

# Understanding quantities in geo-information in terms of amounts, magnitudes, extents, and intents

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## Abstract

The QuAnGIS project<sup>1</sup> builds on fundamental theories of geographic information science (GI), but misses a theory of GI quantities. My PhD project is aimed to address this. Two quantity concepts, amounts and magnitudes, are axiomatized. To better understand the intuition behind their axiomatizations, the notions of extent and intent are investigated.

## Introduction

Ever since its inception the community of geographic information (GI) science has sought to ground the theory of their field, either in terms of operations (e.g., Albrecht's attempt to classify operations bottom-up [1]), technical data types (e.g., the vector versus raster debate [2]) and semantic data types (e.g., the objects versus fields debate [3]). Over the more recent years the community became increasingly dedicated to finding the fundamental building blocks upon which to build comprehensive theoretical structures (e.g., the geo-atom [4] and the core concepts [5]). This dedication was in part fueled by the emergence of big data, the possibilities offered by Artificial Intelligence (AI) and Natural Language Processing (NLP), and a steady growth of societal and academic interest in GI-systems.

Within this context the QuAnGIS project<sup>1</sup> started. It was focused on question-based analysis of geographic information with semantic queries. In the envisioned GI-system a question's phrases are made machine-readable by annotating them with labels embedded in an ontology based on Kuhn's core concepts [6]. The same labels are then also used to annotate the inputs and outputs of geo-analytical operations – such as buffers and intersects –, effectively characterizing their transformational capacities. Geo-analytical questions can thus be matched with relevant transformation workflows by matching annotation labels. However, geo-analyses usually require multiple of such transformations to be chained, which complicates the retrieval task. Given an annotated question and labeled input data a transformation chain can be planned in order to find a meaningful answer to the question. For example, "What is the length of the shortest route from here to the supermarket?" can then be linked to a sequence of first a shortest route analysis and second a distance calculation.


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<sup>1</sup>See <https://questionbasedanalysis.com/>

A conceptual gap in the theoretical grounding of GI-science became apparent early in the project. While much is written about the fundamental concepts of GI science, the semantics of their quantification are underaddressed. Since quantities are ubiquitous in GI-analysis, an adequate theory of geographic quantities is paramount. This finding motivated the initial research question of the PhD project: *How can quantities in geographic information be conceptualized and linked to GI-core concepts?* An observation shared by Simons [7] is that an enumeration – such as a population count – should not be confused with its source – i.e., the population itself. With this in mind, two concepts were added to the project’s working list of core concepts, namely that of *amounts* to represent the phenomenal sources and that of *magnitudes* to represent their enumerations. Consequently, I implicitly started working on a basic question: *What are amounts?* Yet still, underlying these investigations a deeper theme can be discerned, namely that of the meaning of *extents* and *intents* in the context of GI science.

## Amounts and magnitudes

In [8] we define amounts and magnitudes as respectively mereological and arithmetic quantities. Sums and intersects of the former behave as joins and meets of a lattice structure, not unlike set union and intersection, while those of the latter are more akin to vector addition and subtraction, hence the name *magnitude*. These definitions are axiomatic in that the lattice and vector properties are assumed rather than derived from simpler axioms. Based on Sinton’s idea [9] that measurement involves a *control* and a *measure*, we postulate that amounts can be controlled to measure other amounts or magnitudes. For example, one may measure a population in a controlled space and subsequently measure a population count from that population. In context of these notions, we dedicate attention to the distinction between extensive and intensive quantities. In our definition measured quantities are extensive if they should be summed whenever their controlling quantities are summed. Consider how the population count of the aggregation of the Netherlands and Germany is equal to the sum of population counts of the Netherlands and Germany. If the quantities do not meet this criterion, we call them intensive (Consider, e.g., population densities). By classifying quantities by Sinton’s *space*, *time*, and *theme* twelve functions of extensive quantity measurement are derived, such as *capacity measurement*, for instance measuring a population within a region, and *accumulation measurement*, for example measuring an exposure to pollutants over time. The theory is used for specifying two measurement ontologies, namely the Amount and Magnitude Measurement Ontology (AMMO), with only the upper-level notions related to amounts, magnitudes, and measurements, and geo-AMMO, with three semantic classes for space, time, and theme.

## Extent and intent

Although some important progress was made during the work on [8], some uncomfortable uncertainty remained. In particular, quantity domains are assumed to be disjoint and only bridgeable by measurement functions – meaning, e.g., that amounts of water and amounts of sand each exist in unrelated structures – while it seems more intuitive to have a single structure in which quantities co-exist. Also, the axiomatic definition left open the question

of why amounts and magnitudes would have these properties. To address this, the focus was directed to finding a single structure that could explain this.

The first attempt is focused on the theory of conceptual spaces [10]. Specifically, this work is motivated by the observation that a single geometric entity can be understood through mutually-exclusive conceptual interpretations. For example, a spatial region can be considered to represent a spatially-discrete object or a spatially-continuous field. Yet, while views may disagree, they do have a catalyst to agree on, e.g., the geometry of the spatial region. To capture this, the idea of a *transcept* (*trans-* meaning across, and *-cept* emulating *concept*) is introduced. Embedded in a conceptual space theory which can be segmented in terms of viewpoints [11], transcepts can be understood as sets of views on the same focal entity. This work set the basis for a contextualist approach (meaning concepts only exist within a context) to the problem, but the notion of conceptual spaces seems too complex to ground quantification in GI science, since space itself (at least theoretical spaces such as Euclidean space) seems to be the result of a quantification rather than the source of it.

Attention then shifted to Formal Concept Analysis (FCA), a mathematical framework for defining concepts in terms of extents and intents. At the core there is a binary relation called the *incidence relation*. An extent of a set of arguments can be derived by gathering all elements that are related to all arguments towards their right and an intent by gathering all elements that are related to all arguments towards their left. Notably, this framework offers a strongly contextualist approach, since the definition of the incidence relation directly affects which extents and intents can be derived. Furthermore, the framework allows one to derive a *concept lattice* from an incidence relation. In [12] we set out to better understand *homeomerosity* – a thing is homeomeros iff its parts and its whole are of the same kind or class; this is a key property of amounts – and we found that homeomerosity can be defined within a concept lattice if the wholes are assumed to imply the parts and members of a class imply their class. Then, if all implied by some  $x$  either implies or is implied by some  $y$ ,  $x$  is homeomeros to  $y$ . A result of the investigation of FCA is a particular interest in extents and intents.

FCA offers a formal structure which amounts can be embedded in, but the definition of the framework presents an oddity; the framework models extents and intents, but is defined in set theory meaning it adheres to the axiom of extensionality. This axiom states that if extents of sets are the same, then sets are equal, regardless of their intents. Resultingly, intents are simultaneously appreciated and ignored in FCA. Addressing this thus requires a non-set-theoretic definition of FCA. To understand how such a definition could be formed, and to more generally understand the notions, the focus went to the semantic meaning of extents and intents. In [13] we explore whether and how FCA may be used to understand the relation between notions of *place* and *space*, where the former takes the role of extent and the latter the role of intent. We conclude that duality in FCA – formally known as the Galois connection ( $A^\uparrow \leq B \iff A \leq B^\downarrow$ ) – can be used to model notions of place and space. In short, a rich description of place has a small list of examples, while the contents of a large space can only be unified under a simple description. In forthcoming work, we extend the exploration to also objects and fields, and we claim that the characteristics of extents and intents also apply to them. The focus of future work is thus to showcase that extents and intents are fundamental to concept-pair examples within (and perhaps beyond) GI science.

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