strongly continuous** representation π' of G as a group of isometric lattice isomorphisms on the above space of p-integrable sections, and π' is **fiberwise order irreducible**.



Answer to the main question

We can decompose the representation π into order irreducible, strongly continuous representations.