Exercise 4 - Solution

Question a):
To start off, we note that the net outflow of the tank is 0,2 litres/second. We now express this in terms the water level in the tank. For this we use the formule for the volume of a cylinder:

$$V = \pi r^2 H,$$

where r is the radius of the cylinder, H the height and V the volume. Since 0,2 litres = 0,2 dm^3 = 200 cm^3, we get

$$200 = \pi 125^2 \cdot H.$$

Solving for h then gives $H = 0,00407$ cm. So we have now obtained the rate of change of the water level; it decreases 0,00407 cm every second. It is given that at $t = 0$, the water level is 4 metres, so we get

$$h(t) = \max(4 - 4,07 \cdot 10^{-5} t, 0).$$

Strictly speaking the water level cannot drop below zero, so therefore we use the max function. If you did not use this but still got the linear formula for $h(t)$ you will still get all the points.

Question b):
It is given that the output of the tank is closed, but the input is still set at 0,5 litres/second. Moreover, the concentration of the mineral in the input is 40g/litres. The input of the mineral is therefore 20g/second.

We start at a height of 4 metres and a concentration of 20g/litres. Let us first calculate how much of the mineral is present in the cylinder before the mixing process is started. A height of 4 meters means a volume of $\pi \cdot (1,25)^2 \cdot 4 = 19,635$ m^3 = 19635 litres. Hence a concentration of 20g/litres yields a presence of 392700 grams of the mineral before the mixing process is started.

Now 8kg of the mineral is added by the worker, and in addition the mineral is added through the input in the tank at a rate of 20g/second. Therefore, $20 \cdot 20 = 400$ gram of the mineral will be added in 20 seconds through the input in the cylinder. So in total $8000 + 400 = 8400$ grams will be added.

Next to the mineral, the input also adds volume to the tank at a rate of 0,5 litres/second. Therefore a total of $20 \cdot 0,5 = 10$ litres is added to the cylinder. So we have calculated that 10 litres of water is added, and 8400 gram of the mineral. Before the mixing process we had 19635 litres water and 392700 gram of the mineral, so we now have $19635 + 10 = 19645$ litres water and $392700 + 8400 = 401100$ gram of the mineral. The concentration C therefore becomes

$$C = \frac{401100}{19645} = 20,417 \ g/l.$$