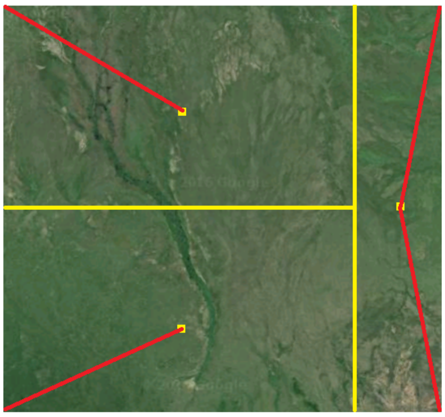
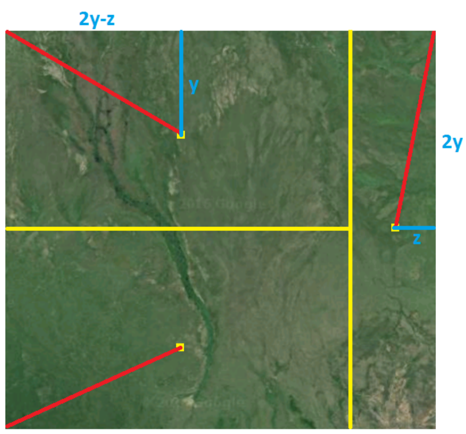
Exercise 3 - Solution

First, we need to realize that the spots that we need to deal with are the corners of the square region, because these are the most outer points. Unfortunately, we only have three food centers to cover the four corners. One food center will have to cover two corners, as is depicted below. The yellow squares are the food centers, and the length of the red lines is our distance x that we need to minimize. Also, three rectangles emerge with the food centers at their center. The horizontal yellow line obviously has to be placed such that it cuts the square in half, but the challenge is to choose the position of the vertical yellow line such that x is minimal. For that, we observe that all the red lines should have equal length. After all, if one of the three red lines would be shorter than the other two, we can just increase the length of that short line to decrease the length of the other two, and thus make x smaller.



Now let's do some mathematics. Assume the sides of the square region have length $4y$, and the width of the long rectangle on the right is $2z$. Then we can deduce the measures depicted below:



By symmetry, it is sufficient to set

$$y^2 + (2y - z)^2 = (2y)^2 + z^2 \tag{1}$$

to make sure that all the red lines have equal length. Note that we used the Pythagorean theorem. We can solve equation (1), and find that

$$\begin{aligned} y^2 + 4y^2 - 4yz + z^2 &= 4y^2 + z^2 \\ y(y - 4z) &= 0 \end{aligned}$$

$$\implies y = 0 \text{ or } y = 4z.$$

$y = 0$ is of course not feasible in this situation, so the solution is $y = 4z$. We know that $4y = 80$ kilometers, so the solution tells us that $z = 5$ km. Hence, the vertical yellow line should be placed at 10 km from the side of the square region. If the food centers are then placed at the centers of the three rectangles imposed by the yellow lines, x is minimized. The exact value for x is then

$$x = \sqrt{40^2 + 5^2} = \sqrt{1625}$$

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