On a conjecture of Sokal concerning roots of the independence polynomial

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Joint work with Viresh Patel and with Han Peters.

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Sokal conjectured about 16 years ago that there exists a region D_Δ in the complex plane that contains the interval $[0, \lambda_c(\Delta))$, where $\lambda_c(\Delta)$ is sp $\lambda_c(\Delta) := \frac{(\Delta-1)^{\Delta-1}}{(\Delta-2)^\Delta}$ for $\Delta \geq 3$, such that for any graph G of maximum degree at most Δ the independence polynomial of G does not vanish on D_Δ. In joint work with Han Peters we have settled this conjecture using complex dynamics. In this talk I will first introduce the independence polynomial of a graph. Then I will explain how zero-free regions for the independence polynomial are closely related to the existence of efficient approximation algorithms for computing evaluations of the independence polynomial and relate this to the solution of Sokal's conjecture. After that I will explain the connection between the location of zeros of the independence polynomial and complex dynamical systems and give some ideas of our proof of the conjecture.