Online Scheduling of Outpatient Procedure Centers

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Outpatient Procedure Centers

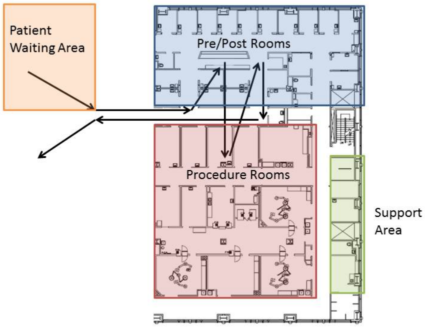
OPCs are a fast growing trend for providing care in the U.S.

Advantages:

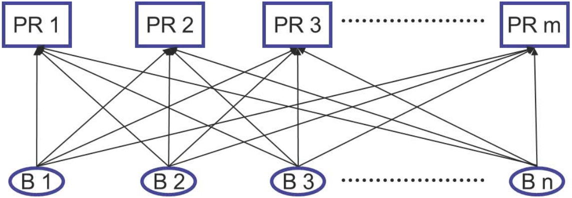
- Safer and lower cost than inpatient stay at hospitals
- Convenient for patients

Challenges:

- Fixed length of day
- High cost of overtime
- Uncertainty in procedure time and procedures per day



Procedure-to-Room Allocation



Decisions:

- How many procedure rooms to plan to open each day?
- Which procedure room to schedule each procedure in?

1. Scheduling Models
2. Exact Methods and Fast Approximations
3. Case Study
4. Conclusions and Other Research

"Bin" Packing

Objective: minimize the number of procedure rooms "opened"

Decisions:

- a subset of m available procedure rooms are "opened"
- n procedures are allocated to the open rooms

Model Formulation:

$$\min \sum_{j=1}^{m} x_j$$

$$\text{s.t.} \quad y_{ij} \leq x_j \qquad i = 1, ..., n, j = 1, ..., m$$

$$\sum_{j=1}^{m} y_{ij} = 1 \qquad i = 1, ..., n$$

$$\sum_{i=1}^{n} d_i y_{ij} \leq S \qquad j = 1, ..., m$$

$$x_j, y_{ij} \in \{0, 1\} \qquad i = 1, ..., n, j = 1, ..., m$$

Extensible "Bin" Packing

Objective: minimize the procedure rooms "opened" plus overtime

Decisions:

- a subset of m available procedure rooms are "opened"
- n procedures are allocated to the open rooms
- overtime is $(\text{total procedure time} - \text{length of day})^{+}$

Model Formulation:

$$\min \sum_{j=1}^{m} c^f x_j + c^v o_j$$

$$\text{s.t. } y_{ij} \leq x_j \qquad i = 1, ..., m, j = 1, ..., n$$

$$\sum_{j=1}^{m} y_{ij} = 1 \qquad i = 1, ..., m$$

$$\sum_{i=1}^{n} d_i y_{ij} - o_j \leq S \qquad j = 1, ..., n$$

$$x_j, y_{ij} \in \{0, 1\} \qquad i = 1, ..., m, j = 1, ..., n$$

A Fast and Easy to Implement Approximation

Dell'Ollmo et al. (1998) showed the LPT heuristic has a worst case performance ratio of 13/12 for a special case ($c^f = c^v S$) of the *extensible bin packing problem*.

LPT Heuristic:

- Sort procedures from longest to shortest
- Allocate procedures one at a time to the least utilized procedure room
- Compute cost of opening procedure rooms and overtime

Stochastic Extensible Bin Packing

Minimize cost of opening procedure rooms and expected overtime given uncertain procedure times:

Model Formulation:

$$\min \sum_{j=1}^{m} c^f x_j + c^v E_\omega[o_j(\omega)]$$

$$\text{s.t. } y_{ij} \leq x_j \qquad i = 1, ..., m, j = 1, ..., n$$

$$\sum_{j=1}^{m} y_{ij} = 1 \qquad i = 1, ..., m$$

$$\sum_{i=1}^{n} d_i(\omega) y_{ij} - o_j(\omega) \leq S \qquad j = 1, ..., n, \forall\omega$$

$$x_j, y_{ij} \in \{0, 1\}, o_j(\omega), \qquad i = 1, ..., m, j = 1, ..., n, \forall\omega$$

Results for LPT

Comparison of the solutions from the *mean value problem* and the *LPT heuristic* with the optimal solution to the stochastic problem:

Instance	1	2	3	4	5	6	7	8	9	10	Avg.
LPT	22%	4%	19%	12%	7%	19%	7%	4%	4%	12%	11%
MV	23%	7%	18%	12%	12%	19%	9%	14%	6%	18%	13%

Table: Error with respect to optimal solution when overtime cost is high (0.5 hours overtime equals cost of opening a new room

Instance	1	2	3	4	5	6	7	8	9	10	Avg.
LPT	0%	0%	0%	0%	0%	1%	1%	3%	1%	0%	1%
MV	0%	0%	0%	0%	1%	1%	3%	3%	2%	0%	1%

Table: Error with respect to optimal solution when overtime cost is low (2 hours overtime equals cost, c^f, of opening a new room)

More About Stochastic Extensible Bin Packing....

Optimal Allocation of Surgery Blocks to Operating Rooms Under Uncertainty

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The allocation of surgeries to operating rooms (ORs) is a challenging combinatorial optimization problem. There is also significant uncertainty in the duration of surgical procedures, which further complicates assignment decisions. In this paper, we present stochastic optimization models for the assignment of surgeries to ORs on a given day of surgery. The objective includes a fixed cost of opening ORs and a variable cost of overtime relative to a fixed length-of-day. We describe two types of models. The first is a two-stage stochastic linear program with binary decisions in the first stage and simple recourse in the second stage. The second is its robust counterpart, in which the objective is to minimize the maximum cost associated with an uncertainty set for surgery durations. We describe the mathematical models, bounds on the optimal solution, and solution methodologies, including an easy-to-implement heuristic. Numerical experiments based on real data from a large health-care provider are used to contrast the results for the two models and illustrate the potential for impact in practice. Based on our numerical experimentation, we find that a fast and easy-to-implement heuristic works fairly well, on average, across many instances. We also find that the robust method performs approximately as well as the heuristic, is much faster than solving the stochastic recourse model, and has the benefit of limiting the worst-case outcome of the recourse problem.

Subject classifications: optimization; stochastic programming; surgery.

Denton, B.T., Miller, A., Balasubramanian, H., Huschka, T., Optimal Surgery Block Allocation Under Uncertainty, *Operations Research* 58(4), 802-816, 2010

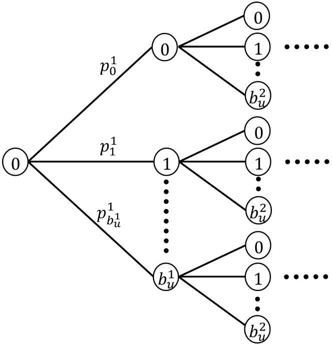
Online Scheduling

Often the number of procedures to be scheduled is not known in advance.

- Procedures are allocated to rooms dynamically as they are requested
- Stochastic Variants:
  - Procedure durations are uncertain
  - Total number and type of procedures is uncertain

Online Scheduling Process

- At the first stage the number of procedure rooms to "open" is decided
- At each stage a batch of procedures arrives to be allocated to procedure rooms; the number of procedures at each stage is a random variable



Related Work about Online Scheduling

- *Best-fit heuristic* has a worst case performance ratio for bin packing of 17/10 (Johnson et al., 1974).
- Online bin packing algorithms cannot have a performance ratio better that 3/2 (Yao, 1980).
- Algorithms for online extensible bin packing with a fixed number of bins cannot have a performance ratio better than 7/6 (Speranza and Tuza, 1999).
    - Further, the *List heuristic* has a worst case performance ratio of 5/4.

Problem Description

Dynamic Scheduling Decisions:

- In the first stage decide how many procedure rooms to open
- In future stages allocate arriving procedures to rooms *online*
- In the final stage random overtime is realized based on outcomes of random procedure times

Stochastic Programming Formulation

Multistage stochastic integer programming formulation:

$$\min_{x}\left\{\sum_{j=1}^{m} c^{f} x_j + \mathcal{Q}_1(x) | \; x_j \in \{0,1\}, \; \forall i\right\}$$

where the stage k recourse function is:

$$\mathcal{Q}_k(y_{k1},...,y_{km}) = \min_{y_{k1},\ldots,y_{km}} \left\{(1-q_{k+1})\left(E_{\omega_k}\left[c^{v}\sum_{j=1}^{m}\max\{0,\sum_{i=1}^{k} d_i(\omega_j)y_{jk} - Sx_i\}\right]\right)\right.$$
$$\left. +q_{k+1}\mathcal{Q}_{k+1}(y_{k+1,1},...,y_{k+1,m}) \mid y_{kj} \leq x_j, \; \forall j; \; \sum_{j=1}^{m} y_{kj} = 1 \; y_{kj} \in \{0,1\}, \; \forall j\right\}.$$

Stochastic List Heuristic

The following heuristic generates a feasible solution to the stochastic programming model.

Data: Set of procedure rooms, $j = 1...m$; scenarios $\omega_k, k = 1, ..., K$; number of procedures, $n(\omega_k)$, and procedure durations for each scenario ω_k.

for $j = 1$ *to* m **do**
  for $\omega_k = 1$ *to* K **do**
    $List(n(\omega_k))$
  $Total\ Cost = E[OTcost] + c^f j$
$Return\ min_j(Total\ Cost)$

Performance Ratio

Definition: the performance ratio (PR) of a heuristic for a problem instance $\mathcal{I}$ is the ratio of $H(\mathcal{I})$ to $Opt(\mathcal{I})$.

The following upper bound on PR for heuristic H is the *worst case performance ratio*:

$$PR^H \leq \max_{\mathcal{I}} \left\{ \frac{H(\mathcal{I})}{Opt(\mathcal{I})} \right\}$$

Worst Case Performance of Stochastic List Heuristic

Theorem

When procedure durations are deterministic:

$$1 + \frac{c^v S}{6c^f} \leq PR^{Stochastic\ \ List} \leq 1 + \frac{c^v S}{4c^f}$$

Worst Case Performance of Stochastic List Heuristic

Theorem

When procedure durations are deterministic:

$$1 + \frac{c^v S}{6c^f} \leq PR^{Stochastic\ \ List} \leq 1 + \frac{c^v S}{4c^f}$$

Theorem

If procedure durations are random and $d_i(\omega) \leq \theta\mu_i, \forall\omega$:

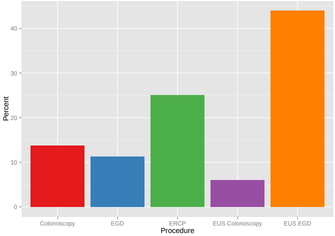
$$1 + \frac{c^v S}{6c^f} \leq PR^{Stochastic\ \ List} \leq 1 + \frac{\theta c^v S}{4c^f} + (\theta - 1)\frac{c^v S}{c^f}$$

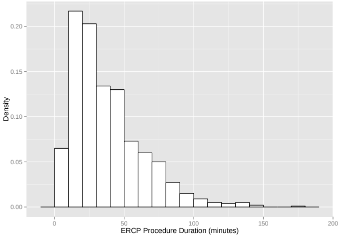
Case Study

- Division of Gastroenterology and Hepatology at Mayo Clinic in Rochester, MN.
- OPC provides minimally invasive procedures to screen, diagnose, and monitor chronic diseases
- Procedure duration distributions and case mix sampled from historical data

Case Study

- Number of routine procedures: $n = 10, 20, 30$
- Number of add-on procedures: $b_u^2 = 0, 5, 10$
- Procedure durations based on historical data
- Overtime estimates: $\frac{c^f}{60c^v} = 1, 2, 4$
- Length of day: $S = 480$ minutes





Special Case (T=3)

The following three stage version of the problem is an important special case at Mayo Clinic

- Number of procedure rooms to be open is decided
- Routine procedures are booked in advance and scheduled as a batch
- An uncertain number of add-on procedures arise on short notice (e.g. 24-48 hours in advance) and are scheduled as a batch

Exact Solution Methods

Extensive Formulation of the Stochastic Program

- Traditional Branch-and-Bound

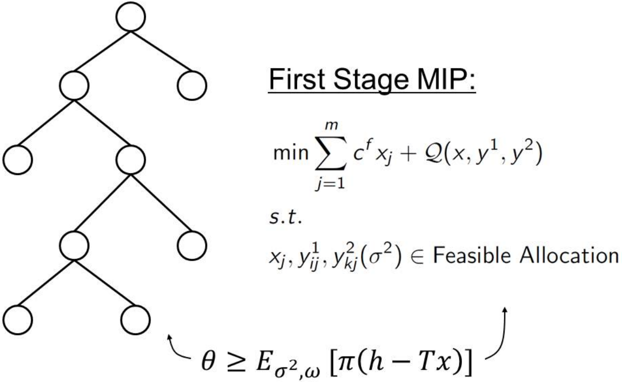
L-shaped Method (Van Slyke and Wets, 1969)

- Reformulate multistage problem as two-stage recourse problem with non-anticipativity constraints
- Approximate the recourse function, $Q(x)$, via outer linearization using *optimality cuts* generated from the second stage dual

Exact Solution Methods

Integer L-shaped Method (Laporte and Louveaux, 1993)

- Incrementally approximate the recourse function, Q(x), using branch-and-cut



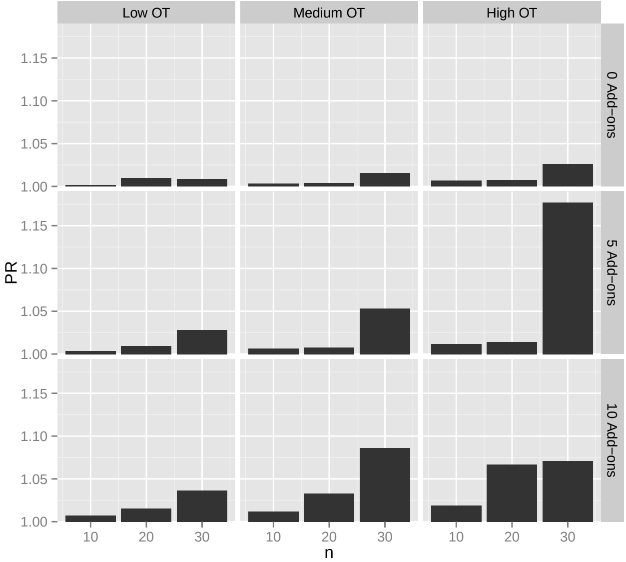
Computational Experiments

Table: Comparison of Solution Methods

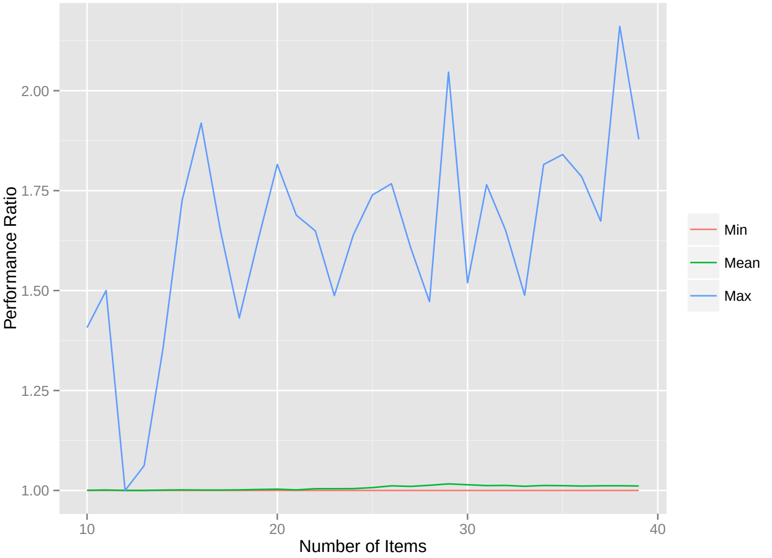
Method	% Optimal (<1%)	% Optimal (<10%)	Average Gap	Max Gap
Extensive Form	66.67%	74.07%	20.80%	288.05%
L-Shaped Method	48.15%	88.89%	3.21%	37.46%
Integer L-Shaped	44.44%	88.89%	4.10%	29.38%

- Results are based on 27 problem instances using 10 random seeds for each instance.
- A maximum runtime of 15k CPU seconds was allowed.

Stochastic List Worst-Case Performance



Stochastic List Worst-Case Performance



Other Heuristics

If procedures arrive in batches, heuristics can sequence procedures prior to the allocation of procedures to rooms.

- **LPT by Mean:** Sort procedures in order of increasing mean and allocate the next procedure to the room with earliest start time.
- **Earliest Start by Variance:** Sort the scheduled procedures by increasing variance and allocate the next procedure to the room with the earliest start time.

Heuristic	% Optimal	Average Gap	Max Gap
LPT by Mean	83.33 %	1.61%	7.75 %
Earliest Start by Variance	88.89%	1.58%	8.09%

Conclusions

1. Fast and very easy to implement approximation methods can provide near optimal solutions with a good worst case performance guarantee

2. High overtime cost and high variance in the number of add-on procedures is associated with longer computation time for exact methods, and weaker performance of approximation methods

Other Research

Bi-Criteria Scheduling of Surgical Services for an Outpatient Procedure Center

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Uncertainty in the duration of surgical procedures can cause long patient wait times, poor utilization of resources, and high overtime costs. We compare several heuristics for scheduling an Outpatient Procedure Center. First, a discrete event simulation model is used to evaluate how 12 different sequencing and patient appointment time-setting heuristics perform with respect to the competing criteria of expected patient waiting time and expected surgical suite overtime for a single day compared with current practice. Second, a bi-criteria genetic algorithm (GA) is used to determine if better solutions can be obtained for this single day scheduling problem. Third, we investigate the efficacy of the bi-criteria GA when surgeries are allowed to be moved to other days. We present numerical experiments based on real data from a large health care provider. Our analysis provides insight into the best scheduling heuristics, and the trade-off between patient and health care provider-based criteria. Finally, we summarize several important managerial insights based on our findings.

Gul, S., Denton, B.T., Huschka, T., Fowler, J.R., Bi-criteria Evaluation of an Outpatient Surgery Clinic via Simulation, Production and Operations Management, 20(3), 406-417, 2011.

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Thank You

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