A MOSUM procedure for change-points analysis of time series data based on ordinal patterns (with an application to sleep stage classification)

Bandt (2020) proposes a method to identify sleep stages from single-channel EEG data. His analysis is based on the so-called turning rate which corresponds to the frequency of local maxima and minima in sliding windows of time series data. The data analysis in Bandt (2020) and related publications is promising in that it aligns with experts' annotations and leaves room for further mathematical exploration. Theoretical analysis of the turning rate can be based on ordinal pattern analysis, i.e., more precisely, on a characterization of time series data by the relative positions of three consecutive data points (see Figure 1 for an illustration). For this, a transition from one sleep stage to

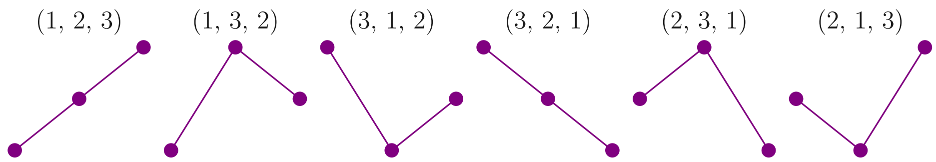

Figure 1: The 6 ordinal pattern of order $p = 3$. The pattern (1, 3, 2), (3, 1, 2), (2, 3, 1), and (2, 1, 3) characterize the turning rate.

another can be interpreted as a structural change in the data-generating process. Since sleep cycles decompose into multiple transitions between sleep stages, change-point analysis allowing for multiple changes in time series data can be considered a suitable tool for a statistical analysis of EEG data recorded for sleep stage classification.

Against this background, the general aim of this master project is to combine the moving sum (MOSUM) procedure proposed in Eichinger and Kirch (2018) for multiple change-point identification with the turning rate analysis proposed in Bandt (2020) and to mathematically and computationally analyze the resulting method.

The proposed project could comprise the theoretical establishment of statistical properties of resulting estimators for the number of change-points or change-point locations, simulation studies substantiating these properties, and the analysis of EEG recordings for sleep stage classification. Depending on the individual preferences of interested master students, the project's focus can be adapted or changed.

In any case, working on the proposed project presupposes basic knowledge of (asymptotic) statistics, probability theory, time series analysis and the willingness to implement numerical analyses in a programming language suitable for statistical applications (ideally `R`).

References

Christoph Bandt. Order patterns, their variation and change points in financial time series and Brownian motion. *Statistical Papers*, 61(4):1565–1588, 2020.

Birte Eichinger and Claudia Kirch. A MOSUM procedure for the estimation of multiple random change points. *Bernoulli*, 24(1):526–564, 2018.