Understanding compositional function classes

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Deep neural networks are the main work horse underlying the success of modern AI technology. The way deep neural networks are structured allows them to learn hierarchical representations. This is one of the major explanations why deep learning performs particularly well on learning problems with a complex underlying structure.

In mathematical terms, 'hierarchical' means that the function is a composition of simpler functions. There are different ways to formalize this. One way is the class of compositional sparse functions that has been recently introduced in Schmidt-Hieber (2020).

The aim of this master thesis project is to study properties of this function space. Compositional sparse function spaces depend on a number of parameters, that characterize for instance the number of hierarchies or compositions. Moreover, to every parameter configuration, one can associate a statistical estimation rate describing the hardness to learn functions in this class.

The master project will study embedding properties of these function classes. Based on the associated statistical estimation rates, we will introduce moreover a notion of irreducibility and try to develop fast algorithms that can find all irreducible spaces. For theoretical guarantees, one also needs to find upper bounds on the number of irreducible compositional sparse function spaces. An application of these results are improved deep Gaussian process priors, see e.g. Finocchio and Schmidt-Hieber (2023).

This is a project for students with a theoretical background who want to contribute to the development of the mathematical foundations underlying modern AI.

References

[1] G. Finocchio and J. Schmidt-Hieber (2023). Posterior contraction for deep Gaussian process priors. *Journal of Machine Learning Research,* Volume 24, Number 66, 1-49.

[2] J. Schmidt-Hieber (2020). Nonparametric regression using deep neural networks with ReLU activation function. *Annals of Statistics,* Volume 48, Number 4, 1875-1897.