Exercise 2 - Solutions

a. We want to know how long it takes for the three men to clean all the windows. Therefore, we make some formulas for George (G), Willem (W) and Jan (J):

$$\begin{aligned} J + W &= 60 \text{ windows per hour} \\ G + W &= 5670 \text{ windows in } 6 \cdot 21 \text{ hours} \\ &= \frac{5670}{126} = 45 \text{ windows per hour} \\ G + J &= 75 \text{ windows per hour} \end{aligned}$$

We rewrite the first two equations in terms of J:

$$\begin{aligned} W &= 60 - J \\ G &= 75 - J \end{aligned}$$

Those equations can be used to calculate J:

$$\begin{aligned} G + W = (60 - J) + (75 - J) &= 45 \\ 135 - 2J &= 45 \\ 2J &= 90 \\ J &= 45 \end{aligned}$$

Now we can fill this in to calculate G and W:

$$\begin{aligned} W &= 60 - 45 = 15 \\ G &= 75 - 45 = 30 \end{aligned}$$

The tree cleaners together clean 15+30+45 = 90 windows per hour. There are 5670 windows in the Carré building, so it takes the three cleaners $\frac{5670}{90} = 63$ hours to clean the Carré building.

b. Jan has to clean all the windows alone. The first 2835 windows, he cleans with a pace of 45 windows per hour. This takes $\frac{2835}{45} = 63$ hours.
The second half of the windows, he gets tired and he cleans 1 second slower every three windows. 45 windows per hour means 45 windows per 3600 seconds, which means that he cleans one window in 80 seconds.

We can make a summation where we use the fact that John cleans one window in 80 seconds, and after 3 windows it will take 1 second longer.

$$\begin{aligned} \sum_{i=1}^{945} 3 \cdot (80 + i) &= 1567755 \text{ seconds} \\ &= 26129 \text{ minutes and } 15 \text{ seconds} \\ &= 435 \text{ hours, } 29 \text{ minutes and } 15 \text{ seconds} \end{aligned}$$

In total, John needs $63 + 435,4875 = 498,4875$ hours (which is a lot!).